Research on Relative Navigation Technology of Air-Borne Missile in Satellite Denial Environment

Haibin Cheng(✉) and Hao Lu

China Airborne Missile Academy, Luoyang 471009, China
chbsian@163.com

Abstract. Aiming at the problem of relative information acquisition in air-borne missile cooperative navigation, a cooperative navigation method in satellite denial environment is proposed. Taking the radar seeker and ranging data link for relative measurement and data transmission, the relative navigation state equation between the leader and the slave is designed employing master-slave structure. The measurement equations derivate based on ranging data link and radar seeker ranging. The relative deviation is estimated by Kalman filter. Finally, the simulation is conducted to verify the algorithm. The simulation results show that the proposed method could estimate the relative deviation between missiles where the relative position estimation deviation is less than 20 m, the relative velocity estimation deviation is less than 5 üm/s, and the relative attitude estimation deviation is less than 8′. The proposed method solves problems of obtaining the relative position, relative velocity and relative attitude error information of multiple missiles, and lays a foundation for realizing the cooperative detection of multiple missiles.

Keywords: Collaborative navigation · Satellite-jamming · Radar seeker · Datalink · Simulation and analysis

1 Overview

With the continuous acceleration of the integration process of information warfare, the confrontation between attack and defense is becoming intensely. The trend is expanding the concept of air-borne missile cooperative operation. The future air combat will change from the confrontation between single combat aircraft to multi-platform cooperative combat. The cooperative air-borne missile will become the main role of future air combat. The cooperative operation of air-borne missile can improve its search ability for moving targets, especially the ability of anti-stealth targets, and effectively improve the comprehensive combat effectiveness.

Current research on air-borne missile cooperative operation mainly focuses on two aspects: The concept and method of air-borne missile cooperative operation; Cooperative guidance model, strategy and control method. Reference [1] proposed a cooperative guidance operation method of air-borne missile; Ref. [2] introduced the technologies of

Chinese Society of Aeronautics and Astronautics: CASTYSF 2022, LNEE 972, pp. 75–86, 2023.
https://doi.org/10.1007/978-981-19-7652-0_9

multi missile cooperative collectivity, formation flight, multi missile data link communication and so on; Refs. [3, 4] studied the multi missile cooperative guidance strategy; Ref. [5] researched on target tracking based on cooperative positioning of two missiles; Ref. [6] studied the multi missile cooperative guidance law. The above research studied the multi missile cooperative guidance law, control model and control method in the hypotheses that the relative information between missiles has been obtained. There has no relevant research on the acquisition method of relative information between missiles. When the relative information between missiles cannot be obtained, the research of multi missile cooperative detection is difficult to achieve.

The sensors that can be used for measurement mainly include lidar [7], vision measurement [8–10], ultra-wideband (UWB) [11], satellite navigation [12–14]. Due to the strictly limited volume of air-borne missiles and the characteristics of high mobility in combat tasks, the optional auxiliary means are very limited. Satellite navigation, radar and infrared seeker and ranging data link can be selected as auxiliary measurement means, among which satellite navigation is the most practical relative navigation means. This paper focus on the relative navigation by using radar seeker and ranging data link as auxiliary measurement in the environment of satellite navigation rejection.

2 Cooperative Guidance Strategy of Air-Borne Missile

2.1 Master-Slave Collaboration Strategy

In this paper, a master-slave collaborative guidance strategy and a coordinated guidance formation are shown in Fig. 1, in which one is the leader and the other is the follower. The pilot in the middle guidance section broadcasts his own inertial navigation data through the high-speed data link. After receiving the data broadcast by the pilot, the follower forms the relative navigation between them, and estimates the relative attitude, relative speed and relative position deviation between missiles through filtering, so as to provide formation basis for multi-missile flight in the middle guidance section. After turning to terminal guidance, the follower uses the obtained relative deviation data to correct the target information measured by the pilot radar that can be used by the follower, finally cooperative detection and cooperative attack can be realized.

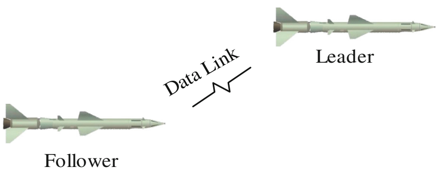

Fig. 1. Multi-missile flying formation

In order to facilitate the derivation of the algorithm, the navigation of the pilot and the follower are unified in the solidified geographic frame(n). The coordinate origin and axis are fixed relative to the earth, and X, Y and Z correspond to the north, up and east.

