



Optimization and Application of Lucy-Richardson Filter on Optical Remote Sensing Image Restoration

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Abstract. In complex space environment, remote sensing imaging process might be affected by different kinds factors. Lucy-Richardson (L-R) filter method is an extensive used method in image reconstruction field. In order to decrease the calculation time and promote the quality of image, a optimal semi-blind remote sensing data reconstruction algorithm based on improved L-R filter is presented. Firstly, self-correlation function is innovated to calculate the point spread function (PSF). To suppress the superposition of noise in L-R iterative process, both full-reference and no-reference evaluation parameters are applied in iteration termination conditions. Finally, Gaussian filter is adopted to filtrate low-pass signal when the noise is constantly amplified. Experiments and simulation results explanation that the improved Lucy-Richardson filter algorithm could gain relatively satisfactory reconstruction effects. And the proposed processing method could be used for the restoring and reconstruction of those remote sensing data degraded by rectilinear motion and random noise interference.

Keywords: Lucy-Richardson filter · Point spread function · Remote sensing image reconstruction · Image evaluation parameter

1 Introduction

As many of you known, remote sensing data is frequently degenerated by various factors under complex imaging conditions in space [1–3]. So the restoration of remote sensing data plays a significant role if it is difficult to control the load platform itself [4].

Classical reconstruction algorithms routinely employ filter models according to the degradation principle, such as Wiener filter, inverse filtering and Laplacian constraint [5–8]. These algorithms are always based on MTF (means modulation transfer function) or PSF (means point spread function) [9]. However, the PSF parameter is frequently unable to obtained exactly. In addition, another widely used method is Richardson–Lucy filter [10, 11].

Recently, machine learning is widely used in image processing field [12], CNN (Convolution Neural Network) in particular. Refs. [13, 14] adopted CNN to revert degenerative images. Hopfield neural network is utilized in Refs. [15–17] to realize image restoration based on optimization reason. Moreover, many other prevalent algorithms

such as U-net, GAN-net and residual net [18–20] are applied in image restoration field. However, neural network or deep learning algorithms need a great number of samples.

Based on the literature above, a optimal semi-blind remote sensing data reconstruction method based on improved L-R filter is presented to reduce the calculation time and improve the image’s quality. Self-correlation function is cited to calculate the point spread function. To suppress the superposition of noise in L-R iterative process, SNR (signal to noise ratio) and GMG (grayscale mean gradient) are applied in iteration termination conditions. And Gaussian filter is adopted to filtrate low-pass signal when the noise is larger than the preset threshold. Experiments and simulation results declare that the proposed Lucy-Richardson filter model could acquire relatively satisfactory reconstruction effects and time consumption.

2 Data Degradation Model and Lucy-Richardson Filter

2.1 Principle of Image Restoration and Degradation

Universal image degeneration and reduction model is presented firstly, as shown in the Fig. 1. Where $f(x, y)$ is the source remote sensing image, $g(x, y)$ is the degeneration data, $h(x, y)$ is the modulation transfer function, and $n(x, y)$ is the random noise.

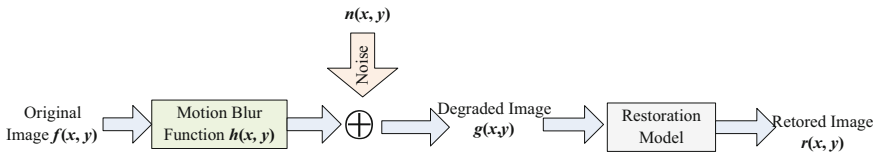


Fig. 1. Universal image restoration and degradation models

To improve the operation speed, the modulation transfer function $h(x, y)$ is often considered to be a space-unchanging and linear equation. And the degeneration process of remote sensing data could be represented by

$$g(x, y) = f(x, y) * h(x, y) + n(x, y) \tag{1}$$

In Fig. 1, $r(x, y)$ represents the restored remote sensing data. Then the goal of image reconstruction is to qualify the following formula.

$$r(x, y) = f(x, y) \tag{2}$$

2.2 Lucy-Richardson Filter Algorithm

Lucy-Richardson filter is an iterative algorithm for image restoration in the background of Poisson distribution, Bayesian theory and maximum likelihood estimation. After the blurred data is convoluted with the given modulation transfer function, and the image without degradation might be obtained, even if the image noise is unknown. Lucy-Richardson filter's principle could be given by the following equation.

