Intuitionistic Fuzzy Metrics and Its Application

Kousik Bhattacharya and Sujit Kumar De

Abstract This article deals with qualitative difference between two intuitionistic fuzzy sets with the help of standard pseudo-metric and metric spaces. Some definitions over metric spaces, pseudo-metric spaces, intuitionistic fuzzy sets, indeterminacy and the formula of measuring metrices have been incorporated. Numerical illustrations, graphical illustrations, area of applications and ranking for decision-making are discussed to show the novelty of this article. Finally, conclusions and scope of future works are mentioned.

Keywords Metric distance · Pseudo-metric distance · Intuitionistic fuzzy set · Ranking

1 Introduction

In traditional set theory (classical), the idea of member and non-member of an element in a set was sudden, i.e. an element either belongs to a set or not belongs to the set. But there was no knowledge about the transition of an element from member to non-member of the set and vice-versa. Zadeh [1] has solved these ambiguities through his new invention, the fuzzy set theory. Since then, numerous research articles have been studied over the fuzzy set itself to explain the real-world phenomenon. Bellman and Zadeh [2] introduced a new concept of decision-making in a fuzzy environment. Piegat [3] gives us a new definition of fuzzy set. The concepts of dense fuzzy set studied by De and Beg [4, 5] to discuss the frequent learning effect of the fuzzy parameters. Analysing the behaviour of human thinking process, De [6] developed a new inexact set which is known as triangular dense fuzzy lock set and its new defuzzification method. After this invention many articles have been made by eminent researchers (Maity et al. [7, 8], De and Mahata [9], etc.) to control the individual or group decision-making problems on pollution sensitive inventory modelling. Baez-Sancheza et al. [10] discussed polygonal fuzzy sets and numbers

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extensively. Trapezoidal approximations of fuzzy numbers and their existence as well as uniqueness and continuity are exclusively discussed by Ban and Coroianu [11]. Chutia et al. [12] contribute to find membership function of a fuzzy number. De and Mahata [13] designed a fuzzy backorder model where demand rate is considered as cloudy inexactness. Decision-making in a bi-objective inventory problem was discussed by eminent researchers like De and Pal [14]. Mahanta et al. [15] made a new approach of fuzzy arithmetic without using alpha cuts. Mao et al. [16] extensively analysed about the relation between cloud aggregation operators and multiattribute group decision-making in interval valued hesitant fuzzy linguistic environment.

However, Atanassov [17, 18] introduces a new approach of fuzzy set, namely intuitionistic fuzzy sets in terms of membership and non-membership function. Some notable works over the EOQ models on fuzzy environments as well as IFS may be pointed out over here. A step order fuzzy approach is discussed by Das et al. [19]. Recently, Maity et al. [20] studied an intuitionistic dense fuzzy model where the learning–forgetting or agreement–disagreement is considered. De and Sana [21] discussed a stochastic demand model under aggregation with Bonferroni mean in intuitionistic fuzzy environment. Deli and Broumi [22] worked in neutrosophic soft matrices and NSM-decision-making. Recently, Kaur et al. [23] contributed to find relation between interval type intuitionistic trapezoidal fuzzy sets and decision-making with incomplete weight information. Liang and Wang [24] considered a linguistic intuitionistic cloudy fuzzy model with sentiment analysis in E-commerce. Xu [25] studied about intuitionistic fuzzy aggregation operators.

From the above discussion, it is observed that none of the researchers have been studied over the metric distances of intuitionistic fuzzy numbers. In this study, we develop the theory of distances between two nonlinear intuitionistic fuzzy sets (numbers) with respect to (pseudo) metrics. We give some definitions of metric spaces and the formula of distance measure of two different sets via cumulative aggregated formula. To show the novelty of this article, a numerical illustration has been analysed through the ranking of distances with the existing metrics.

2 Preliminaries

2.1 Here, We Shall Introduce Some Definitions Over Metric and Pseudo-Metric Spaces

Definition 2.1.1 Let A be a non-empty set. A function $d : A \times A \to \mathbb{R}$ is said to be a 'metric' or a distance function on A if it satisfies the following properties:

i. $d(x, y) \geq 0$ *for all* $x, y \in A$;
ii. $d(x, y) = 0$ *if and only if* $x = y$ *;*
iii. $d(x, y) = d(y, x)$ *for all* $x, y \in A$ *;*
iv. $d(x, z) = d(x, y) + d(y, z)$ *for all* $x, y, z \in A$.

