

Parallel Kinematics-Based Mechanism and Its Industrial Application in CNC Machine Tool Development



Mayur Singh, Priyanka Duarah, Sourabh Narnaware,
and Shrikrishna Nandkishor Joshi

1 Introduction

A parallel kinematic manipulator is a closed-loop kinematic chain mechanism that consists of one or multiple closed kinematic loops. The end effector and fixed base are connected by two or more kinematic chains, which create each loop. In recent times, because of its high mechanical rigidity, high acceleration, and high workload, the adoption of these types of manipulators, in the industries, is growing very fast. Various practical applications of these manipulators are found significant in sectors like space, medical science, flight and automobile motion simulators, high-speed multi-axis machining and assembly, medical parallel robot, general pick-and-place operation, robots in food industries, etc. The PKM-based machine was firstly shown at the 1994 International Manufacturing Technology in Chicago by two American machine tool companies: Giddings & Lewis and Ingersoll [1]. Hexapods and Tripods are terms used to describe PKMs having six and three degrees of freedom (DOF), respectively [2, 3]. In 1965, D. Stewart proposed the six-DOF parallel manipulators, and hence, the six-DOF PKMs are also named as Stewart platform. However, this six-DOF platform was initially proposed by Gough in the year around 1956 to 1957 to test tire [4]. Therefore, this mechanism is also referred to as the Stewart–Gough platform. One of the pioneers in the subject of parallel robotics, Klaus Cappel, is an American engineer who created an octahedral hexapod manipulator as a motion simulator and patented it in 1967 [5].

This paper presents a literature review of the existing PKM industrial applications as well as a Monte Carlo-based approach to obtaining a well-conditioned workspace of a six-degree-of-freedom parallel manipulator. The region in three-dimensional Cartesian space that can be reached by a point on the moving platform can be thought

M. Singh · P. Duarah · S. Narnaware · S. N. Joshi (✉)
Department of Mechanical Engineering, Indian Institute of Technology Guwahati, Guwahati,
Assam 781039, India
e-mail: snj@iitg.ac.in

of as the parallel robot's workspace, with the point of interest being the center of the moving platform. The main advantage of using the Monte Carlo method involves solving only the inverse kinematics problem for parallel manipulators.

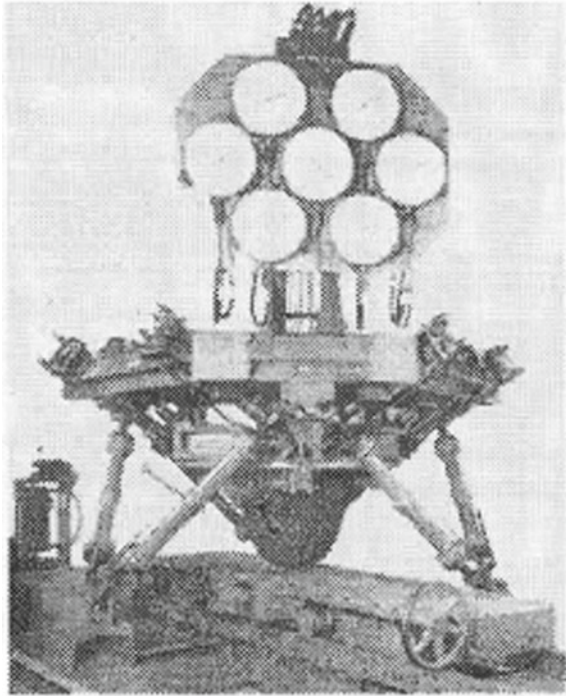
2 Comparison of Serial Manipulator and Parallel Manipulator

Serial and parallel manipulators both have a wide range of industrial applications due to their unique characteristics. Serial Kinematic Manipulators offer a large dexterous workspace, and hence, they are widely adopted in the industries [6]. But, due to their cantilever kinematic structure, they are prone to bending at high load and they vibrate at high speed. This leads to a lack of precision and lesser load capacity. In parallel controllers, since the load is shared with all parallel links, there is a high load capacity. Also, these types of manipulators show high structural stiffness, high precision, high acceleration, and low inertia. In a parallel kinematic, multi-axis system, all actuator acts without delay on one moving platform. However, link interferences and limitations brought on by universal and spherical joints reduce the workspace's dexterity. On the other hand, the use of multi-DOF universal and spherical joints in PKM avoids the adverse effect of bending and torsion forces by limiting the load into compressive and tensile only [5]. Unlike serial manipulator, in case of PKM, presence of passive joint also can be found. Ultimately, this makes the kinematic analysis of PKM far different than serial ones. The inverse kinematic analysis of a PKM is quite simple and straightforward, and forward kinematic analysis is very much complex, while, in case of serial manipulator, it has been seen opposite. Another great advantage of PKM is that its error does not get accumulated rather it gets averaged. And hence, PKM-based robots are found to be more accurate than serial manipulator-based robots.

3 History of Industrial Applications of PKM

It has been observed that the majority of robotic arms used in industrial manufacturing utilize serial technology. But in the last few decades, parallel manipulators have been explored and developed extensively, both theoretically and practically, in academics as well as in industries. After reviewing a number of academic papers, it appears that Gwinnett, who received a patent in 1931 for a motion platform based on a spherical parallel, created the first real-world use for a parallel manipulator. The motion platform developed by Gwinnett was designed for use in the entertainment business [7].

Fig. 1 Universal tire testing machine of Gough [<https://www.parallemic.org/Reviews/Review007.html> (as seen on 17.5.2022)]



Willard L.V. Pollard received a patent in 1942 for a creative industrial parallel robot design that is regarded as the first PKM-based spatial industrial robot [6]. The design was intended to build a spray painting robot, but it was never built.