$$r_{k+1}(x, y) = r_k(x, y) \left[h(-x, -y) * \frac{g(x, y)}{h(x, y) * r_k(x, y)} \right] \quad (3)$$

where $r_k(x, y)$ represents the restored image after k -th iteration, $h(x, y)$ is point spread function, $g(x, y)$ is the degraded image. And the symbol $*$ represents convolution operation. To start the iteration process, the initial value of $r_0(x, y)$ is set as follow.

$$r_0(x, y) = g(x, y) \quad (4)$$

And the remaining questions are the calculation of point spread function and the determination of iteration stop condition.

3 Design of Remote Sensing Image Restoration Method

3.1 Estimation of Point Spread Function

According to the motion characteristics of the satellite, degraded remote sensing data influenced by linear motion mode is take into account. And the modulation transfer function of degraded system meets with the following formula.

$$h(x, y) = \frac{1}{d}, \quad 0 \leq x \leq d \cos \theta, \quad y = x \sin \theta \quad (5)$$

where d is fuzzy scale, θ is the fuzzy direction. To simplify operation steps, the fuzzy scales of horizontal direction and vertical direction are calculated respectively. In horizontal direction, $h(x)$ is given as

$$h(x) = \frac{1}{d}, \quad 0 \leq x \leq d - 1 \quad (6)$$

In this paper, the self-correlation function is applied to calculate the fuzzy scale d . The detailed calculation procedures are as below.

(1) Implement first-order difference on the vertical orientation of blurred image $g(x, y)$.

$$g_y(i, j) = g(i, j) - g(i, j - 1) \quad (7)$$

(2) Perform first-order difference operation on the horizontal orientation.

$$g_{yx}(i, j) = g_y(i, j) - g_y(i - 1, j) \quad (8)$$

- (3) Compute the degenerative image's self-correlation function on every row. In case there are l pixels on the j -th row, the self-correlation function would be given by

$$r(k) = \sum_{i=l}^{h-k-l} \sum_{m=-l}^l g(i+k+m, j)g(i+m, j) \quad k \in [-1, 1] \quad (9)$$

where h is the height scale of the source image.

Estimate the average value of self-correlation equation and delineate the self-correlation equation profile. The minimal value of the profile location the fuzzy scale.

Similarly, the fuzzy scale in vertical direction could be computed soon. Then the point spread function $h(x, y)$ is estimated.

3.2 Noise Suppression and Iteration Termination Condition

During the iterative process of image restoration, noise information might be amplified. So the value of SNR (Signal to Noise Ratio) is quoted here and evaluate the random noise level of restored remote sensing images. The expression of SNR could be given as

$$SNR = 10 \lg \left[\frac{\sum_{x=1}^w \sum_{y=1}^h f(x, y)^2}{\sum_{x=1}^w \sum_{y=1}^h [f(x, y) - f_p(x, y)]^2} \right] \quad (10)$$

where $f(x, y)$ is the image, $f_p(x, y)$ is the denoised data, h and w are the height and width of the data, respectively. In the process of Lucy-Richardson filter, there is no reference image. In this paper, the degraded image $g(x, y)$ is firstly preprocessed with Gaussian filter. And the result is $g_G(x, y)$. Then the SNR could be calculated by

$$SNR = 10 \lg \left[\frac{\sum_{x=1}^w \sum_{y=1}^h g_G(x, y)^2}{\sum_{x=1}^w \sum_{y=1}^h [g_G(x, y) - r(x, y)]^2} \right] \quad (11)$$

The larger the SNR_G , the smaller the noise, and vice versa.

Another index of image restoration evaluation is GMG (Grayscale Mean Gradient). And in one sense, the value GMG parameter delivers the image's texture contrast and features.

$$GMG = \frac{1}{(w-1)(h-1)} = \sum_{i=1}^w \sum_{j=1}^h \sqrt{\frac{[f(i, j+1) - f(i, j)]^2 + [f(i+1, j) - f(i, j)]^2}{2}} \quad (12)$$

Ideally, both *GMG* and *SNR* become larger as the iteration progresses. When this condition cannot be satisfied, the minimum threshold of *SNR* is fixed in the proposed method. On the other hand, the specified threshold of *GMG* is also assigned to reduce iteration time.

