

Chapter 5

Optimal Inventory Management Policies for Substitutable Products Considering Non-instantaneous Decay and Cost of Substitution



Ranu Singh and Vinod Kumar Mishra

Abstract Substitution is an important key of real retail business market. Nowadays, most of retailers offer similar product to the customer in out-of-stock period. This study presents an inventory management approach for substitutable non-instantaneous decaying products under joint ordering policy. This study deals all the possible cases: when a product is completely depleted due to requirement/demand and decay, its demand is partially met by other products, and remain unsatisfied demand is lost. In this article, demand and deterioration rate are taken as deterministic type. The purpose of this article is to find the optimal ordering quantities to optimize the total cost. This article presents solution methodology and provides a numerical simulation to illustrate the model's application. The numerical result shows a significant reduction in overall cost in the case of with substitution over without substitution case. Finally, the sensitivity of key parameters is examined and concludes this study with some future plans.

Keywords Substitutable product · Joint replenishment · Inventory management · Non-instantaneous decay

Introduction

In current retail business market, the incidence of partial stock-outs in the types of usually purchasing products is a very common phenomenon. In certain situations, it is regular for purchasers who are looking to purchase a particular product to buy a similar product in order to save time, rather of traveling to another supermarket/retail shop to find the first product. A product is replacement for other product only if it

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can be used in just the same way and follows the same requirements. In the case of alternatives, an increment in the cost of one of the products rises the substitute demand. Anupindi et al. [1] presented a study result in which he observed that 82–88% of the customer would be interested to purchase substitute goods if the requested goods are out of stock. In retail shops, substitution is a regular happening event for example between different brand of toothpaste, Coke and Pepsi, milk, etc. The effect of deterioration is very common in products, which play crucial role in the decision of ordering quantity. Hence, it would be beneficial for retailers to consider the effect of deterioration with substitution into account, when making inventory policies. To avoid extra transportation cost (ordering cost), and supplier cost, several organizations try to refill products jointly to take profit of joint ordering. In this research work, we present an inventory model having two non-instantaneous substitutable decaying products under joint ordering policy for a retailer perspective.

There is large number of literature available on product substitution, deterioration with joint replenishment separately. McGillivray and Silver [2] was the earliest researcher to establish an inventory model with similar cost of substitute product and shortage cost. Drezner et al. [3] presented an economic order quantity (EOQ) model with substitution and compared the output with no substitution. Salameh et al. [4] developed an inventory strategy by considering substitute product with joint ordering policy and show that substitution between items can be saved in a fixed order price, which is free from the quantity of items involved in the order. Krommyda et al. [5] expanded the previous model by incorporating a joint ordering strategy and a substitution rate ranging from 0 to 1. Many researchers categorized substitution into different parts. Tang and Yin [6] divided substitution into some categories: stock-out, price, and assortment-based substitution. They investigate how under variable and fixed pricing policies, the order size and selling price of two substitute products could be jointly determined by the retailer. A brief literature review on substitutable products in the inventory is given by [7–11].

Whitin [12] first investigated the inventory problem of deteriorating products, considering fashion products deteriorating at the end of storage interval. [13–18] presented extensive literature on decaying/deteriorating items in inventory system. Mishra [19] generalized Salameh et al. [4] model by taking deterioration of product into account. Tiwari et al. [20] developed an ordering policy inventory model for non-instantaneous deteriorating item with different realistic situation.

Based on the above literature review and our knowledge, no study has considered the effect of non-instantaneous deterioration for substitutable products. To fill this literature gap, authors have considered two mutually substitutable products considering non-instantaneous decay and the cost of substitution. The cost of substitution includes the promotional effects like (cost comes from advertisements and sales promotions). The objective of this study is to obtain the optimal ordering quantities to minimize the overall costs. A comparative study has been done in the with substitution and without substitution cases. A solution methodology is developed to determine the optimal values of decision parameters. The nature of the overall cost function is also discussed. The comparative study and gap analysis of the relevant paper are presented in Table 5.1.

