Gino Fano (1871–1952)

The Scientific Trajectory of an Italian Geometer Between Internationalism and Persecution

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Abstract Gino Fano is the first of the important group of Corrado Segre's disciples and, when he began his university studies in Turin in 1888, various circumstances favored his scientific maturation. The purpose of this essay is to highlight some less known aspects of his life and work taking into consideration the manuscripts and other unpublished documents kept in various archives in Italy and abroad. Three aspects are specially dealt with, namely:

- Fano's research, teaching, and dissemination activities in the wake of Segre's mastership and legacy, both on the national and on the international scene;
- His exile experience in Switzerland after racial discrimination;
- Fano's material and immaterial heritage in his works on threefolds.

Keywords Fano's scientific apprenticeship with C. Segre · Fano in Göttingen with Klein · Epistemological vision and teaching of mathematics · Racial laws · Swiss exile · Fano's material and immaterial heritage · Fano threefolds

Acronyms and Abbreviations

ACT	Archivio Colonnetti di Torino
AMS	American Mathematical Society
ASUT	Archivio Storico dell'Università di Torino

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AUL	*Papers of Professor William Henry Young and Grace Chisholm Young*. Special Collections and Archives, University of Liverpool
BSMT	Biblioteca Speciale di Matematica, Università di Torino
CUI	Campo Universitario Italiano di Losanna
FESE	Fond Européen de Secours aux Étudiants
GF	Gino Fano
ICM	International Congress of Mathematicians
MIT	Massachusetts Institute of Technology
r	recto
SPSL	Society for the Protection of Science and Learning
UMI	Unione Matematica Italiana
v	verso

1 Introduction

Gino Fano (Mantua, January 5, 1871—Verona, November 8, 1952) is the first of the important group of Corrado Segre's disciples and, when he began his university studies in Turin in 1888, various circumstances favored his scientific maturation. Segre had just obtained the professorship of higher geometry and was broadening his research horizons under the influence of the German School, in particular of Alexander Brill, Max Noether, and Felix Klein.[1] As Fano wrote many years later, Segre was able to rapidly assimilate and make his students appreciate the research of the most important foreign mathematicians, allowing him to set going that scientific project that was to make him the new leader in the field of Italian geometry:

> He [Segre] became so, just in the moment in which Cremona's scientific activity had completely ceased, the new leader of Ital[ian] geometry, the founder of a new school. He was also able to learn, to make his own, and to let estimate by his pupils all that, for the development of his programme, was to be got from the most important foreign mathematicians (Klein, Noether, Lie, Cayley, Zeuthen, Darboux, ...); and by means of his 35 years of teaching, about all most various branches of geometry, diff[erential]. and enumerative geom[etry] (abzähl[ende] Geom[etrie]) included, he had a very great influence on the development of all geometry in Italy. ([3], c. 63*r*)

At that time in Turin, there was also Guido Castelnuovo, called by Segre in 1887 as assistant to Enrico D'Ovidio. Both soon realized Fano's mathematical skills, took his training to heart, and immediately oriented him towards the most topical research themes.

Fano came from a wealthy Jewish Mantua family,[2] and after starting his university studies as a student engineer at the University of Turin, he soon moved on to mathematical studies (Fig. 1).

[1] On Corrado Segre's work, see the recent volume by Casnati et al. [1]; on the birth of the Italian School of algebraic geometry, see [2].

[2] For Fano's biography, see inter alia [4–12].

Fig. 1 Gino Fano in 1887

While still a student, under Segre's prompting, Fano translated Klein's Erlangen program and published his first works. In 1892, he graduated[3] with a dissertation in hyperspace geometry supervised by his teacher, but also stimulated by Castelnuovo's research on curves of the highest genus in a projective space.[4]

[3] See the degree certificate in ASUT XD 193, 36.

[4] The memoir by Fano [13] is taken from the thesis. See Segre's report in *Atti dell'Accademia delle Scienze di Torino* 28 (1892–93), 865–866. Fano's mathematical work can be accessed at http://www.bdim.eu/item?id=GM_Fano.

The following year, Segre sent him to Göttingen for a period of study with Klein to complete his training. Back in Italy, in 1894, he went to Rome as an assistant to Castelnuovo, who in 1891 had obtained the professorship in that city. In 1899, he won the competition for the professorship in Messina, but in 1901, again following a competition, he obtained the chair of projective and descriptive geometry with drawing at the University of Turin, where he taught continuously until 1938. For Fano, this was a period of intense work in the field of research, teaching, and scientific dissemination. In particular, in 1904, he published his first article on three-dimensional algebraic varieties, a theme that was to occupy him throughout his life. On account of the racial laws enacted in 1938, he was forced to emigrate to Switzerland and, after returning to Italy, was to reside alternately in Italy and in the United States because his sons Ugo and Robert lived and worked overseas. The three values that inspired his whole life were, as his son Robert recalls, "his family, his country, his profession" [6].

The purpose of our essay is to highlight some lesser known aspects of Fano's life and work taking into consideration the manuscripts and other unpublished documents kept in various archives in Italy and abroad. The three points we intend to develop are the following:

- From Segre's School to achievements on the international scene: research, teaching, and dissemination of ideas;
- Racial laws, Swiss exile, and the return to Italy;
- Fano's material and immaterial heritage: the case study of his works on three-folds.

In order better to highlight the most significant features of these aspects of Fano's life and work, three different historical methodologies have been adopted. The category of "School" has been considered the most appropriate to address the issues developed in Sect. 2 and specifically to highlight the influence of Segre on Fano's work.[5] To suitably investigate the late period of Fano's trajectory, we decided to place it within a general phenomenon, i.e., the emigration of Jewish mathematicians from racist Italy. Consequently, Sect. 3 is to be considered as a case study in social history of mathematics, which narrative adopts the categories developed for the study of political emigration on racial ground from central and eastern Europe.[6] The last part (Sect. 4), in applying the heritage investigation approach, both in material and cultural dimensions, leads to a reassessment of some of the best mathematical contributions by Fano: the three-dimensional algebraic varieties today called Fano threefolds.[7]

[5] See Rowe, D.: Mathematical Schools, Communities, and Networks. In Jo Nye, M. (ed.) The Cambridge History of Science, 5, Modern Physical and Mathematical Sciences, pp. 113–132. Cambridge University Press, Cambridge (2003).

[6] See Ash, M. and Söllner, M.: Forced Migration and Scientific Change. Emigré German-Speaking Scientists and Scholars After 1933. Cambridge University Press, Cambridge (1996).

[7] See [14].

2 From Segre's School to Achievements on the International Scene

During the training phase, three mentors, Segre, Castelnuovo,[8] and Klein, oriented Fano's research in three main directions: projective geometry in a higher dimensional space, birational geometry, and Lie's theory of groups.

The sectors in which he made significant contributions are manifold: foundations of hyperspace projective geometry; algebraic curve theory; K3 surfaces and Enriques surfaces; continuous groups of transformations in projective and birational geometry; line geometry; algebraic varieties defined by linear differential equations; research on three-dimensional algebraic varieties; and birational geometry in dimension three.

Alongside this research work, Fano made a valuable contribution as a writer of treatises and with the dissemination activity that characterized the various moments in his life.

It is not one of our objectives to investigate all his varied scientific activity, on which significant literature is already available.[9] We will take into consideration only those aspects that are useful for illustrating the points we intend to develop.

2.1 Early Research as a Student

As already mentioned, Segre devoted himself to Fano's training starting from the second year of his university studies. In 1899, he entrusted his pupil with the translation of Klein's Erlangen program into Italian, having long recognized its relevance for the development of geometric research.[10] On that occasion, he wrote to Klein:

> I would like, for the benefit of the Italian geometers who hardly know it, to publish an Italian version [of Klein's Erlangen Program] which I would have done by one of my students (who has even already sketched it out) and which I would correct myself with the utmost care.[11]

Fano's translation, the first of the Erlangen program, was published in 1890 in the *Annali di Matematica Pura ed Applicata* [21] and was the first contact between the young researcher and Klein, a contact that was to prove important from various points of view.

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8 See the letters from Fano to Castelnuovo (60 documents from 1889 to 1903) in [15].

9 On Fano's scientific work, see, inter alia, [4, 16–18] pp. 251–260.

10 According to [16] (p. 187), the memoir by Segre [19] is "*the earliest study of geometry in the spirit of the* Erlanger Programm."

11 C. Segre to F. Klein, Turin 19 November 1889 ([20], p. 151): *Je voudrais, pour l'avantage des géomètres italiens qui ne le connaissent presque pas, en publier une version italienne que je ferais faire par un de mes élèves (qui l'a même déjà ébauché) et que je corrigerais moi-même avec les plus grands soins.*

In that same year, during the famous course [22] in which he dealt with geometry on an algebraic curve from the triple point of view, hyperspatial, algebraic, and functional, Segre proposed the problem of assigning a system of postulates for projective hyperspace geometry:

> Define the space S_r not by means of coordinates, but rather by a series of properties from which the representation with coordinates can be deduced as a consequence.[12]

Both Fano and Federico Amodeo, who attended the course as an auditor, addressed the problem and both published separate works [23, 24], despite Segre's invitation to collaborate. Federigo Enriques, who in November 1892 had come to Turin to meet Segre, also dealt with the same problem [25]. While Fano's and Amodeo's approaches are completely abstract, i.e., make no reference to the experimental, psychological, or physiological origin of the postulates, the approach adopted by Enriques, as he himself points out, is to establish the postulates deriving from the experimental intuition of space, which are presented as the simplest ones for defining the object of projective geometry. Subsequently, his contacts with Fano, who at the time was in Göttingen, led to the publication of their correspondence on the subject [26, 27]. There thus began that interaction and comparison of methods within the School to which Segre aspired.[13]

This group of works is part of a fertile field of research, the foundations of geometry, cultivated in Italy at the time both in Segre's and Peano's Schools [29]. Fano illustrates them in a letter to Klein, who had asked him for information on the matter, and also presents a comparison with the studies of Mario Pieri, which came shortly after, highlighting differences and innovations.[14]

Fano's 1892 article, in addition to testifying to Segre's role as a mentor, is particularly significant: Fano, indeed, in order to demonstrate the independence of the postulates he established (of the n-th from the preceding $n - 1$) uses a number of geometric models and so he comes to discover examples of finite projective spaces. Although these examples never became the starting points to develop a new field of research, they represent the early sources for those finite geometries that were to be developed various years later by the American School of Oswald Veblen, and in the 1930s by the German mathematicians R. Baldus, H. Liebmann, and M. Steck, authors to whom Fano was to refer in two subsequent notes on this subject published in *Rendiconti dell'Accademia dei Lincei* [30, 31].

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[12] [23], p. 107: *Definire lo spazio* $\mathrm{S_r}$ *non già mediante coordinate, ma con una serie di proprietà dalle quali la rappresentazione con coordinate si possa dedurre come conseguenza*.

[13] Segre cites these papers in his 1893–1894 course, and again in his 1898–1899 course, where he presents them at greater length, also referring to an article by Mario Pieri devoted to the same subject. See BSMT, Fondo Segre, Quaderni. 5, p. 13 and Quaderni. 12, p. 3 in [28].

[14] G. Fano to F. Klein, Rome, 9 April 1897 in [20].

2.2 Fano in Göttingen

At the end of the nineteenth century, the favorite destinations of Italian mathematicians to perfect their studies were the German universities, which constituted an important reference point for young researchers. From 1883, Segre had constant correspondence with Klein, who a few years later was to move from the University of Leipzig to that of Göttingen. Segre himself visited the main German institutes in 1891, so it is not surprising that he sent Fano to Göttingen to strengthen his mathematical training. In his letter recommending Fano to Klein, Segre wrote:

> He is gifted with a great memory, and has a lively mind. But his tendencies are essentially directed toward geometry, pure geometry. And even though I have repeatedly encouraged him to cultivate analysis too, and in my courses, I have shown not only the synthetic methods but also analytical methods, he has remained up to now too exclusively a geometer [. . .] I believe that it is possible to strengthen him a great deal as a geometer if you can make him fully acquire the analytical tools.[15]

Fano arrived in Göttingen in mid-October 1893. Teaching at that university and German universities in general was characterized by *Lern-und Lehrfreiheit*, that is, by freedom of teaching and study and by courses on specific research topics [32].

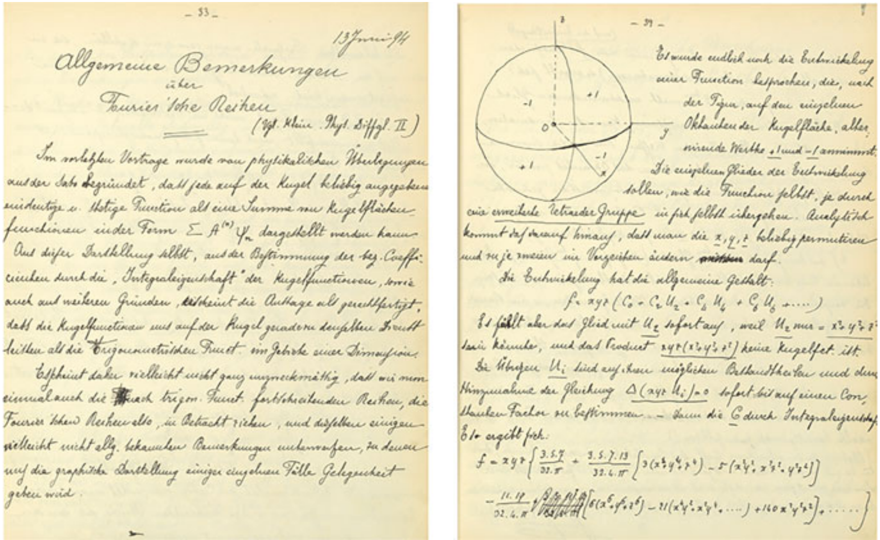
In the winter semester and the following summer semester, Fano attended three courses held by Klein, on hypergeometric function, on second-order linear differential equations, and on elementary geometry. The latter was to converge in the volume *Vorträge über ausgewählte Fragen der Elementargeometrie* (1895), which tackles issues regarding the possibility or impossibility of performing certain geometric constructions with ruler and compass and the relative character of the concept of solving a problem, adopting a historical approach.

The courses alternated with seminars which, as Fano writes, constituted a "very important complement,"[16] because, in addition to the professors, the students—including many foreigners, especially English and American ones—were required to present a topic related or otherwise with the course. Klein devoted his summer-semester seminars to spherical functions and their applications in mathematical physics, and Fano developed two topics, one on Fourier series and another on spherical functions: *Allgemeine Bemerkungen über Fourier'sche Reihen* (June 13, 1894) and *Kugelfunktionen* (June 20, 1894)[17] (Fig. 2).

[15] C. Segre to F. Klein, Turin 4 October 1893: *È dotato di molta memoria ed ha un ingegno vivace. Ma le sue tendenze sono essenzialmente geometriche, per la pura geometria. E quantunque io l'abbia eccitato ripetutamente a coltivare anche l'analisi, e nei miei corsi gli abbia fatto vedere non solo i metodi sintetici ma anche quelli analitici, egli finora è rimasto troppo esclusivamente geometra* [. . .] *credo che si possa rinforzarlo di molto come geometra se si riesce a fargli acquistare pienamente gli strumenti analitici* (in [20], pp. 164–165).

[16] [32], p. 183: *complemento importantissimo*.

[17] All the presentations made in seminars from 1872 until 1913, when Klein retired, are carefully noted in his *Protocollbuch*, usually by the speaker himself See [101]. All of these reports are available online and are a valuable document regarding Klein's ambitious research and teaching



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Allgemeine Bemerkungen über Fourier'sche Reihen

Fig. 2 The seminar by Fano in Klein's *Protocollbuch*, pp. 33 and 39

Among the students attending the seminars were Virgil Snyder, Emanuel Beke, Wilhelm Lorey, and Grace Chisholm—among other things, the latter was to come to Turin with her husband William Young in 1898–1899 to attend Segre's courses. All of them were to remain in contact with Fano, as evidenced by the various offprints they sent him over the years [97]. Relations between students and professors were favored not only by seminars, but also by extra-university meetings at the *Mathematische Gesellschaft*, where original research could be presented. Fano held two lectures here, one on recent research by Italian geometers, and the other on line geometry: *Ueber die neuesten Untersuchungen der italienischen Geometer* and *Ueber eigene Untersuchungen im Gebiete der Liniengeometrie*. Probably in the latter lecture, there was the starting point of the research on the subject that he was to expound in the 1895–1896 course in Rome and on which he was to publish important articles, where line geometry is seen, in line with Klein, as the geometry of a quadratic manifold in five-dimensional projective space (that is, the Grassmannian in a five-dimensional projective space). As David Rowe notes:

> Line geometry served here as a bridge between the older world of geometrical research and the newer style that soon led to the many more familiar achievements of Italian algebraic geometry. ([34], p. 261)

In Göttingen, Fano thus began the work of spreading Italian geometry abroad, which was to continue over the years.

program. Fano's seminars can be accessed at https://www.uni-math.gwdg.de/aufzeichnungen/klein-scans/klein/V12-1894-1896/V12-1894-1896.html.