3 Relative Navigation Error Model

3.1 Inertial Navigation Error Equation

Owning to the short flight time and unpredictable trajectory, the estimation of position error, velocity error and misalignment angle can be applied to reduce the complexity of the algorithm without regard to device error. The pilot error equation can be seen as follows:

$$\begin{gathered}\dot{\boldsymbol{\varphi}}_l = -\boldsymbol{\omega}_{ie}^n \times \boldsymbol{\varphi}_l - \delta\boldsymbol{\omega}_{ib}^n \\ \delta\dot{\boldsymbol{v}}_l^n = \boldsymbol{C}_b^n \boldsymbol{f}^b \times \boldsymbol{\varphi}_l - 2\boldsymbol{\omega}_{ie}^n \times \delta\boldsymbol{v}_l^n + \delta\boldsymbol{f}^n \\ \delta\dot{\boldsymbol{R}}_l^n = \delta\boldsymbol{V}_l^n\end{gathered} \tag{1}$$

3.2 Relative Navigation State Equation

The state equation of the pilot and follower are:

$$\dot{\boldsymbol{x}}_l = \boldsymbol{A}_l\boldsymbol{x}_l + \boldsymbol{G}_l\boldsymbol{w}_l \tag{2}$$

$$\dot{\boldsymbol{x}}_f = \boldsymbol{A}_f\boldsymbol{x}_f + \boldsymbol{G}_f\boldsymbol{w}_f \tag{3}$$

We can get the Eq. (4) by take the pilot as the benchmark when subtract Eq. (2) from Eq. (3) to obtain:

$$\dot{\boldsymbol{x}}_l - \dot{\boldsymbol{x}}_f = \boldsymbol{A}_l\boldsymbol{x}_l - \boldsymbol{A}_f\boldsymbol{x}_f + \boldsymbol{G}_l\boldsymbol{w}_l - \boldsymbol{G}_f\boldsymbol{w}_f \tag{4}$$

where:

$$\boldsymbol{x}_{lf} = \boldsymbol{x}_l - \boldsymbol{x}_f \quad \boldsymbol{w}_{lf} = \boldsymbol{w}_l - \boldsymbol{w}_f \quad \boldsymbol{G}_{lf} = \boldsymbol{G}_l - \boldsymbol{G}_f$$

Equation (4) contains both pilot state and follower state by transforming to eliminate follower state variables.

$$\begin{aligned}\dot{\boldsymbol{x}}_{lf} &= \boldsymbol{A}_l\boldsymbol{x}_l - \boldsymbol{A}_f\boldsymbol{x}_l + \boldsymbol{A}_f\boldsymbol{x}_l - \boldsymbol{A}_f\boldsymbol{x}_f + \boldsymbol{G}_l\boldsymbol{w}_l - \boldsymbol{G}_f\boldsymbol{w}_l + \boldsymbol{G}_f\boldsymbol{w}_l - \boldsymbol{G}_f\boldsymbol{w}_f \\ &= (\boldsymbol{A}_l - \boldsymbol{A}_f)\boldsymbol{x}_l + \boldsymbol{A}_f(\boldsymbol{x}_l - \boldsymbol{x}_f) + (\boldsymbol{G}_l - \boldsymbol{G}_f)\boldsymbol{w}_l + \boldsymbol{G}_f(\boldsymbol{w}_l - \boldsymbol{w}_f) \\ &= (\boldsymbol{A}_l - \boldsymbol{A}_f)\boldsymbol{x}_l + \boldsymbol{A}_f\boldsymbol{x}_{lf} + \boldsymbol{G}_{lf}\boldsymbol{w}_l + \boldsymbol{G}_f\boldsymbol{w}_{lf}\end{aligned} \tag{5}$$