Any non-empty set A together with a metric d defined on it is said to be a 'metric space'.

Definition 2.1.2 Let A be a non-empty set. A function $d^p : A \times A \to \mathbb{R}$ is said to be a 'pseudo-metric' on A if it satisfies the following properties:

i. $d^p(x, y) \geq 0$ *for all* $x, y \in A$ *;*
ii. $x, y \in A$ and $x = y \Rightarrow d^p(x, y) = 0$ *;*
iii. $d^p(x, y) = d^p(y, x)$ *for all* $x, y \in A$ *;*
iv. $d^p(x, z) = d^p(x, y) + d^p(y, z)$ *for all* $x, y, z \in A$.

Any non-empty set A together with a pseudo-metric d^p defined on it is said to be a 'pseudo-metric space'.

Definition 2.1.3 [17] An intuitionistic fuzzy set A defined in the universe of discourse X is given by $A = \{x, \mu_A(x), \nu_A(x)|x \in X\}$, where $\mu_A, \nu_A : X \to [0, 1]$ denote the degree of membership and non-membership of x in A , respectively, satisfying the condition $0 \leq \mu_A(x) + \nu_A(x) \leq 1$. The indeterminacy degree $\psi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ expresses the lack of knowledge of whether x belongs to A or not, also $0 \leq \psi_A(x) \leq 1$ for $x \in X$. An intuitionistic fuzzy number $\omega = (\mu_\omega, \nu_\omega)$ is an ordered pair which satisfies the conditions: $0 \leq \mu_\omega \leq 1$, and $0 \leq \mu_\omega + \nu_\omega \leq 1$, where and ν_ω are called membership degree and non-membership degree, respectively.

2.2 *Pseudo-Metrics in Intuitionistic Fuzzy Set*

Let us consider F be the set of all intuitionistic fuzzy numbers and each $\omega = (\mu_\omega, \nu_\omega) \in F$ is called a point of F. However, $\omega = (\mu_\omega, \nu_\omega, \psi_\omega)$ only has two degrees of freedom because $\mu_\omega + \nu_\omega + \psi_\omega \equiv 1$. So, we observe such a system by keeping one variable constant when the other variable is changing.

Definition 2.2.1 [25] Given two intuitionistic fuzzy numbers (IFN) ρ and σ, $\rho \cap \sigma = \left(min\left(\mu_\rho, \mu_\sigma\right), max\left(\nu_\rho, \nu_\sigma\right)\right)$ and $\rho \cup \sigma = \left(max\left(\mu_\rho, \mu_\sigma\right), min\left(\nu_\rho, \nu_\sigma\right)\right)$.

Lemma 1 If $\left(\mu_\rho - \mu_\sigma\right)\left(\nu_\rho - \nu_\sigma\right) \geq 0$, $\rho \cap \sigma \in F$ and $\rho \cup \sigma \in F$.

Proof

$$\mu_{\rho\cap\sigma} + \nu_{\rho\cap\sigma} = \mu_\rho + \nu_\sigma \leq \mu_\sigma + \nu_\sigma \leq 1,$$

$$\mu_{\rho\cup\sigma} + \nu_{\rho\cup\sigma} = \mu_\sigma + \nu_\rho \leq \mu_\sigma + \nu_\sigma \leq 1, if\, \mu_\rho \leq \mu_\sigma;$$

$$\mu_{\rho\cap\sigma} + \nu_{\rho\cap\sigma} = \mu_\sigma + \nu_\rho < \mu_\rho + \nu_\rho \leq 1,$$

$$\mu_{\rho\cup\sigma} + \nu_{\rho\cup\sigma} = \mu_\rho + \nu_\sigma\langle\mu_\rho + \nu_\rho \leq 1, if\mu_\rho\rangle\mu_\sigma;$$

Definition 2.2.2 An ordered pair $\left(F, d_\psi^p\right)$ is called the intuitionistic fuzzy indeterminacy pseudo-metric space on F F, where $d_\psi^p : F^2 \to \mathbb{R}$ $d_\psi^p : F^2 \to \mathbb{R}$ is the indeterminacy pseudo-metric, for any $\rho, \sigma \in F$, $d_\psi^p(\rho, \sigma) = \frac{|\psi_\rho^2 - \psi_\sigma^2|}{2}$ $d_\psi^p(\rho, \sigma) = \frac{|\psi_\rho^2 - \psi_\sigma^2|}{2}$.