Klein had had the opportunity to appreciate the work methods of the young Italian mathematician, which aimed at exploiting geometric intuition, in the style of Segre's School, and mentioned him for the 1894 prize of the Berlin Academy for research on linear differential equations. While warmly thanking him, Fano replied that at the moment his studies had different objectives:

> I am now pursuing a different but not insignificant goal, that is, the case in which, among the fundamental solutions, there are algebraic relations with constant coefficients, that is, where the periodic projective curve lies on a well determined algebraic variety (in particular, it is itself algebraic). By contrast, the Berlin Academy wants the function Z of certain variables $\frac{u_2}{u_1}, \ldots, \frac{u_n}{u_1}$ to be the object of in-depth studies.[18]

In the article that he was to publish the following year [35], Fano thanks Klein for having oriented him towards that kind of research. The German mathematician's appreciation of him was manifested again in 1899, when he offered him the chair of geometry in Göttingen previously occupied by Arthur M. Schönflies with these words:

> I conceive the chair essentially as a *geometric* chair, that is, I wish the one who holds it to extol geometric intuition and develop geometric studies in all directions. But now you know the decline of geometry in the younger German generation. I have reached the conclusion that you are precisely the man for us![19]

Fano replied very diplomatically that he was honored by such an offer but preferred a chair at an Italian university;[20] moreover, as his son Ugo recalls, he did not want to "be Germanized" ([7], p. 178). Indeed, in that same year, following a competition, he obtained a professorship at the University of Messina,[21] but his aspiration was to return to Turin. In 1901, again following a competition, he obtained the chair of Projective and Descriptive Geometry with drawing in the Piedmontese capital and here he put the multifaceted experiences of Göttingen to good use.

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[18] G. Fano to F. Klein, Rome, 20 April 1895: *Ich jetzt ein nicht unbedeutend verschiedenes Ziel, den Fall nämlich wo zwischen den Fundamentallösungen algebraische Relationen mit const. Coeff. bestehen, d. h. wo die projectiv-periodische Curve auf einer bestimmter algebraischen Mannifaltigkeit liegt (bezw. selbst algebraisch ist. Dagegen wünscht die Berliner Akademie dass die Function Z gewisser Variablen* $\frac{u_2}{u_1}, \ldots, \frac{u_n}{u_1}$ *... eigehend untersucht* (in [20], pp. 178–179).

[19] F. Klein to G. Fano, Göttingen 5 February 1899: *Ich fasse die Professur wesentlich als eine geometrische Professur, d. h. ich wünsche, dass der Neuzuberufende die geometrische Anschauung hervorkehrt und nach allen Richtungen die geometrischen Studien belebt. Nun kennen Sie aber den Niedergang der Geometrie in der jüngeren deutschen Generation. Ich bin also auf den Gedanken gekommen, ob nicht Sie der geeignete Mann für uns wären*! (in [20], pp. 195–196). This letter can be accessed at https://www.corradosegre.unito.it/fondofano/lettera9.pdf.

[20] G. Fano to F. Klein, Turin, 10 February 1899 (in [20], pp. 197–198).

[21] On Fano's courses and on his stay in Messina, see for example his letters to Castelnuovo: 20 November 1899, 3 December 1899, and 7 February 1900, in [15].

2.3 Research, Epistemological Vision, and Teaching

The constant reference to Segre, whom he met periodically,[22] and interactions with the other members of the School, in particular Castelnuovo and Enriques, contributed to Fano's full scientific maturation, but equally important was the period of postgraduate studies spent in Göttingen with Klein, which left a significant imprint on his research, epistemological vision, and teaching.

As is well known, the Erlangen program, which set the concept of group at the basis of the study of geometry, favored the flourishing of research on the connections between Lie's group theory and geometry and, as Hawkins ([16], p. 186) observes, Italian geometers played an especially important role. In 1893, Enriques, who among other things had spent a study period in Turin with Segre in the winter of 1893–1894, had faced the problem of determining surfaces in three-dimensional space left invariant by a continuous group of projective transformations [36], unaware that Lie had already published a work on this subject and had presented the results again in 1893 in the third volume of *Theorie der Transformationsgruppen*. On his return from Göttingen, Fano published two works [37] and [38] on the problem of determining algebraic varieties in four-dimensional space left invariant by a group of projective transformations. This research was not a mere extension of that of Enriques and Lie, because, as Hawkins writes:

> The earlier work of Lie and Enriques involved methods peculiar to 3-dimensional space and huge amounts of computation. Fano therefore looked to what would now be called the theory of the structure and representation of Lie algebras for more general methods. ([16], p. 190)

Fano therefore made use of the so-called theory of representations of Lie's algebras about 14 years before the work of Élie Cartan. Subsequently, together with Enriques, he published a work [39] on the determination of all birationally distinct types of Cremonian finite continuous groups of space and in 1897 presented the results obtained in Zurich during the International Congress of Mathematicians [103]. This work was integrated by another one [40] published in 1898.

It is not surprising that Klein invited Fano to write two articles for the *Encyklopädie der mathematischen Wissenschaften* [41, 104], one of them [41] precisely on continuous groups, which had a significant influence on Cartan, who edited a revised and expanded version in French.

We have focused on this particular line of Fano's research, which originates from Klein's work, for its scientific importance and because it is indicative of both master-disciple interactions within the School and the opening up of the School towards the outside world. Indeed, it was Segre who oriented Fano towards Klein's research and presented his 1896 and 1898 memoirs for publication in the *Memorie dell'Accademia delle Scienze di Torino*, with extensive reports in which he emphasizes the "geometric acumen" and "patient care" and highlights his

[22] See G. Fano to G. Castelnuovo, Mantova, 24 December 1901, in [15].

"considerable scientific importance."[23] It was also Segre that devoted three courses in higher geometry to continuous groups of transformations, in 1897–1898, 1906–1907, and 1911–1912,[24] where he expounded the fundamental parts of the theory, also dwelling on their classification, with an approach that privileged geometric aspects, and did not fail to refer to the contributions of his disciples.

2.3.1 The Epistemological Vision

Another aspect that clearly shows Klein's influence is Fano's epistemological vision. As is well known, for Klein, construction of mathematical knowledge takes place in three phases: gathering information derived from experience, putting the data obtained into mathematical form and proceeding to a purely mathematical treatment of the problem, and, finally, translating the mathematical results into the form most suitable for applications. Fano, like the other members of the Italian geometric School [42], takes this point of view. For the choice of the postulates at the basis of a theory, he accepts the criterion of expediency, in line with Klein, who considers postulates *vernünftige Sätze* (reasonable propositions induced by spatial intuition), and rejects the nominalist approach of Henri Poincaré, who considers them as simple conventions that are convenient but completely arbitrary as long as they are compatible with each other.

Fano also shares with Klein the unitary conception of science, the enhancement of discovery processes through intuition, "the antivenom to logic,"[25] the role of experimental procedures in mathematics too, and the distinction between *Präzisionsmathematik*, precision mathematics, and *Approximationsmathematik*, seen by Klein as the exact mathematics of approximate relations [44]. This consonance of thought is confirmed by the numerous references to the German mathematician that appear in Fano's articles on epistemology, teaching, and popularization of mathematics, in which he interprets Klein's teachings and makes them his own.

As is well known, Klein distinguished between naïve intuition and refined intuition, and highlighted the fact that the former is important in the discovery phase of a theory, while the latter intervenes in the elaboration of data furnished by naïve intuition, and in the rigorous logical development of the theory itself ([45], pp. 41–42). In harmony with this point of view, Fano attributes to intuition the role of guiding the reasoning, predicting its conclusion, and checking and experimentally confirming the result obtained ([46], p. 122). Furthermore, he distinguishes various types of intuition, all valuable: "the vision by spirit" of the Greeks; "a form of memory, developed as a consequence of past scientific work"; "the intuition of the

[23] See *Atti della R. Accademia delle Scienze di Torino*, 31, 623–624 (1895–1896) and 33, 796–797 (1897–1898): *acume geometrico, la paziente cura, la notevole importanza.*

[24] The handwritten notebooks of Segre's lessons can be found in [28], Quaderni. 11, Quaderni. 20, and Quaderni. 25.

[25] [43], p. 25: *contravveleno alla logica.*

formalist"; and "the intuitional skillfulness of a refined analyst" ([47], p. 15). As regards the relationship between intuition and logical rigor, he writes:

> With intuition [the mathematician] discovers; with logic the new discovery is broken down into the single elements, links in a chain to be reviewed in the light of criticism; with intuition [the mathematician] makes the synthesis again, precisely as the living organism is the synthesis of its cells and the former and the latter are both rich in interest; but even minute knowledge of cells is not enough to make the whole individual known. Mathematics is also a living organism, of which it could be said that logical connections constitute the skeleton; but the organism is not just a skeleton. The skeleton alone does not live![26]

For Fano, mathematics, as well as being the logical science par excellence, can be counted among the experimental sciences:

> Without the recognition of the possibility, for ourselves, of repeating the same operation several times, no mathematical knowledge would be possible; not even the greatest genius would perhaps have arrived at the notion of the natural numbers two, three, etc.[27]

By experimental procedures in mathematics, Fano means thorough and complete examination of special examples that the mathematician produces in his "intellectual laboratory" that can lead to new ideas and are a powerful method of error checking. Among experiments, he also includes the use of physics and drawing:

> Certain theories, certain calculations, can, I will say, be mentally accompanied by a geometric image, or a physical image: thanks to these, we sometimes manage to embrace in a single glance what logical deduction or calculation would show us only later.[28]

To exemplify his views, Fano mentions Klein, for whom physics was a tool of discovery: for example, to demonstrate the existence of the so-called Abelian integrals, he thought of resorting to electrical experiences; likewise, the origin of most of Riemann's ideas lies at least partially in considerations of a physical nature. The physical sciences, says Fano, have oriented mathematical research that has thus escaped the danger of becoming pure symbolism closed in on itself ([48], p. 28).

If it is true that for Fano mathematics can and must be an end in itself, almost a work of art due to its characteristics of simplicity and harmony, he is also well aware that recent developments in applied mathematics make it essential to compare

[26] [43], p. 27: *Coll'intuizione si scopre; colla logica si decompone il nuovo trovato nei singoli elementi, anelli di una catena per ripassarli al lume della critica; coll'intuizione di nuovo si sintetizza: appunto come l'organismo vivente è la sintesi delle sue cellule e quello e queste sono entrambi ricchi di interesse; ma la conoscenza anche minuta delle cellule non basta a far conoscere l'individuo. La matematica è anch'essa un organismo vivente, di cui si potrebbe dire che le connessioni logiche costituiscono lo scheletro; ma l'organismo non è solo scheletro. Lo scheletro da solo non vive!*

[27] [43], p. 22: *Senza la constatazione della possibilità, per noi stessi, di ripetere una medesima operazione più volte, nessuna conoscenza matematica sarebbe possibile; nemmeno il più gran genio sarebbe forse arrivato alla nozione dei numeri naturali due, tre, ecc*.

[28] [43], pp. 13–14: *Certe teorie, certi calcoli, si possono, dirò così, accompagnare mentalmente con un'immagine geometrica, o con un'immagine fisica: grazie a queste, riusciamo talvolta ad abbracciare in un sol colpo d'occhio ciò che la deduzione logica, o il calcolo ci mostrerebbero soltanto successivamente*.

"exact mathematics", which derives from a work of abstraction and idealization, and "approximate mathematics", which actually is useful in applications. The fact is that every measurement, performed with our senses or with any instruments, has a degree of approximation or precision beyond which we cannot go. In a 1911 article, Fano explicitly refers to Klein, who in his 1901 lectures in *Anwendung der Differential-und Integralrechnung auf Geometrie, eine Revision der Prinzipien*[29] stresses the importance of considering "mathematische Wissenschaft als ein zusammengehöriges Ganzes" (mathematical science as a coherent whole) and addresses the problem of the relationships between the two types of mathematics in order to bring pure mathematicians closer to technicians dealing with applications, without neglecting the importance of this approach in teaching. In his article, Fano takes up many of the examples presented by Klein, drawn from analysis, geometry, and geodesy, and concludes by emphasizing the educational value of bringing teaching closer to reality:

> it would be desirable ... to give, as soon as possible, in schools, a clear idea of the relationships between the physical world and the mathematical world, of the degree of accuracy with which mathematical concepts and formulas can represent entities and physical dependencies, and of the different ways in which the same entity or physical relationship can sometimes be idealized...; and on the other hand, if we carefully highlight... the practical value of a concept or a mathematical result, we will bring teaching close to reality, eliminating a divergence and filling a gap that are among the most fatal defects at any school level.[30]

Similar themes are also addressed by Segre in his lectures at the *Scuola di Magistero* (teacher training college) (Quaderni. 40 in [28]), which Fano had attended during his university studies; by Castelnuovo, in particular in his 1913–1914 higher geometry course [15]; by Enriques, in his epistemological writings; and by other members of the School of Segre. Among those that Fano mentions most frequently is Enriques, and in particular his dynamic vision of science seen as a process of successive approximations ([48], p. 23). Being a man broadly read in mathematics, history, and philosophy, Fano also brings in precise references to various other scholars (mathematicians, physicists, chemists, philosophers) outside the School, in addition to Klein. Among these, there are Ernst Mach, whose vision of science he shares as a minimum problem which consists of presenting the facts following the criteria of simplicity and economy of thought ([48], p. 21), as well as Borel,

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[29] Fano carefully read this work by Klein, as we can see from the annotations on his copy of the lithograph preserved in the Biblioteca Speciale di Matematica "G. Peano" of the University of Turin. See also the letter by G. Fano to G. Castelnuovo, Colognola ai Colli, 15 July 1902 in [15].

[30] [44], p. 126: *sarebbe desiderabile che* [...] *fosse data il più presto possibile, nelle scuole, un'idea chiara delle relazioni che intercedono fra il mondo fisico e il mondo matematico, del grado di esattezza con cui i concetti e le formule matematiche possono rappresentare enti e dipendenze fisiche, dei diversi modi in cui uno stesso ente o rapporto fisico può talvolta essere idealizzato* [...]*; e d'altra parte, precisando bene* [...] *il valore pratico di un concetto o di un risultato matematico, accosteremo l'insegnamento alla realtà, eliminando una divergenza e colmando una lacuna che sono fra i difetti più fatali a ogni ordine di scuola*.

Einstein, Helmholtz, Maxwell, Peirce, Poincaré, Rignano, Riemann, and many others.

2.3.2 "Fighting Prejudices Against the Supposed Mysteries of Mathematics."[31] Fano's Commitment to Education

Fano's vision of mathematics teaching, as is natural, is closely linked to his epistemological vision. He expounds his reflections on the subject in various contexts: in the congresses of Mathesis, the national association of mathematics teachers (Florence 1908, Padua 1909, Genoa 1912, Trieste 1919, Naples 1921, Milan 1925); in the *Conferenze Matematiche torinesi* (Turin mathematical lectures) promoted by Giuseppe Peano; in various dissemination lectures, including those at the *Gabinetto di Cultura della Scuola di Guerra* (Culture Cabinet of the War School) and at the *Società di Cultura* (Culture Society) in Turin; in his institutional roles as president of the *Scuola Operaia femminile* (School of Female Workers) (1909–1937) and president of the Piedmont section (1913), later the Turin section, of Mathesis; and finally through his collaboration with the "Enciclopedia delle matematiche elementari." He was attentive to the legislative measures proposed by the government: in 1913, he commented on the syllabuses of the *Liceo moderno* (Modern High School);[32] in 1919, he expressed himself in favor of maintaining the positive aspects of the curricula of the provinces of Trento and Trieste, which had just been annexed to the Kingdom of Italy;[33] in 1921, he took sides against the suppression of the *Scuola di Magistero*; and in 1923, he highlighted positive and negative aspects of the Gentile Reform ([50], pp. 23–25).

His interest in problems connected with mathematics teaching arose from his vision of mathematics as a discipline that educates the character and accustoms people to a sense of economy of work, and to precision and clarity, but it also originated, as for Castelnuovo, from social concerns, as is evident from his commitment to tackling inequalities and illiteracy, which emerges from his work in the *Scuola Operaia femminile*. Fano writes:

> for the working classes too, education must aim not only at allowing people to acquire certain types of knowledge, but also at getting them used to a moderate intellectual discipline.[34]

[31] [49], p. 367: *Combattere le prevenzioni contro* [...] *i presunti misteri della matematica*.