It can be seen from Eq. (5) that in order to obtain the estimation of the state parameters of cooperative navigation, it is necessary to obtain the estimation of the follower's state first. Therefore, the final cooperative navigation state equation is as follows, in which the follower's state equation is combined:

$$\begin{aligned}&\dot{\boldsymbol{X}} = \boldsymbol{A}\boldsymbol{X} + \boldsymbol{G}\boldsymbol{W} \\ &\begin{bmatrix}\dot{\boldsymbol{x}}_l \\ \dot{\boldsymbol{x}}_{lf}\end{bmatrix} = \begin{bmatrix}\boldsymbol{A}_l & \boldsymbol{0} \\ \boldsymbol{A}_{lf} & \boldsymbol{A}_f\end{bmatrix}\begin{bmatrix}\boldsymbol{x}_l \\ \boldsymbol{x}_{lf}\end{bmatrix} + \begin{bmatrix}\boldsymbol{G}_l & \boldsymbol{0} \\ \boldsymbol{G}_{lf} & \boldsymbol{G}_f\end{bmatrix}\begin{bmatrix}\boldsymbol{w}_l \\ w_{lf}\end{bmatrix}\end{aligned} \tag{6}$$

4 Measurement Model

4.1 Data Link Distance Measurement Model

The data link uses pseudo code to measure the distance, which can obtain the distance information between missiles. The true position vector of the pilot $\boldsymbol{R}_l^n$ and follower $\boldsymbol{R}_f^n$ can be expressed as follows.

$$\boldsymbol{R}_l^n = \begin{bmatrix} R_{lx} & R_{ly} & R_{lz} \end{bmatrix}^T \quad \boldsymbol{R}_f^n = \begin{bmatrix} R_{fx} & R_{lf} & R_{fz} \end{bmatrix}^T$$

The position vector of the navigator and follower inertial navigation output is:

$$\hat{\boldsymbol{R}}_l^n = \boldsymbol{R}_l^n + \delta R_l \tag{7}$$

$$\hat{\boldsymbol{R}}_f^n = \boldsymbol{R}_f^n + \delta R_f \tag{8}$$

The position Eq. (7) of missile inertial navigation output minus Eq. (8) is:

$$\hat{\boldsymbol{R}}_{lf}^n = \hat{\boldsymbol{R}}_l^n - \hat{\boldsymbol{R}}_f^n = \boldsymbol{R}_l^n - \boldsymbol{R}_f^n + \delta\boldsymbol{R}_l - \delta\boldsymbol{R}_f = \begin{bmatrix} R_{lx} - R_{fx} \\ R_{ly} - R_{fy} \\ R_{lz} - R_{fz} \end{bmatrix} + \begin{bmatrix} \delta R_{lx} - \delta R_{fx} \\ \delta R_{ly} - \delta R_{fy} \\ \delta R_{lz} - \delta R_{fz} \end{bmatrix}$$

where:

$$\boldsymbol{R}_{lf}^n = \boldsymbol{R}_l^n - \boldsymbol{R}_f^n \quad \delta\boldsymbol{R}_{lf}^n = \delta\boldsymbol{R}_l - \delta\boldsymbol{R}_f$$

Then we can get:

$$\hat{\boldsymbol{R}}_{lf}^n = \begin{bmatrix} R_{lfx} \\ R_{lfy} \\ R_{lfz} \end{bmatrix} = \boldsymbol{R}_{lf}^n + \delta\boldsymbol{R}_{lf}^n = \begin{bmatrix} \hat{R}_{lfx} \\ \hat{R}_{lfy} \\ \hat{R}_{lfz} \end{bmatrix} + \begin{bmatrix} \delta R_{lfx} \\ \delta R_{lfy} \\ \delta R_{lfz} \end{bmatrix} \tag{9}$$

The distance between bullets can be expressed as:

$$\hat{L} = \left| \hat{R}_{lf}^n \right| = \sqrt{\hat{R}_{lfx}^2 + \hat{R}_{lfy}^2 + \hat{R}_{lfz}^2}$$

The formula can be obtained by expanding the Taylor series at $\left(R_{lfx} \ R_{lfy} \ R_{lfz} \right)$ and removing the error of the first term:

$$\hat{L} = L + \frac{R_{lfx}}{L}\delta R_{lfx} + \frac{R_{lfy}}{L}\delta R_{lfy} + \frac{R_{lfz}}{L}\delta R_{lfz} \tag{10}$$