Table 5.1 Comparative study and gap analysis

Researcher (s)	Model	Substitutable item	Cost of substitution	Non-instantaneous decaying product	Joint replenishment	Objective function
McGillivray and Silver [2]	EOQ	✓				Cost
Anupindi et al. [1]	–	✓				–
Gurler and Yilmaz [8]	–	✓				Profit
Maity and Maiti [10]	EOQ	✓				Profit
Krommyda et al. [5]	EOQ	✓			✓	Cost
Mishra [19]	EOQ	✓	✓		✓	Cost
Tiwari et al. [20]	EOQ			✓		Cost
Mishra and Mishra [9]	EOQ	✓	✓			Cost
Singh and Mishra [21]	EOQ	✓		✓	✓	Cost
This article	EOQ	✓	✓	✓	✓	Cost

The remaining part of this article is ordered as follows: The assumptions and notations of the study, model description and formulation, solution method, numerical application, and sensitivity of key variables are given in section-wise, respectively. The last section represents the conclusion of the research.

Assumptions and Notations

Assumptions

- i. Lead time is assumed to be zero.
- ii. Two substitutable products are taken, and each are ordered jointly in every cycle.
- iii. Deterioration and demand rates are constant, and each product has distinct deterioration rates.
- iv. Both products are partially substitutable, if one of them is out of stock, then the other can be a substitute for that product.
- v. Substitution cost includes promotional effects like (cost comes from advertisements and sales promotions) are considered.

Notations

Symbol	Description
D_1, D_2	Demand rate of product 1 and product 2
C_1, C_2	Unit purchase cost of product 1 and product 2
θ_1, θ_2	Deterioration rate of product 1 and product 2
A_1, A_2	Setup cost for product 1 and product 2
$Q_i, i = 1, 2$	Stock level of product 1 and product 2
h	Holding cost of each product
$\alpha_i, i = 1, 2$	Substitution rate for product 1 and product 2
$\pi_i, i = 1, 2$	Unit lost sale cost of product 1 and product 2
CS_{12}, CS_{21}	Unit substitution cost for product 1 and product 2
l	Stock level of product when other product is completely depleted
s	Part of period when substitution occur
$I_i^1(t), i = 1, 2$	Stock level of product i when first product finishes before second
$I_3^1(t)$	Stock level of product 2 when first product finishes before second and substitution takes place
$TC_i(Q_1, Q_2), i = 1, 2$	Average total cost for case 1 and case 2
$TC_{NS}(Q_1, Q_2)$	Average total cost for case 3

Model Description and Formulation

Based on assumptions, we assume two substitutable non-instantaneous decaying products. At starting, the retailer’s orders Q_1 and Q_2 quantity (unit) of product 1 and 2 jointly. The stock level of each product consumes due to demand in time period $[0, t_1]$ and deplete due to deterioration and demand in period $[t_1, t_2]$ and after that substitution occurs. Both products replenish jointly after consumption to acquire the benefits of joint ordering. At $t = t_2$, three possibilities arise as follows.

Case 1: Product 1 fully consumes before product 2, i.e., if product 1 is out of stock at time t_2 , as seen in Fig. 5.1, then requirement of product 1 is partially satisfied by product 2 with substitution rate α_1 and rest demand for product 1 is lost with rate of $(1 - \alpha_1)$. The stock level of products is depicted by the given below differential equation, as seen in Fig. 5.1.