[32] *Bollettino della Mathesis* 5.1, 46–48 (1913).

[33] *Bollettino della Mathesis* 12, 62 (1920).

[34] [46], p. 12: *anche per le classi operaie l'istruzione deve tendere, non soltanto a far acquistare determinate cognizioni, ma da abituare a una pur moderata disciplina intellettuale*.

For the multifaceted work he carried out as president and for the donations made to this school, in 1928 he was awarded the Gold Medal of Merit of Public Education.[35]

The methodological approach that guided his work concerning mathematics teaching was openly influenced by Klein, who, as is well known, was the promoter of an important teaching reform movement in Germany [53].

Like Klein, Fano repeatedly stresses the importance of establishing a bridge between secondary and university education through early introduction of the concepts of function and transformation in mathematics teaching in secondary schools ([48], p. 27). For this teaching to be profitable, it is also necessary to establish links between mathematics and reality and between mathematics and applications and to valorize experimental procedures while trying to find the right balance between rigor and intuition. In this regard, Fano takes up the simile of the tree already used by Klein:

> To ask whether ... intuition or reasoning matters most, would be like asking whether, for a tree, roots or branches and leaves, are more important and needful: the question would be badly put, because the life of the tree rests on the reciprocal action of the different organs. ([47], p. 16)

The problem of teacher training, dear to Klein, is also central to Fano. Convinced that "it is worthless to know *more* than one teaches, if this does not make the things to be taught better known,"[36] at the time of the suppression of teacher training colleges, Fano strongly supported the importance of instituting courses of "Higher views on elementary mathematics" with emphasis on the historical, critical, methodological, and teaching aspects, citing by way of example the lectures of Segre and Enriques. He also invited the faculties to accept dissertations in complementary mathematics as graduate theses and urged his colleagues to start practical training in secondary schools for prospective teachers, without waiting for ministerial decrees ([54], pp. 103 and 109).[37]

A particularly significant aspect of Fano's commitment to mathematics education is given by the numerous treatises he wrote for his university courses (in Rome and Turin, at the University and at the Polytechnic). Many of these were written in the late nineteenth and early twentieth centuries and then revised and perfected in subsequent editions, on various sectors of geometry: line geometry (1896); non-Euclidean geometry (1898, 1935); descriptive geometry (1903, 1910, 1914, 1926, 1932, 1935, 1944); projective geometry (1902, 1903, 1907); analytical geometry

[35] See G. Fano to the president of the Reale Accademia Virgiliana, Turin, 24 November 1935, in [8], pp. 149–150. For an in-depth study on this aspect of Fano's work, see [51]. For the historical background, see [52].

[36] [54], p. 102: *a nulla vale saper più di ciò che si insegna, se questo di più non fa conoscer meglio le cose da insegnar*e.

[37] On the problem of teacher training in Italy and on the contribution of Italian geometers, see [55].

(1944); analytical and projective geometry (1926, 1930, 1940, and 1957 [with Terracini]); and complements of geometry (1935).[38]

Each of these treatises deserves to be studied in depth, but here we will limit ourselves to underlining the common traits that characterize them: clarity of presentation; alternation of analytical and synthetic approaches for educational purposes; presence of historical hints or a real historical approach as in those on non-Euclidean geometries; attention to applications to other scientific fields such as shadow theory, perspective, photogrammetry, and theory of relativity; and the tendency, learned from his teacher Segre, to highlight links with research in order to "allow people to presage future developments."[39]

It is sufficient to mention the example of the lesser known treatise on line geometry that originates from the free course of projective geometry that Fano held at the University of Rome in the year 1895–1896. The treatise opens with a documented historical introduction in which he illustrates the origins of line geometry from research in three different fields—geometry, mechanics, and physics—to arrive at Klein's approach, which considers the *lines* of ordinary space as *points* of Klein's quadric, which is just a smooth quadric in the five-dimensional projective space. Hence, he introduces the study of line congruences, considered as the surfaces of this quadric, and mentions the research, only sketched out, on third-order congruences, also referring to his own recent studies. Among others, he cites the contributions of Segre, who had dealt with line congruencies since his graduation dissertation and who generously made his notes on this subject available to him[40] ([57], p. 142).

Fano's lectures, as Terracini recalls, were "solemn lectures, which were prepared in every detail, but found the most effective spontaneity in the power of the arguments, in their concatenation and in the emphasis given to the fundamental ideas!"[41] Like Segre, he was a strict and demanding professor, so it is not surprising that in 1902, a few months after his return to Turin from Messina, some students attacked him in a highly controversial article that appeared in the satirical newspaper *La Campana degli studenti*.[42]

38 See *Publications of Gino Fano 1890–1953*, in [4], pp. 127–137.

39 [56], p. VI: *far presagire sviluppi futuri*.

40 See Quaderni. 4 1891–1892: C. Segre, *Geometria della retta*, pp. 18–42, in [28].

41 [12], p. 708: *lezioni togate, preparate in ogni particolare, ma che nella potenza delle argomentazioni, nel loro concatenamento e nel rilievo dato alle idee fondamentali ritrovavano la più efficace spontaneità!*

42 See "La questione Fano." *La Campana degli studenti*, 27 November 1902: Fano was accused of *strage degli innocenti* (slaughter of the innocents) during the exams with rather irreverent tones. For details on the affair, see the letters: G. Fano to G. Castelnuovo, Turin, 11 and 29 November 1902 and 14 December in [15].

2.3.3 Scientific Dissemination

From the outset, Fano combined significant scientific dissemination activity in two main directions, with scientific and teaching activity. First of all, there was dissemination of the research of the Italian geometric School through various channels: stays abroad, international congresses of mathematicians, correspondence, and exchange of offprints. Fano always accompanied this activity with work of dissemination in a broad sense through various channels: the more traditional ones, such as *Mathesis*, but also the *Scuola Operaia femminile* and the *Scuola di Guerra*, articles for the "Enciclopedia Italiana," as well as journals with different readerships—teachers, philosophers, and intellectuals in general.[43] Among the themes he most often addresses are interactions between intuitive and logical aspects in the history of science, the formative value of mathematics, its role in other sciences, and the foundations of geometry. What is striking is the style he generally adopts: very clear and simple language that does not prevent him from referring to modern research in mathematics and connections with physics; a historical approach; many examples; and extensive use of metaphors to illustrate the relationship between logic and intuition and their respective roles. For Fano, metaphors serve to make a wide audience understand what he is saying by referring to aspects of real life familiar to everyone, such as chess, parts of the human body, parts of the tree, maneuvers of an army, and work of the surgeon.[44]

According to Fano, you can talk to anyone about mathematics: you just have to find the right language, talk about its applications, and make use of the history of science [102]. Only in this way can prejudices against this discipline be combated. For example, he discusses experimental demonstrations or applications of mathematics to astronomy in the *Scuola Operaia femminile*, or the educational value of mathematics, relations with physics, and the scientist's process of discovery in the *Gabinetto di Cultura della Scuola di Guerra*.

Besides, due to his sense of belonging to the Italian geometric School, Fano did not miss opportunities to disseminate its results. In 1894, still very young, he had illustrated the recent research work of Italian geometers in Göttingen and then again in 1897 during the International Congress of Mathematicians in Zurich. In 1923, the contacts with Grace Chisholm and the fame acquired earned Fano an invitation to hold a course on Italian geometry at the University College of Wales in Aberystwyth. Other cycles of lectures and conferences were later to be held in Louvain (1925), in Kazan (1926), and then in Lausanne (1942–1944).[45]

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[43] Fano wrote articles in the following journals: *Rivista di matematica* (2), *Bollettino di bibliografia e storia delle scienze matematiche* (1), *Rivista d'Italia* (1), *Scientia* (4), *Bollettino della 'Mathesis'* (3), *Periodico di Matematiche* (1), *Nuova Antologia* (1), *Rivista di filosofia* (1), *Alere Flammam* (1), *Nuovo Cimento* (1), *Conferenze di fisica e di matematica*, then *Rendiconti del seminario matematico. Università e Politecnico di Torino* (2).

[44] See, for example [47], pp. 12 and 16, and [43], pp. 24–25, 26, 27.

[45] See Sect. 3 in this paper, and [58].

In Aberystwyth, Fano held about 20 lectures on eight themes starting from the contributions of Cremona and his successors, down to the most recent research of the Italian School of algebraic geometry, highlighting the connections with international research (Fig. 3). The themes were the following:

1. A short account of the mathematical work of Cremona [. . .]
2. Some notions concerning Clebsch and Noether
3. Cremona's first successors in Italy [. . .]
4. Summary of concepts and notions on more dimensional projective geometry (Veronese, Segre, etc.)
5. Groups, especially continuous groups of in the plane and space. [. . .] Birational contact-transformations in the plane
6. Geometry on an algebraic manifold, especially on an algebraic curve. Different methods [. . .]
7. Geometry on algebraic surfaces [. . .] A short account of the methods used and results obtained by Italian Geometers: their connection with Picard's theory of integrals of total differentials on surfaces
8. A short account of the new essential differences we meet with in the theory of algebraic M_3s. Irrational involutions in S_3 ([47], *Preface*)

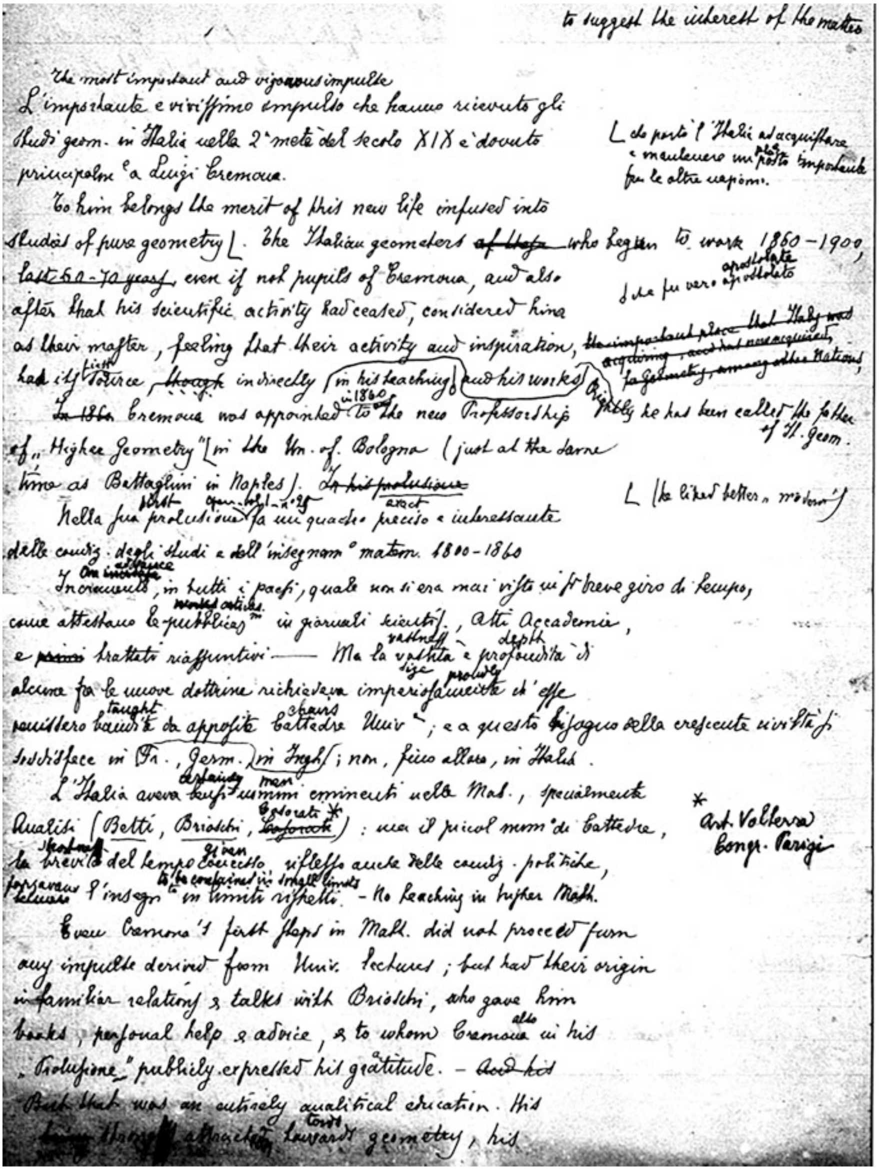
Given the audience's poor basic knowledge of the subjects, Fano was forced to take longer than expected to explain the methods of the Italian geometers, so he was unable to deal with points 5 and 8. Perhaps some difficulties also arose from his English, defined as "eccentric" by a student who attended the courses, whose testimony we have,[46] although he was helped in the translation by his sister Maria Ettlinger, Grace Chisholm, and George A. Schott—the latter taught applied mathematics at that university.

In the Archives of the University of Liverpool (AUL), among the *Papers* of William Young and Grace Chisholm, only the typewritten lectures relating to Cremona held in February 1923 are preserved, but in the *Fondo Fano* of the Biblioteca Speciale di Matematica in Turin, there is a substantial manuscript that collects notes from different periods.[47] These notes are precious because they contain various English versions of the lectures and sometimes also the Italian version, with afterthoughts and additions, and thus show us what Fano defines as "the intellectual laboratory" (*laboratorio intellettuale*) which will be discussed later.[48] They also offer a vivid testimony to the enthusiasm that permeated the group of Segre's students in the 20 so-called golden years, to their awareness of belonging to a School, and to the collective work which Fano often refers to here and elsewhere. He writes:

[46] The person in question was the student Graham Sutton, who was to become the general director of the Meteorological Observatory [59].

[47] See *Appunti vari*, BSMT, Fondo Fano, Scritti. 4. The manuscript can be accessed at the website [28]. See also [106].

[48] See Sect. 4 in this paper.


to suggest the interest of the matter

The most important and vigorous impulse
L'importante e vivissimo impulso che hanno ricevuto gli
studi geom. in Italia nella 2ª metà del secolo XIX è dovuto
principalm.te a Luigi Cremona.
To him belongs the merit of this new life infused into
studies of pure geometry. The Italian geometers of these who began to work 1860–1900,
last 60–70 years, even if not pupils of Cremona, and also
after that his scientific activity had ceased, considered him
as their master, feeling that their activity and inspiration,
had its first source, though indirectly in his teaching and his works.
In 1860 Cremona was appointed in 1860 to the new Professorship
of „Higher Geometry" in the Un. of. Bologna (just at the same
time as Battaglini in Naples). In his prolusione
Nella sua prolusione fa un quadro preciso e interessante
delle condiz. degli studi e dell'insegnam.º matem. 1800–1860
Incremento, in tutti i paesi, quale non si era mai visto in sì breve giro di tempo,
come attestano le pubblicaz.ni in giornali scientif., atti Accademie,
e trattati riassuntivi — Ma la vastità e profondità di
alcune fra le nuove dottrine richiedeva imperiosamente ch'esse
venissero bandite da apposite cattedre Univ.; e a questo bisogno della crescente civiltà si
soddisface in Fr., Germ., in Ingh.; non, fino allora, in Italia.
L'Italia aveva bensì uomini eminenti nella Mat., specialmente
Analisti (Betti, Brioschi, Casorati *): ma il piccol num.º di cattedre,
la brevità del tempo concesso, riflesso anche delle condiz. politiche,
forzavano l'insegn.to in limiti ristretti. – No teaching in higher Math.
Even Cremona's first steps in Math. did not proceed from
any impulse derived from Univ. lectures; but had their origin
in familiar relations & talks with Brioschi, who gave him
books, personal help & advice, & to whom Cremona in his
„Prolusione" publicly expressed his gratitude. – And his
But that was an entirely analitical education. His
attraction towards geometry, his

che portò l'Italia ad acquistare e mantenere un posto importante fra le altre nazioni.
che fu vero apostolato
the important place that Italy was acquiring, and has now acquired, in geometry, among other Nations,
Rightly he has been called the father of It. Geom.
(he liked better „modern")
* Art. Volterra Congr. Parigi

Fig. 3 Fano's draft of the lecture in Aberystwyth on Cremona's work ([3], c. 18*r*)

> It was indispensable that everything be treated and digested, that it became the blood of our blood, that we had it at our fingertips in order to be able to use it in the most advanced research ... Fecundity![49]

> Collective research—Segre-Castelnuovo: 1890–91 in Turin—Castelnuovo Enr[iques] (1896–900) afterwards Severi for surfaces (irreg[ular] 1904–05). Energies of investigators are summed. Their discoveries follow each other rapidly. ([3], c. 84*v*)

In addition to university lectures, Fano also held two popularizing lectures, where he sums up the various themes that characterize his vision of science.