The distance measured by the data link ranging system can be discribed as:

$$\tilde{L} = L + \delta L \tag{11}$$

Subtract Eq. (11) from Eq. (10) to obtain:

$$Z_L = \hat{L} - \tilde{L} = \frac{R_{lfx}}{L}\delta R_{lfx} + \frac{R_{lfy}}{L}\delta R_{lfy} + \frac{R_{lfz}}{L}\delta R_{lfz} - \delta L \tag{12}$$

where: $\boldsymbol{M}_L = \begin{bmatrix} \frac{R_{lfx}}{L} & \frac{R_{lfy}}{L} & \frac{R_{lfz}}{L} \end{bmatrix}$ $\boldsymbol{H}_L = \begin{bmatrix} \boldsymbol{0}_{1\times 3} & \boldsymbol{0}_{1\times 3} & \boldsymbol{0}_{1\times 3} & \boldsymbol{M}_L \boldsymbol{0}_{1\times 3} & \boldsymbol{0}_{1\times 3} \end{bmatrix}$

And: $Z_L = \boldsymbol{H}_L \boldsymbol{X} + \upsilon_L \ \upsilon_L = -\delta L$.

4.2 Radar Relative Position Measurement

The position vector between the leader and follower shells has.

The position vector $\hat{\boldsymbol{R}}_{fl}^{n}$ between the leader and follower can be discribed:

$$\hat{\boldsymbol{R}}_{lf}^{n} = \boldsymbol{R}_{l}^{n} - \boldsymbol{R}_{f}^{n} = \boldsymbol{R}_{lf}^{n} + \delta\boldsymbol{R}_{lf}^{n} \tag{13}$$

The vector b_f is formed when projected into the projectile body coordinate system of the slave projectile:

$$\hat{\boldsymbol{R}}_{lf}^{b_f} = \hat{\boldsymbol{C}}_{b_l}^{b_f}\hat{\boldsymbol{C}}_{n}^{b_l}\hat{\boldsymbol{R}}_{lf}^{n} \tag{14}$$

The relationship between the attitude misalignment angle of the leading projectile and the subordinate projectile and the attitude matrix is:

$$\hat{\boldsymbol{C}}_{b_l}^{n} = (\boldsymbol{I} - \boldsymbol{\Phi}_l)\boldsymbol{C}_{b_l}^{n} \tag{15}$$

$$\hat{\boldsymbol{C}}_{b_f}^{n} = \left(\boldsymbol{I} - \boldsymbol{\Phi}_f\right)\boldsymbol{C}_{b_f}^{n} \tag{16}$$

where $\boldsymbol{\Phi}$ is the antisymmetric matrix composed of misalignment angle $\boldsymbol{\varphi}$.

$$\boldsymbol{\Phi} = \begin{bmatrix} 0 & -\varphi_z & \varphi_y \\ \varphi_z & 0 & -\varphi_x \\ -\varphi_y & \varphi_x & 0 \end{bmatrix}$$

The conversion matrix from the lead bomb system to the slave bomb system can be written as:

$$\hat{\boldsymbol{C}}_{b_l}^{b_f} = \hat{\boldsymbol{C}}_{n}^{b_f}\hat{\boldsymbol{C}}_{b_l}^{n} = \boldsymbol{C}_{n}^{b_f}\left(\boldsymbol{I} + \boldsymbol{\Phi}_f\right)(\boldsymbol{I} - \boldsymbol{\Phi}_l)\boldsymbol{C}_{b_l}^{n} = \boldsymbol{C}_{b_l}^{b_f} - \boldsymbol{C}_{n}^{b_f}\left(\boldsymbol{\Phi}_l - \boldsymbol{\Phi}_f\right)\boldsymbol{C}_{b_l}^{n} - \boldsymbol{C}_{n}^{b_f}\boldsymbol{\Phi}_f\boldsymbol{\Phi}_l\boldsymbol{C}_{b_l}^{n}$$

Ignore small product terms $\boldsymbol{C}_{n}^{b_f}\boldsymbol{\Phi}_f\boldsymbol{\Phi}_l\boldsymbol{C}_{b_l}^{n}$, Let $\boldsymbol{\Phi}_{lf} = \boldsymbol{\Phi}_l - \boldsymbol{\Phi}_f$, and:

$$\hat{\boldsymbol{C}}_{b_l}^{b_f} \approx \boldsymbol{C}_{b_l}^{b_f} - \boldsymbol{C}_{n}^{b_f}\boldsymbol{\Phi}_{lf}\boldsymbol{C}_{b_l}^{n} \tag{17}$$