It is easy to verify that d_ψ^p d_ψ^p satisfies the properties of pseudo-metric. Similarly, we can define the intuitionistic fuzzy membership pseudo-metric space on F F, where $d_\mu^p : F^2 \to \mathbb{R}$ $d_\mu^p : F^2 \to \mathbb{R}$ is the membership pseudo-metric, for any $\rho, \sigma \in F$, $d_\mu^p(\rho, \sigma) = \frac{|\mu_\rho^2 - \mu_\sigma^2|}{2}$ $d_\mu^p(\rho, \sigma) = \frac{|\mu_\rho^2 - \mu_\sigma^2|}{2}$ and the intuitionistic fuzzy non-membership pseudo-metric space on F F, where $d_\nu^p : F^2 \to \mathbb{R}$ $d_\nu^p : F^2 \to \mathbb{R}$ is the non-membership pseudo-metric, for any $\rho, \sigma \in F$, $d_\nu^p(\rho, \sigma) = \frac{|\nu_\rho^2 - \nu_\sigma^2|}{2}$.

It is also easy to verify that d_μ^p and d_ν^p satisfies the properties of pseudo-metric.

Definition 2.2.3 An ordered pair $\left(F, d_\psi\right)$ is called the intuitionistic fuzzy indeterminacy metric space on F, where $d_\psi : F^2 \to \mathbb{R}$ is the indeterminacy metric, for any $\rho, \sigma \in F$, $d_\psi(\rho, \sigma) = \left|\psi_\rho - \psi_\sigma\right|$.

It is easy to verify that d_ψ satisfies the properties of metric. Similarly, we can define the intuitionistic fuzzy membership metric space on F, where $d_\mu : F^2 \to \mathbb{R}$ is the membership metric, for any $\rho, \sigma \in F$, $d_\mu(\rho, \sigma) = \left|\mu_\rho - \mu_\sigma\right|$ and the intuitionistic fuzzy non-membership metric space on F , where $d_\nu : F^2 \to \mathbb{R}$ is the non-membership metric, for any $\rho, \sigma \in F$, $d_\nu(\rho, \sigma) = \left|\nu_\rho - \nu_\sigma\right|$.

It is also easy to verify that d_μ and d_ν satisfies the properties of metric.

Lemma 2 If $\alpha, \beta \in F$, $d_\Sigma(\alpha, \beta) = max\left(d_\mu(\alpha, \beta), d_\nu(\alpha, \beta), d_\psi(\alpha, \beta)\right)$ satisfies the four properties of pseudo-metric as well as metric.

Lemma 3 Let $\tilde{P}$ and $\tilde{Q}$ be two intuitionistic fuzzy sets defined over the interval $[L, R]$, then the summative metric distance (summative pseudo-metric distance) between $\tilde{P}$ and $\tilde{Q}$ is denoted by $d\left(\tilde{P}, \tilde{Q}\right)$ (for pseudo-metric $d_p\left(\tilde{P}, \tilde{Q}\right)$ and defined as.

$$d\left(\tilde{P}, \tilde{Q}\right) = \frac{1}{R - L} \int_L^R d_\Sigma\left(\left(\mu_{\tilde{P}}(x), \nu_{\tilde{P}}(x)\right), \left(\mu_{\tilde{Q}}(x), \nu_{\tilde{Q}}(x)\right)\right)dx \tag{1}$$