$$\frac{dI_1^1(t)}{dt} = -D_1; \quad 0 \leq t \leq t_1 \tag{5.1}$$

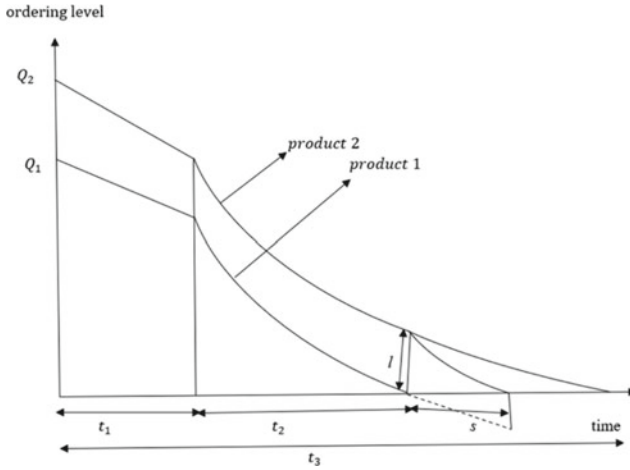


Fig. 5.1 Stock size for case 1 at time t

$$\frac{dI_1^1(t)}{dt} + \theta_1 I_1^1(t) = -D_1; \quad t_1 \leq t \leq t_2 \tag{5.2}$$

With $I_1^1(0) = Q_1, I_1^1(t_2) = 0$.

$$\frac{dI_2^1(t)}{dt} = -D_2; \quad 0 \leq t \leq t_1 \tag{5.3}$$

$$\frac{dI_2^1(t)}{dt} + \theta_2 I_2^1(t) = -D_2; \quad t_1 \leq t \leq t_2 \tag{5.4}$$

With $I_2^1(0) = Q_2, I_2^1(t_1) = Q_2 - D_2 t_1, I_2^1(t_2) = l$.

$$\frac{dI_3^1(t)}{dt} + \theta_2 I_3^1(t) = -(D_2 + \alpha_1 D_1); \quad t_2 \leq t \leq t_2 + s \tag{5.5}$$

With $I_3^1(t_2) = l, I_3^1(t_2 + s) = 0$.

The result of Eqs. (5.1–5.5) are

$$I_1^1(t) = Q_1 - D_1 t; \quad 0 \leq t \leq t_1 \tag{5.6}$$

$$I_1^1(t) = \frac{D_1}{\theta_1} (e^{\theta_1(t_2-t)} - 1); \quad t_1 \leq t \leq t_2 \tag{5.7}$$

$$I_2^1(t) = Q_2 - D_2 t; \quad 0 \leq t \leq t_1 \tag{5.8}$$

$$I_2^1(t) = -\frac{D_2}{\theta_2} + e^{\theta_2(t_1-t)} \left(Q_2 - D_2 t_1 + \frac{D_2}{\theta_2} \right); \quad t_1 \leq t \leq t_2 \quad (5.9)$$

$$I_3^1(t) = \frac{D_2 + \alpha_1 D_1}{\theta_2} (e^{\theta_2(t_2+s-t)} - 1); \quad t_2 \leq t \leq t_2 + s \quad (5.10)$$

Total cost of product 1 and product 2 during each cycle consists purchase, setup, and holding cost with their emissions cost and can be symbolized as

$$TC11(Q_1, Q_2) = A_1 + C_1 Q_1 + h \left(Q_1 t_1 + \frac{D_1}{\theta_1} (t_1 - t_2) + \frac{D_1}{\theta_1^2} (e^{\theta_1(t_2-t_1)} - 1) - \frac{D_1 t_1^2}{2} \right) \quad (5.11)$$

$$\begin{aligned} TC12(Q_1, Q_2) &= A_2 + C_2 Q_2 + (h) \left(Q_2 t_1 + \frac{D_2}{\theta_2} (t_1 - t_2) \right. \\ &\quad \left. + \frac{(Q_2 \theta_2 - D_2 t_1 \theta_2 + D_2)}{\theta_2^2} (1 - e^{\theta_2(t_1-t_2)}) - \frac{D_2 t_1^2}{2} \right) \end{aligned} \quad (5.12)$$