In the first one, *Intuition in mathematics* ([47], pp. 5–17), he proposes to examine the question of whether the work of the mathematician is merely logical work. He adopts a historical approach in order to show that a rigorous treatment can only be applied to well-defined concepts and therefore to materials that our mind has already processed. He cites various examples starting from Euclid's "Elements" which with their deductive logical arrangement appear as the final result of a long period of discovery and elaboration, down to the critical studies of the nineteenth century, which led to the creation of mathematical logic and to its symbolism. He then focuses on another trend that emerged at the same time in which a "philosophic and intuitive spirit" prevails, mentioning the most representative figures, Riemann and Klein. It is clear, however, that among mathematical inventors and mathematical demonstrators, whom he defines as skilled technicians, his preferences go to the former because in his opinion it is intuition, "a pioneer of progress," that opens the way to logical developments. As Terracini observes, Fano reserved a checking function for mechanical calculation procedures, to which he attributed considerable importance. This fact "confirms that he must, consciously or not, have attributed to conceptual demonstrations such a profound intuitive origin, as to make it advisable to resort to checks of another nature."[50]

The second lecture, *All geometry is theory of relativity*, also starts from two questions: What is a geometric figure? What is a geometric property of this figure? To answer the first one, Fano observes that the geometric objects (a point, a straight line, etc.) that make up a figure are abstractions obtained from reality. To answer the second one, he introduces the concept of transformation group and cites Klein's Erlangen program, which classifies geometries according to the invariant properties for a particular group of transformations.

Hence, the meaning of the title is clear: a geometry is something related to a group of transformations. To illustrate this statement, he presents examples for the various branches of geometry—elementary, projective, and topology—and also

[49] [3], c. 69*r*: *era indispensabile che fossero trattate e digerite, che diventassero sangue del ns* [nostro] *sangue, averle sulla p*[unta] *delle dita, p*[er] *valersene in ricerche + elevate ... Fecondità!*

[50] [10], p. 486: *conferma che alle dimostrazioni concettuali egli doveva, consciamente o no, attribuire una così profonda origine intuitiva, da rendere consigliabile il ricorso a controlli di altra natura.*

cites non-Euclidean geometries, and to make himself clear he resorts to commonly used objects like rubber bands, ribbons, and pancakes.

The trip to Aberystwyth dates from 1923, a year after the march on Rome which marked the rise of Mussolini.

3 The Late Fano

The racial laws, which for Italian Jews determined the loss of civil and political rights and banishment from the scientific and academic arenas, triggered a series of institutional, epistemic, and social upheavals in high culture. Emigration was one of these. Unable to tolerate what G. Mortara defined "the reduction to a caste of pariahs," by the end of 1941, about 6000 Jews "packed up and left"; another 4000 would repair to Switzerland after the armistice. Among these, there were many intellectuals and scholars, about 30 men of science and 16 mathematicians, including Gino Fano, the eldest among the mathematicians uprooted from racist Italy, who left Turin for Switzerland in the winter of 1938, at the age of 70, settling in Lausanne. He would remain here until 1945, engaging in three areas: solidarity, helping Jewish aid and rescue associations to "track down people who have unfortunately disappeared forever";[51] geometric teaching, in the courses organized for Italian university students interned in the Lausanne and Huttwil camps; and dissemination, through a series of conferences on Italian algebraic geometry that he held at the Cercle Mathématique. The 7 years that he spent in Switzerland are generally dismissed as the unpleasant and unjust epilogue of a highly successful scientific life, and instead are definitely not, in the measure that they yield a broader narrative: that of Jewish scholars, trained in the Belle Époque of scientific internationalism, who saw the principles of the rule of law denied by race theories and who witnessed the perversion of collective consciences under totalitarian regimes [58, 60].

3.1 *Discrimination and the Collapse of the Three Pillars of Life: Family, Country, and Profession*

Born into a family where the patriotic tradition was alive and which had instilled in him "high feelings of Italianness,"[52] Fano covered the whole pathway that links Risorgimento patriotism, interventionism, and nationalism. His own, however, was not only that Garibaldian sentiment common to many Jews of the first post-Risorgimento generations. In the Great War, as an interventionist of the first hour,

[51] [10], p. 487: *rintracciare persone purtroppo scomparse per sempre*.

[52] [9], p. 262: *alti sentimenti di italianità.*

he took off his civilian clothes for the uniform and committed himself personally to directing the Industrial Mobilization Regional Committee in Piedmont [61] and making his contribution to the "spiritual assistance of the nation."[53] Disgusted by the Versailles camarilla led by "those who administrated European politics, and in particular Italy, as it was hardly conceivable that the Austria of the Holy Alliance and the prince of Metternich could do,"[54] Fano did not tolerate the barbarization of the political debate during the red 2-year period either. In the face of rising fascism:

> he was displeased but much calmer. He was involved in his scientific work, which the fascists did not disturb. He was certainly strongly nationalistic, Italy was "my country right or wrong", and my impression is that he considered Mussolini and his cohorts like a childhood disease of a very young nation, a terrible nuisance but a stage that would pass. ([7], pp. 183–184)

Perhaps convinced that there was nothing to fear as long as one "kept one's nose clean of politics", he maintained this feeling until he failed to be elected as an Academician of Italy. In fact, Mussolini himself opposed Fano's candidacy, proposed by his former pupil F. Severi, alleging Fano's belonging to the Judeo-Pluto-Masonic conspiracy. For him, it was a shame, which to some extent prepared him for the storm to come.

Of Jewish descent from either the father or the mother's lineages, but unobservant and alien to living Judaism in the communities of Mantua and Turin (Fig. 4), Fano was dismissed from his posts at the University and the Polytechnic, from November 29 and October 7, 1938, respectively; he was removed from the direction of the Special Mathematics Library and expelled from all the academic and scientific societies to which he belonged (Lincei, UMI, Virgiliana, Academy of the Sciences of Turin, etc.).

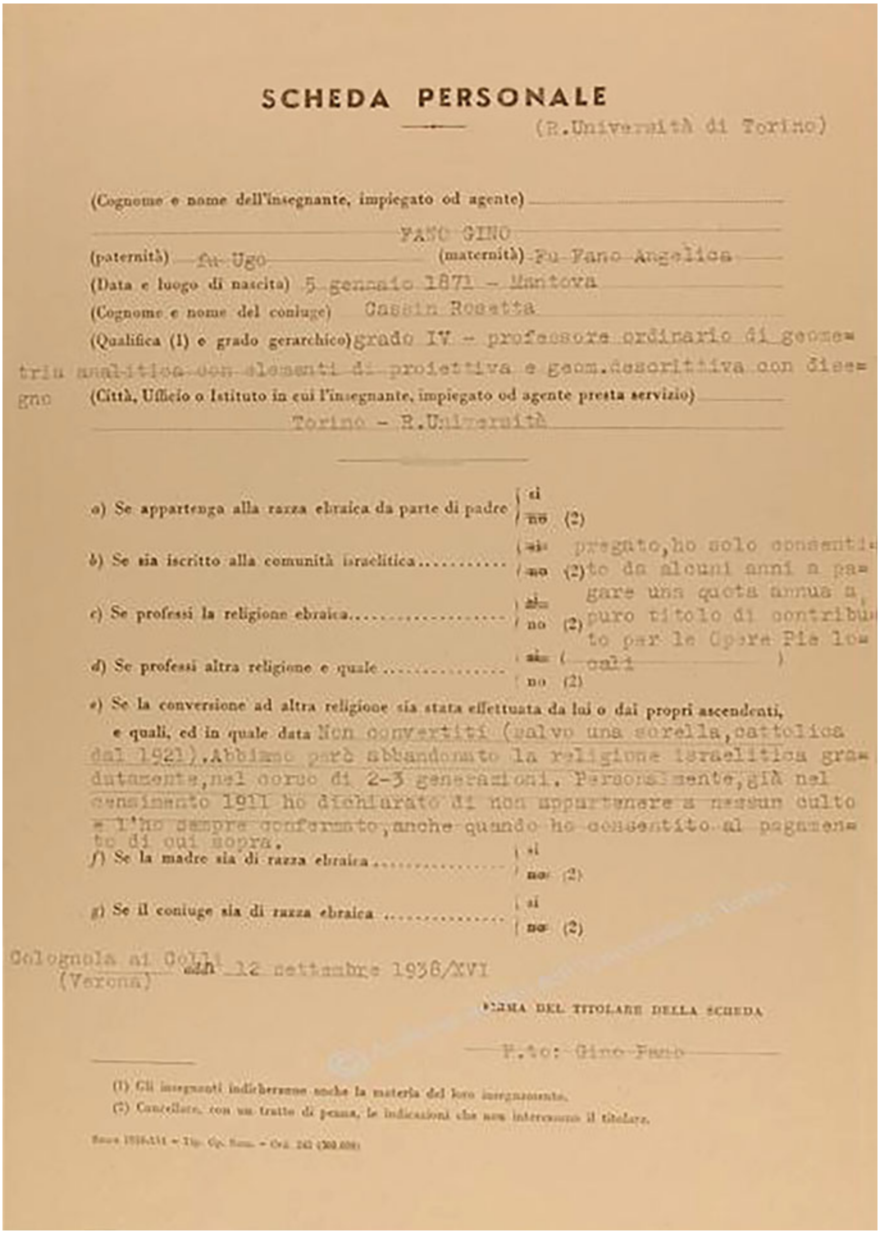
The shock faced with professional demotion, social exclusion, and complete marginalization from academia was traumatic, even if Fano did not show it outside the context of family and friends, even responding to the letter of forced resignation from the Polytechnic:

> I warmly thank the Board of this Faculty of Engineering for the mindful greeting that it was pleased to address to me through you. I always felt and I feel particularly attached to this Institute, for having lived through all its phases, from the very initial practices for its construction, to the most recent years of your enlightened and energetic Direction. The organization and gradual improvement of the two geometry courses for the first two-year level were certainly one of the performances of my long career as a teacher.[55]

[53] [62], p. 1: *assistenza spirituale della nazione.*

[54] [62], p. 10: *coloro che hanno trattato la politica europea, e in particolare l'Italia, come appena appena si può comprendere che la trattasse l'Austria della santa alleanza e del principe di Metternich.*

[55] Historical Archive of the Polytechnic of Turin, file Gino Fano: G. Fano to G. Vallauri, Colognola ai Colli, 29 October 1938: *Ringrazio vivamente il Consiglio di codesta Facoltà di Ingegneria del memore saluto che per mezzo Vostro si è compiaciuto rivolgermi. A codesto Istituto mi sono sempre sentito e mi sento particolarmente legato, per averne vissute tutte le fasi, dalle pratiche iniziali per la sua prima costruzione agli anni più recenti della illuminata ed energica Vostra Direzione.*



SCHEDA PERSONALE

(R.Università di Torino)

(Cognome e nome dell'insegnante, impiegato od agente)

FANO GINO

(paternità) fu Ugo (maternità) fu Fano Angelica

(Data e luogo di nascita) 5 gennaio 1871 - Mantova

(Cognome e nome del coniuge) Cassin Rosetta

(Qualifica (1) e grado gerarchico) grado IV - professore ordinario di geome= tria analitica con elementi di proiettiva e geom.descrittiva con dise= gno

(Città, Ufficio o Istituto in cui l'insegnante, impiegato od agente presta servizio)

Torino - R.Università

a) Se appartenga alla razza ebraica da parte di padre { sì / ~~no~~ (2)

b) Se sia iscritto alla comunità israelitica .... { ~~sì~~ / ~~no~~ (2) pregato, ho solo consentito da alcuni anni a pagare una quota annua a puro titolo di contributo per le Opere Pie locali

c) Se professi la religione ebraica .... { ~~sì~~ / no (2)

d) Se professi altra religione e quale .... { ~~sì~~ ( ) / no (2)

e) Se la conversione ad altra religione sia stata effettuata da lui o dai propri ascendenti, e quali, ed in quale data Non convertiti (salvo una sorella, cattolica dal 1921). Abbiamo però abbandonato la religione israelitica gra= datamente, nel corso di 2-3 generazioni. Personalmente, già nel censimento 1911 ho dichiarato di non appartenere a nessun culto e l'ho sempre confermato, anche quando ho consentito al pagamen= to di cui sopra.

f) Se la madre sia di razza ebraica .... { sì / ~~no~~ (2)

g) Se il coniuge sia di razza ebraica .... { sì / ~~no~~ (2)

Colognola ai Colli (Verona) addì 12 settembre 1938/XVI

FIRMA DEL TITOLARE DELLA SCHEDA

F.to: Gino Fano

(1) Gli insegnanti indicheranno anche la materia del loro insegnamento.

(2) Cancellare, con un tratto di penna, le indicazioni che non interessano il titolare.

Roma 1938-XVI - Tip. Op. Rom. - Ord. 242 (300.000)

Fig. 4 Fano's racial census form, August 1938

As a matter of fact, his situation was simpler than that of other colleagues. Firstly, in 1938, before him Fano had only 3 years left before retirement. This means that he was paid with an indemnity near to the maximum amount. His family, moreover, held a large fortune and extensive assets and estates, which allowed them to live with dignity even without income from dependent work,[56] and enjoyed such a good intellectual and social standings as to trust in the success of the reverse discrimination procedure. Fano's sister Alina Regina (1874–?) was the widow of Leo Wollenborg, senator of the Historical Left and ex-minister of Economy and Finance of the Zanardelli government; another sister, Maria Fano Ettlinger (1871–1966), was an accredited translator of novels and other texts from English; his wife Rosa was a relative of the economist Robert Michels. The Fanos' applications for reverse discrimination would indeed all be accepted, that of Gino in 1940, but the news of the positive outcome of the procedure was to arrive when the family had already dispersed.

On the other hand, it was above all young people, i.e., the sons, to be affected by racial legislation, which excluded them from any prospect of professional fulfillment. Fano had two sons: Ugo (1912–2001) and Roberto (1917–2016). The latter, on paper, had better prospects, as he was to start his fourth year at the Turin Polytechnic to graduate in 1939, which was guaranteed by law. The elder one, Ugo, graduated in Mathematics and Physics in 1934, a pupil of E. Persico and E. Fermi, and was a promising atomic physicist but was at the beginning of his career. There will be Ugo—the "Colossal Fanaccio" as he was affectionately nicknamed by the so-called Via Panisperna boys—and Roberto the first to make up their minds about emigration, encouraged by their cousin Giulio Racah and by Ugo's girlfriend, Camilla Lattes.

3.2 *"So the Time Came to Flee Turin": Emigration*

In one of his last interviews, Gino Fano's son, Robert, recalled as follows the family debate around the choice—staying or leaving?—which racial laws forced them to make:

FANO: And at that point my family, with the exception of my father, decided it was time to go.
INTERVIEWER: And your father?

L'organizzazione e il graduale perfezionamento dei due corsi di geometria del biennio sono stati certo una delle esplicazioni della mia non breve carriera di insegnante.

56 Filling up the questionnaire to be submitted to the SPSL (SPSL, *GF*, c. 297), at the item "Sources of Income before dismissal," Fano stated: "About L. it. 3000 monthly from Univ. position, further, a good family position." Their properties were confiscated by a special public agency, the *Ente di Gestione e Liquidazione Immobiliare* (Agency for Real Estate Management and Liquidation), in 1943.

FANO: Well, we persuaded my father to move too.
INTERVIEWER: So you all got out?
FANO: We all got out. Yes.
INTERVIEWER: You were lucky.
FANO: [. . .] Basically there was a family emergency reunion, on my birthday as a matter of fact—November 17—in our country home near Verona. And basically we decided that we had to scramble, because that war was coming and God only knows what's going to happen.[57]

To flee from Italy, three prerequisites were needed: adequate financial assets, a network of international relationships, and a certain mentality too. The Fano family had the resources because, with highly dangerous smuggling activity, Roberto had succeeded in transferring the family's patrimony to Switzerland, and in particular the money necessary for his parents to settle in Lausanne, and for him and his brother to obtain visas for America, without asking for the help of international Jewish rescue committees (Society for the Protection of Science and Learning and Emergency Committee in Aid of Displaced Foreign Scholars).

The courage, the inner strength to rebuild one's existence and career as a stranger in a foreign country, is not for everyone. Fano, who was a nineteenth-century old gentleman, did not have it, nor, unlike some colleagues, such as Fubini, was he willing to accept any solution to keep the family together. Disagreement between him, his wife, and children mainly pivoted around "the American road." Fano could have recourse to many personal contacts in the United States, constructed along his long-term professional trajectory (J. Coolidge, V. Snyder, E.B. Stouffer, O. Veblen, S. Lefschetz), and had sojourned in America a few times: the first in St. Louis in 1904 and the last in Los Angeles in 1932. However, he belonged to the generations which identified Paris, Berlin, and Göttingen as traditional destinations of academic mobility and did not understand the young researchers, who grew up considering English as a lingua franca and looking to the United States as a new mecca for scientific studies.[58] The pragmatism, the tacit but pervasive anti-Semitism that he had perceived in some American environments, disgusted him. The desire to appropriate the so-called American way of life was inconceivable for him. Finally, there was also an ideological reason: Fano refused to take into consideration the idea of emigrating to any country likely to be at war with Italy ([6], p. 3).