After introducing Eqs. (13), (15) and (17) into Eq. (14), we get:

$$\begin{aligned}\hat{\boldsymbol{R}}_{lf}^{b_f} &= \hat{\boldsymbol{C}}_{b_l}^{b_f}\hat{\boldsymbol{C}}_{n}^{b_l}\hat{\boldsymbol{R}}_{lf}^{n} = \left[\boldsymbol{C}_{b_l}^{b_f} - \boldsymbol{C}_{n}^{b_f}\boldsymbol{\Phi}_{lf}\boldsymbol{C}_{b_l}^{n}\right]\left[\boldsymbol{C}_{n_l}^{b} + \boldsymbol{C}_{b_l}^{n}\boldsymbol{\Phi}_l\right]\left(\boldsymbol{R}_{lf}^{n} + \delta\boldsymbol{R}_{lf}^{n}\right) \\ &= \boldsymbol{R}_{lf}^{b_f} + \boldsymbol{C}_{n}^{b_f}\boldsymbol{\Phi}_l\boldsymbol{R}_{lf}^{n} - \boldsymbol{C}_{n}^{b_f}\boldsymbol{\Phi}_{lf}\boldsymbol{R}_{lf}^{n} + \boldsymbol{C}_{n}^{b_f}\delta\boldsymbol{R}_{lf}^{n} - \boldsymbol{C}_{n}^{b_f}\boldsymbol{\Phi}_{lf}\boldsymbol{\Phi}_l\boldsymbol{R}_{lf}^{n} \\ &\quad + \left(\boldsymbol{C}_{n}^{b_f}\boldsymbol{\Phi}_l + \boldsymbol{C}_{n}^{b_f}\boldsymbol{\Phi}_{lf} + \boldsymbol{C}_{n}^{b_f}\boldsymbol{\Phi}_{lf}\boldsymbol{\Phi}_l\right)\delta\boldsymbol{R}_{lf}^{n}\end{aligned}$$

After ignoring the small product term, we can get:

$$\hat{\boldsymbol{R}}_{lf}^{b_f} = \boldsymbol{R}_{lf}^{b_f} - \boldsymbol{C}_{n}^{b_f}\left(\boldsymbol{R}_{lf}^{n}\times\right)\varphi_l + \boldsymbol{C}_{n}^{b_f}\left(\boldsymbol{R}_{lf}^{n}\times\right)\varphi_{lf} + \boldsymbol{C}_{n}^{b_f}\delta\boldsymbol{R}_{lf}^{n} \tag{18}$$

The range $\tilde{d}$, azimuth $\tilde{\alpha}$ and elevation angle $\tilde{\beta}$ of the lead-in missile measured by the radar seeker of the slave missile are:

$$\tilde{d} = d + \varepsilon_d \quad \tilde{\alpha} = \alpha + \varepsilon_\alpha \quad \tilde{\beta} = \beta + \varepsilon_\beta$$

where: d, α, β are the true values of distance and azimuth; $\varepsilon_d, \varepsilon_\alpha, \varepsilon_\beta$ refers to ranging error and angle measurement error respectively.

The projection of the relative position vector of the lead-in missile measured by the radar seeker of the slave missile under the missile body system a of the slave missile is:

$$\tilde{r}_{lf}^{b_f} = \begin{bmatrix} \tilde{r}_x^{b_f} \\ \tilde{r}_y^{b_f} \\ \tilde{r}_z^{b_f} \end{bmatrix} = \begin{bmatrix} \tilde{d}\cos\tilde{\alpha}\sin\tilde{\beta} \\ \tilde{d}\cos\tilde{\alpha}\cos\tilde{\beta} \\ \tilde{d}\sin\tilde{\alpha} \end{bmatrix} \tag{19}$$

Owning to the ranging error and angle measurement error are small, the small product term is ignored and the sine and cosine of small angle are simplified as:

$$\begin{aligned} \tilde{r}_x^{b_f} &= (d + \varepsilon_d)\cos(\alpha + \varepsilon_\alpha)\sin\big(\beta + \varepsilon_\beta\big) \\ &\approx d\cos\alpha\sin\beta + d\varepsilon_\beta\cos\alpha\cos\beta - \varepsilon_\alpha\sin\alpha\sin\beta + \varepsilon_d\cos\alpha\sin\beta \end{aligned}$$

$$\begin{aligned} \tilde{r}_y^{b_f} &= (d + \varepsilon_d)\cos(\alpha + \varepsilon_\alpha)\cos\big(\beta + \varepsilon_\beta\big) \\ &\approx d\cos\alpha\cos\beta - d\varepsilon_\beta\cos\alpha\sin\beta - d\varepsilon_\alpha\sin\alpha\cos\beta + \varepsilon_d\cos\alpha\cos\beta \end{aligned}$$