The lost sale costs of product 1 per length cycle

$$= \pi_1 (1 - \alpha_1) D_1 \frac{\ln \left(\frac{((-D_2 t_1 + Q_2) \theta_2 + D_2) e^{\theta_2(t_1-t_2)} + \alpha_1 D_1}{D_2 + \alpha_1 D_1} \right)}{\theta_2} \quad (5.13)$$

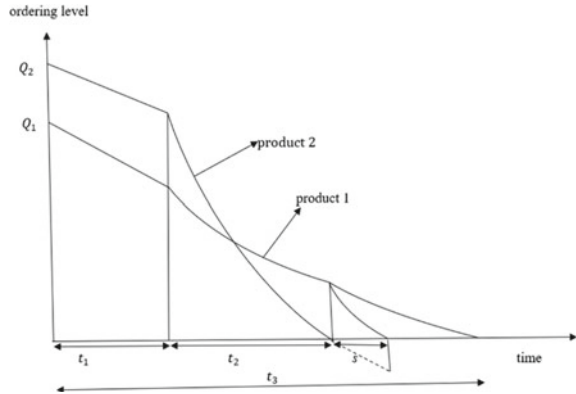
Substitution cost due to product 1 partially replaced by product 2 per length cycle

$$= CS_{12} \alpha_1 D_1 \frac{\ln \left(\frac{((-D_2 t_1 + Q_2) \theta_2 + D_2) e^{\theta_2(t_1-t_2)} + \alpha_1 D_1}{D_2 + \alpha_1 D_1} \right)}{\theta_2} \quad (5.14)$$

Thus, the average total costs $TC(Q_1, Q_2)$ for case 1 per unit time

$$\begin{aligned} TC1(Q_1, Q_2) &= \frac{1}{t_1 + t_2 + s} \left[A_1 + A_2 + C_1 Q_1 + C_2 Q_2 + h \left\{ Q_1 t_1 + \frac{D_1}{\theta_1} (t_1 - t_2) \right. \right. \\ &\quad \left. \left. + \frac{D_1}{\theta_1^2} (e^{\theta_1(t_2-t_1)} - 1) - \frac{D_1 t_1^2}{2} + Q_2 t_1 + \frac{D_2}{\theta_2} (t_1 - t_2) \right. \right. \\ &\quad \left. \left. + \frac{(Q_2 \theta_2 - D_2 t_1 \theta_2 + D_2)}{\theta_2^2} (1 - e^{\theta_2(t_1-t_2)}) - \frac{D_2 t_1^2}{2} \right. \right. \\ &\quad \left. \left. + \frac{(D_2 + \alpha_1 D_1)}{\theta_2^2} (e^{\theta_2 s} - 1 - \theta_2 s) \right\} \right. \\ &\quad \left. + \pi_1 (1 - \alpha_1) D_1 \frac{\ln \left(\frac{((-D_2 t_1 + Q_2) \theta_2 + D_2) e^{\theta_2(t_1-t_2)} + \alpha_1 D_1}{D_2 + \alpha_1 D_1} \right)}{\theta_2} \right. \\ &\quad \left. + CS_{12} \alpha_1 D_1 \frac{\ln \left(\frac{((-D_2 t_1 + Q_2) \theta_2 + D_2) e^{\theta_2(t_1-t_2)} + \alpha_1 D_1}{D_2 + \alpha_1 D_1} \right)}{\theta_2} \right] \end{aligned} \quad (5.15)$$

Fig. 5.2 Stock size for case 2 at time t



Case 2: Product 2 fully consumes before product 1, that means, if product 2 is out of stock at time t_2 , as seen in Fig. 5.2, then requirement of product 2 is partially met by product 1 with substitution rate α_2 and rest demand for product 2 is lost with rate $(1 - \alpha_2)$. The mathematical formation of case 2 is identical to case 1. The models are depicted in Fig. 5.2.