Faced with his determination, his wife and children could do nothing but give up and to be contented with persuading him to take refuge in Switzerland. The choice of destination was largely against the trend. Indeed, Switzerland was only to become the "frontier of hope" for victims of persecution after the armistice of September 8, 1943 [63, 64]. In 1938, when the Fanos relocated there, very few considered it

[57] Transcript of the Interview MIT 150 | Robert M. Fano '41, ScD '47, p. 1, in https://infinite.mit.edu/video/robert-m-fano-%E2%80%9941-scd-%E2%80%9947.

[58] In the questionnaire form submitted to the SPSL (SPSL, *GF,* c. 298), as far as language knowledge is concerned, Fano assesses that he is fluent in French and German while he can read, write, and speak English "rather well."

a good country to expatriate to, and generally it was emigrants for political, not racial, reasons. The Swiss Confederation represented at most a free transit for those who intended to route to other lands. For Fano, by contrast, it was not a temporary solution, but the only one that he could accept.

Once the decision had been made, all Fano's family left—further evidence of the fact that Jewish intellectual emigration from racist Italy was largely "a family matter"—and dispersed. The first to flee Italy were Gino and Rosa, who entered Switzerland in December 1938, settling in Lausanne, at the Hotel Élite, a quite familiar environment, where in the past they had been used to sojourn for business trips or on vacation. The first son, Ugo, fled to Paris in February 1939, and at first managed to obtain a visa for Argentina; embarking in Bordeaux, he reached Buenos Aires in July 1939, thence went to New York, and finally landed in Washington, where he was offered a position at the Carnegie Institution's Department of Terrestrial Magnetism. Fano's grandson, Giulio Racah, a former professor of theoretical physics at the University of Pisa up to the time of racial measures, settled in Palestine in September 1939, after an intermediate stage in London. Roberto postponed his departure for a few months, to finish his exams, and the delay was almost fatal. Due to the outbreak of the war, he was no longer able to reach Bordeaux, one of the main French ports of embarkation to the United States. In Lausanne, where he went to say goodbye to his parents before fleeing Europe, he met his cousin Leo Wollenborg, a writer and a journalist, who, thanks to the intermediation of a high Prelate at the French embassy in Zurich, succeeded in obtaining two passes for France. He was thus to arrive in the United States in October 1939.

In the meantime, since the beginning of 1939, difficulties multiplied both for those who intended to leave Italy and for those who already lived abroad. In fact, the Fanos could not access the assets they had moved to Switzerland and deposited at the Union de Banques Suisses. Thus, at the suggestion of Giulio Racah, who paid a visit to the SPSL offices, Ugo wrote to the Society proposing a sort of agreement: an anonymous Swiss person (alias his father himself) would have issued a bank transfer to the organization, which the SPSL would pay to him in the form of grants in order to finance his research.[59] The SPSL accepted and sent Gino Fano the questionnaire to be filled in so that his case could fall under the administrative laws concerning asylum seekers.[60] Compiling these documents, although it was a mere formality, was unpleasant, even humiliating for a scholar who had prepared the last curriculum vitae for a competition 32 years earlier and for which fields such as "the name of the religion to which you belong: Jewish Orthodox or Jewish Reformed?" were incomprehensible (Fig. 5).[61] On March 6, 1939, the Union de

[59] SPSL, *GF*, fol. 303, 304, 305: U. Fano to SPSL, Paris 15 February 1939; SPSL to U. Fano, London, 17 February 1939; U. Fano to SPSL, Paris 19 February 1939.

[60] SPSL, *GF*, fol. 306, 307: SPSL to U. Fano, London, 20 February 1939, G. Fano to SPSL, Lausanne, 14 March 1939.

[61] SPSL, *GF*, fol. 296–298, 302: General & Confidential Information, Curriculum Vitae.


GENERAL INFORMATION
ALLGEMEINE AUSKUNFT

CONFIDENTIAL INFORMATION
VERTRAULICHE AUSKUNFT

Fig. 5 Fano's questionnaire submitted to SPSL

Banques Suisses informed the SPSL that one of its clients who did not want to be named had allocated 15,000 francs to Gino Fano to enable him to continue his scientific activity in Lausanne.[62] So, a practice began of monthly scholarships which, albeit with various obstacles due to Foreign Exchange Control, would allow the Fanos to pass through the war years with relative peace of mind.[63]

3.3 The Latest Studies on Varieties

Although it was a fictitious grant, the SPSL financed Fano to continue his geometric research and he was required to report annually on his outputs. As a matter of fact, despite his age, Fano was productive: between 1939 and 1945, he published 12 works in *Commentarii Mathematici Helvetici*, *Revista de la Universidad Nacional de Tucumán*, and *Atti della Pontificia Academia Scientiarum.*[64] The themes are those typical of Fano's scientific portfolio: algebraic curves, non-rationality and birational geometry in dimension three, cubic threefolds, Fano threefolds, Fano-Enriques threefolds, Enriques surfaces and their automorphisms. However, the fact that production renews in the wake of continuity was normal for senior mathematicians exiled from Italy, such as Fubini and B. Levi, and in some ways, it was mandatory,

[62] SPSL, *GF*, fol. 308: Union Bank of Switzerland to SPSL, Lausanne, 6 March 1939.

[63] Fano's dossier kept in the SPSL Archive includes extensive correspondence between the Foreign Exchange Control, the SPSL, and the Union de Banques Suisses concerning the installment of the grant, which had not gone unnoticed by the checks on foreign deposits in 1940 and 1941.

[64] References are listed in [4], pp. 135–136.

since these scholars had been forced to separate themselves from their libraries, collections, and manuscripts. From this point of view, Fano was advantaged by the fact that up to the summer of 1939, his son Roberto forwarded to him the books and offprints received in Turin. After his departure for the United States and the outbreak of the war, this possibility ceased to exist and he had to "make do with" what he could find in Swiss libraries.[65]

Although there is no solution of continuity between the works issued before and after 1938, nor any trace of Fano's insertion into the new Swiss mathematical scene, which counted leading names such as G. De Rham, P. Finsler, A. Speiser, M. Plancherel, and R. Fueter, those of the Swiss period are not merely remakes or translations of publications that appeared before Fano's emigration. For example, the note *Sulle curve ovunque tangenti a una quintica piana generale* ([65], submitted to the *Commentarii Mathematici Helvetici* on October 10, 1939, and here published in vol. 12, 1940, pp. 172–190) gives evidence of a web of relationships between Fano and the Cambridge geometric School (J.S. Milne, H.F. Baker, ...) that followed his departure from Turin.[66]

But it was mainly in the field of threefolds that Fano achieved the most significant successes, arriving in Lausanne at some important results on classification and rationality problems for cubic varieties. The paper, ready since 1942, was presented by Severi to the Pontifical Academy in February 1943, but it would only come out in 1947. Its contents, however, were known both at national and international levels. For example, Beniamino Segre, John A. Todd, Lucien Godeaux, and Guido Castelnuovo discussed them in their letters:

> I was very glad to receive your letter, and I thank you for your friendly appreciation of my two notes. These will be followed in the Journal by other two. In one of them I prove the result quoted by Mordell in his note just appeared (but presented *after* mine!), while in the remaining one I put the final touch to theorem 1 of my note I, by solving parametrically the cubic Diophantine equation $z^2 = f(x, y)$, where f is any rational cubic polynomial in x, y which cannot be written as a polynomial in a single linear function of x and y. I have already obtained several additional results on cubic surfaces. One of them, by means of which theorem VIII of note I follows at once from theorem VII of the same note, is that "a non-singular cubic surface contains no homaloid linear system of complete intersections." An extension of this result to the non-regular V_3^3, would obviously prove its irrationality. I was told by Fano that this irrationality has been very recently proved by him, on considering the linear system of surfaces of genera 1 lying on V_3^3, but I have not seen the proof. I feel that one should be able to obtain the result also by my methods, but I have not yet had time of thinking seriously about this. (Caltech Archives, B. Segre Papers: B. Segre to J.A. Todd, Manchester 8 October 1943)

> During the war, I had some relations with M. Fano, a refugee in Lausanne; he managed to demonstrate the irrationality of the cubic variety of four-dimensional space, but I don't know his proof yet.[67] (Caltech Archives, B. Segre Papers: L. Godeaux to B. Segre, Liège 13 August 1945)

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[65] Lincei National Academy, Levi-Civita Archive: G. Fano to T. Levi-Civita, Lausanne, 9 February 1939.

[66] See Sect. 4.3 in this paper.

> Prof. Fano was ill in Boston, but now is well; he wrote me a long letter, and I have frequent news of him from my daughter Gina. His address is 3510 Rodman str., Washington D.C. N.W. The Memoir on the cubic variety which must be published by the Pontifical Academy has not yet come out; you will have seen the short summary published in the Lincei Rendiconti. In these days a very intelligent young man working here communicated to me a very simple and brief demonstration of the irrationality of the cubic variety based on topological considerations. But I need to think again about the matter.[68] (Caltech Archives, B. Segre Papers: G. Castelnuovo to B. Segre, Rome 19 December 1946)

For Fano, the work on V_3^3 would substantially be the last original article, since he was to stop active research in the middle of the war, because he felt himself inefficient, as "too often he had to read papers for a second time" ([6], p. 3).

3.4 The Return to Teaching in the Italian University Camp in Lausanne

In less than a month, between 8 September and 1 October 1943, Switzerland saved about 20,000 Italians fleeing German occupation and the Italian Social Republic [66–69]. Among them were the engineers Gustavo Colonnetti and Franco Levi; the mathematicians Modesto Dedò, Bruno Tedeschi, and Bonaparte Colombo; and the mathematics teachers Nedda Friberti, Ernesto Carletti, and Bianca Ottolenghi [70].

Most refugees were shunted off first to transit camps (*campi di smistamento*), where civilians were separated from the military, then to quarantine centers, and finally to labor camps, where they remained until repatriation. The life of the inmates, although different in various realities, was generally miserable. Refugees, especially simple soldiers, were crammed into stables and sheepfolds and forced to engage in penal labor.

To improve this situation, in September 1943, the Fond Européen de Secours aux Etudiants involved the Eidgenossisches Kommissariat für Internierung und Hospitalisierung in the formulation of a support program, aimed at providing refugees with "that intellectual and moral help that represents an imperative

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[67] *Pendant la guerre, j'ai eu quelques relations avec M. Fano, réfugié à Lausanne; il a réussi à démontrer l'irrationalité de la variété cubique de l'espace à quatre dimensions, mais je ne connais pas encore sa démonstration.*

[68] *Il Prof. Fano è stato malato a Boston, ma ora sta benino, mi ha scritto egli stesso una lunga lettera, e di lui ho frequenti notizie dalla mia figlia Gina. Il suo indirizzo è 3510 Rodman str., Washington D.C. N.W. La Memoria sulla varietà cubica che deve esser pubblicata dall'Ac. Pontificia non è ancora uscita; avrà visto il breve estratto pubblicato nei Rendiconti dei Lincei. In questi giorni un giovane di qua, molto intelligente, mi ha comunicato una dimostrazione molto semplice e breve della irrazionalità della varietà cubica fondata su considerazioni topologiche. Ma ho bisogno di pensare ancora alla cosa.*

necessity and constitutes an indispensable complement to material aid."[69] The secretary of the Fond Européen, A. de Blonay, circulated a questionnaire in all the about 150 internment camps located in Switzerland, to register university students. On the date of November 13, 1140 questionnaires duly filled in had already been received by FESE. Seeing the census results, a Comité d'aide aux universitaires italiens en Suisse was set up in Lausanne, chaired by Colonnetti and by P. Bolla, the vice president of the federal court, which selected 540 applications. Those admitted were divided into four camps (Lausanne, Freiburg, Neuchâtel, and Geneva) in which parallel university courses were organized, so as to allow interned soldiers to enroll in Swiss universities and finish their studies. In favor of the excluded applicants, University Studia (*Studi Universitari*) for officers and subofficers were created in Mürren and Huttwil afterwards [71, 72]. Lausanne was chosen as the seat of the main Italian University Campus (CUI), inaugurated on January 26, 1944, and operating until May 1945, which hosted about 200 students. Its professors were prominent scholars: Colonnetti (dean and teacher in the courses of construction science), L. Einaudi, A. Fanfani, M.G. Levi, F. Levi, and L. Szegö [70].

For mathematical courses, Colonnetti immediately recruited Fano (the two had been colleagues at the Polytechnic of Turin for almost 20 years, from 1919 to 1938), who immediately agreed to resume his teaching in the Campus of Lausanne and Huttwil, with such energy and genuine enthusiasm as to deserve a personal letter of thanks from Colonnetti at the end of the first semester:

> At the moment when the university camp in Lausanne is near to closure, I would like to express to you all my gratitude for the service you have performed as a teacher of Analytical, Projective and Descriptive Geometry, with such a high sense of patriotism and solidarity for young soldiers interned.[70]

In addition to lecturing in Lausanne, Fano also committed himself in the University Studium of Mürren and Huttwil where, thanks to him and to Dedò, many "valid disciples, such as Aldo Andreotti,[71] were attracted to geometric studies."[72] In Huttwil, he was both teacher and president of the examination boards for all mathematics courses, and in just a semester, he set and held 166 oral tests, helped by two colleagues only: Alessandro Levi (philosopher of law) and Paolo D'Ancona

[69] Swiss Federal Archives, Bern: *Les universitaires italiens internés en Suisse*, November 1943, memorandum signed by de Blonay, E5791, 1, 18/1, fol. 4.

[70] ACT: G. Colonnetti to G. Fano, Lausanne, 24 July 1944: *Nel momento in cui sta per chiudersi il campo universitario di Losanna io desidero esprimerle tutta la mia riconoscenza per l'opera da Lei prestata in qualità di docente di Geometria Analitica Proiettiva e Descrittiva, con così alto spirito di patriottismo e di solidarietà per i giovani militari internati.*

[71] After taking shelter in Lausanne in October 1942, to avoid deportation to labor camps in Germany, Andreotti attended the courses of De Rham and B. Eckmann at the University and those of Fano in the CUI. Returning to Italy at the end of the war, in 1951, he took over from Fano the chair of analytical and projective geometry.

[72] ACT: E. Carletto to G. Fano, Mürren, 15 November 1943 and M. Dedò to G. Colonnetti, Mürren-Münchenbuchsee, 8 January 1944: *molti discepoli valorosi, come Aldo Andreotti, sono attirati verso gli studi geometrici*.

(historian of art). Of the geometric teaching imparted by Fano, there remains an evocative textual trace in two volumes of handouts: the *Lezioni di Geometria descrittiva* written by Roberto Ballarati and Franco Brindisi, and the lecture notes in analytical and projective geometry compiled by an anonymous student [73, 74]. Fano's expertise in this field was enormous. Suffice it to notice that in Turin, he had been the only full professor of projective and descriptive geometry from 1901 to the merger of the two courses. At the Polytechnic, he had taught projective geometry from the foundation of the School of Engineering, in 1908, up to the time of the racial laws. Equally impressive was his output as a textbook author, which included many treatises on descriptive, projective, and analytical geometry, two of which were co-authored by Terracini.[73] The second edition of *Lezioni di geometria analitica e proiettiva* had been their last shared commitment before the departures of Fano for Lausanne and Terracini for Tucumán (Argentina).[74]

The handouts of Fano's lessons at CUI reflect his mastery in this sector. In those of analytic and projective geometry, a sub-discipline to which he had always attributed special value for its mirrors on the development of geometric studies in Italy, one can seize the clarity of his didactic style, his way of proceeding "from a few premises to the construction of large theories, [a way] that left the students full of admiration and almost amazed, especially after the more complicated apparatus of elementary geometry, learned in secondary schools."[75] Equally evident are the legacy of his classical texts in descriptive geometry, specially the third edition of the *Lezioni di geometria descrittiva date nel R. Politecnico di Torino*, lectures which he based on his own theory chapters, with the exception of the section on quadrics, and for the applications (instruments for construction machinery and equipment, photogrammetry, etc.).

3.4.1 Keynote Lectures at the Cercle Mathématique

Fano is credited as one of the main popularizers of Italian geometric culture abroad, a kind of mathematics communication exercise which he began at the Mathematische Gesellschaft in Göttingen in 1893, continued in Aberystwyth in

[73] G. Fano, *Lezioni di geometria descrittiva*, Torino, litogr. 1903; *Lezioni di geometria descrittiva date nel R. Politecnico di Torino*, Turin, Paravia, 1909, 1914^2, 1926^3; *Geometria Proiettiva. Lezioni raccolte da D. Pastore e E. Ponzano*, Turin, litogr. 1907; G. Fano, A. Terracini, *Lezioni di geometria analitica e proiettiva*, Turin, litogr. 1926, then Turin, Paravia, 1930, 1940^2, 1948^3. See Sect. 2.3.2 in this paper.

