Chapter 10
Optimization of an Inventory Model with Demand Dependent on Selling Price and Stock, Nonlinear Holding Cost Along with Trade Credit Policy

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Abstract The demand for a product is influenced by a number of factors, including the selling price and the displayed stock level, among others. Considering this scenario, an EOQ inventory model is developed where demand is a function of both selling price and the inventory level which is one of the main contributions of this research work. Holding cost is assumed to be nonlinearly dependent on stock. Besides that, supplier grants a full trade credit policy to the retailer. This policy is very advantageous for both the counterpart—the supplier as well as the retailer. The supplier can attract more customers by offering a delay period whereas the latter enjoys the benefit of getting goods without instant payment. The proposed mathematical model aims to find out the optimal selling price and optimal length of the replenishment cycle so as to maximize the total profit of the retailer per unit time. Several theorems are well-established in order to reach to the optimal solution. A numerical example is also presented to demonstrate the suggested inventory model, and a sensitivity analysis is executed to highlight the findings of the inventory model and put forward valuable managerial insights. This research work can be helpful to the business communities facing nonlinear demand patterns. Businesses that want to offer trade credit policies but are dealing with nonlinear holding costs may also find it helpful. Sensitivity analysis can be useful in determining the impact of various cost parameters on the total generated profit.

Keywords Inventory · Selling price · Stock-dependent · Trade credit

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Introduction

Majority of the inventory models presume demand rate to be constant throughout the inventory cycle. In real life, demand is observed to be dependent on many factors such as selling price and displayed stock level. Customers show a strong desire to buy more products when supermarkets showcase them in huge quantities in the showroom according to Levin et al. [15]; this frequently increases product demand. Hence, displayed stock plays a significant role in the analysis of inventory. The inventory model presented by Datta and Pal [7] assumes that demand is a function of inventory level. Wu and his co-researchers [19] presented an inventory model for non-instantaneous deteriorating products with partial backlogging as well as stock level dependent demand.

Pricing is another key aspect that influences the demand for a product. A vital question that arises in the inventory model is what should be an item selling price so that the seller gains maximum benefits while satisfying customer needs. Various researchers and academicians have formulated different inventory models with price-dependent demand. Alfares and Ghaithan [1] formulated an inventory model with price-dependent demand as well as quantity discounts where holding cost was allowed to vary with time. Chang and his collaborators [4] portrayed an integrated model of inventory with the policy of trade credit tied with order size and price-dependent demand. In that same year, an integrated inventory system was framed with trade credit to be dependent on the size of the order [18].

One another assumption that is prevalent in economic order quantity (EOQ) inventory models is that the buyer should pay the seller upon receipt of goods. Small businesses that do not have enough money to pay for the ordered goods instantly suffer a lot due to this policy. Trade credit policy provides a chance to the buyer to buy goods without paying instantly. The buyer is granted a trade credit period in order to settle accounts. The trade credit policy is often utilized to stimulate demand. Haley and Higgins [10] were the first to frame an EOQ inventory model considering allowable delay in payments. Following that, Goyal [9] investigated an EOQ model of inventory with trade credit where the buyer is exempted from clearing the payment and earns interest throughout the credit period. Various trade credit policies have been developed as a result of diversification of trade and changes in the business environment. Musa and Sani [17] constructed a model of inventory for decaying items allowing delay in payments. After that, Khanra and his collaborators [13] formulated an inventory model considering shortages and permissible delay in payments for a single item where demand of customers is a quadratic function of time. An inventory model was developed with demand dependent on stock, partial backlogging, and nonlinear holding cost by Cárdenas-Barrón et al. [3]. The retailer was allowed a full trade credit period to clear the debt by the supplier. Ghosh et al. [8] constructed an inventory model with multiple advanced and delayed payment policies along with complete backordering for perishable items. Similary, other authors have made valuable contributions to the existing literature [5, 6, 11, 12, 14].

Some of the researchers have formulated inventory models where demand is dependent on both selling price and displayed stock level. By taking into account the exhibited level of stock and selling price-dependent demand, Hsieh and Dye [11] formulated an inventory model for deteriorating items. To find out the optimal solution, particle swarm optimization is applied. An EOQ inventory model with demand dependent on stock and selling price for deteriorating items was analyzed by Mishra et al. [16] where an estimation of the optimal order quantity, selling price, and preservation technology investment was made from the retailer's perspective.

The objectives of this study are as follows:

- Majority of the existing works are based on the assumption that demand always remains constant which is far from reality. Many factors influence demand, notably selling price, displayed goods, and so on. The aim of this study is to build an inventory model that considers demand as a function of selling price and displayed stock.
- Recent review of the literature suggests that holding cost is considered to be linearly dependent on stock which need not to be always. It totally depends on the nature of the item. In this paper, we have considered holding cost to be nonlinearly dependent on stock. Besides that, the supplier-retailer relationship is analyzed where the supplier offers a full trade credit period to the retailer.
- The main goal of the proposed study is to find out the optimal selling price at which goods are to be sold and the optimal replenishment length of the cycle so as to maximize the retailer's total generated profit considering demand to be nonlinear, nonlinear holding cost along with the trade credit policy. A numerical example is also presented to demonstrate the findings, and a sensitivity analysis is done to highlight the results.