Therefore, the average total costs $TC2(Q_1, Q_2)$ of case 2 is

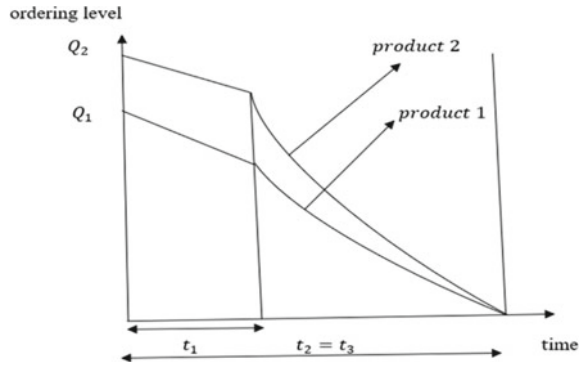
$$\begin{aligned}
 TC2(Q_1, Q_2) = & \frac{1}{t_1 + t_2 + s} \left[A_1 + A_2 + C_1 Q_1 + C_2 Q_2 + h \left\{ Q_1 t_1 + \frac{D_1}{\theta_1} (t_1 - t_2) \right. \right. \\
 & + \frac{D_2}{\theta_2^2} (e^{\theta_2(t_2-t_1)} - 1) - \frac{D_1 t_1^2}{2} + Q_2 t_1 + \frac{D_2}{\theta_2} (t_1 - t_2) \\
 & + \frac{(Q_1 \theta_1 - D_1 t_1 \theta_1 + D_1)}{\theta_1^2} (1 - e^{\theta_1(t_1-t_2)}) - \frac{D_2 t_1^2}{2} \\
 & \left. \left. + \frac{(D_1 + \alpha_2 D_2)}{\theta_1^2} (e^{\theta_1 s} - 1 - \theta_1 s) \right\} + \pi_2 (1 \right. \\
 & - \alpha_2) D_2 \frac{\ln \left(\frac{((-D_1 t_1 + Q_1) \theta_1 + D_1) e^{\theta_1(t_1-t_2)} + \alpha_2 D_2}{D_1 + \alpha_2 D_2} \right)}{\theta_1} \\
 & \left. \left. + CS_{21} \alpha_2 D_2 \frac{\ln \left(\frac{((-D_1 t_1 + Q_1) \theta_1 + D_1) e^{\theta_1(t_1-t_2)} + \alpha_2 D_2}{D_1 + \alpha_2 D_2} \right)}{\theta_1} \right] \quad (5.16)
 \end{aligned}$$

Case 3 (without substitution): The stock level of each product is totally consumed at t_2 , i.e., no substitution happens, as shown in Fig. 5.3. The stock level of each product is depleted at $t_2 = t_3$, i.e., $Q_1/D_1 = Q_2/D_2$ as displayed in Fig. 5.3.

Therefore, the average total costs $TC_{NS}(Q_1, Q_2)$ for case 3 is

$$TC_{NS}(Q_1, Q_2) = \frac{1}{t_1 + t_2} [A_1 + A_2 + (C_1)Q_1 + (C_2)Q_2 + (h)((Q_1 + Q_2)t_1$$

Fig. 5.3 Stock size for case 3 at time t



$$\begin{aligned}
 &+ \left(\frac{D_1}{\theta_1} + \frac{D_2}{\theta_2} \right) (t_1 - t_2) - (D_1 + D_2) \frac{t_1^2}{2} + \frac{D_1}{\theta_1^2} (e^{\theta_1(t_2-t_1)} - 1) \\
 &+ \frac{D_2}{\theta_2^2} (e^{\theta_2(t_2-t_1)} - 1) \Big] \tag{5.17}
 \end{aligned}$$

Solution Procedure

To obtain most desirable ordering quantity from overall cost function in the subsequent phase due to highly nonlinear function, we have graphically shown that the nature of overall cost expression is strictly convex nature. We develop an algorithm to achieve the most desirable ordering quantity.