[74] Fano returned to Turin a few times, up to 1940, to make arrangements with Paravia in view of the second edition of *Lezioni.* Cfr. BSMT, A. Terracini Papers: G. Fano to G. Sacerdote Terracini, 16 December 1947; G. Fano to A. Terracini, New York, 6 February 1948; Mantua 7 April 1948, 9 May 1948, 20 May 1948.

[75] [11], p. 325: *da poche premesse alla costruzione di teorie di larga portata che lasciava gli studenti ammirati e quasi meravigliati, soprattutto dopo l'apparato più macchinoso della geometria elementare, appresa nelle scuole secondarie.*

1923 and Kazan in 1929, and ended in Lausanne. Here, in the span of 2 years, from May 1942 to February 1944, at the Cercle Mathématique, Fano held five invited lectures dedicated to Italian algebraic geometry,[76] from a historical and "School" perspective, which met with success and gave rise to lively discussions with his new Swiss friends and colleagues De Rham, G. Dumas, P.G. Javet, and J. Marchand.[77] The history of the Italian geometric Risorgimento from Cremona to the Veronese-Segre-Bertini triad and Castelnuovo and Enriques' classification of algebraic surfaces and contributions on threefolds and on birational transformations (including the research carried out in the last period) allowed Fano to promote (and celebrate) the results of a tradition, of a School to which he strongly felt he belonged.

In the first conversation, *Quelques aperçus sur le développement de la géométrie algébrique en Italie pendant le dernier siècle* (evening of May 4, 1942), Fano traces the recent and contemporary history of algebraic geometry by L. Cremona, who set the foundations of the Italian School, up to the arrival of the great masters: G. Veronese, C. Segre, and E. Bertini (Fig. 6). The second, *Géométrie sur les surfaces algébriques* (May 11, 1942), focuses on one of the masterpieces of the Italian School: the theory of algebraic surfaces by Castelnuovo and Enriques. The third, *Aperçu général sur les surfaces du 3ème ordre* (February 2, 1943), deals with a classic topic: the existence of the 27 lines on a cubic surface. The fourth lecture, *Les surfaces du 4ème ordre* (May 13, 1943), published posthumously by Aldo Andreotti [75], represents the natural continuation of the previous one, focusing on the quartic surfaces. In the last one, *Transformations de contact birationelles dans le plan* (February 10, 1944), Fano outlines his contributions concerning the birational geometry in dimension three. On this occasion, he had the opportunity to recapitulate some of his past studies, presented at the International Congress of Mathematicians in Bologna in 1928, and to highlight two important issues, on which he had resumed working "in recent times": systems ∞^2 of curves that correspond, under a birational contact transformation, to points of the plane or to straight lines, and the determination of the simplest operations with which to obtain, as products, the totality of birational transformations.

There exists a common thread that runs through these five conferences: the desire to exhibit, to display in front of foreign colleagues the best achievements of his own school, a desire that is not the nostalgic recollection of an old Italian geometer at the end of his career, but on the contrary permeates all his experiences of dissemination, from the end of the nineteenth century onwards.[78]

In this perspective, the first conversation held by Fano at the Cercle Mathématique, *Quelques aperçus sur le développement de la géométrie algébrique en Italie pendant le dernier siècle*, particularly stands out, as it nicely expresses what the

[76] The manuscript drafts of the five lectures are kept in BSMT, Fondo Fano, and are digitized in [28].

[77] Minutes of the sittings of the Cercle register the presence of at least 20 participants in each conversation, both Cercle associates and external guests.

[78] See Sect. 2.3.1 in this paper.

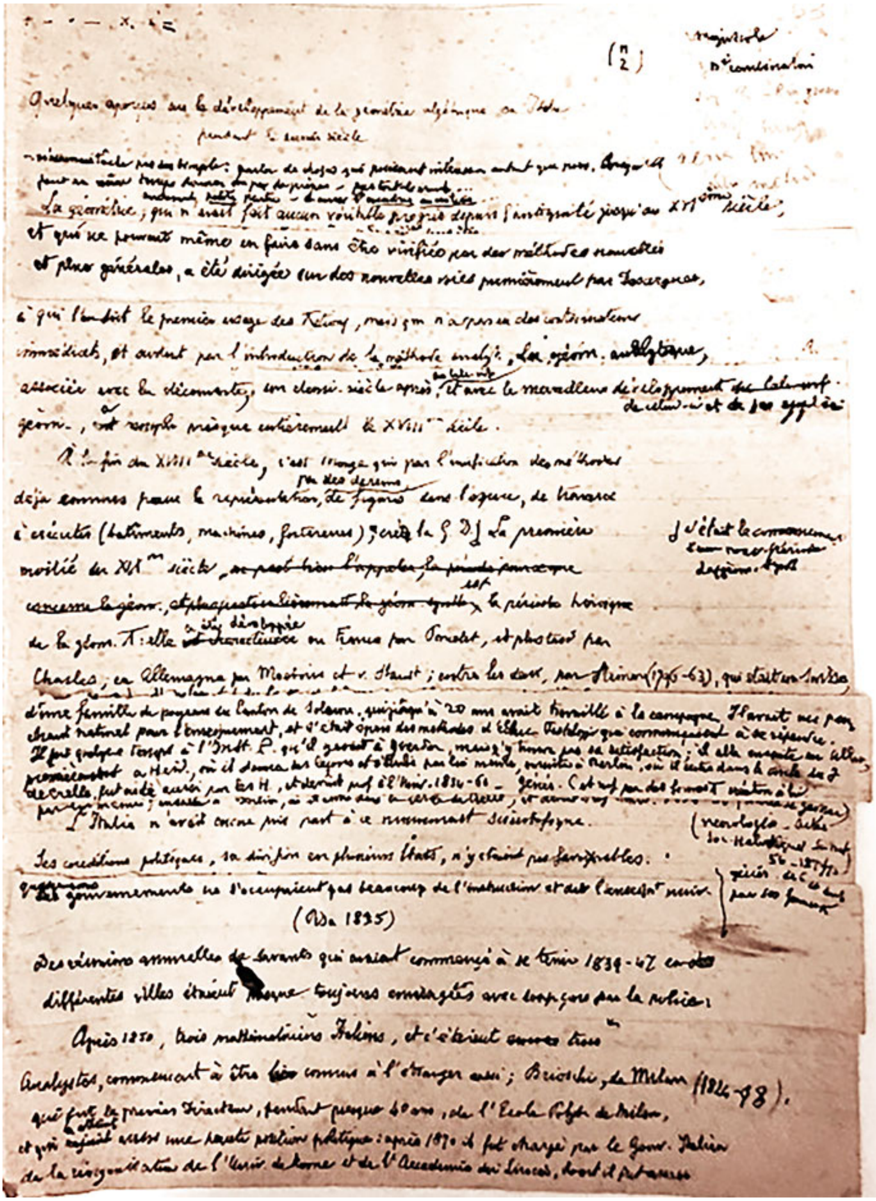

Fig. 6 Manuscript draft of the lecture *Quelques aperçus sur le développement de la géométrie algébrique en Italie pendant le dernier siècle* ([3], c. 53*r*)

Italian School of geometry was for its members, what it meant to be part of it, and how the birth, image, and evolution of this research group were declined abroad. The notion of School according to Fano and the main characters of the historical fresco offered by him in the *Aperçus* can be summarized in the following points:

- Continuity and stability of the development path of Italian geometry from Cremona to Severi;
- Importance of the social dimension for the flourishing, competitiveness, and attractiveness of research schools;
- Emergence of a national identity in mathematical studies only after the Risorgimento;
- Need for "grafts" (*innesti*) of foreign trends of studies (primarily from Germany) on the trunk of Italian geometric production;
- Scientific and moral role of the masters, both in the moment of creation of a school and in the successive phases of development, expression, and affirmation of the working group beyond geopolitical borders;[79]
- Added value of the collaboration: the school is not only a team of scholars who share a research project or a workplace, but is a network, in a way a family.

These are aspects which Fano had often thought about in the past and on which he would have continued to reflect until the last months of his life when, preparing the Lyncean commemoration of Castelnuovo, he

> lingered in thought on the schools that in the last decades of the 19th century had determined the revival of geometric studies in Italy, and on men, in the first place Luigi Cremona who had made it possible. One of the many clues to the continuity of Italian geometric thought,[80]

a continual tradition in the wake of which he had been able to harmoniously place his own contribution.

Pronounced at a terrible historical juncture (when Turin was being bombed and when mass deportations to concentration camps had already begun), the lecture *Quelques aperçus sur le développement de la géométrie algébrique en Italie* is characterized by some national markers which at first reading would appear singular, almost bizarre. They are definitely not: they similarly shape other series of seminars and conversations held by Italian refugees, for example the *Correrías en la logica matématica* by Levi (Tucumán, autumn 1942, then [77], pp. 13–78). As Levi "brought to Tucumán" the voices of Peano, Pieri, and Burali-Forti, Fano intended to revive in front of his colleagues at the Cercle Mathématique the studies and figures of those who had been his masters and friends (Cremona, Segre, Castelnuovo, Klein), in a word that golden season of Italian geometric research, which he had experienced first as an observer, and later as a player. The two contexts are

[79] On the notion and role of research Schools, according to Italian geometers (Segre, Castelnuovo, etc.), see [2, 20, 76].

[80] [12], pp. 702–703: *indugiato col pensiero sulle scuole che negli ultimi decenni del secolo scorso avevano determinato il rifiorire degli studi geometrici in Italia, e sugli uomini, in primo luogo Luigi Cremona* [. . .]. *Uno dei molteplici indizi della continuità del pensiero geometrico italiano.*

obviously very different: Lausanne is not Tucumán, the audience is different, but the underlying spirit and basic aim are analogous: to claim, even during the experience of exile, their own belonging to two mathematical Schools that had led Italy to achieve an international *Führende Stellung* (leading position).

3.5 Between Turin and the United States

Fano was both among the first to leave Italy and among the first to come back. That in Switzerland was a parenthesis for him, to be closed as soon as possible in view of the future, which Fano foresaw in Italy, where he returned immediately after the liberation. What Terracini defines "el dilema de la vuelta" did not touch him.[81]

The case of his sons was quite different. Italian Jews who arrived in America on the eve of the war can be somewhat schematically divided into two groups: those who had come as immigrants and who tried to integrate into the new world, and those who had come as refugees and were always ready to return to their homeland ([78], p. 357): the young Fanos fall into the former category. Both were now settled, married, and with children; they had obtained American citizenship in 1946 and had completed the path from denationalization to renationalization. The desire to establish themselves in their new homeland and to contribute to determining its technological-scientific primacy prevailed over the sense of being dispossessed that they had experienced after racial discrimination ([79], p. 363).

Faced with their determination to stay in America, this time it was the father who gave up, agreeing to spend the last few years of his life partly in Italy and partly in the United States, whence he came over for the first time after the war in August 1946. Thus, although reinstated in service, Fano only nominally resumed teaching from May to November 1946, when he retired.[82] Aware that his return to the chair was "fictitious," he immediately told his colleagues that the time had come to think about his succession, a succession that—as he confided to his friends Tricomi and Castelnuovo—he hoped would be ensured by Beniamino Segre or Alessandro Terracini, who "were meritorious of the country, having resumed their university positions."[83]

On the other hand, Fano continued to maintain his contacts with the Italian and American milieu and made a decisive contribution to the reconstruction of the heritage of the Special Mathematics Library of Turin, which had been partially

[81] SPSL Archive: SPSL to G. Fano, London, 15 May 1947.

[82] ASUT: Dean M. Allara to G. Fano, Turin, 5 November 1945; G. Fano to M. Allara, 8 December 1945.

[83] Caltech Archives, B. Segre Papers: G. Fano to B. Segre, New York, 21 March 1945: *si sono resi veramente benemeriti del Paese, riprendendo le loro cattedre universitarie*. See also G. Castelnuovo to C. Segre, Rome, 3 October 1946; F. Tricomi to C. Segre, Turin, 26 October 1946.

destroyed in the bombings of 1942–1943, incidentally by donating his entire collection of offprints (over 5000 items).[84] Declared an emeritus in the summer of 1948,[85] he dedicated the last years of his life to a project proposed to him by B. Segre: the monograph edition, under the auspices of the Italian Mathematical Union, of his writings concerning the demonstration of the irrationality of the V_3^3 of $\mathbb{P}^4$. His last talk, held at the Mathematical Seminar of the University and Polytechnic of Turin in February 1950 and published in his *Festschrift* volume [105], focused on this subject.[86]

4 On Fano's Material and Immaterial Heritage

In this part of the essay, we will focus on some Fano's mathematical contributions from the historiographical perspective of "mathematical heritages," referring with this term not only to material collections (libraries, miscellanies, museums, and archives) but also to the immaterial aspects of crystallization, transmission, and circulation of knowledge.[87] Such an approach can provide a new key to understanding a mathematical community, a well-characterized social group such as the Italian School of algebraic geometry. By encompassing both components, Fano's case study allows us to detect the close link between material and immaterial dimensions.

It is now well established that Fano's mathematical activity addressed different research topics, which are analyzed in ([4], pp. 115–122). Fano is best known for his pioneering work on three-dimensional algebraic varieties (or "threefolds")—to which his name is inextricably linked—and the problems of their classification and rationality. Within the literature in this field, two complementary and sometimes opposite trends emerge. On the historical side, it is agreed that Fano's research, especially that of the last period, was carried out in the declining phase of the Italian School, when results were "intuited" rather than "demonstrated." Fano's original papers are "very obscure and criticizable from the point of view of rigor," thus being indicative of the limits of Italian tradition, and "they were considered so even in the period of the full flowering of the School" ([86], p. 129). On the other hand, on the

[84] See [80, 81].

[85] ASUT: Dean M. Allara to the Ministry of Public Instruction, Turin, 18 June 1948; the Ministry of Public Instruction to M. Allara, Rome, 19 July 1948; M. Allara to G. Fano, Turin, 24 July 1948; G. Fano to M. Allara, Colognola ai Colli, 1 August 1948. The proposal to declare Fano Emeritus came from Terracini, who also drew up the report concerning "his merits as teacher and scientist" (*meriti come maestro e come scienziato*). The report, dated 7 June1948, is kept in ASUT, in Fano's personal file.

[86] ASUT, Corrispondence of BSMT: A. Terracini to G. Fano, Turin, 15 February 1950; G. Fano to A. Terracini, New York, 5 January 1951, 23 January 1951; A. Terracini to G. Fano, Turin, 29 January 1951.

[87] For some examples of such historiographical approach, see, inter alia, [14, 82–85].

mathematical side, the originality of Fano's ideas is widely recognized. As J. Murre pointed out, Fano's mathematical creativity enabled him

> to tackle these problems almost empty-handed because there was no foundation for higher dimensional algebraic varieties. The modern development has shown that Fano was essentially right and, once the foundations were available, his methods were correct and effective. ([18], p. 224)

A further aspect to be considered is the fact that Fano's research has been a source of inspiration for modern studies since the 1980s. Our aim is to reconsider this set of elements from a new perspective, adopting as investigation lens that of "patrimonialization," also in the light of some unpublished documents and manuscripts, through which the material and immaterial dimensions intertwine in a significant way. In the case of Fano's contributions on threefolds, this conception can be articulated on three different levels which will be investigated in the following paragraphs.

4.1 Fano's Work Within the Cultural Heritage of the Italian Geometric Tradition

Firstly, Fano's work is positioned within a specific cultural heritage, that of the Italian School of algebraic geometry [107]. From the point of view of research alone, it is characterized by the commonality not only of mathematical questions and research themes but also of the following:

- Method, which can be briefly described as a prevalently synthetic approach based on projective tools and a widespread use of so-called geometrical intuition;
- Sources where, alongside the works of Italian geometers, there are the German contributions of the late nineteenth and early twentieth centuries;
- Way of writing and presenting the mathematical findings, i.e., "in progress" and as a result of "experimental work".[88]

The starting point of Fano's research on threefolds is fully in line with the Italian tradition: as is well known, he started from Lüroth's problem in higher dimensions—asking whether every unirational variety is rational—with the aim of extending the results of rationality and classification of algebraic surfaces achieved by Castelnuovo and Enriques a few years earlier.