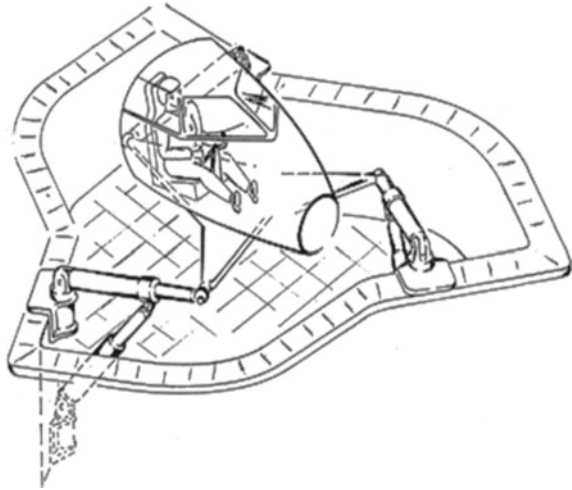
In 1954, Dr. Eric Gough who was known to be an employee of the Dunlop Rubber Company in England had designed and developed a six-degree-of-freedom octahedral hexapod for the universal tire testing machine for the first time [4]. As depicted in Fig. 1, this parallel manipulator was employed as a universal tire testing machine.

Later, Mr. Stewart had proposed the use of such a framework for flight simulators through his paper, which was published in the year 1965 [7]. The same architecture was concurrently proposed by Klaus Cappel as a motion simulator mechanism [7]. The Clavel-proposed Delta robot is one of the most successful parallel robot designs, besides the Gough platform [8].

Karl-Erik Neumann invented and manufactured a new type of robot in 1987 called the parallel kinematic robot (PKR) [9]. This kind of robot has three or more parallel linear axes that move in the same direction. It has three prismatic actuators: two of which regulate the rotational axis and the third controls the translational degree of freedom of the mobile platform.

Applications of parallel manipulators are quite promising in the field of medical science also. There are many applications in medical fields that involve robotics technology, such as injecting electrodes, drilling bone, medical transportation, and performing various other tests. Wall painting, shipbuilding, bridge construction,

Fig. 2 Original Stewart platform proposed as a flight simulator [<https://www.parallelemic.org/Reviews/Review007.html>] (as seen on 17.5.2022)]



aircraft maintenance, ship-to-ship freight handling, and other applications require service robots that can move in a vertical plane, and hence, PKM can be a potential technology for these applications. Due to developments in computer technology and the development of advanced control systems, it can be said that parallel manipulators have recently become more practical and efficient technology for various high-demand industrial products like CNC-based multi-axis manufacturing machines, robots used in automobile industries, etc.

4 Computer Numerical Control (CNC)

Computer numerical control (CNC) machines arose from the advancement of numerical control (NC) machines. The machine control unit (MCU) understands and processes numerical control (NC), which is a series of coded instructions consisting of alphanumeric letters and symbols. To carry out machining operations on a workpiece, the machine's motors and controllers convert these commands into electrical current pulses. The machine tool's numbers, letters, and symbols serve as coded instructions that refer to certain lengths, positions, functions, or motions that the machine tool may decode as it processes the workpiece. NC and CNC machines are currently widely employed in a wide range of sectors, from small to large [10].

Modern CNC systems use computer-aided design (CAD) and computer-aided manufacturing (CAM) software to automate the entire component design process. These applications generate a computer file that is processed by a post-processor to extract the commands needed to run a specific machine and then fed into CNC machines for manufacturing. These machines perform numerous processes such as

milling, drilling, sawing, and other operations with several tools in a single cell, saving time and increasing output rate and efficiency.

4.1 Stewart Platform and CNC Machine

The Stewart–Gough platform, often known as the hexapod, is one of the most well-known six-degree-of-freedom parallel kinematics manipulators. The mechanism is made up of a fixed base plate and a moving platform which are linked by six prismatic actuators via spherical and universal joints. The six actuators can be either linear or angular, and they connect the robot's bottom, which has no mobility, with the component that does the end effector [1]. Because of the closed-loop architecture, not all of the joints can be independently actuated. Inverse kinematics analysis is used to determine the lengths of the connections in terms of position, orientation, translation, and angular velocities. When compared to solving the forward kinematics equations, the inverse kinematics equation is much simpler to solve. By changing the link lengths, the orientation and positions of the moving plate can be controlled. The Stewart platform can be used to construct a CNC hexapod machine tool. A hexapod CNC machine can be an excellent machining tool that addresses various demands of the modern machining processes and can be modified according to the needs. Nowadays, CNC machines are widely employed in several sectors like 3D printing, graphics printing on tee shirt, building construction, food sorting, etc. End-to-end component design is highly automated in modern CNC systems, using computer-aided design (CAD) and computer-aided manufacturing (CAM) programs.

4.2 Advantages of Stewart Platform-Based CNC

The Stewart platform is gaining favor in the design of multi-degree freedom machine tools due to its accurate movements. Machines with high speed, accuracy, rigidity, and multi-axial capabilities, as well as high-quality produced parts, are required for today's new generation. CNC hexapod machining has a lot of potential for transforming manufacturing processes. As listed below, there are various advantages to such a machine.

Six Degrees of Freedom

The hexapod is made up of two platforms joined by six struts that move back and forth between the mobile and fixed platforms. The coordinated motion of these six struts enables the spindle, which is mounted on the movable platform, to move in any direction. In addition to the traditional motion in orthogonal axis movements x , y , z (lateral, longitudinal, and vertical), this device can also move in the rotary α , β , γ (pitch, roll, and yaw) coordinates. This allows the spindle to reach unusual angles and geometrical features and provides a significant advantage over conventional CNC machining.