3.3 Remote Sensing Data Restoration Algorithm

TO reduce the superposition of noise, Gaussian filter is adopted to filtrate low-pass signal. The remote sensing image t restoration process is shown in the Fig. 2.

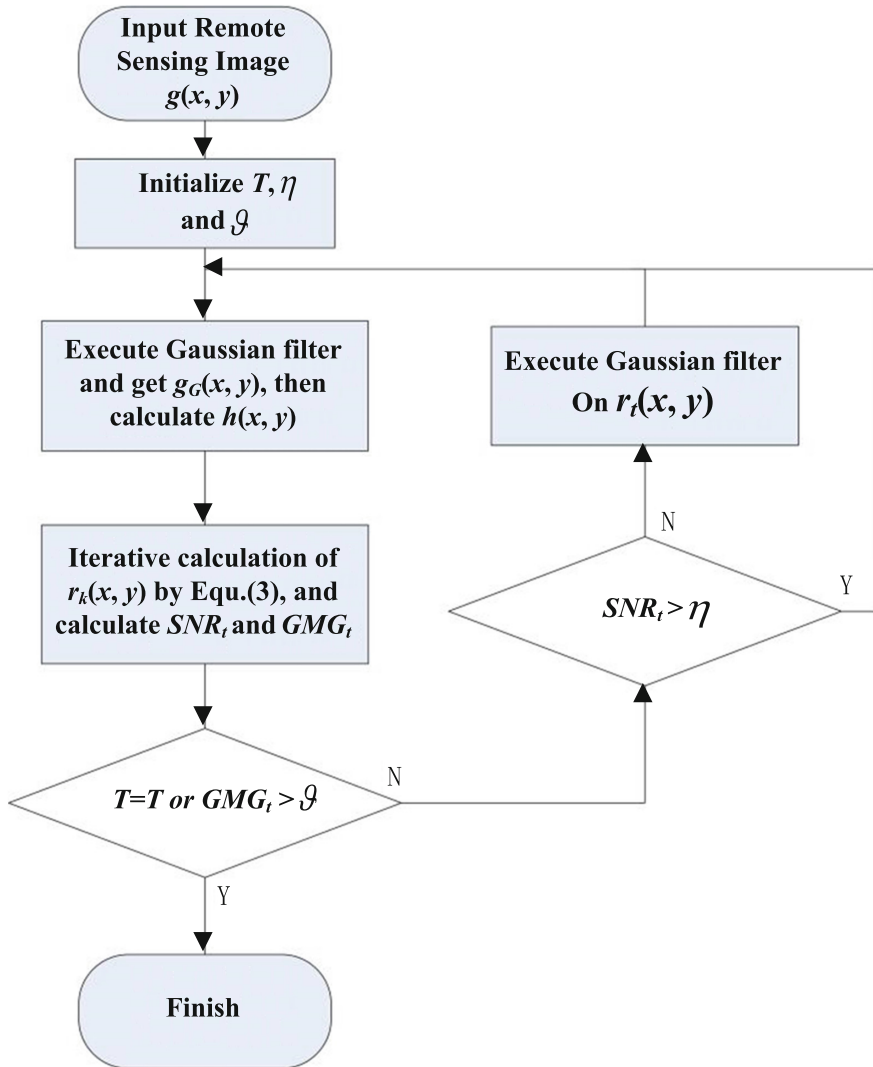


Fig. 2. Remote sensing data restoration process

The implementation steps are as below.

Step 1: Perform initialization operation of these values of T , η and ϑ , here T is the maximal iteration number, η is the minimum threshold of SNR , and ϑ is the specified threshold of GMG . And $r_0(x, y) = g(x, y)$. The number of iterations $t = 0$.

Step 2: Carry out Gaussian filter on $g(x, y)$, assume the result is $g_G(x, y)$.

Step 3: Determine the point spread function $h(x, y)$ through autocorrelation function.

Step 4: Execute the iterative Eq. (3). The number of iterations $t = t + 1$. Calculate SNR_t by Eq. (11). Calculate GMG_t by Eq. (12).