The paper is further organized as follows: Section "Assumptions and Notations" proposes the assumptions and notations required to establish the inventory model mathematically. Section "Mathematical Model" constructs the mathematical model considering demand to be nonlinear, nonlinear holding cost, and trade credit. Section "Theoretical Results and Optimization Procedure" discusses some important theoretical results and optimization procedures necessary to reach to the optimal solution. Section "Numerical Example" illustrates a numerical example. In Section "Sensitivity Analysis", sensitivity analysis is done by changing one parameter at a time and keeping rest of the parameter's constant. Section "Managerial Insights" highlights some important results. Finally, Section "Conclusion" concludes the findings as well as suggests some important future research directions.

Assumptions and Notations

Assumptions

1. The inventory system's planning horizon is infinite.
2. Demand is assumed to be a function of stock level and selling price given by

$$D(t) = \alpha(a - bp)[q(t)]^{\beta} \quad \text{when } q(t) \geq 0$$

3. Holding cost is nonlinearly dependent on stock. It is represented as follows:

$$H(t) = h[q(t)]^{\gamma} \quad \text{when } \gamma > 0$$

   If $\gamma = 1$, then holding cost is considered to be linearly dependent on stock.
4. Rate of replenishment is instantaneous with negligible lead-time.
5. Retailer is granted a full trade credit period by the supplier.

Notations

See Table 10.1.

Table 10.1 Notations used to establish the inventory model

Parameter	Unit	Description	Parameter	Unit	Description
c	\$/unit	Purchasing price per unit	M	unit time	Trade credit period offered by the supplier to the retailer
o	\$/order	Ordering cost per order	I_p	%/unit time	Interest paid by the retailer
h	\$/unit/unit time	Holding cost per unit item per unit time	γ		Holding cost elasticity; $\gamma > 0$
β		Demand elasticity rate; $0 \leq \beta < 1$	α		Demand rate scale parameter
			Decision variables		
I_e	%/unit time	Interest earned by the retailer	p		Selling price per unit item
TP(p, T)	\$/unit time	Total profit per unit time	T		Replenishment cycle length

Mathematical Model

An inventory model is formulated where the cost of holding goods varies nonlinearly with stock. Initially, Q units of product exist in the inventory of retailer. Demand is dependent upon the level of stock carried in the inventory and selling price. The level of inventory decreases due to demand during the interval $[0, T]$. At time $t = T$, it drops down to zero. After that, again a replenishment order of Q units is placed. The beginning of the next inventory cycle is marked by the arrival of products. The supplier also offers a full trade credit period M to the retailer additionally. The inventory situation is best described by the following differential equations:

$$\frac{dq(t)}{dt} = -\alpha(a - bp)[q(t)]^\beta, \qquad 0 \leq t \leq T \tag{10.1}$$

With the following boundary conditions: $q(0) = Q$ and $q(T) = 0$. Solving the differential equation (10.1) with the above-mentioned boundary conditions, the following results are obtained:

$$q(t) = \left[Q^{1-\beta} - \alpha(a - bp)(1 - \beta)t\right]^{\frac{1}{1-\beta}}, \qquad 0 \leq t \leq T \tag{10.2}$$

$$Q = [\alpha(a - bp)(1 - \beta)T]^{\frac{1}{1-\beta}} \tag{10.3}$$

Various costs related with the proposed inventory model are as follows:

(1)

$$\text{Ordering cost} = o \tag{10.4}$$

(2)

$$\text{Purchasing cost} = c[\alpha(a - bp)(1 - \beta)T]^{\frac{1}{1-\beta}} \tag{10.5}$$

(3

$$\text{Sales revenue collected} = p[\alpha(a - bp)(1 - \beta)T]^{\frac{1}{1-\beta}} \tag{10.6}$$

(4)

$$\text{Inventory holding cost} = h \int_0^T [q(t)]^\gamma \, dt$$

$$= h \int_0^T \left[Q^{1-\beta} - \alpha(a - bp)(1 - \beta)t\right]^{\frac{\gamma}{1-\beta}} dt$$

$$= h\frac{[\alpha(a-bp)(1-\beta)T]^{\frac{\gamma+1-\beta}{1-\beta}}}{\alpha(a-bp)(\gamma+1-\beta)} \tag{10.7}$$

Since the supplier offers a full trade credit period to the retailer, following two sub-cases arise:

Case-1: $M \leq T$
Case-2: $M \geq T$.