High Precision and Accuracy

When compared to traditional multi-axis positioning tools, the hexapod requires all six struts to change their lengths even if only one axis needs to be changed. If only one strut changes its length, all six coordinates (x , y , z) need to be adjusted. This common motion of movable platforms, which is dependent on all struts, ensures great precision and accuracy.

High Stiffness

A hexapod's high stiffness and rigidity of its components and all moving parts, such as joints and drive screws, are another important feature. It produces extremely high natural frequencies (500 Hz @ 10 kg load), which enables for extremely high-speed cutting operations and other machining goals. Furthermore, all of a hexapod's struts are solely subjected to forces in the longitudinal direction, preventing any bending effects in the struts and providing a significant advantage.

High Load to Weight Ratio

The hexapod's high nominal load to weight ratio is a key benefit. The weight of a load on the platform is spread about evenly among the six parallel links, implying that each link bears just one-sixth of the entire weight. The hexapod's struts function longitudinally, and the load applied to them exerts either tension or compression forces.

5 Workspace Analysis By Monte Carlo Method

The Monte Carlo approach uses random sampling to solve mathematical problems numerically. Any phenomenon that is influenced by random variables can be simulated using the Monte Carlo approach [11]. The primary disadvantage of parallel manipulators is their limited workspace. In serial manipulators, by using the geometry of the manipulator, its Denavit–Hartenberg parameters and the actuated joint limits, the workspace could be easily determined. But in parallel or hybrid manipulators, in addition to the actuated joint values, the values of the passive joints are determined by solving the forward kinematics. The real solutions for the forward kinematics may not exist which in turn prohibits the parallel manipulator to assemble for given joint variables [12]. Additionally, the self-collision of the links and singularities may split the workspace and increase the complexity of determining the workspace. The approach is search based—a region in space is discretized and the discrete points are generated, the inverse kinematics is solved at generated discretized points, and then the joint variables are checked for violation of limits. The best result is obtained by discretizing the points finer. To get a well-conditioned workspace of a six-degree-of-freedom parallel manipulator, Monte Carlo-based technique is applied [12].

5.1 Monte Carlo Method

The integrals of any arbitrary functions (vector or scalar functions of smooth or non-smooth type) over any arbitrary domain could be evaluated using the Monte Carlo approach [12]. The integral

$$I = \int_{[0,1]^d} f(x)dx \tag{1}$$

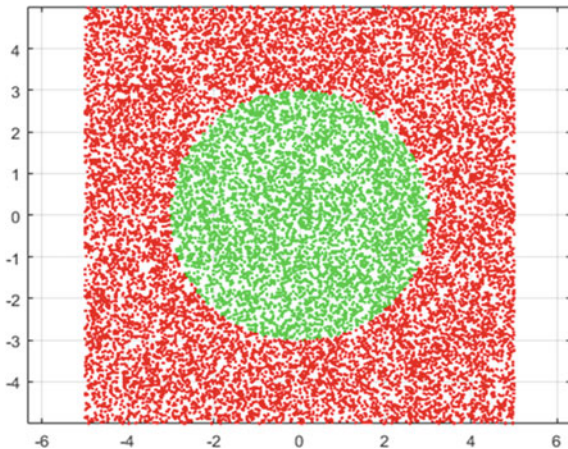
where $f(\cdot)$ is a bounded real-valued function, can be obtained as $E(f(U))$, where $E(\cdot)$ is the expectation of a variable taking a particular probabilistic value and $U = [u_1, u_2, \dots, u_d]^T$ a $1 * d$ vector taking random values of $u_i \in [0, 1] \quad \forall i = 1, 2, \dots, d$ (Fig. 3).

The Monte Carlo method has been used to obtain the well-conditioned and reachable workspace of a closed-loop mechanism or parallel manipulator, by recognizing that the volume (or area) of a manipulator is an integration problem in $R^3(R^2)$.

It can be implemented by formulating a function $f(j) = \begin{cases} 0 \\ 1 \end{cases}$, where $j = \{\theta_i, \phi_j\}^T, \quad \forall i = 1, 2, \dots, n$ actuated joint variables and $\forall j = 1, 2, \dots, m$ passive joint variables and $m + n = d$.

The function f takes 0 or 1 depending on whether the given position and orientation of the end effector of manipulator are well conditioned, and the condition for inverse kinematic of the manipulator is satisfied at that position and orientation with all the joint values within permissible joint limits. The main advantage of using Monte Carlo method involves solving only the inverse kinematics problem for parallel manipulator, and the other conditions could be incorporated to ensure that the workspace is well conditioned and does not violate any restrictions.

Fig. 3 Visualization of Monte Carlo search



5.2 Solution of Inverse Kinematics of Stewart Platform

A Stewart platform is shown in Fig. 4, where P denotes the position of the spherical joint on the top plate and B denotes the position of universal joint on the base plate. Let l denote the actuator length.

A leg of a Stewart platform is shown in Fig. 5, where a vector equation in this leg is applied in order to get the kinematics of a loop of a Stewart platform.