3 Representation of IFSs $\tilde{P} and\ \tilde{Q}$ Over the Interval $[a_1, a_3]$ [20]

$$\mu_{\tilde{P}}(x) = \begin{cases} \left(\frac{x-a_1}{a_2-a_1}\right)^m & if a_1 \pounds\ x \pounds\ a_2 \\ \left(\frac{a_3-x}{a_3-a_2}\right)^m & if a_2 \pounds\ x \pounds\ a_3 \\ 0 & \text{otherwise} \end{cases} \quad \text{and} \quad \mu_{\tilde{Q}}(x) = \begin{cases} \left(\frac{x-a_1}{a_2-a_1}\right)^n & if a_1 \le x \le a_2 \\ \left(\frac{a_3-x}{a_3-a_2}\right)^n & if a_2 \le x \le a_3 \\ 0 & \text{otherwise} \end{cases}$$

$$\nu_{\tilde{P}}(x) = \begin{cases} \left(\frac{a_2-x}{a_2-a_1}\right)^m & if a_1 \pounds\ x \pounds\ a_2 \\ \left(\frac{x-a_2}{a_3-a_2}\right)^m & if a_2 \pounds\ x \pounds\ a_3 \\ 0 & \text{otherwise} \end{cases} \qquad \nu_{\tilde{Q}}(x) = \begin{cases} \left(\frac{a_2-x}{a_2-a_1}\right)^n & if a_1 \le x \le a_2 \\ \left(\frac{x-a_2}{a_3-a_2}\right)^n & if a_2 \le x \le a_3 \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

Now as per Eq. (1), the pseudo-metric between two IFSs is given by

$$\begin{aligned}
\mathrm{d}^p\left(\tilde{P}, \tilde{Q}\right) &= \frac{1}{a_3 - a_1} \int_{a_1}^{a_3} \mathrm{d}_\Sigma\left(\left(\mu_{\tilde{P}}(x), \nu_{\tilde{P}}(x)\right), \left(\mu_{\tilde{Q}}(x), \nu_{\tilde{Q}}(x)\right)\right)\mathrm{d}x \\
&= \frac{1}{a_3 - a_1} \int_{a_1}^{a_2} \mathrm{d}_\Sigma\left(\left(\mu_{\tilde{P}}(x), \nu_{\tilde{P}}(x)\right), \left(\mu_{\tilde{Q}}(x), \nu_{\tilde{Q}}(x)\right)\right)\mathrm{d}x \\
&+ \frac{1}{a_3 - a_1} \int_{a_2}^{a_3} \mathrm{d}_\Sigma\left(\left(\mu_{\tilde{P}}(x), \nu_{\tilde{P}}(x)\right), \left(\mu_{\tilde{Q}}(x), \nu_{\tilde{Q}}(x)\right)\right)\mathrm{d}x \\
&= \frac{1}{a_3 - a_1} \int_{a_1}^{a_2} \mathrm{d}_\Sigma\left(\left(\left(\frac{x - a_1}{a_2 - a_1}\right)^m, \left(\frac{a_2 - x}{a_2 - a_1}\right)^m\right), \left(\left(\frac{x - a_1}{a_2 - a_1}\right)^n, \left(\frac{a_2 - x}{a_2 - a_1}\right)^n\right)\right)\mathrm{d}x \\
&\frac{1}{a_3 - a_1} \int_{a_2}^{a_3} \mathrm{d}_\Sigma\left(\left(\left(\frac{a_3 - x}{a_3 - a_2}\right)^m, \left(\frac{x - a_2}{a_3 - a_2}\right)^m\right), \left(\left(\frac{a_3 - x}{a_3 - a_2}\right)^n, \left(\frac{x - a_2}{a_3 - a_2}\right)^n\right)\right)\mathrm{d}x \\
&+ \frac{1}{a_3 - a_1} \\
&\int_{a_2}^{a_3} \max\left\{ \begin{array}{c} \frac{1}{2}\left|\left(\frac{x-a_2}{a_3-a_2}\right)^{2m} - \left(\frac{x-a_2}{a_3-a_2}\right)^{2n}\right|, \frac{1}{2}\left|\left(\frac{a_3-x}{a_3-a_2}\right)^{2m} - \left(\frac{a_3-x}{a_3-a_2}\right)^{2n}\right|, \\ \frac{1}{2}\left|\left(1 - \left(\frac{x-a_2}{a_3-a_2}\right)^m - \left(\frac{a_3-x}{a_3-a_2}\right)^m\right)^2 - \left(1 - \left(\frac{x-a_2}{a_3-a_2}\right)^n - \left(\frac{a_3-x}{a_3-a_2}\right)^n\right)^2\right| \end{array} \right\}\mathrm{d}x
\end{aligned} \tag{3}$$