Case-1: Trade Credit Period Is Less Than or Equal to the Cycle Length ($M \leq T$)

The trade credit period granted by the supplier to the retailer is less than or equal to the cycle length in this scenario. After the end of credit period, the retailer has to bear the interest charges and needs to pay interest during the interval $[M, T]$. Consequently, the amount of interest paid is computed as follows:

$$\text{IP} = \frac{cI_p\left[[\alpha(a-bp)(1-\beta)(T-M)]^{\frac{2-\beta}{1-\beta}}\right]}{\alpha(a-bp)(2-\beta)} \tag{10.8}$$

The retailer earns interest during the credit period up to $t = M$.

$$\text{IE} = pI_e\left[M[\alpha(a-bp)(1-\beta)T]^{\frac{1}{1-\beta}} + \frac{1}{\alpha(a-bp)(2-\beta)}\right.$$
$$\left.\{\alpha(a-bp)(1-\beta)(T-M)\}^{\frac{2-\beta}{1-\beta}} - \{\alpha(a-bp)(1-\beta)T\}^{\frac{2-\beta}{1-\beta}}\}\right] \tag{10.9}$$

The total profit per unit time is calculated as follows:

$$\text{TP}_1(p, T) = \frac{\text{SR} + \text{IE} - o - \text{PC} - \text{HC} - \text{IP}}{T}$$

Therefore,

$$\text{TP}_1(p, T) = \left[\frac{1}{T}\right]\left[p[\alpha(a-bp)(1-\beta)T]^{\frac{1}{1-\beta}}\right.$$
$$+ pI_e\left[M[\alpha(a-bp)(1-\beta)T]^{\frac{1}{1-\beta}}\right.$$
$$+ \frac{1}{\alpha(a-bp)(2-\beta)}\{\alpha(a-bp)(1-\beta)(T-M)\}^{\frac{2-\beta}{1-\beta}}$$
$$\left.\left.- \{\alpha(a-bp)(1-\beta)T\}^{\frac{2-\beta}{1-\beta}}\}\right]\right] - o$$

$$
\begin{aligned}
&- c[\alpha(a - bp)(1 - \beta)T]^{\frac{1}{1-\beta}} - h\frac{[\alpha(a - bp)(1 - \beta)T]^{\frac{\gamma+1-\beta}{1-\beta}}}{\alpha(a - bp)(\gamma + 1 - \beta)} \\
&- \frac{cI_p\left[[\alpha(a - bp)(1 - \beta)(T - M)]^{\frac{2-\beta}{1-\beta}}\right]}{\alpha(a - bp)(2 - \beta)}
\end{aligned}
\Bigg]
\tag{10.10}
$$

Problem 1

Maximize $\mathrm{TP}_1(p, T) = \frac{S_1}{T}$

where $S_1 = \mathrm{SR} + \mathrm{IE} - o - \mathrm{PC} - \mathrm{HC} - \mathrm{IP}$

subject to $M \leq T$.

Case-2: Trade Credit Period Is Greater Than or Equal to the Cycle Length $(M \geq T)$

In this case, the trade credit period granted to the retailer by the supplier is greater than or equal to the cycle length. In this scenario, the retailer does not need to pay interest. Therefore,

$$
\mathrm{IP} = 0 \tag{10.11}
$$

The retailer earns interest during the credit period up to $t = M$.

$$
\mathrm{IE} = pI_e\left[M[\alpha(a - bp)(1 - \beta)T]^{\frac{1}{1-\beta}} - \frac{[\alpha(a - bp)(1 - \beta)T]^{\frac{2-\beta}{1-\beta}}}{\alpha(a - bp)(2 - \beta)} \right] \tag{10.12}
$$

The total profit per unit time is computed as follows:

$$
\mathrm{TP}_2(p, T) = \frac{\mathrm{SR} + \mathrm{IE} - o - \mathrm{PC} - \mathrm{HC} - \mathrm{IP}}{T}
$$

Therefore,

$$
\begin{aligned}
\mathrm{TP}_2(p, T) = &\left[\frac{1}{T}\right]\Bigg[p[\alpha(a - bp)(1 - \beta)T]^{\frac{1}{1-\beta}} + pI_e\left[M[\alpha(a - bp)(1 - \beta)T]^{\frac{1}{1-\beta}} \right. \\
&\left. - \frac{[\alpha(a - bp)(1 - \beta)T]^{\frac{2-\beta}{1-\beta}}}{\alpha(a - bp)(2 - \beta)} \right] - o - c[\alpha(a - bp)(1 - \beta)T]^{\frac{1}{1-\beta}} \\
&- h\frac{[\alpha(a - bp)(1 - \beta)T]^{\frac{\gamma+1-\beta}{1-\beta}}}{\alpha(a - bp)(\gamma + 1 - \beta)} \Bigg]
\end{aligned}
\tag{10.13}
$$

Problem 2

Maximize $\text{TP}_1(p, T) = \frac{S_2}{T}$

where $S_2 = \text{SR} + \text{IE} - o - \text{PC} - \text{HC} - \text{IP}$

subject to $M \geq T$.