Rotation matrix can be written as

$$R_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \phi_1 & -\sin \phi_1 \\ 0 & \sin \phi_1 & \cos \phi_1 \end{bmatrix}$$

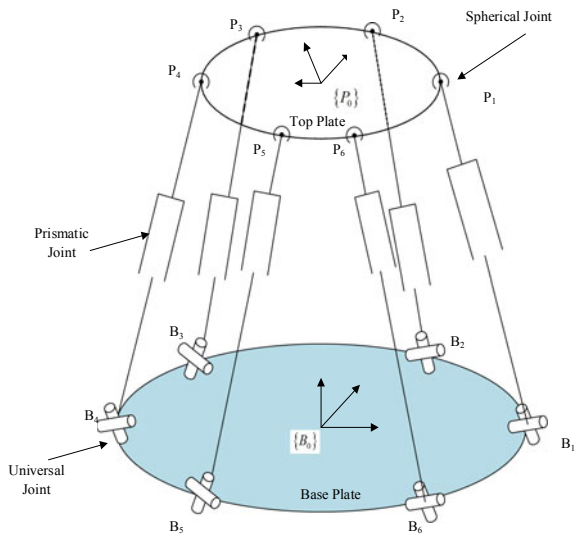
$$R_y = \begin{bmatrix} \cos \phi_1 & 0 & \sin \phi_1 \\ 0 & 1 & 0 \\ -\sin \phi_1 & 0 & \cos \phi_1 \end{bmatrix}$$

$$R_z = \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Let

- $B_0 t$ denote the location of the origin $\{P_0\}$ with respect to $\{B_0\}$,
- $B_0 P_0 [R]$ Orientation of top platform with respect to fixed base,
- Ψ Euler rotation about X -axis,

Fig. 4 Stewart platform



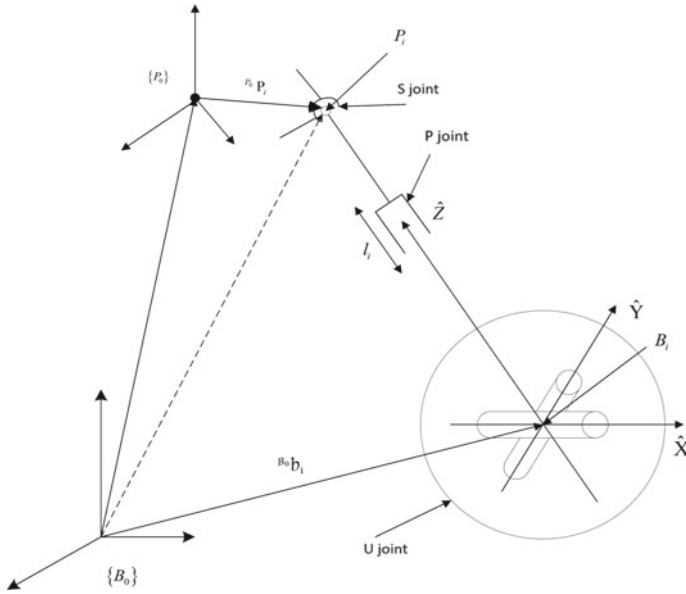


Fig. 5 Schematic diagram of a leg of Stewart platform

- ϕ Euler rotation about Y -axis,
- θ Euler rotation about Z -axis.

From Fig. 5, an arbitrary platform point P_i can be written in $\{B_0\}$ as

$${}^{B_0} P_i = {}^{B_0} t [R] {}^{P_0} P_i + {}^{B_0} t \tag{2}$$

The ${}^{P_0} P_i$ is a known constant vector in $\{P_0\}$.

The location of the base connection points ${}^{B_0} b_i$ is known.

From the known ${}^{B_0} t [R]$ and translation vector ${}^{B_0} t$, ${}^{B_0} P_i$ is obtained.

$$\left[R(\hat{Z}, \gamma_i) \right]^T [(x, y, z)^T - {}^{B_0} b_i] = \left[R(\hat{Y}, \phi_i) \right] \left[R(\hat{X}, \varphi_i) \right] (0, 0, l_i)^T \tag{3}$$

$$\left[R(\hat{Z}, \gamma_i) \right]^T [(x, y, z)^T - {}^{B_0} b_i] = l_i \begin{Bmatrix} \sin \phi_1 \cos \varphi_1 \\ -\sin \phi_1 \\ \cos \phi_1 \sin \varphi_1 \end{Bmatrix} \tag{4}$$

After solving the above Eq. (4), the leg length for an actuator can be obtained as follows:

$$l_i = \pm \sqrt{[(x, y, z)^T - {}^{B_0} b_i]^2} \tag{5}$$

By performing the same operation for each leg, the individual leg length for each actuator can be obtained.

5.3 Workspace Generation

The workspace for the parallel robot can be defined as the region in three-dimensional Cartesian space that can be reached by a point on the moving platform, the point being considered is the center of moving platform. The constraints are simply the link lengths that have been selected for a given workpiece and are the only constraints considered. The actuated joints calculated by solving the equation

$$[(x, y, z)^T - B_0 b_1]^2 = l_1^2 (\sin^2 \varphi_1 + \cos^2 \varphi_1) \quad (6)$$

are then checked for joint limits while using Monte Carlo method for workspace generation.

An algorithm has been developed for calculation of workspace and work volume for various configurations of PKM-based CNC machine tool by using Monte Carlo method.

Following are the results obtained by simulating different models of Stewart platform. The random number was generated, and a number of random points taken were in order of 10^3 to 10^6 . Following results are for 10^5 sampling points. The orientation of the top plate is considered to be horizontal.

Configuration 1

In this configuration, the diameter of base plate is 600 mm and size of top plate is 470 mm. The actuator joints are placed at 10 mm from the edge of the plate as shown in Fig. 6. On base plate, the actuators are located at 60° and are evenly spaced. The first actuator is at 30° with the horizontal reference axis as shown in Fig. 6 followed by the second actuator at 60° to the first and so on the next actuators are placed. While using Monte Carlo method, the joint constraint equation has been applied as the link length should fall in between 324 and 625 mm as these are the limits for our actuators.

On the top plate, the spherical joint is located at 60° and is evenly spaced. The first spherical joint is at 30° with the reference and followed by the second joint which is at 60° to the first and the third as shown in Fig. 7.