Similarly, for the metric distance we have

$$\begin{aligned}
d\left(\tilde{P}, \tilde{Q}\right) &= \frac{1}{a_3 - a_1} \int_{a_1}^{a_3} d_\Sigma\left(\left(\mu_{\tilde{P}}(x), \nu_{\tilde{P}}(x)\right), \left(\mu_{\tilde{Q}}(x), \nu_{\tilde{Q}}(x)\right)\right)\mathrm{d}x \\
&= \frac{1}{a_3 - a_1} \int_{a_1}^{a_2} d_\Sigma\left(\left(\mu_{\tilde{P}}(x), \nu_{\tilde{P}}(x)\right), \left(\mu_{\tilde{Q}}(x), \nu_{\tilde{Q}}(x)\right)\right)\mathrm{d}x \\
&+ \frac{1}{a_3 - a_1} \int_{a_2}^{a_3} d_\Sigma\left(\left(\mu_{\tilde{P}}(x), \nu_{\tilde{P}}(x)\right), \left(\mu_{\tilde{Q}}(x), \nu_{\tilde{Q}}(x)\right)\right)\mathrm{d}x \\
&= \frac{1}{a_3 - a_1} \int_{a_1}^{a_2} d_\Sigma\left(\left(\left(\frac{x - a_1}{a_2 - a_1}\right)^m, \left(\frac{a_2 - x}{a_2 - a_1}\right)^m\right), \left(\left(\frac{x - a_1}{a_2 - a_1}\right)^n, \left(\frac{a_2 - x}{a_2 - a_1}\right)^n\right)\right)\mathrm{d}x
\end{aligned}$$

$$+\frac{1}{a_3-a_1}\int_{a_2}^{a_3} d_{\Sigma}\left(\left(\left(\frac{a_3-x}{a_3-a_2}\right)^m, \left(\frac{x-a_2}{a_3-a_2}\right)^m\right), \left(\left(\frac{a_3-x}{a_3-a_2}\right)^n, \left(\frac{x-a_2}{a_3-a_2}\right)^n\right)\right)\mathrm{d}x$$

$$=\frac{1}{a_3-a_1}\int_{a_1}^{a_2} \max\left\{\begin{array}{l}\left|\left(\frac{x-a_1}{a_2-a_1}\right)^m-\left(\frac{x-a_1}{a_2-a_1}\right)^n\right|,\\ \left|\left(\frac{a_2-x}{a_2-a_1}\right)^m-\left(\frac{a_2-x}{a_2-a_1}\right)^n\right|,\\ \left|\left(1-\left(\frac{x-a_1}{a_2-a_1}\right)^m-\left(\frac{a_2-x}{a_2-a_1}\right)^m\right)-\left(1-\left(\frac{x-a_1}{a_2-a_1}\right)^n-\left(\frac{a_2-x}{a_2-a_1}\right)^n\right)\right|\end{array}\right\}\mathrm{d}x$$

$$=\frac{1}{a_3-a_1}\int_{a_1}^{a_2} \max\left\{\begin{array}{l}\left|\left(\frac{x-a_1}{a_2-a_1}\right)^m-\left(\frac{x-a_1}{a_2-a_1}\right)^n\right|,\\ \left|\left(\frac{a_2-x}{a_2-a_1}\right)^m-\left(\frac{a_2-x}{a_2-a_1}\right)^n\right|,\\ \left|\left(1-\left(\frac{x-a_1}{a_2-a_1}\right)^m-\left(\frac{a_2-x}{a_2-a_1}\right)^m\right)-\left(1-\left(\frac{x-a_1}{a_2-a_1}\right)^n-\left(\frac{a_2-x}{a_2-a_1}\right)^n\right)\right|\end{array}\right\}\mathrm{d}x$$