Theoretical Results and Optimization Procedure

The problem is solved using the following theorem of generalized concave functions. If $f(x)$ is non-negative, differentiable, and strictly concave, and $g(x)$ is positive, differentiable and convex, then the real-valued function

$$z(x) = \frac{f(x)}{g(x)} \tag{10.14}$$

is strictly pseudo-concave in nature. The detailed proof can be seen in Cambini and Martein [2].

$M \leq T$

Theorem 1 *For any given p,*

(a) $\text{TP}_1(p, T)$ *is a strictly pseudo-concave function in T, hence there exists a unique maximum solution T_1^*.*
(b) *If $M \leq T_1^*$, then $\text{TP}_1(p, T)$ subject to $M \leq T$ is maximized at T_1^*.*
(c) *If $M \geq T_1^*$, then $\text{TP}_1(p, T)$ subject to $M \leq T$ is maximized at M.*

Proof See Appendix 1.

To find T_1^*, for any given p, taking the first-order derivative $\text{TP}_1(p, T)$ with respect to T, setting the result to zero and rearranging terms we get

$$
\begin{aligned}
\frac{\partial \text{TP}_1(p, T)}{\partial T} = {} & \left[\frac{1}{T}\right]\Big[(p - c)\alpha(a - bp)[\alpha(a - bp)(1 - \beta)T]^{\frac{\beta}{1-\beta}} \\
& - h[\alpha(a - bp)(1 - \beta)T]^{\frac{\gamma}{1-\beta}} - cI_{\text{p}}[\alpha(a - bp)(1 - \beta)(T - M)]^{\frac{1}{1-\beta}} \\
& + pI_{\text{e}}\Big[\alpha(a - bp)M[\alpha(a - bp)(1 - \beta)T]^{\frac{\beta}{1-\beta}} \\
& + [\alpha(a - bp)(1 - \beta)(T - M)]^{\frac{1}{1-\beta}} - [\alpha(a - bp)(1 - \beta)T]^{\frac{1}{1-\beta}}\Big]\Big] \\
& - \left[\frac{1}{T^2}\right]\Big[p[\alpha(a - bp)(1 - \beta)T]^{\frac{1}{1-\beta}} + pI_{\text{e}}
\end{aligned}
$$

$$\left[M[\alpha(a-bp)(1-\beta)T]^{\frac{1}{1-\beta}} + \frac{1}{\alpha(a-bp)(2-\beta)} \right.$$

$$\left\{ \{\alpha(a-bp)(1-\beta)(T-M)\}^{\frac{2-\beta}{1-\beta}} - \{\alpha(a-bp)(1-\beta)T\}^{\frac{2-\beta}{1-\beta}} \right\} \right]$$

$$- o - c[\alpha(a-bp)(1-\beta)T]^{\frac{1}{1-\beta}} - h\frac{[\alpha(a-bp)(1-\beta)T]^{\frac{\gamma+1-\beta}{1-\beta}}}{\alpha(a-bp)(\gamma+1-\beta)}$$

$$\left. - \frac{cI_{\mathrm{p}}\left[[\alpha(a-bp)(1-\beta)(T-M)]^{\frac{2-\beta}{1-\beta}} \right]}{\alpha(a-bp)(2-\beta)} \right] = 0 \qquad (10.15)$$

Similarly, for any given T, taking the first-order derivative of $\mathrm{TP}_1(p, T)$ with respect to p, setting the result equal to zero and rearranging terms we get

$$\frac{\partial \mathrm{TP}_1(p, T)}{\partial p} = \left[\frac{1}{T} \right] \left[[\alpha(a-bp)(1-\beta)T]^{\frac{1}{1-\beta}} \right.$$

$$- b\alpha(p-c)T[\alpha(a-bp)(1-\beta)T]^{\frac{\beta}{1-\beta}}$$

$$- h\left[-\frac{bT[\alpha(a-bp)(1-\beta)T]^{\frac{\gamma}{1-\beta}}}{(a-bp)} \right.$$

$$\left. + \frac{b[\alpha(a-bp)(1-\beta)T]^{\frac{\gamma+1-\beta}{1-\beta}}}{\alpha(\gamma+1-\beta)(a-bp)^2} \right]$$

$$- cI_{\mathrm{p}}\left[-\frac{b(T-M)[\alpha(a-bp)(1-\beta)(T-M)]^{\frac{1}{1-\beta}}}{(a-bp)} \right.$$

$$\left. + \frac{b[\alpha(a-bp)(1-\beta)(T-M)]^{\frac{2-\beta}{1-\beta}}}{\alpha(2-\beta)(a-bp)^2} \right]$$

$$+ I_{\mathrm{e}}\left[M[\alpha(a-bp)(1-\beta)T]^{\frac{1}{1-\beta}} + \frac{1}{\alpha(a-bp)(2-\beta)} \right.$$

$$\left. \{\alpha(a-bp)(1-\beta)(T-M)\}^{\frac{2-\beta}{1-\beta}} - \{\alpha(a-bp)(1-\beta)T\}^{\frac{2-\beta}{1-\beta}} \} \right]$$

$$+ pI_{\mathrm{e}}\left[-b\alpha MT[\alpha(a-bp)(1-\beta)T]^{\frac{\beta}{1-\beta}} \right.$$