Step 5: If $t = T$ or $GMG_t = \vartheta$, finish processing. Otherwise, compare the size of SNR_t and η . Go to Step 6.

Step 6: If $SNR_t > \eta$, return to Step 4. Else, implement Gaussian filter on $r_t(x, y)$. Return to Step 4.

4 Experiments and Simulations

4.1 Simulations

To testify the availability and significance of the proposed algorithm, the checkboard figure with the size 512×512 is picked as the test data, which is shown in Fig. 3a. And Fig. 3b shows the degenerate image. And the image is influenced by linear motion modulation transfer function and Gaussian noise. And the value of fuzzy scale is 15.

Figure 3c–f are the restoration results when the number of iterations equals to 5, 10, 20, and 50. It can be concluded from these figures that the higher the number of iterations, the better the image restoration effect. However, after iterating to a certain extent, the improvement effect is not obvious at all. Furthermore, L-R algorithm has good noise suppression effect at the beginning of filtering. With the increase of filtering times, the sides of the data are getting clearer and clearer, and the noise is becoming more and more obvious.

Conventional classic Wiener filter method, constrained least squares method and blind deconvolution method are chosen to compare to the proposed L-R filter method, the test results are shown in Fig. 4.

Through human vision, it is clearly that Wiener filter's process effects have lower noise and more fuzzy details, as shown in Fig. 4a. And the restoration result of constrained least squares method has higher noise and more clear details. Figure 4c shows the treatment results of blind deconvolution. It seems that new noises and uncertain factors are brought into the data.

Table 1 displays two assess arguments of the initial data, degenerate data and four methods' processing results. As analyzed above, Wiener filter method can obtain larger SNR , but the GMG value is relatively low. It seems that the L-R filter method has more balanced performance between SNR and GMG when the number of iterations reaches to 20.

4.2 Experimental Tests

To further test the advantages of the algorithm, this paper chooses five more degenerative remote sensing data to implement reconstruction testing, as displayed in the five