Algorithm to achieve the optimal overall cost and ordering quantity.

Step 1: Solve the constraint-based problem

$$\min_{Q_1, Q_2} TC1(Q_1, Q_2) \text{ Subject to, } \frac{Q_1}{D_1} \leq \frac{Q_2}{D_2}$$

Step 2: Solve the constraint-based problem

$$\min_{Q_1, Q_2} TC2(Q_1, Q_2) \text{ Subject to, } \frac{Q_1}{D_1} \geq \frac{Q_2}{D_2}$$

Step 3: To get optimum value, choose minimum between above two steps, i.e.,

$$\min[TC1(Q_1, Q_2), TC2(Q_1, Q_2)].$$

Numerical Example and Sensitivity Analysis

In this section, we first establish numerical experiment that demonstrates the implementation of proposed inventory model. The value of parameter of the numerical example is $C_1 = 3, C_2 = 2.5, A_1 = 300, A_2 = 250, h = 2, t_1 = 0.20, \theta_1 = 0.50, \theta_2 = 0.55, D_1 = 200, D_2 = 40, \alpha_1 = 0.2, \pi_1 = 0.4,$ and $CS_{12} = 2$. We solve the above optimization problem with the above-developed algorithm and software maple. The outcomes of step 1 are $TC1(Q_1, Q_2) = 1288.56, Q_1 = 92.41,$ and $Q_2 = 91.07,$ and the outcomes of step 2 are $TC2(Q_1, Q_2) = 1350.79, Q_1 = 171.21,$ and $Q_2 = 27.27$. On comparing both of outcomes in step 3 of the algorithm, step 1 outcomes dominance to the optimal values. Hence, the optimal total costs for with substitution is 1288.56 and optimal ordering quantities is $(Q_1^*, Q_2^*) = (92.41, 91.07)$. The optimal ordering quantities and overall cost for without substitution case is $(Q_1, Q_2) = (163.29, 32.65)$ and $TC_{NS}(Q_1, Q_2) = 1358.39$.

Comparing the outcomes of with substitution and without substitution, the beneficial difference in optimal total cost is 69.83 and the benefits of substitution over without substitution in percentage is 5.14%.

To investigate the essence of the total cost, we draw the overall cost function for product 1 and product 2 with different order quantities. Figure 5.4 shows the optimal total cost expression is strictly convex nature. Decision-making situations rarely remain fixed in the real world; thus, the parameters of the model change so, we perform sensitivity analysis numerically by varying the value of fixed parameter. Table 5.2 shows the numerical output of sensitivity analysis. The results of changes in parameters and percentage change are illustrate in Figs. 5.5, 5.6, 5.7 and 5.8.