Assembly is done by connecting the top and base plate with the help of linear actuator. The connections are symmetrical, the first U-joint is connected to the first S-joint and so on as shown in Fig. 8.

Figure 9 shows the workspace for spatial Stewart platform. The orientation of the top plate is considered to be horizontal. Triangulation of the spatial plot of Fig. 10 is shown in Fig. 9.

Fig. 6 Base plate for configuration 1

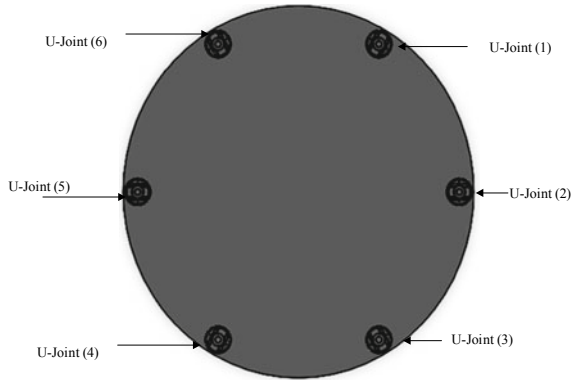


Fig. 7 Top plate for configuration 1

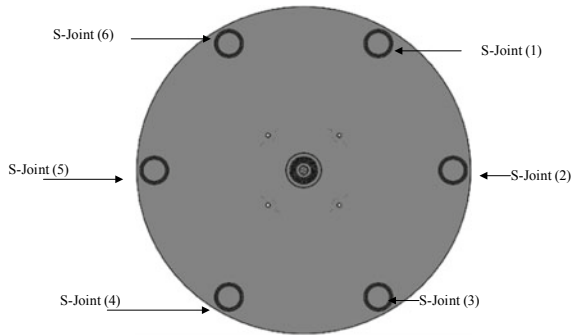


Fig. 8 Assembly for configuration 1 (isometric view)

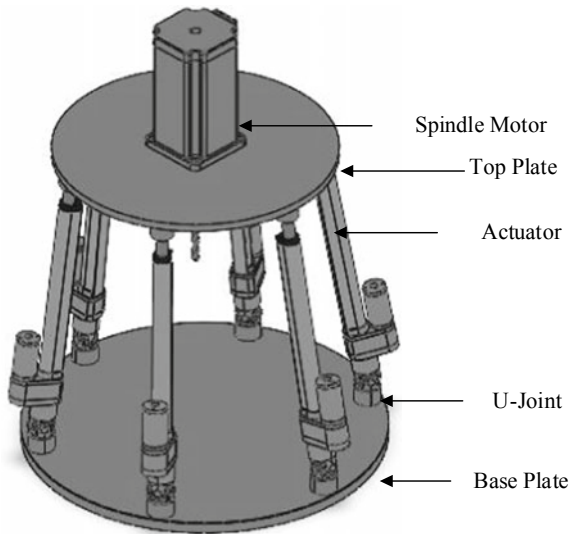


Fig. 9 Available workspace for configuration 1 (with triangulation)

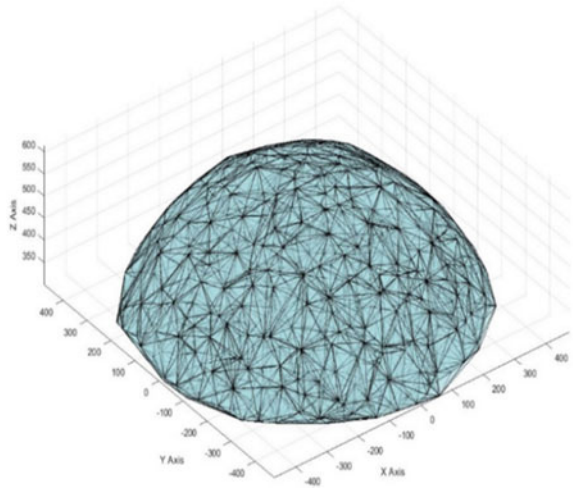
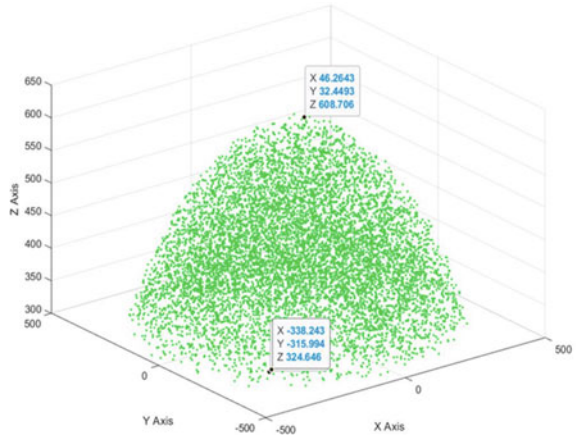


Fig. 10 Available workspace for configuration 1 (spatial Stewart platform)



This configuration of Stewart platform gives the maximum volume for the given dimensions of top and base plate and for given actuator stroke. Since Monte Carlo algorithm is a random number generation approach, we have done maximum number of simulation, but the variation in the workspace volume was found to be approximately same.

Following are the results obtained by simulation of MATLAB code for configuration 1 (Table 1).