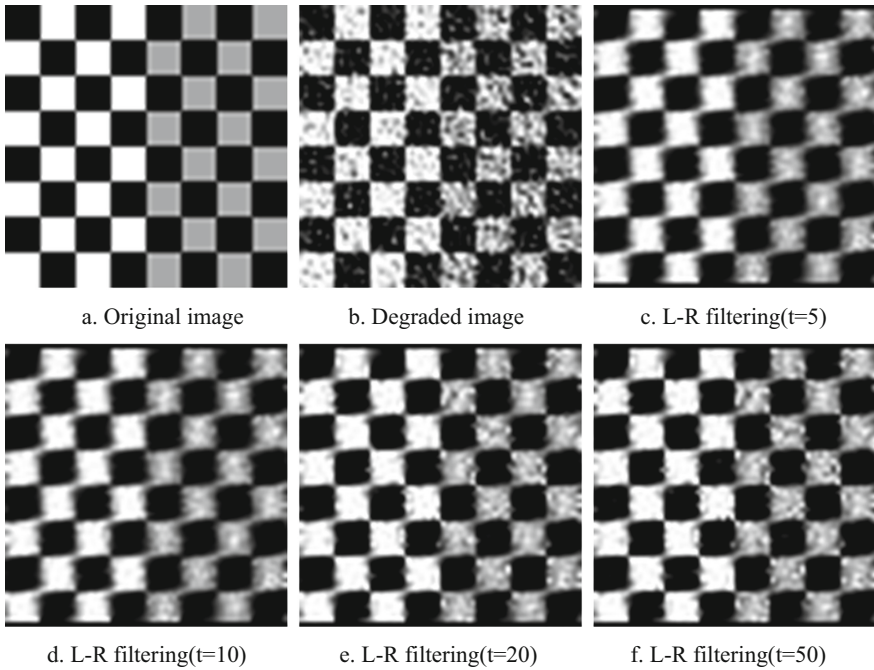


Fig. 3. Restoration results of L-R filter on checkboard

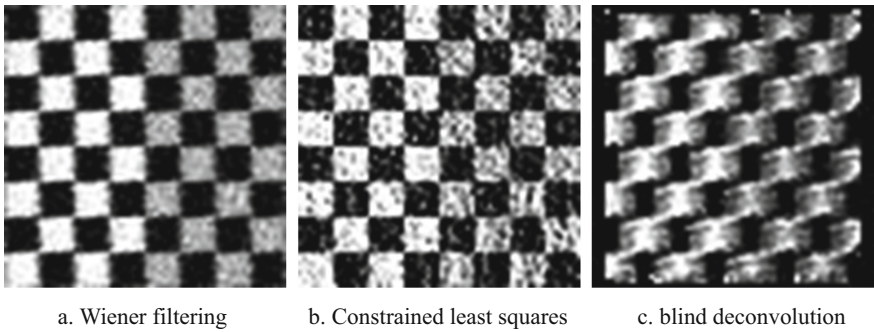


Fig. 4. Processing effects of three methods

subgraphs on the left side of Fig. 5. These remote images' size is 4096×4096 . And the initial calculation parameters are as follows, $T = 15$, $\eta = 16$ and $\vartheta = 5$.

The right five images of Fig. 5 are recovered data. By comparing five groups of data, it is apparent that the details and contours of the recovered data is clearer, and the features of surface seem more notable. By implementing the presented L-R filter, these remote sensing data's GMG features are promoted to some extent, which is shown in Table 2. The SNR index could be limited within the specified scope. And the number of iterations is no larger than T .

Table 1. Checkboard's processing results

	SNR	GMG
Initial image	/	32.46
Degraded image	/	26.92
L-R filtering(t = 5)	9.92	34.02
L-R filtering(t = 10)	8.95	35.36
L-R filtering(t = 20)	7.21	36.84
L-R filtering(t = 50)	6.96	37.57
Wiener filtering	9.96	33.56
Constrained least squares	3.71	35.03
Blind deconvolution	-68.9	34.86

4.3 Analysis of Experimental Results

Lucy-Richardson filter is based on Poisson distribution, Bayesian theory and maximum likelihood estimation. Theoretically, as the number of iterations increases, the image quality will be better and better without considering the noise. After integrating SNR and GMG factors, the presented improved L-R model could contain more useful details. Therefore, the algorithm would achieve more approving remote sensing restored results.

5 Conclusions

To decrease the calculation time and improve the level of image, this paper puts forward a novel semi-blind remote sensing data reconstruction algorithm on the basis of an improved L-R filter. Compared with other methods, our presented method has two following advantages: (I) Autocorrelation functions are quoted to calculate the point spread function, which simplify the calculation steps. (II) Gaussian filter and SNR/GMG parameters are applied in iteration termination conditions to suppress the superposition of noise in L-R iterative process. Experiments and simulation results explanation that this modified Lucy-Richardson filter algorithm can gain relatively satisfactory reconstruction results.