$$+\frac{1}{a_3-a_1}\int_{a_2}^{a_3} \max\left\{\begin{array}{l}\left|\left(\frac{x-a_2}{a_3-a_2}\right)^m-\left(\frac{x-a_2}{a_3-a_2}\right)^n\right|,\\ \left|\left(\frac{a_3-x}{a_3-a_2}\right)^m-\left(\frac{a_3-x}{a_3-a_2}\right)^n\right|,\\ \left|\left(1-\left(\frac{x-a_2}{a_3-a_2}\right)^m-\left(\frac{a_3-x}{a_3-a_2}\right)^m\right)-\left(1-\left(\frac{x-a_2}{a_3-a_2}\right)^n-\left(\frac{a_3-x}{a_3-a_2}\right)^n\right)\right|\end{array}\right\}\mathrm{d}x$$

$$=\frac{1}{a_3-a_1}\int_{a_1}^{a_2} d_{\Sigma}\left(\left(\mu_{\tilde{P}}(x), \nu_{\tilde{P}}(x)\right), \left(\mu_{\tilde{Q}}(x), \nu_{\tilde{Q}}(x)\right)\right)\mathrm{d}x$$

$$+\frac{1}{a_3-a_1}\int_{a_2}^{a_3} d_{\Sigma}\left(\left(\mu_{\tilde{P}}(x), \nu_{\tilde{P}}(x)\right), \left(\mu_{\tilde{Q}}(x), \nu_{\tilde{Q}}(x)\right)\right)\mathrm{d}x \tag{4}$$

3.1 Numerical Example

Let us consider the interval valued fuzzy set like $[a_1, a_3]$=[10, 20], and we consider two IFSs $\tilde{P} and \tilde{Q}$ developed by using (2) where $\langle a_1, a_2, a_3\rangle = \langle 10, 15, 20\rangle$.. Then utilizing (4) and (5), we have the following result stated in Table 1.

Table 1 shows that for each criteria, the pseudo-metric distances between two IFSs as (pseudo) metric space is always less than the distances with respect to the usual metric. Also, it is seen that criteria *B* and criteria *C* gives exactly same results. Naturally, it comes because of the symmetric property of pseudo-metric spaces and metric spaces also.

Table 2 shows the ranking of distances over IFSs consisting of m–n exponts fuzzy ss under various criteria (integer-fraction). The ranking is getting ascending ord when the exponents are both integer (fraction) and keeps descending order when exponents of two different IFSs assume integer-fraction values. But in each case ranking type is same for pseudo-metric distance and metric distance. The distance under pseudo-metric spaces gives ranking of each distances over several criteria extensively, whereas the metric distance gives a broader sense of ranking. This means that for any kind of decision-making problems, if we want to measure a qualitative

Table 1 (Pseudo) metric distances under IFSs

Criteria	Exponents			Pseudo-metric distance	Metric distance
		m	n		
A	A_1	2	4	0.146	0.267
(m, n integer)	A_2	3	5	0.108	0.167
	A_3	4	6	0.079	0.114
B	B_1	2	1/2	0.150	0.7
(m integer n fraction)	B_2	3	1/3	0.228	1.000
	B_3	4	1/4	0.278	1.200
C	C_1	1/2	2	0.150	0.667
(m fraction n integer)	C_2	1/3	3	0.228	1.000
	C_3	1/4	4	0.278	1.200
D	D_1	1/2	1/4	0.126	0.267
(m, n fraction)	D_2	1/3	1/5	0.096	0.167
	D_3	1/4	1/6	0.074	0.114

difference between two subjects under study the pseudo-metric distance is more popular (user friendly) and easy to interpret for a decision-maker instead of metric distance only.

Figure 1 shows that the distances of two IFSs under metric space is higher than the distances under pseudo-metric when both the exponents of fuzzy numbers are integers. The minimum distance gap ranges from 0.04 to 0.14 approximately whenever we consider the exponents of fuzzy numbers as incremental indices

Figure 2a shows that the distances of two IFSs under metric is higher than the distances under pseudo-metric when one of the exponents of fuzzy numbers is integer and another one is fraction number.

Figure 2b shows the symmetricity of distances of (pseudo) metric spaces whenever the values of exponents of fuzzy numbers are getting interchanged.