$$- \frac{b(T-M)[\alpha(a-bp)(1-\beta)(T-M)]^{\frac{1}{1-\beta}}}{(a-bp)}$$

$$+ \frac{b[\alpha(a-bp)(1-\beta)(T-M)]^{\frac{2-\beta}{1-\beta}}}{\alpha(2-\beta)(a-bp)^2}$$

$$+ \frac{bT[\alpha(a-bp)(1-\beta)T]^{\frac{1}{1-\beta}}}{(a-bp)}$$

$$-\frac{b[\alpha(a-bp)(1-\beta)T]^{\frac{2-\beta}{1-\beta}}}{\alpha(2-\beta)(a-bp)^2}\Bigg]\Bigg] = 0 \qquad (10.16)$$

Theorem 2 *For any given* $T > 0$, *if* $Z_1 = \frac{\partial^2 f}{\partial p^2} < 0$ *where* $f = \Big[p[\alpha(a-bp)(1-\beta)T]^{\frac{1}{1-\beta}} + pI_{\mathrm{e}}\Big[M[\alpha(a-bp)(1-\beta)T]^{\frac{1}{1-\beta}} + \frac{1}{\alpha(a-bp)(2-\beta)}\Big\{\{(a-bp)(1-\beta)(T-M)\}^{\frac{2-\beta}{1-\beta}} - [\alpha(a-bp)(1-\beta)T]^{\frac{2-\beta}{1-\beta}}\Big\}\Big] - o - c[\alpha(a-bp)(1-\beta)T]^{\frac{1}{1-\beta}} - h\frac{[\alpha(a-bp)(1-\beta)T]^{\frac{\gamma+1-\beta}{1-\beta}}}{\alpha(a-bp)(\gamma+1-\beta)} - \frac{cI_{\mathrm{p}}\Big[[\alpha(a-bp)(1-\beta)(T-M)]^{\frac{2-\beta}{1-\beta}}\Big]}{\alpha(a-bp)(2-\beta)}\Big]$,

then $\mathrm{TP}_1(p,T)$ *is a strictly pseudo-concave function in* p, *hence there exists a unique maximum solution* p^*.

$M \geq T$

Theorem 3 *For any given* p,

(a) $\mathrm{TP}_2(p, T)$ *is a strictly pseudo-concave function in* T, *hence there exists a unique maximum solution* T_2^*.
(b) *If* $M \geq T_2^*$, *then* $\mathrm{TP}_2(p, T)$ *subject to* $M \geq T$ *is maximized at* T_2^*.
(c) *If* $M \leq T_2^*$, *then* $\mathrm{TP}_2(p, T)$ *subject to* $M \geq T$ *is maximized at* M.

Proof See Appendix 2.

To find T_2^*, for any given p, taking the first-order derivative $\mathrm{TP}_2(p, T)$ with respect to T, setting the result to zero and rearranging terms we get

$$\frac{\partial \mathrm{TP}_2(p, T)}{\partial T} = \Big[\frac{1}{T}\Big]\Big[(p-c)\alpha(a-bp)[\alpha(a-bp)(1-\beta)T]^{\frac{\beta}{1-\beta}}$$
$$- h[\alpha(a-bp)(1-\beta)T]^{\frac{\gamma}{1-\beta}} + pI_{\mathrm{e}}[\alpha(a-bp)$$
$$M[\alpha(a-bp)(1-\beta)T]^{\frac{\beta}{1-\beta}} - [\alpha(a-bp)(1-\beta)T]^{\frac{1}{1-\beta}}]\Big]$$
$$- \Big[\frac{1}{T^2}\Big]\Big[p[\alpha(a-bp)(1-\beta)T]^{\frac{1}{1-\beta}}$$
$$+ pI_{\mathrm{e}}\Big[M[\alpha(a-bp)(1-\beta)T]^{\frac{1}{1-\beta}}$$
$$- \frac{[\alpha(a-bp)(1-\beta)T]^{\frac{2-\beta}{1-\beta}}}{\alpha(a-bp)(2-\beta)}\Big] - o - c[\alpha(a-bp)(1-\beta)T]^{\frac{1}{1-\beta}}$$
$$- h\frac{[\alpha(a-bp)(1-\beta)T]^{\frac{\gamma+1-\beta}{1-\beta}}}{\alpha(a-bp)(\gamma+1-\beta)}\Big] = 0 \qquad (10.17)$$