After a preliminary work published in 1904 [87] devoted to the cubic hypersurface of $\mathbb{P}^4$, in 1908 Fano began to study threefolds having the plurigenera equal to zero, stating that "for three-dimensional algebraic varieties the nulling of all

[88] For a detailed discussion of the latter aspect, see [76], pp. 196–197.

genera is not yet a sufficient condition for their biunivocal representation" on $\mathbb{P}^3$, i.e. the projective space of dimension three. His research then became directed towards showing "the existence—which occurs for the first time in the case of three-dimensional varieties—of birationally distinct types of varieties having all the genera equal to zero."[89]

From this moment on, Fano's studies unfolded in several directions, among which the introduction of the today called "Fano threefolds" definitely stands out: in modern language, they are three-dimensional varieties whose anticanonical bundle is ample. In order to understand Fano's printed and unpublished writings, it is necessary to introduce some fundamental elements of his notation. Denoting by p the genus of curve sections, Fano identified the families of threefolds M_3^{2p-2} of order $2p - 2$, embedded in the projective space $\mathbb{P}^{p+1}$. Such three-dimensional varieties contain the surfaces $F^{2p-2} \subseteq \mathbb{P}^p$ as hyperplane sections, having the same order of the starting varieties and all the plurigenera equal to one; from a modern point of view, they are K3 surfaces. Canonical curve sections $C_p^{2p-2} \subseteq \mathbb{P}^{p-1}$ of genus p and order $2p - 2$ are obtained from the intersection of two generic hyperplane sections. These particular varieties were not systematically introduced by Fano until 1928, during the International Congress of Mathematicians in Bologna. Here during the iconic plenary lecture on the Italian algebraic geometry by Castelnuovo, who had taken on the role of leader of the School after Segre's death, the centrality of the issues addressed by Fano was underlined with these words:

> How can we decide whether an assigned equation with four unknowns represents a rational or a semi-rational variety? We know nothing about this, not even for the lowest values of the degree, higher than 2. Actually, research that Fano has been carrying out for several years, and which he will present during his communication, shows how complex the question is. He examines the varieties that have all the genera and plurigenera equal to zero and he distributes them into a finite number of families: the first one is composed of rational varieties, the second of semi-rational varieties, and the others of varieties that become more detached from rationality. An accurate classification of these types would shed light on a question that needs to be solved for the future development of algebraic geometry.[90]

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[89] [88], p. 973: [...] *per le varietà algebriche a tre dimensioni l'annullarsi di tutti i generi (analoghi ai precedenti) non è ancora condizione sufficiente perché esse possano rappresentarsi biunivocamente sullo spazio* S_3*; e scopo di questa breve Nota è appunto di assodare l'esistenza – che si presenta per la prima volta nel caso di varietà a tre dimensioni – di tipi birazionalmente distinti di varietà aventi tutti i generi nulli.*

[90] [89], p. 200: *Come decidere se una equazione assegnata a quattro incognite rappresenti una varietà razionale o semirazionale? Nulla sappiamo in proposito, nemmeno per i più bassi valori del grado, superiori a 2. Anzi, ricerche che il Fano prosegue da vari anni, e di cui vi parlerà in una sua comunicazione, fanno vedere quanto la questione sia complessa. Egli prende in esame le varietà che hanno nulli tutti i generi e i plurigeneri e le distribuisce in un numero finito di famiglie, di cui la prima si compone di varietà razionali, la seconda di varietà semirazionali e le altre di varietà che si staccano sempre più dalla razionalità. Una classificazione accurata di questi tipi getterebbe molta luce sopra una questione che è necessario risolvere per lo sviluppo futuro della geometria algebrica.*

However, some manuscript papers recently identified within the *Appunti vari* of the Fondo Fano (BSMT) show that Fano's research in this direction originated a few years earlier, starting not from this construction though, but from the idea of undertaking a complete analogy with the study of surfaces. This emerges from the drafts[91] of some lectures dedicated to the main differences encountered in the study of algebraic surfaces and in that of threefolds that Fano had planned to give during his cycle of lectures at the University College of Wales in Aberystwyth in 1923.[92] As he himself declared ([47], p. 3), these topics were not covered for reasons of time. The predominantly didactic approach adopted in this context further emphasizes Fano's position within the heritage of methods and approaches typical of the Italian School, among which proceeding by analogy plays a fundamental role.

At the beginning, Fano refers to Severi's research on higher dimensional varieties dating back to the years 1906–1909, introducing some fundamental notions: geometric and arithmetic genus (denoted by P_g and P_a, respectively), three-dimensional irregularity $q_1 = P_g - P_a$, surface irregularity q_2, linear connection $2q_2 + 1$, and sum of irregularities $q_1 + q_2$. At this juncture, especially in the introduction, Fano took almost verbatim some passages from Severi,[93] who in those years began to assume a central role in Italian mathematics. However, he also refers to the studies of Marino Pannelli, a mathematician specializing in algebraic geometry and a secondary figure in the Italian geometrical scenery, considered by Severi "one of the best of the Italian scholars of this discipline who were not appointed to a professorial chair."[94] In 1906, Pannelli determined some relations between the numerical characters of threefolds which are invariant under birational transformations, including Ω_2, the virtual arithmetic genus of the canonical surface, to which we will return in the next section.

This first manuscript already helps to shed light on Fano's positioning within the wide immaterial heritage of the Italian School of algebraic geometry, composed of the works by both great masters like Severi and minor authors like Pannelli. This is confirmed by an analysis of the citational network of Fano's papers on Fano threefolds:[95] indeed, almost all the cited works are publications in the "glorious" Italian geometric tradition (88% of citations), for a total of 22 Italian authors (Fig. 7). Among them, with at least ten references, besides Severi's (with 33

[91] See [3], cc. 125–130.

[92] See Sect. 2.3.3 in this chapter.

[93] Compare [3], c. 125*r* with [90], p. 337.

[94] [91], p. 83: *[Il Severi lo considerava come] uno dei migliori fra gli studiosi italiani di quella disciplina non pervenuti ad una cattedra universitaria.*

[95] To perform this analysis, we considered the references contained in Fano's 16 publications on three-dimensional varieties, published between 1904 and 1950. Out of a total of 165 citations, only 19 papers are signed by foreign authors: 9 works were from Germany (F. Klein, M. Noether, T. Reye, G. Salmon, A. Voss, E. Weber), 6 from the United Kingdom (D.W. Babbage, A. Cayley, P. Du Val, L. Roth, J.A. Todd), 2 from Denmark (H.G. Zeuthen), and 1 each from Austria (K. Zindler) and Switzerland (L. Schläfli).

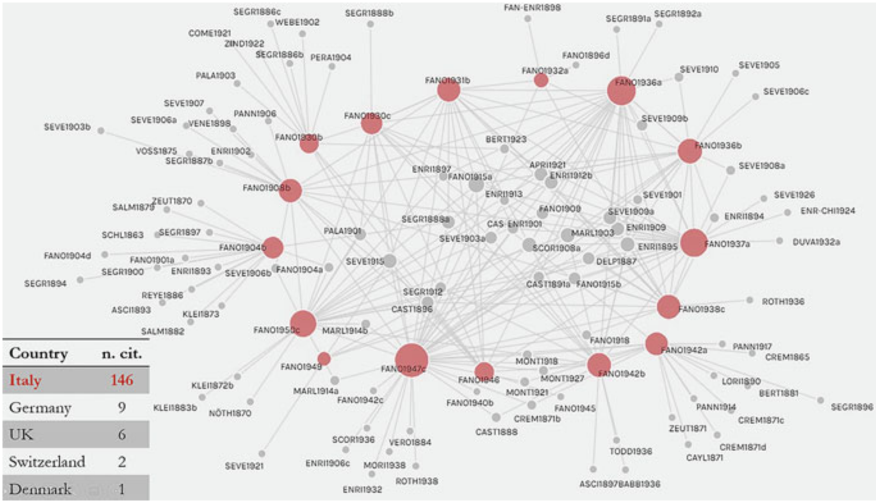


Fig. 7 Citational network of Fano's works on Fano threefolds

citations), the names of Enriques (30), C. Segre (18), Castelnuovo (12), and G. Marletta (10) stand out.

Besides the quoted works, at least a further aspect deserves to be considered in order to grasp Fano's commitment in the wake of the Italian geometric tradition. The opening of his talk in Bologna is representative in terms of both expository style and research methodology. Connecting up with Castelnuovo's talk, Fano introduced the subject as follows:

> The distinction, which seems traditional, between sciences of reasoning and experimental sciences is now outdated. In every science, experience and reasoning play a role; the distinction concerns only their reciprocal proportions. In mathematics, the part reserved for experience, small and limited to the stage of discovery, consists essentially in the careful examination of some particular cases. I propose to set forth here the result of a little experimental work, and of some further conjectures, regarding an arduous and important question, which has been waiting in vain for a solution for a long time.[96]

[96] [92], p. 115: *La distinzione, che pareva tradizionale, tra scienze di ragionamento e scienze sperimentali è ormai sorpassata. In ogni scienza hanno parte l'esperienza e il ragionamento; la distinzione concerne solo le reciproche proporzioni. In matematica la parte riservata all'esperienza, piccola e limitata alla fase di scoperta, consiste essenzialmente nell'esame accurato di qualche caso particolare. Io mi propongo appunto di esporre qui il risultato di un po' di lavoro sperimentale, e di qualche congettura ulteriore, riguardo a una questione ardua e importante, che da tempo attende invano la soluzione.*

4.2 A Specific Mathematical Heritage: Methods and Results

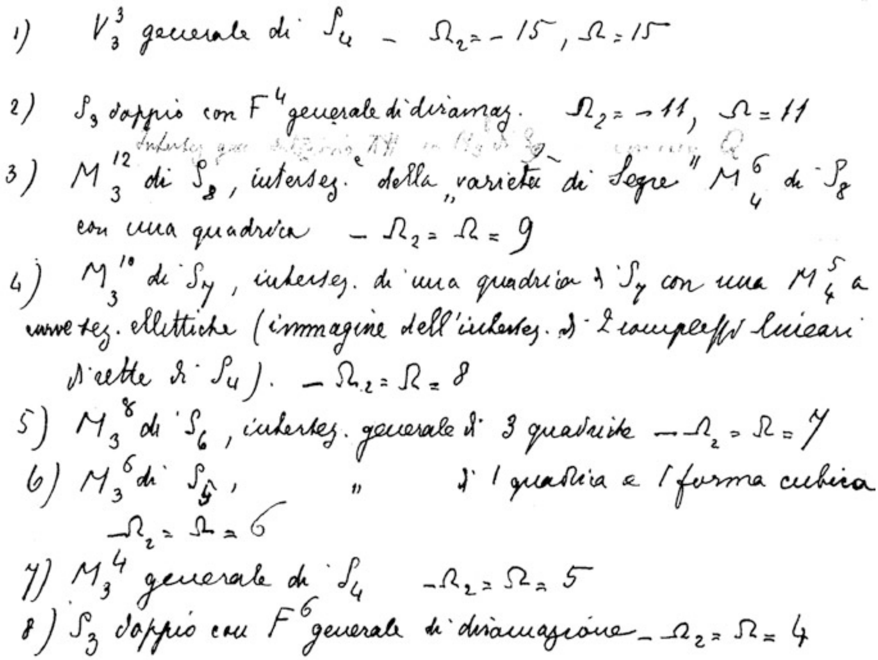
A second manuscript,[97] entitled *Appunti e vedute concernenti le varietà algebriche a tre dimensioni aventi tutti i generi nulli*, contains the preliminary studies of the communication that Fano presented at the International Congress in Bologna. It leads to a second level of investigation, that of immaterial heritage conceived as a specific set of contents, including the results obtained but also (and above all) the techniques and instruments developed to achieve them and the language used and/or coined for this purpose. In this sense, Fano's manuscripts and printed works constitute a true gold mine for historians of mathematics. While the results achieved or, at least, "suggested" by Fano were described in the aforementioned historical papers, the methods and tools of research deserve to be considered more carefully.

Returning to the invariant Ω_2, Fano's unpublished notes brought to light a first classification of Fano threefolds based on this numerical character (Fig. 8). Here, Fano addressed the issue starting from the need to identify an arithmetic character analogous to the Castelnuovo-Enriques invariant ω for surfaces: it is a relative invariant whose maximum value for a class of birationally equivalent surfaces is an absolute invariant, i.e. the virtual linear genus $p^{(1)}$ of the canonical curve. He thus identified the analogue of this character with the virtual arithmetic genus Ω_2 of the canonical surface of a threefold, whose maximum absolute value Ω turns out to be an absolute invariant. Fano then gave a first classification of these threefolds depending on such invariant, which essentially coincides with that subsequently presented at the ICM of Bologna, with the sole exception of the case $p = 8$. In the communication of 1928, Fano instead classified these varieties according to the value of p, the geometric genus of the curve-sections. However, in the paper published in the ICM proceedings, he again introduced Ω_2: differently from the manuscript, he added that for a variety M_3^{2p-2} this invariant is equal to $-(p+2)$ and coincides with the dimension of the systems of surfaces of genus one contained within such threefold increased by one unit. Taking into account both the handwritten notes and the 1931 printed work, Fano's first systematic classification of M_3^{2p-2} can be summarized as in Table 1.[98]

It seems natural to wonder why Fano reintroduced the invariant Ω_2, despite having already provided a classification of Fano threefolds based on the value of p. On one side, this invariant allowed him to classify not only these $M_3^{2p-2} \subseteq \mathbb{P}^{p+1}$, but also a second category of three-dimensional varieties. These are singular threefolds M_3^n of order n, immersed in $\mathbb{P}^4$ and containing a line of multiplicity $n-2$. On the other side, it seems emblematic of the author's desire to give the

[97] See [3], cc. 45–46 and 52.

[98] Here, the following notations are used: * = this threefold does not appear within the manuscript; $\Longleftrightarrow$ = birationally equivalent varieties; $V(d) \subseteq \mathbb{P}^N$ = hypersurface of degree d in $\mathbb{P}^N$; $V(d_1, \ldots, d_k) \subseteq \mathbb{P}^N$ = intersection of k hypersurfaces of degree $d_1, \ldots, d_k$ in $\mathbb{P}^N$; $Gr(n, k)$ = Grassmannian of $(k+1)$-dimensional vector subspaces of an $(n+1)$-dimensional vector space; H_i = hyperplane.


1) V_3^3 generale di S_4 — $\Omega_2 = -15$, $\Omega = 15$

2) S_3 doppio con F^4 generale di diramaz. $\Omega_2 = -11$, $\Omega = 11$

3) M_3^{12} di S_8, intersez. della „varietà di Segre" M_4^6 di S_8 con una quadrica — $\Omega_2 = \Omega = 9$

4) M_3^{10} di S_7, intersez. di una quadrica di S_7 con una M_4^5 a curve sez. ellittiche (immagine dell'intersez. di 2 complessi lineari di rette di S_4). — $\Omega_2 = \Omega = 8$

5) M_3^8 di S_6, intersez. generale di 3 quadriche — $\Omega_2 = \Omega = 7$

6) M_3^6 di S_5, " di 1 quadrica e 1 forma cubica — $\Omega_2 = \Omega = 6$

7) M_3^4 generale di S_4 — $\Omega_2 = \Omega = 5$

8) S_3 doppio con F^6 generale di diramazione — $\Omega_2 = \Omega = 4$

Fig. 8 Manuscript draft of Fano's first classification of M_3^{2p-2} ([3], c. 52*v*)

Table 1 Fano's classification of Fano threefolds

p	$\|\Omega_2\|$	M_3^{2p-2}	Basic modern description
13	15	$V_3^3 \subseteq \mathbb{P}^4 \Longleftrightarrow M_3^{24} \subseteq \mathbb{P}^{14}$	Cubic threefold $V(3) \subseteq \mathbb{P}^4$
9	11	'double' $\mathbb{P}^3 \Longleftrightarrow M_3^{16} \subseteq \mathbb{P}^{10}$	Double cover of $\mathbb{P}^3$ with branched surface of degree 4
8	*	$M_3^{14} \subseteq \mathbb{P}^9$	$Gr(1,5) \bullet H_1 \bullet H_2 \bullet H_3 \bullet H_4 \bullet H_5 \subseteq \mathbb{P}^9$
7	9	$M_3^{12} \subseteq \mathbb{P}^8$	Intersection of a quadric hypersurface in $\mathbb{P}^8$ with the image of $\mathbb{P}^2 \times \mathbb{P}^2$ via the Segre embedding
6	8	$M_3^{10} \subseteq \mathbb{P}^7$	$Gr(1,4) \bullet V(2) \bullet H_1 \bullet H_2 \subseteq \mathbb{P}^7$
5	7	$M_3^8 \subseteq \mathbb{P}^6$	$V(2,2,2) \subseteq \mathbb{P}^6$
4	6	$M_3^6 \subseteq \mathbb{P}^5$	$V(3,2) \subseteq \mathbb{P}^5$
3	5	$V_3^4 = M_3^4 \subseteq \mathbb{P}^4$	Quartic threefold $V(4) \subseteq \mathbb{P}^4$
2	4	'double' $\mathbb{P}^3$	Double cover of $\mathbb{P}^3$ with branched surface of degree 6

"heritage status" to a harvest of discoveries and achievements in a new geometric field, where there was still much to explore and develop. This consideration is corroborated by Fano's introduction of new mathematical terminology. As noted in the margins of the manuscript, he defined varieties of the first type "semi-rational" (*semirazionali*) since they are "intermediate between rational entities and those having at least one of the genera and plurigenera greater than zero."[99] Threefolds of the second type, having "as an analogue, in the field of surfaces, something intermediate between rational surfaces and irrational ruled surfaces,"[100] are instead called "pseudo-rational" (*pseudorazionali*).