Configuration 2

In this configuration, the diameter of base plate is 600 mm and size of top plate is 470 mm. The actuator joints are placed at 10 mm from the edge of the plate as shown in Fig. 11. On base plate, the actuators are located at 60° and are evenly spaced. The

Table 1 Simulation results for configuration 1

Sample points satisfying constraints	Volume of workspace in mm ³
8876	124,260,000
8949	125,286,000
8830	123,620,000
8908	124,710,000
8932	125,048,000
8856	123,984,000
8851	123,910,000
8862	124,068,000
8843	123,802,000
8962	125,468,000

first actuator is at 30° with the horizontal reference axis as shown in Fig. 11 followed by the second actuator at 60° to the first and so on the next actuators are placed. While using Monte Carlo method, the joint constraint equation has been applied as the link length should fall in between 324 and 625 mm as these are the limits for our actuators.

On the top plate, the spherical joint is located at 40° and 80° apart and is not evenly spaced. The first spherical joint is at 20° with the reference and followed by the second joint which is at 80° to the first and 40° to the third as shown in Fig. 12.

Assembly is done by connecting the top and base plate with the help of linear actuator. The connections are not symmetrical in this case; here the first U-joint is connected to the first S-joint which are located at 30° and 20° with the reference axis which is Y-axis on X–Y plane. This can be seen in Fig. 13.

Figure 14 shows the workspace for spatial Stewart platform. The orientation of the top plate is considered to be horizontal. Triangulation of the spatial plot of Fig. 15 is shown in Fig. 14.

Fig. 11 Base plate for configuration 2

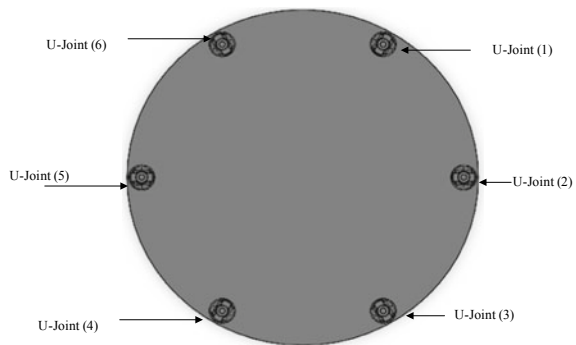


Fig. 12 Top plate for configuration 2

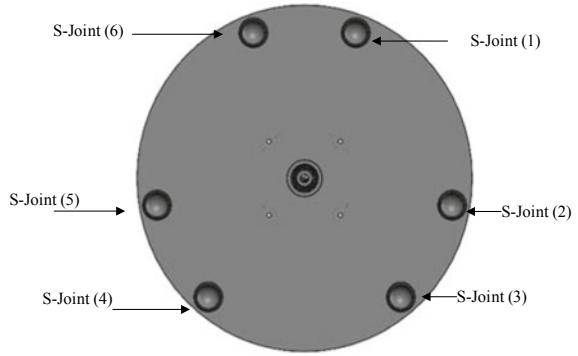
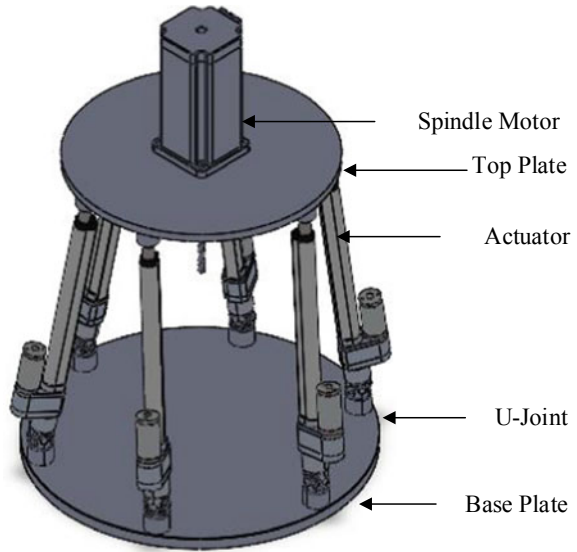


Fig. 13 Assembly for configuration 2 (isometric view)



Volume obtained in this configuration is less than that of configuration 1, where all the joints were equally spaced.

Following are the results obtained by simulation of MATLAB code for configuration 2 (Table 2).

Configuration 3

In this configuration, the diameter of base plate is 600 mm and size of top plate is 470 mm. The actuator joints are placed at 10 mm from the edge of the plate as shown Fig. 16. On base plate, the actuators are located at 40° and 80° and are not evenly spaced. The first actuator is at 40° with the horizontal reference axis as shown in Fig. 16 followed by the second actuator at 40° to the first and 80° to the third universal joint or actuator.