Figure 3 expresses the distances of two IFSs under metric is higher than the distances under pseudo-metric when both the exponents are considered as fraction numbers

Figure 4shows that the variations of distances in various criteria of exponents, i.e. integer, fraction, both, etc., under (pseudo) metric. Also, it is clear from the graph that the distances of two IFSs as (pseudo) metric space are not intersecting. The gaps are getting increased when one index is integer and another is fraction than both indices are integer or fractions exclusively

Table 2 Ranking of (pseudo)metric distances

Criteria	Pseudo-metric distance	Metric distance
m, n integer	$A_1 > A_2 > A_3$	$A_1 > A_2 > A_3$
m integer n fraction or m fraction n integer	$B_1 < B_2 < B_3$ $C_1 < C_2 < C_3$	$B_1 < B_2 < B_3$ $C_1 < C_2 < C_3$
m, n fraction	$D_1 > D_2 > D_3$	$D_1 > D_2 > D_3$
Pseudo-metric distance	$B_3 > B_2 > B_1 > A_1 > D_1 > A_2 > D_2 > A_3 > D_3$	
Metric distance	$B_3 > B_2 > B_1 > A_1 > A_2 > A_3$	

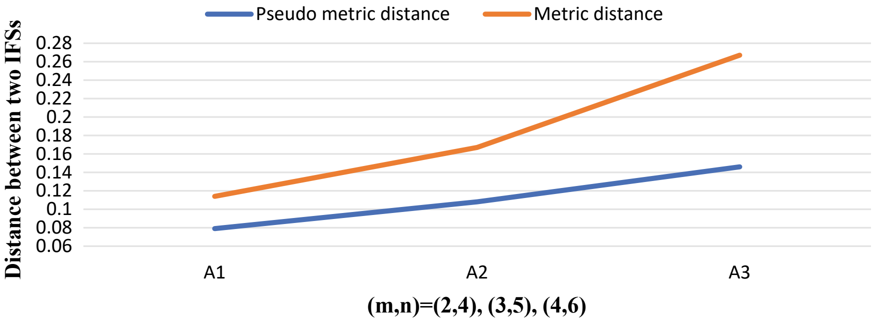

Fig. 1 Comparative study of distances when both exponents are integer

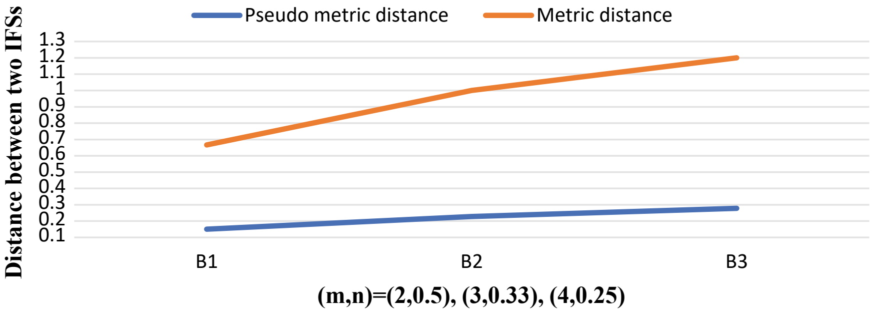

Fig. 2 **a**Comparative study of distances when both exponents is integer and anthor is fraction, **b** Comparative study of distances when both exponents is fraction and anthor is integer

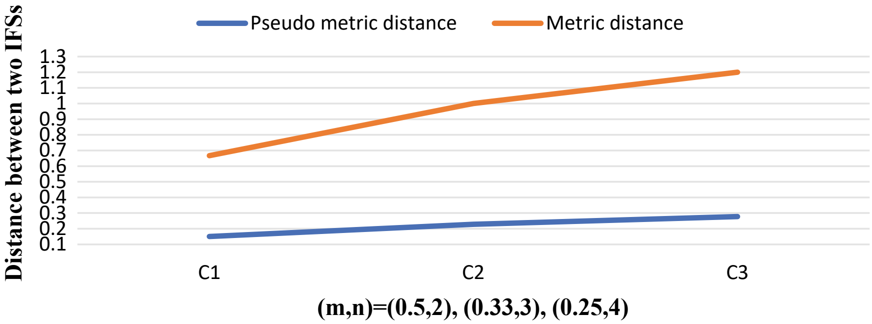

Fig. 2 (continued)