Although the work published in the ICM proceedings is the only printed work in which Fano followed an approach also based on Ω_2, the handwritten notes contain the germ of further ideas and techniques later extended and refined in various publications. Fano thus began to work out some fundamental notions that, developed and expanded during research activity of over 40 years, would become an integral part of the heritage of classical studies on three-dimensional varieties.

Regarding the problem of rationality, Fano claimed that "examining these different varieties, one gets the impression that, if they are not rational, they come closer to rationality as p increases, despite some restrictions."[101] Moreover, these threefolds carry an additional property: each of them can be projected from the lower order curves contained in them (and therefore from a straight line, if it exists) into a variety of the same type that corresponds to lower values of p (namely a $M_3^{2p-6} \subseteq \mathbb{P}^{p-1}$) and contains a ruled cubic surface as an image of the projection center. Lastly, Fano observed that "from the birational point of view (with some restrictions), each of the enumerated varieties includes the following threefolds as particular cases (corresponding to higher values of p) [. . .]; so that the increase of p implies, in principle, a progressive particularization of M_3."[102]

An interesting aspect is that in both manuscripts, unlike the publications, Fano explicitly stated his "work plan" for dealing with the issue of three-dimensional varieties. Indeed, he opened a window on his "intellectual laboratory,"[103] declaring that his research

> aimed at demonstrating, as far as possible, the irrationality of some of these varieties, was essentially directed at studying:
>
> (a) the linear systems at least ∞^2 of regular surfaces having all the genera =1;
> (b) the set (group) of possible birational transformations;

[99] [92], p. 120: *come intermedie fra gli enti razionali e quelli aventi almeno uno dei generi e plurigeneri maggiore di zero.*

[100] [92], p. 121: *come analogo, nel campo delle superficie, qualcosa di intermedio fra le superficie razionali e le rigate irrazionali.*

[101] [92], p. 118: *esaminando queste diverse varietà, si ha l'impressione che esse, qualora non siano razionali, tuttavia, al crescere di p, pur con qualche restrizione, vadano gradatamente accostandosi alla razionalità.*

[102] [92], p. 119: *dal punto di vista birazionale (con qualche limitazione), ciascuna delle varietà enumerate comprende come casi particolari le successive (corrispondenti a valori più elevati di p) [. . .]; sicché il crescere di p implica, in massima, una progressiva particolarizzazione della* M_3.

[103] See Sect. 2.3.1 in this chapter.

> and trying to find in the systems a) and in the transformations b)—in turn, naturally related to each other—some properties that are different from those of the space S_3, so that we can conclude that they are birat[iona]lly distinct entities.[104]

However, the apparatus of tools developed by Fano to address the problem of threefolds is not limited to the study of the relative invariant Ω_2, to the analysis of linear systems of K3 surfaces contained in M_3^{2p-2} (corresponding to the point *a)* of the manuscript), or to the comparison between the group of birational transformations on threefolds and that of $\mathbb{P}^3$ (point *b)*) with the aim of showing that $Bir\left(M_3^{2p-2}\right) \neq Bir\left(\mathbb{P}^3\right)$. Noteworthy is the study of homaloidic systems of surfaces to prove the irrationality of the quartic hypersurface of $\mathbb{P}^4$ and of the threefold obtained as the complete intersection of a quadric and a cubic hypersurface in $\mathbb{P}^5$. To examine the systems of surfaces contained in a certain threefold, Fano exploited the properties of canonical curves obtained as prime sections of such surfaces which are transmitted to the variety under investigation. After 1928, he began to extend the method of projection of a given threefold from a line, exploring the effects of projecting the variety from different vertices. In this way, the projection of M_3^{2p-2} from a conic of itself is a M_3^{2p-8} of canonical curve sections that contains a rational quartic ruled surface, the image of the starting conic; again, M_3^{2p-2} projects from the tangent space at a general point into a M_3^{2p-10} of canonical curve sections, on which the neighborhood of the vertex is mapped by a Veronese surface. But Fano did not extend the classical tool of projection from a line only in this direction, through the choice of an appropriate vertex. In the extensive memoir of 1937 [93], he introduced the method of so-called double projection that makes it possible birationally to refer each Fano threefold to a "simpler" (but not more general) variety of the same type, immersed in the projective space of dimension $p-6$. Indeed, after having projected $M_3^{2p-2} \subseteq \mathbb{P}^{p+1}$ from a line of itself into a $M_3^{2p-6} \subseteq \mathbb{P}^{p-1}$ which contains a ruled cubic surface spanning $\mathbb{P}^4$, it is possible to project M_3^{2p-6} from this $\mathbb{P}^4$ into a Fano threefold $M_3^{2p-18} \subseteq \mathbb{P}^{p-6}$. Within this vast immaterial heritage of techniques and tools, it is finally worth mentioning the analysis of involutions on threefolds, an area in which Fano's research intertwined with the contemporary studies by Enriques and G. Aprile.

In addition, Fano's way of writing and presenting his achievements is paradigmatic of the process of patrimonialization of mathematical knowledge, as emerges from his 1950 work. Invited at the Turin Mathematical Seminar to deliver a lecture

[104] [3], c. 45*r*: *[Le mie ricerche], intese a dimostrare, per quanto possibile, la irrazionalità di alcune fra queste varietà, sono state essenzialmente dirette a studiare:*

(a) *i sistemi lineari almeno* ∞^2 *di superficie regolari aventi tutti i generi* $=1$*;*
(b) *l'insieme (gruppo) delle eventuali trasformazioni birazionali;*

e a cercare di trovare nei sistemi a) e nelle trasformazioni b) – naturalmente, a loro volta, legati fra loro – qualche proprietà che sia diversa da quelle dello spazio S_3*, in modo da poterne concludere che si tratta di enti bir[azionalmen]te distinti.*

on the occasion of his appointment as an emeritus professor, Fano did not merely describe his main scientific contributions during 40 years of academic activity but paid close attention to retracing the main stages of his research path.

4.3 Fano's Legacy in the Short and Long Terms

The third aspect that emerges from adoption of the heritage investigation lens is linked to the concepts of legacy and influence of a certain line of research, over a long or short period. Beyond the ideas and insights provided to modern algebraic geometry, Fano's studies gave an important impulse to research in different contexts but in close dialogue with the Italian School, even in the years of its decline, when at the international level different traditions and directions—such as those outlined by topology and abstract algebra—were becoming widespread. This is the case of the English School of geometry[105] on which the research of Italian geometers exerted a guiding action, at least until the 1930s. In particular, Fano's heritage was positively welcomed and continued in Cambridge. He maintained a regular exchange with this mathematical community, as evidenced by the correspondence with H.F. Baker who, in December 1931, wrote to Fano:

> Dear Sir,
> I was very honoured by, and very grateful to you for, your letter of 2 Dec., telling me that you had written further about my little Note of the Del Pezzo ψ^5. I shall look forward to the privilege of an offprint, when the paper is published. And, as soon as possible, I shall study your letter in detail, which I have not been able to do as yet.
> Our students in Cambridge read many of your published papers, and find them very helpful—so that I am particularly grateful to you for writing to me. (BSMT, Fondo Fano, letter no. 22: H.F. Baker to G. Fano, Cambridge, 14 December 1931)[106]

It is therefore not surprising that Fano's classical line of research on threefolds was taken up by Leonard Roth, who published it in an organic form in the treatise *Algebraic Threefolds: With Special Regard to Problems of Rationality* [95]. It is relevant that Roth had spent a year in Rome with Severi as the winner of a Rockefeller fellowship in 1930–1931. Fano's legacy was hence taken up by Roth, whose profound knowledge of Fano's studies on three-dimensional varieties shines through this volume. The scientific correspondence between the two mathematicians, which continued at least until Fano's relocation to Switzerland, shows far-reaching sharing both in terms of research themes and of geometric tools adopted. Favorite topics are certainly the results of rationality and unirationality, but there is also a focus on some specific results of Fano on threefolds such as those that led Roth to state: "I must say how satisfying it is to know that the series of the V_3^{2p-2} [should read

[105] For an insight into the development of geometry at Cambridge in these years, see [94], pp. 340–349.

[106] This letter can be accessed at https://www.corradosegre.unito.it/fondofano/lettera22.pdf.

M_3^{2p-2} in our notation] ends for $p = 37$ and if $p > 10$ they are rational."[107] As far as methods are concerned, Roth skillfully handled the tools pioneered by Fano, such as the study of homaloidic systems of surfaces and the analysis of the complete surface sections of a given threefold, but he also provided new ideas for advancing research. He combined in an original way the results obtained by English geometers with classical Italian techniques, like successive projections, for instance in the following terms:

> In a recent note—not yet published—I determined that a quartic form of S_4 [should read $\mathbb{P}^4$ in our notation] cannot have more than 45 isolated nodes and, after all, it is well known that this limit is reached, since a rational V_3^4 of this kind exists (see Todd, Quarterly Journal, Oxford 1936). Perhaps we might use this result to show, via subsequent projections, that the V_3^{2p-2} of the first species do not exist for $p > 23$.[108]

However, it should not be thought of as a "one-way" interaction, from the Italian to the English School. A broad conception of patrimonialization, conceived as a process of recognition and appropriation of an articulated set of elements, includes exchange and dialogue with other traditions with which there is a pooling of principles and fundamental aspects. The group of five writings cited by Fano in his works on threefolds, signed by the English geometers P. Du Val, D. Babbage, J. Todd, and Roth and published between 1932 and 1938, thus appears particularly significant. From these publications, Fano drew both specific results, such as those on the quartic hypersurface of $\mathbb{P}^4$, and certain procedures, like those adopted by Roth for the study of M_3^{14}. The fact that all the English geometers mentioned by Fano were students of Baker in Cambridge is not accidental. Besides Roth, Du Val too, thanks to a Trinity fellowship, visited Rome (1930–1932) where he worked with Enriques, specializing in the theory of algebraic surfaces according to the Italian study orientation. The outcome of this period of study abroad is the two works cited by Fano [33, 96], which represent Du Val's first two papers, written in Italian, devoted to the classification of surfaces.

If we look at Fano's contributions on another family of three-dimensional varieties, the so-called Fano-Enriques threefolds (i.e., special Fano threefolds whose general hyperplane section is an Enriques surface), a parallel argument can be made for the work of Lucien Godeaux—who, unsurprisingly, is the most represented

[107] BSMT, Fondo Fano, letter no. 23: L. Roth to G. Fano, London, 18 February 1937: [. . .] *ma devo dire quanto è soddisfacente sapere che la serie delle* V_3^{2p-2} *termina per* $p = 37$ *e che per* $p > 10$ *esse sono razionali*. This letter can be accessed at https://www.corradosegre.unito.it/fondofano/lettera23.pdf.

[108] BSMT, Fondo Fano, letter no. 23: L. Roth to G. Fano, London, 18 February 1937: *[. . .] in una Nota recente –non ancora pubblicata– ho stabilita che una forma quartica di* S_4 *non può aver più di 45 nodi isolati, e del resto si sa che tale limite è raggiunto, perché esiste una* V_3^4 *razionale di questa natura (ved. Todd, Quarterly Journal, Oxford 1936). Forse si potrebbe usare questo risultato per dimostrare, mediante proiezioni successive, che le* V_3^{2p-2} *della prima specie non esistono per* $p > 23$.

author within Fano's miscellany[109]—and the Belgian School of geometry. It must be remembered that Godeaux, in turn, spent a period of advanced training in algebraic geometry in Bologna with Enriques[110] between 1912 and 1914; he was greatly influenced by the work of the Italian School, and his future research directions were to a great extent laid down at this time. Godeaux's 1933 result about linear systems of surfaces over special threefolds ([98], p. 134) represents the starting point for Fano's introduction and classification of this completely new class of three-dimensional varieties 5 years later [99]. Looking at the citational network of Fano's papers on this theme, Godeaux's works make up 25% of the citations (excluding Fano's self-citations).[111] If on the one hand Fano drew directly from Godeaux's research, on the other hand the legacy of Fano's studies on Godeaux is manifest. Indeed, even though Fano's mathematical activity in this direction was not so successful and had a very limited reception, Godeaux's last works on the topic, dating back to the 1960s, were still within the classical research path traced by Fano.

5 Conclusive Remarks

The significant use of archival sources has made it possible to trace out a more "nuanced" vision of Fano's scientific biography than the one provided by the existing historiography (Fig. 9).

Manuscripts preserved in Turin, Göttingen, and Liverpool have contributed to placing him in the context of the Italian School of algebraic geometry, conceived as a scholarly community characterized by specific fields of research, a geometrical style, and some epistemological and linguistic patterns. Fano's research contributions were deeply rooted in the works of Italian masters and in Klein's ideas, as were many of Fano's assumptions in the methodological and teaching fields (use of intuition, dynamic teaching, role of *Approximationsmathematik*, emphasis on the concepts of function and group of transformation, importance of teacher training, ...). His dissemination activity too testifies to his looking back to his cultural roots. This action of promotion, developed in several contexts, reached its peak in the Aberystwyth lectures and ended in Lausanne talks at the Cercle Mathématique where, even from the pain of exile, Fano continued to proudly declare his belonging to the Italian geometric tradition.

[109] For a complete list of this library heritage, see [97] which can be accessed at https://www.corradosegre.unito.it/doc/fano_miscellanea.pdf. For an analysis of such patrimony, see [81].

[110] Also a pupil of Godeaux, Pol Burniat (1902–1975) went to Italy in 1929–1930 to study with Enriques, who had moved to Rome in the meantime.

[111] Excluding citations of his own work, in the two papers on Fano-Enriques threefolds, Fano referred mainly to the publications of the Italian School (15 citations), followed by those by Godeaux (9) and by three other foreign mathematicians (G. Darboux, S. Junski, T. Reye).

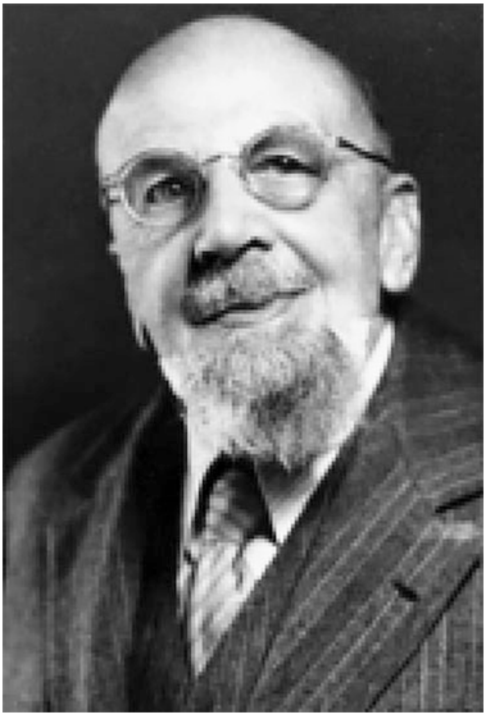


Fig. 9 Gino Fano in the 1930s

Documents preserved in the SPSL and Caltech archives, in addition to unpublished correspondence, have made it possible to document a short parenthesis in Fano's scientific life, generally dismissed as a sad epilogue of a successful career. By way of contrast, the contours of his experience in Switzerland, as reconstructed through such sources, are so peculiar that we can reasonably state that no other refugee from racist Italy had an analogous experience.

Fano's handwritten notes on threefolds have shed new light on his research activity. While perfectly appropriating the cultural heritage of the Italian School of algebraic geometry (a certain way of doing geometry and a set of readings and cultural references), Fano's contributions represented a heritage, in terms of questions dealt with, tools and techniques developed, and language used. In this sense, his manuscripts and publications on threefolds are paradigmatic of the process of patrimonialization of mathematical knowledge, which started at the end of the nineteenth century within the Italian context, continued in some contemporary geometric Schools (such as the English and Belgian ones), and is still living nowadays in some trends of contemporary algebraic geometry.[112]

[112] See [100] for an example regarding the Fano-Enriques threefolds.

In conclusion, this chapter, strongly grounded in archival research, can be considered as a first, but not provisional, contribution to the biography of an Italian geometer belonging to the School of Segre.

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