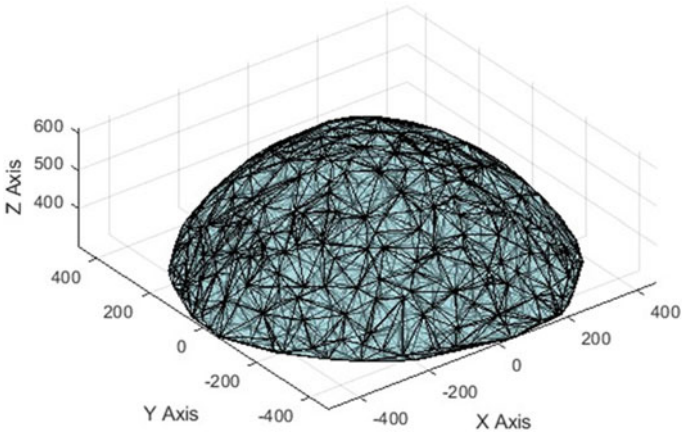


Fig. 14 Triangulation on the plot for configuration 2

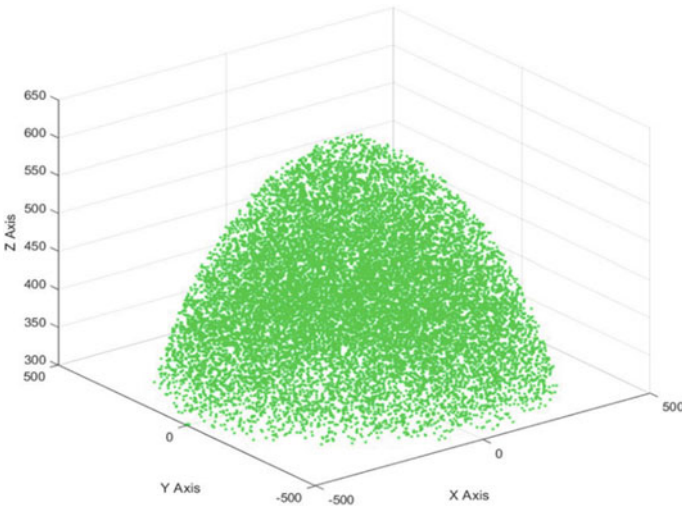


Fig. 15 Available workspace for configuration 2 (spatial Stewart platform)

On the top plate, the spherical joint is located at 40° and 80° apart and is not evenly spaced. The first spherical joint is at 20° with the reference and followed by the second joint which is at 80° to the first and 40° to the third as shown in Fig. 17.

Assembly is done by connecting the top and base plate with the help of linear actuator. The connections are not symmetrical in this case; here the first U-joint is connected to the first S-joint which are located at 20° and 40° with the reference axis which is Y-axis on X–Y plane. This can be seen in Fig. 18.

Table 2 Simulation results for configuration 2

Sample points satisfying constraints	Volume of workspace in mm ³
8534	119,476,000
8645	121,030,000
8652	121,128,000
8738	122,332,000
8683	121,562,000
8676	121,464,000
8615	120,610,000
8788	123,032,000

Fig. 16 Base plate for configuration 3 (isometric view)

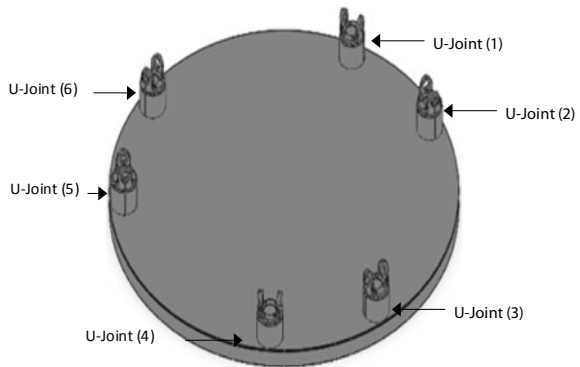


Fig. 17 Top plate for configuration 3

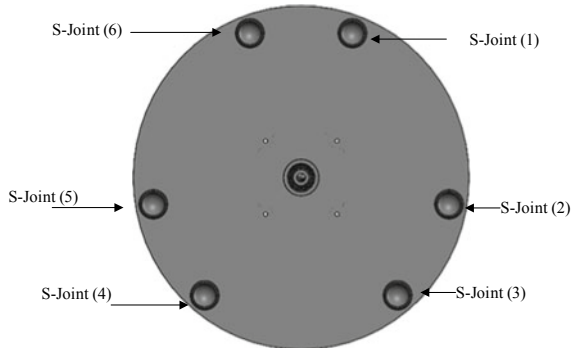


Figure 19 shows the workspace for spatial Stewart platform. The orientation of the top plate is considered to be horizontal. Triangulation of the spatial plot of Fig. 20 is shown in Fig. 19.

The volume for configuration 3 is least of all the three configurations.

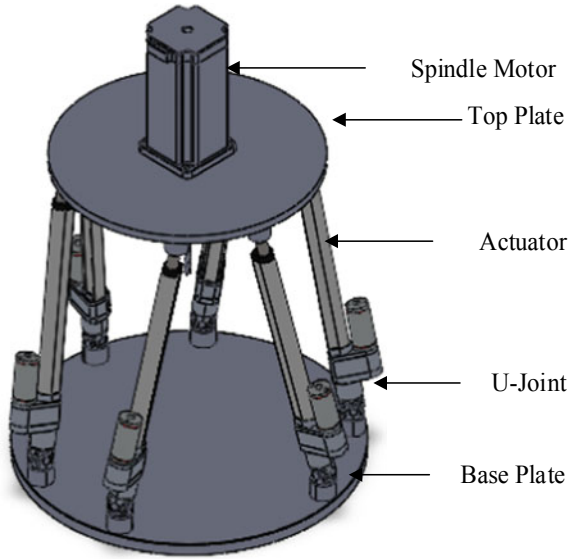


Fig. 18 Assembly for configuration 3 (isometric view)