Similarly, for any given T, taking the first-order derivative of $\text{TP}_2(p, T)$ with respect to p, setting the result equal to zero and rearranging terms we get

$$
\begin{aligned}
\frac{\partial \text{TP}_2(p, T)}{\partial p} = \left[\frac{1}{T}\right] & \Big[[\alpha(a - bp)(1 - \beta)T]^{\frac{1}{1-\beta}} \\
& - b\alpha(p - c)T[\alpha(a - bp)(1 - \beta)T]^{\frac{\beta}{1-\beta}} \\
& - h\left[-\frac{bT[\alpha(a - bp)(1 - \beta)T]^{\frac{\gamma}{1-\beta}}}{(a - bp)} \right. \\
& \left. + \frac{b[\alpha(a - bp)(1 - \beta)T]^{\frac{\gamma+1-\beta}{1-\beta}}}{\alpha(\gamma + 1 - \beta)(a - bp)^2} \right] \\
& + I_e\Big[M[\alpha(a - bp)(1 - \beta)T]^{\frac{1}{1-\beta}} \\
& - \frac{[\alpha(a - bp)(1 - \beta)T]^{\frac{2-\beta}{1-\beta}}}{\alpha(a - bp)(2 - \beta)} \Big] \\
& + pI_e\Big[-b\alpha MT[\alpha(a - bp)(1 - \beta)T]^{\frac{\beta}{1-\beta}} \\
& + \frac{bT[\alpha(a - bp)(1 - \beta)T]^{\frac{1}{1-\beta}}}{(a - bp)} \\
& - \frac{b[\alpha(a - bp)(1 - \beta)T]^{\frac{2-\beta}{1-\beta}}}{\alpha(2 - \beta)(a - bp)^2} \Big] \Big] = 0
\end{aligned} \qquad (10.18)
$$

Theorem 4 *For any given* $T > 0$, *if* $Z_1 = \frac{\partial^2 f}{\partial p^2} < 0$ *where* $f = \left[p[\alpha(a - bp)(1 - \beta)T]^{\frac{1}{1-\beta}} + pI_e\Big[M[\alpha(a - bp)(1 - \beta)T]^{\frac{1}{1-\beta}} - \frac{[\alpha(a-bp)(1-\beta)T]^{\frac{2-\beta}{1-\beta}}}{\alpha(a-bp)(2-\beta)} \right] - o - c[\alpha(a - bp)(1 - \beta)T]^{\frac{1}{1-\beta}} - h\frac{[\alpha(a-bp)(1-\beta)T]^{\frac{\gamma+1-\beta}{1-\beta}}}{\alpha(a-bp)(\gamma+1-\beta)} \Big]$, then $\text{TP}_2(p, T)$ is a strictly pseudo-concave function in p, hence there exists a unique maximum solution p^*.*

For any given p, let us define the discriminant term

$$
\begin{aligned}
\Delta = \left[\frac{1}{M}\right] & \Big[(p - c)\alpha(a - bp)[\alpha(a - bp)(1 - \beta)M]^{\frac{\beta}{1-\beta}} \\
& - h[\alpha(a - bp)(1 - \beta)M]^{\frac{\gamma}{1-\beta}} \\
& + pI_e\Big[\alpha(a - bp)M[\alpha(a - bp)(1 - \beta)M]^{\frac{\beta}{1-\beta}} \\
& - [\alpha(a - bp)(1 - \beta)M]^{\frac{1}{1-\beta}} \Big] \Big] - \left[\frac{1}{M^2}\right] \\
& \Big[p[\alpha(a - bp)(1 - \beta)M]^{\frac{1}{1-\beta}} + pI_e
\end{aligned}
$$

$$\left[M[\alpha(a-bp)(1-\beta)M]^{\frac{1}{1-\beta}} - \frac{[\alpha(a-bp)(1-\beta)M]^{\frac{2-\beta}{1-\beta}}}{\alpha(a-bp)(2-\beta)} \right]$$

$$-o - c[\alpha(a-bp)(1-\beta)M]^{\frac{1}{1-\beta}}$$

$$-h\frac{[\alpha(a-bp)(1-\beta)M]^{\frac{\gamma+1-\beta}{1-\beta}}}{\alpha(a-bp)(\gamma+1-\beta)} \right] \tag{10.19}$$

Theorem 5 *For any given p,*

(1) *If $\Delta > 0$, then the total profit is maximized at T_1^*.*
(2) *If $\Delta = 0$, then the total profit is maximized at M.*
(3) *If $\Delta < 0$, then the total profit is maximized at T_2^*.*

Proof See Appendix 3.

Numerical Example

The proposed inventory model is illustrated with a numerical example. The aim is to find out the optimal selling price p^* and optimal replenishment length of the cycle T^* so as to maximize the overall profit earned by the retailer per unit time. The input values of various parameters are as follows:

$a = 90$; $b = 0.5$; $I_p = 12\%$/year; $I_e = 7\%$/year; $M = 180/365$ year; $\alpha = 0.4$; $o = \$200$/order; $h = \$10$/unit/year; $c = \$50$/unit; $\gamma = 1.1$; $\beta = 0.4$.

Since $\Delta > 0$, it falls into the category 4.1. Hence, the optimal solution is

$p^* = 113.696$; $T^* = 3.07143$; $Q^* = 205.796$; $TP^*(p, T) = 2708.96$. The profit function is concave in nature as shown in Fig. 10.1.