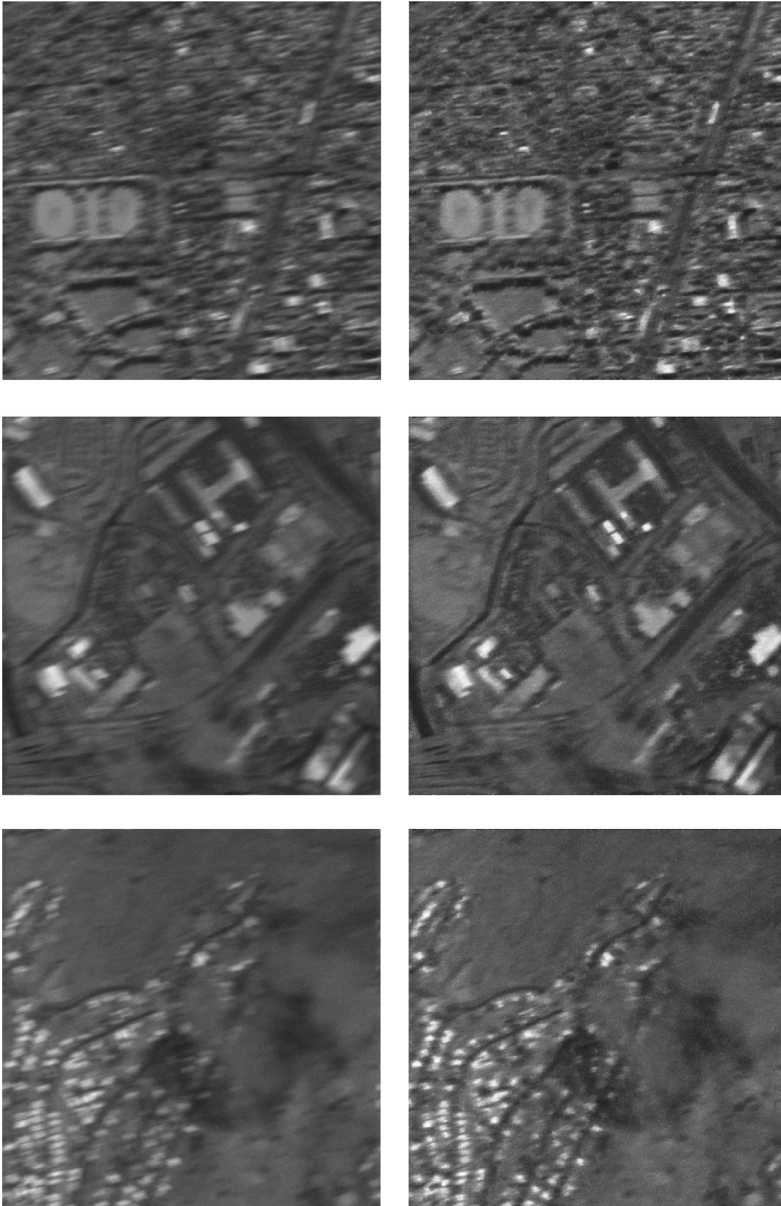


Fig. 5. Restoration effects of proposed L-R algorithm

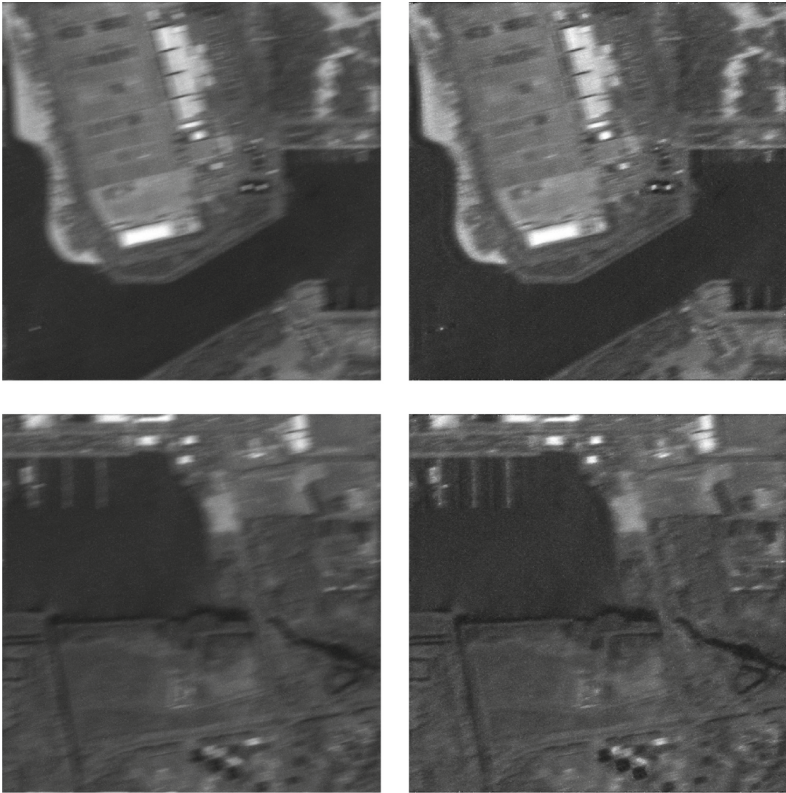


Fig. 5. (continued)

Table 2. Restoration parameters of proposed L-R method

Image No.	SNR	GMG		Number of iterations
		Before	After	
1	16.92	4.56	5.23	12
2	17.98	4.12	5.81	13
3	17.67	4.16	6.13	12
4	20.39	4.12	5.47	11
5	18.62	4.07	5.66	14

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