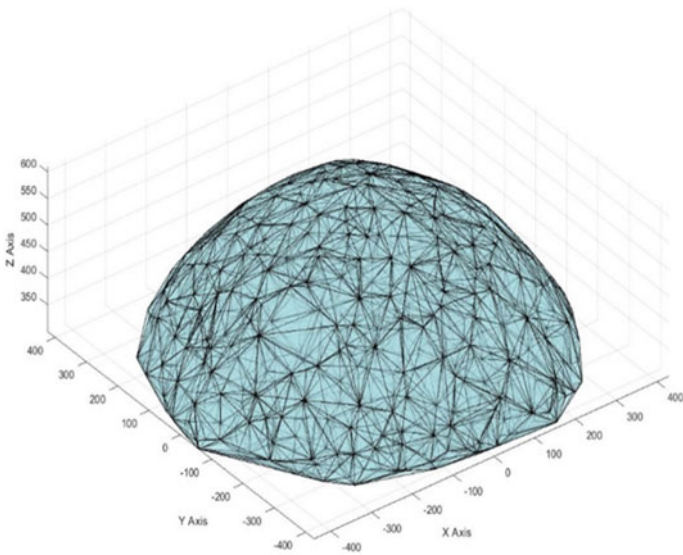


Fig. 19 Triangulation of the workspace for configuration 3

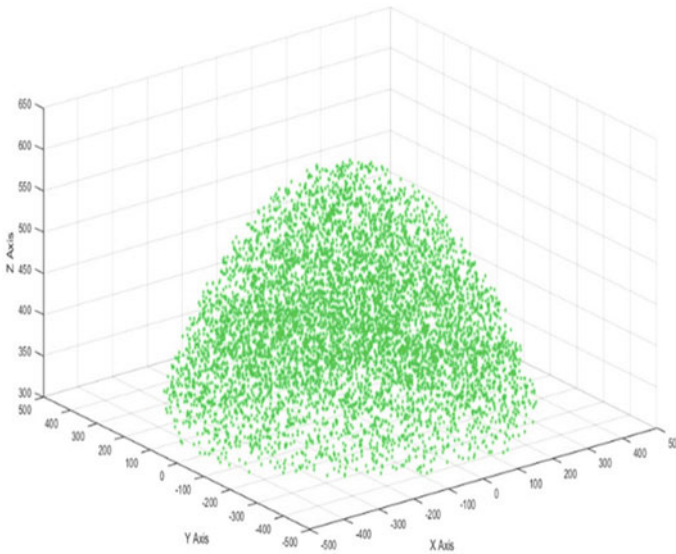


Fig. 20 Available workspace for configuration 3 (spatial Stewart platform)

Following are the results obtained by simulation of MATLAB code for configuration 3 (Table 3).

The volume obtained by configuration 1 is maximum, but due to its instability, configuration 2 will be selected for the PKM-based CNC machine tool. Depending on the maximum volume computed for a particular configuration, the orientation and position of the linear actuator on top and bottom plate have been selected, which comes out to be configuration 2, i.e., 60° equally spaced universal joint and 40° and 80° spacing of spherical joint on top plate. The first S-joint should be placed at 20° from the reference axis, and the first U-joint should be placed at 30° from the reference axis.

Table 3 Simulation results for configuration 3

Sample points satisfying constraints	Volume of workspace in mm ³
6942	97,188,000
6883	96,362,000
6771	94,794,000
6892	96,488,000
6957	97,398,000
6910	96,740,000
6757	94,598,000
6947	97,258,000

6 Conclusion

This paper presents Parallel Kinematics Manipulator and its advantages over Serial Kinematic Manipulator. A literature review on the existing industrial applications of PKM has been done. A detailed discussion about the computer numerical control (CNC) and Stewart platform-based CNC machines has also been brought off in this research paper. The advantages of Stewart platform have also discussed. Moreover, three different configurations of Stewart platform have been compared and simulated using MATLAB coding to calculate the workspace volume and position and orientation of the linear actuators, thereby selecting the most feasible configuration required for our suggested tabletop CNC machine configuration. Monte Carlo-based method to compute the workspace using inverse kinematics analysis and work volume for different configurations has been carried out in this paper. It is effective to solve for the conditioned workspace area and manipulator volumes using the Monte Carlo approach. The Monte Carlo-based algorithm and computed volumes for three-dimensional workspaces will aid to the design and fabrication of the suggested simple and low-cost tabletop CNC machine configuration.

References

1. Stan SD, Maties V, Balan R (2008) Optimal design of parallel kinematics machines with 2 degrees of freedom. In *Parallel Manipulators, Towards New Applications*. IntechOpen
2. Herve JM, Sparacino F (1991) Structural synthesis of 'parallel' robots generating spatial translation. Fifth international conference on advanced robotics robots in unstructured environments, Pisa, Italy, vol. 1, 808–813. <https://doi.org/10.1109/ICAR.1991.240575>
3. Kong X, Gosselin CM (2002) Type synthesis of linear translational parallel manipulators. *Advances in robot kinematics—Theory and applications*, Lenarc'ic J. and Thomas F. Eds., Kluwer Academic Publishers, pp. 411–420
4. Stewart D (1965) A platform with six degrees of freedom. *Proc Inst Mech Eng* 180(1):371–386
5. Patel YD, George PM (2012) Parallel manipulators applications—a survey. *Mod Mech Eng* 02(03):57–64
6. Alazard D, Chretien JP (1994) Dexterous manipulation in space: Comparison between serial and parallel concepts, *IFAC Proceedings Volumes*, 27(13): 411–418
7. Gallardo-Alvarado J (2016) Kinematic analysis of parallel manipulators by algebraic screw theory, Springer Cham, pp 1–377. <https://doi.org/10.1007/978-3-319-31126-5>
8. Staicu S, Carp-Ciocardia DC (2003) Dynamic analysis of Clavel's delta parallel robot. *Proc IEEE Int Conf Robot Autom* 3(May):4116–4121
9. Angeles J, Morozov A, Bai S (2005) A novel parallel-kinematics machine tool, *Proceedings of The fifth international workshop on advanced manufacturing technologies AMT2005 At: London, Ontario, Canada* 1–6
10. Rolland L (2015) Path planning kinematics simulation of CNC machine tools based on parallel manipulators. *Mech Mach Sci* 29:147–192
11. Rastegar J (1990) Manipulation workspace analysis using the Monte Carlo Method. *Mech Mach Theory* 25(2):233–239
12. Chaudhury AN, Ghosal A (2017) Optimum design of multi-degree-of-freedom closed-loop mechanisms and parallel manipulators for a prescribed workspace using Monte Carlo method. *Mech Mach Theory* 118:115–138