Fig. 5.4 Optimal overall cost versus optimal total quantities $(Q1^*, Q2^*)$

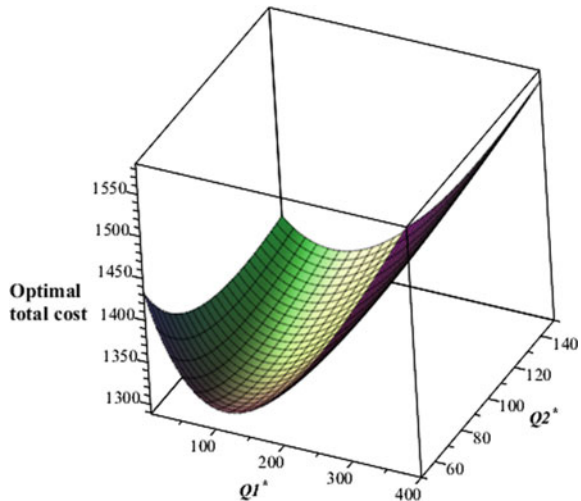


Table 5.2 Sensitivity analysis when key parameters change

Parameter	Data of parameter	With substitution			Without substitution			% Improvement in optimal total cost
		Q_1^*	Q_2^*	TC_d^{2*}	Q_1^*	Q_2^*	TC_d^{NS*}	
C_1	2.5	134.01	70.13	1243.71	173.91	34.78	1269.72	2.05
	3	92.41	91.07	1288.56	163.29	32.65	1358.39	5.14
	3.5	57.34	105.90	1316.47	153.50	30.70	1445.17	8.91
	4	27.36	116.14	1331.71	144.41	28.88	1530.13	12.97
C_2	2	84.84	101.56	1252.56	165.34	33.06	1340.81	6.58
	2.5	92.41	91.07	1288.56	163.29	32.65	1358.39	5.14
	3	100.26	81.24	1322.42	161.27	32.25	1375.89	3.89
	3.5	108.42	71.90	1354.07	159.28	31.85	1393.32	2.82
h	1	129.61	105.12	1222.37	201.83	40.36	1272.25	4.17
	2	92.41	91.07	1288.56	163.29	32.65	1358.39	5.14
	3	72.16	80.83	1341.08	139.24	27.84	1428.26	6.03
	4	59.43	73.09	1385.19	122.51	24.50	1487.33	6.85
A_1	250	92.41	80.04	1248.91	151.16	30.23	1303.32	3.92
	300	92.41	91.07	1288.56	163.29	32.65	1358.39	5.14
	350	92.41	101.43	1325.81	174.87	34.97	1410.89	6.10
	400	92.41	111.25	1361.12	185.97	37.19	1461.21	6.87
π_1	3	38.65	118.90	1189.72	163.29	32.65	1358.39	12.42
	4	92.41	91.07	1288.56	163.29	32.65	1358.39	5.14
	5	146.32	51.65	1346.32	163.29	32.65	1358.39	0.89
	6	166.01	33.20	1351.67	163.29	32.65	1358.39	0.49
CS_{12}	2	92.41	91.07	1288.56	163.29	32.65	1358.39	5.14
	3	105.87	82.47	1307.46	163.29	32.65	1358.39	3.75
	4	119.34	73.10	1323.65	163.29	32.65	1358.39	2.56
	5	132.82	62.88	1336.78	163.29	32.65	1358.39	1.59
α_1	0.2	92.41	91.07	1288.56	163.29	32.65	1358.39	5.14
	0.4	125.42	71.23	1339.41	163.29	32.65	1358.39	1.40
	0.6	165.86	33.17	1351.80	163.29	32.65	1358.39	0.49
	0.8	165.95	33.19	1351.97	163.29	32.65	1358.39	0.47
t_1	0.10	84.88	126.09	1407.07	201.05	40.21	1547.97	6.69
	0.20	92.41	91.07	1288.56	163.29	32.65	1358.39	4.54
	0.30	100.58	59.70	1174.95	135.36	27.07	1198.46	2.64
	0.40	109.35	31.22	1064.56	116.01	23.20	1066.48	1.08

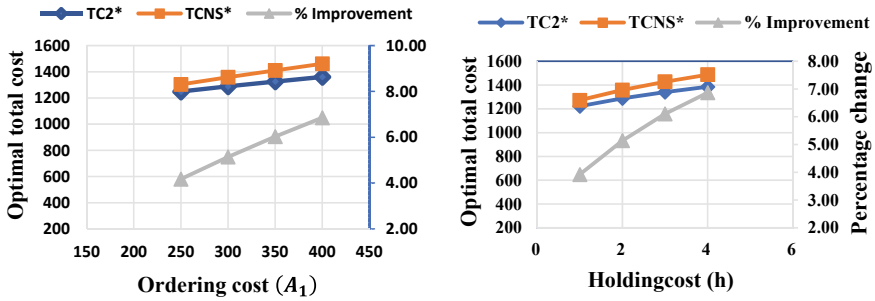


Fig. 5.5 Sensitivity analysis with variation in ordering cost (A_1) and holding cost (h)