Sensitivity Analysis

The illustrated numerical example is used to study the impact of under or overestimation of input parameters on the optimal values of selling price (p^*), replenishment cycle length (T^*), order quantity (Q^*), and the total profit per unit time $(TP^*(p, T))$ of the inventory system. The sensitivity analysis is carried out by changing the input parameters from -20 to 20%. It is done by changing the input parameters one at a time and keeping the other parameters constant. The results are presented in Table 10.2. From Table 10.2, following results are obtained:

(1) With the increase in purchasing price c, the optimal selling price (p) increases. It is also observed that the optimal replenishment cycle length (T), optimal order quantity (Q), and the total profit per unit time $(TP(p, T))$ decreases. It
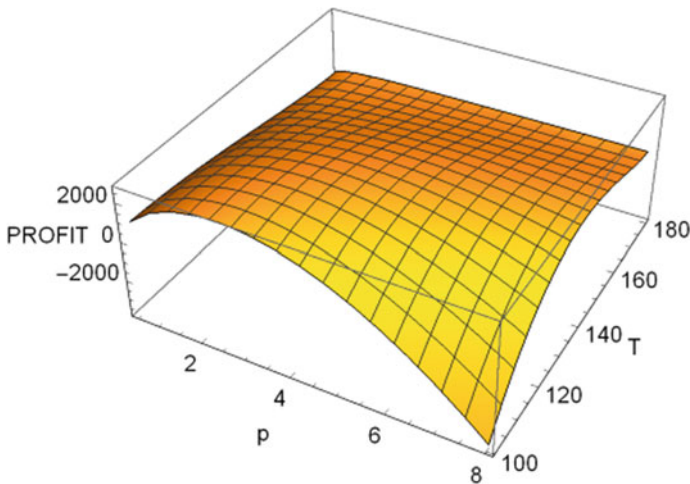
Fig. 10.1 Change in the profit function with respect to p and T

is obvious that if the purchasing price of goods increases, the selling price will also increase.

(2) As holding cost h increases, optimal replenishment cycle length (T), optimal order quantity (Q), and the total profit per unit time (TP(p, T)) decreases. Selling price (p) remains constant up to some time and then increases. With the increase in holding cost, inventory carrying cost increases, and hence profit decreases.

(3) With the increase in the value of γ, the optimal replenishment cycle length (T), optimal order quantity (Q), and the total profit per unit time (TP(p, T)) decreases whereas selling price (p) increases. With the increase in the value of γ, holding cost increases, hence retailer's order quantity decreases, and total profit decreases.

(4) As a increases, it is observed that optimal selling price (p), optimal replenishment cycle length (T), optimal order quantity (Q), and the total profit per unit time (TP(p, T)) increases. With the rise in the value of b, optimal selling price (p), optimal replenishment cycle length (T), optimal order quantity (Q), and the total profit per unit time (TP(p, T)) decreases.

(5) As the value of β increases, optimal replenishment cycle length (T), optimal order quantity (Q), and the total profit per unit time (TP(p, T)) increases. As α increases, order quantity (Q) and the total profit per unit time (TP(p, T)) increases. It is observed that as ordering cost increases, total profit per unit time decreases.

(6) As I_p increases, total profit per unit time decreases. It is observed that as the value of I_e increases, total profit per unit time increases. As M increases, the retailer has the chance to sell more goods and collect sales revenue. The retailer

Table 10.2 Sensitivity analysis with respect to input parameters

Parameter	% Change in parameter	Change in optimal values			
		p^*	T^*	TP*	Q^*
c	− 20	109.464	3.39286	3507.8	269.317
	− 10	110.875	3.23214	3088.72	240.166
	10	116.518	2.91071	2363.57	175.005
	20	117.929	2.75	2050.39	153.346
h	− 20	113.696	3.55357	2990.11	262.408
	− 10	113.696	3.23214	2840.04	224.054
	10	113.696	2.91071	2591.97	188.163
	20	115.107	2.75	2486.34	165.14
γ	− 20	113.696	7.08929	4410.35	829.608
	− 10	113.696	4.51786	3466.79	391.522
	10	115.107	2.10714	2163.59	105.955
	20	116.518	1.625	1778.25	66.2433
a	− 20	101	2.75	917.614	83.1708
	− 10	105.232	2.91071	1669.58	145.258
	10	123.571	3.39286	4099.84	294.543
	20	132.036	3.71429	5902.13	418.713
b	− 20	136.268	3.875	4873.87	339.656
	− 10	123.571	3.39286	3608.74	258.273
	10	106.643	2.75	2049.78	155.914
	20	101	2.58929	1556.68	126.736
β	− 20	115.107	2.58929	1702.19	99.6341
	− 10	115.107	2.75	2112.68	132.749
	10	115.107	3.39286	3602.23	305.972
	20	113.696	3.71429	5024.96	511.158
α	− 20	115.107	3.23214	1879.38	149.027
	− 10	113.696	3.07143	2280.27	172.653
	10	113.696	3.07143	3163.33	241.226
	20	113.696	3.07143	3642.28	278.872
o	− 20	113.696	3.07143	2721.98	205.796
	− 10	113.696	3.07143	2715.47	205.796
	10	113.696	3.07143	2702.45	205.796
	20	113.696	3.07143	2695.94	205.796
I_p	− 20	113.696	3.23214	2769.02	224.054
	− 10	113.696	3.07143	2738	205.796
	10	113.696	3.07143	2679.93	205.796