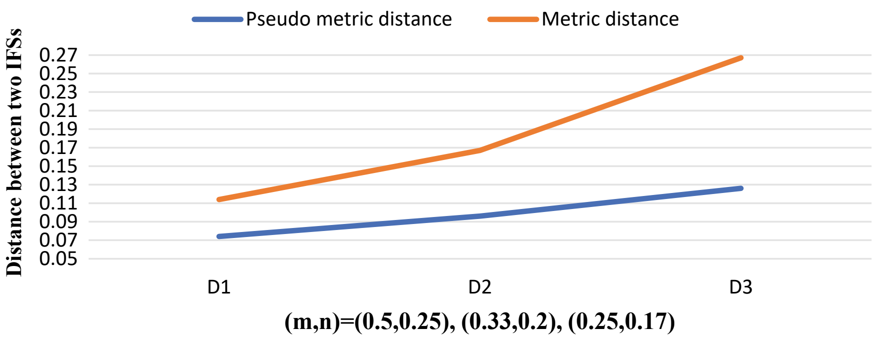

Fig. 3 Comparative study of distances when both exponents are fraction

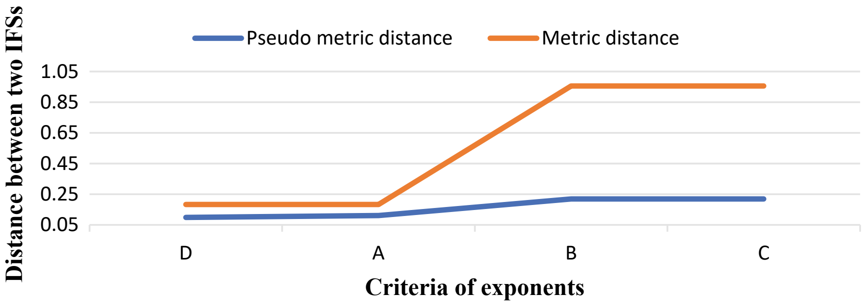

Fig. 4 Comparative study of distances for each criteria of exponents

3.2 Graphical Illustration

4 Area of Application

The area of applications of this proposed approach are stated below:

i. It is used to measure the qualitative differences among various subjects like any supply chain modelling or decision-making problem.
ii. To know the degrees of membership, non-membership and indeterminacy, this method can be applied.
iii. Any kind of ranking of different subjects over several disciplines is possible with its help.

4.1 Merits and Demerits

After the study of numerical and graphical illustrations, we see that there exists some merits and demerits of the proposed approach. They are stated as follows:

4.1.1 Merits

i. This approach is very useful to find difference between two IFSs whose membership, non-membership and indeterminacy functions are nonlinear.
ii. This method told us that the pseudo-metric distance is more user friendly for decision-making.
iii. This method helps us to study different nature of intuitionistic fuzzy sets drawn over physical problems with more detailing.

4.1.2 Demerits

i. This method is not applicable for the IFSs which are defined in a discrete space.
ii. This method is silent for higher dimensional intuitionistic fuzzy sets.
iii. This method might be complicated to handle when the membership function is complicated.
iv. The results may vary when the IFSs are assumed to be different.

5 Conclusion

In this study, we have discussed about the qualitative differences of the subjects of real-world problem by means of metric distances between two nonlinear intuitionistic fuzzy sets. Here, we see that the distance under pseudo-metric is more effective and extensive rather than the distance under conventional standard metric. The ranking of (pseudo) metric distances gives us a clear idea of quality measurement of two IFSs with nonlinear membership and non-membership function. This idea will be very useful to solve a decision-making problem. Also, graphical illustrations show the fluctuation of differences in several criteria of exponents in membership and non-membership function.

Scope of future work

This study of IFS with the help of metric and pseudo-metric is innovative. In future, various types of works can be done using this approach. This method can be applied for decision-making problems such as supply chain modelling or inventory modelling.

6 Conflicts of Interest

It is declared by the authors that there is no conflict of interest regarding the publication of this article.

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