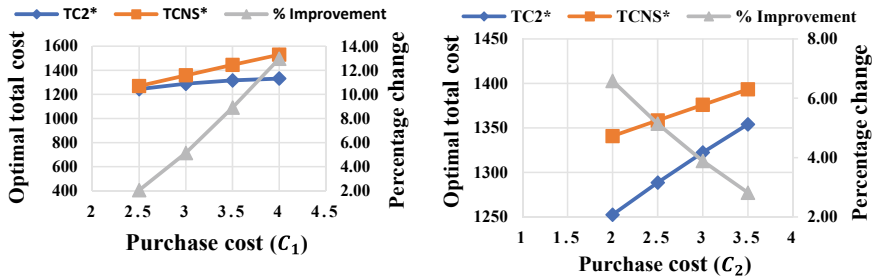


Fig. 5.6 Sensitivity analysis with variation in purchase cost (C_1, C_2)

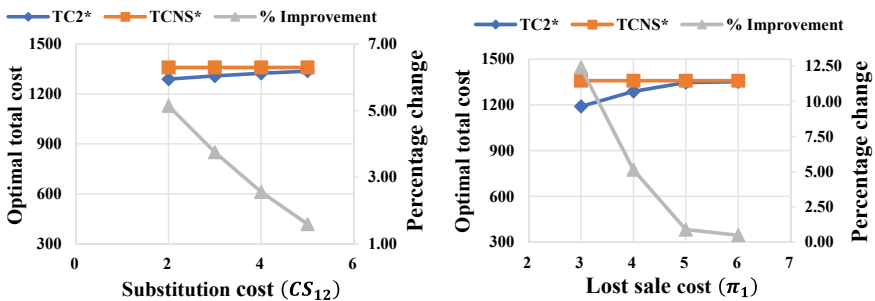


Fig. 5.7 Sensitivity analysis with variation in substitution cost (CS_{12}) and lost sale cost (π_1)

Conclusion

This study presents an inventory management model for two non-instantaneous decaying substitute products under joint ordering policy in each ordering cycle. At any time, if one of product is out of stock, a fraction of its demand may be met using the stock of the other product and other undesirable demand goes to loss. The suggested resulting model and solution method for determining the optimal

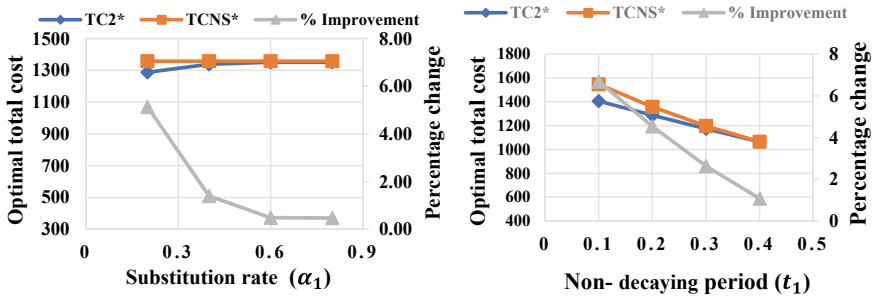


Fig. 5.8 Sensitivity analysis with variation in substitution rate (α_1) and non-decaying period (t_1)

ordering strategy is easy to understand and enforce. We first derive the overall cost expressions and illustrate graphically that they are convex. The numerical example demonstrates the application of the model, and the numerical output provides the percentage profitable improvement in overall cost, comparing with substitution and without substitution. This suggested that the impact of substitution with joint replenishment is a significant consideration that a retailer should consider into account, when picking inventory selections.

As the present model is restricted with two products which can be extended for multi-products. Another expansion of this study may be the consideration of trade credit systems and coordination between manufacturers and retailers. This study can also be extended for fuzzy stochastic environment, screening process, quality discounts, promotional effect, trade credit policies, and several other practical situations [22, 23].

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