(continued)

Table 10.2 (continued)

Parameter	% Change in parameter	Change in optimal values			
		p^*	T^*	TP*	Q^*
	20	113.696	2.91071	2652.47	188.163
I_e	− 20	113.696	3.07143	2702.18	205.796
	− 10	113.696	3.07143	2705.57	205.796
	10	113.696	3.07143	2712.35	205.796
	20	113.696	3.07143	2715.74	205.796
M	− 20	113.696	3.07143	2666.35	205.796
	− 10	113.696	3.07143	2687.58	205.796
	10	113.696	3.07143	2730.5	205.796
	20	113.696	3.07143	2752.19	205.796

has to pay interest charges for a lesser number of goods, hence total profit per unit time increases.

Managerial Insights

In order to compete in the business era, it is very important to decide the selling price of the item since it directly impacts customer choice. The retailer must appropriately decide the selling price in order to generate profit rather than suffer loss. Another aspect is to decide efficiently the replenishment cycle length so as to avoid shortages and run the business smoothly. To provide a better insight, demand is supposed to be a function of selling price as well as the inventory level. It is also important to efficiently manage the total holding cost. Trade credit policy allows the buyer to purchase goods without paying instantly. While this policy looks lucrative, a better insight can only be provided if the effect of this policy on the buyer's profit is analyzed from all aspects which is one of the intentions of this research work. This research work can also help the managers in analyzing the impact of the important parameters like purchasing price, ordering cost, demand parameters, etc., and improve the efficacy of the supply chain.

Conclusion

With the increase in globalization, the demand for an efficient and effective supply chain management has increased. Selling price plays a significant role in deciding the demand of a product. Along with selling price customer's demand is also determined by exhibited stock. In this research work, an inventory model is presented considering demand to be a function of selling price as well as displayed stock along with trade

credit policy. In general, the common perception is that cost of holding goods in the inventory is always linearly dependent on stock which need not to be. In this paper, holding cost is considered to be nonlinearly dependent on stock. Trade credit policy allows the buyer to purchase goods without paying instantly. In this research work, the supplier grants a full trade credit period to the retailer. The main aim of the proposed model of inventory is to determine the optimal selling price, optimal replenishment cycle length so as to maximize the total profit earned by the retailer per unit time.

This research work can be extended along many directions such as: partial trade credit policy, inflation, fuzzy-valued inventory costs, credit-dependent demand function among others.

Appendix 1

$$
\begin{aligned}
f(T) = \Big[& p[\alpha(a - bp)(1 - \beta)T]^{\frac{1}{1-\beta}} \\
& + pI_e \Big[M[\alpha(a - bp)(1 - \beta)T]^{\frac{1}{1-\beta}} \\
& + \frac{1}{\alpha(a - bp)(2 - \beta)} \Big\{ [(a - bp)(1 - \beta)(T - M)]^{\frac{2-\beta}{1-\beta}} \\
& - [\alpha(a - bp)(1 - \beta)T]^{\frac{2-\beta}{1-\beta}} \Big\} \Big] \\
& - o - c[\alpha(a - bp)(1 - \beta)T]^{\frac{1}{1-\beta}} \\
& - h \frac{[\alpha(a - bp)(1 - \beta)T]^{\frac{\gamma+1-\beta}{1-\beta}}}{\alpha(a - bp)(\gamma + 1 - \beta)} \\
& - \frac{cI_p \Big[[\alpha(a - bp)(1 - \beta)(T - M)]^{\frac{2-\beta}{1-\beta}} \Big]}{\alpha(a - bp)(2 - \beta)} \Big]
\end{aligned}
$$

$$
g(T) = T
$$

If $f''(T) < 0$, then by using the theoretical result in (10.14) it can be proved that $TP_1(p, T)$ is a strictly pseudo-concave function in T, which completes the proof of Part (1). The proof of Part (2) and Part (3) follows immediately from the proof of Part (1) of Theorem 1.

Appendix 2

Similar to Appendix 1.

Appendix 3

Since $\text{TP}_1(p, T)$ is a strictly pseudo-concave function in T, we know that $\frac{\partial \text{TP}_1(p,T)}{\partial T}$ is a decreasing function in T. If $\Delta > 0$, then $\lim_{T \to \infty} \frac{\partial \text{TP}_1(p,T)}{\partial T} < 0$.

By applying mean value theorem, we know that there exists a unique $T_1^* \in (M, \infty)$ such that $\frac{\partial \text{TP}_1(p,T)}{\partial T} = 0$. By this, we complete the proof of $\Delta > 0$. Similarly, other theorems of Theorem 5 can be proved.

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