$$\tilde{r}_z^{b_f} = (d + \varepsilon_d)\sin(\alpha + \varepsilon_\alpha) \approx d\sin\alpha + d\varepsilon_\alpha\cos\alpha + \varepsilon_d\sin\alpha$$

where:

$$\varepsilon_r = \begin{bmatrix} d\varepsilon_\beta\cos\alpha\cos\beta - \varepsilon_\alpha\sin\alpha\sin\beta + \varepsilon_d\cos\alpha\sin\beta \\ -d\varepsilon_\beta\cos\alpha\sin\beta - d\varepsilon_\alpha\sin\alpha\cos\beta + \varepsilon_d\cos\alpha\cos\beta \\ d\varepsilon_\alpha\cos\alpha + \varepsilon_d\sin\alpha \end{bmatrix}$$

$$R_{lf}^{b_f} = \begin{bmatrix} r_x^{b_f} \\ r_y^{b_f} \\ r_z^{b_f} \end{bmatrix} = \begin{bmatrix} d\cos\alpha\sin\beta \\ d\cos\alpha\cos\beta \\ d\sin\alpha \end{bmatrix}$$

Then, Eq. (19) can be expressed as:

$$\tilde{\boldsymbol{r}}_{lf}^{b_f} = \boldsymbol{R}_{lf}^{b_f} + \varepsilon_r \tag{20}$$

Subtracting Eq. (20) from Eq. (18), the position measurement equation can be expressed as:

$$\boldsymbol{Z}_r = \hat{\boldsymbol{R}}_{lf}^{b_f} - \tilde{\boldsymbol{r}}_{lf}^{b_f} = \boldsymbol{H}_r\boldsymbol{X} + \boldsymbol{v}_r = -\boldsymbol{C}_n^{b_f}\left(\boldsymbol{R}_{lf}^n\times\right)\boldsymbol{\varphi}_l + \boldsymbol{C}_n^{b_f}\delta\boldsymbol{R}_{lf}^n + \boldsymbol{C}_n^{b_f}\left(\boldsymbol{R}_{lf}^n\times\right)\boldsymbol{\varphi}_{lf} - \boldsymbol{\varepsilon}_r \tag{21}$$

Then:

$$\begin{aligned} \boldsymbol{H}_r &= \left[\boldsymbol{0}_{3\times3}\ \boldsymbol{0}_{3\times3}\ -\boldsymbol{C}_n^{b_f}\left(\boldsymbol{R}_{lf}^n\times\right)\ \boldsymbol{C}_n^{b_f}\ \boldsymbol{0}_{3\times3}\ \boldsymbol{C}_n^{b_f}\left(\boldsymbol{R}_{lf}^n\times\right)\right] \\ \boldsymbol{v}_r &= -\boldsymbol{\varepsilon}_r \end{aligned}$$

4.3 Radar Relative Velocity Measurement Model

The velocity measured by radar is the velocity in the direction of line of sight, and the tangential velocity of line of sight has missed, so the angular projection method cannot be used to decompose the velocity into other coordinate systems, see Fig. 2.

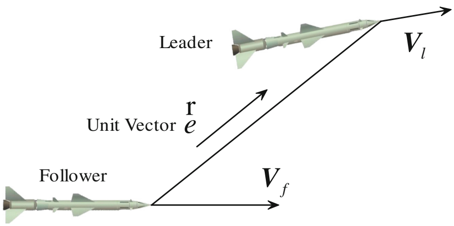

Fig. 2. Relative velocity

The output speeds of the pilot and the following inertial navigation are $\hat{\boldsymbol{V}}_l^n$ and $\hat{\boldsymbol{V}}_f^n$ respectively, which can be expressed as:

$$\hat{\boldsymbol{V}}_l^n = \boldsymbol{V}_l^n + \delta \boldsymbol{V}_l \tag{22}$$

$$\hat{\boldsymbol{V}}_f^n = \boldsymbol{V}_f^n + \delta \boldsymbol{V}_f \tag{23}$$

where $\boldsymbol{V}_l^n$, $\boldsymbol{V}_f^n$ are the true velocity, $\delta \boldsymbol{V}_l$, $\boldsymbol{V}_f$ are the errors of velocity.

$\vec{e}$ is the unit vector of the line of sight direction, which can be expressed as

$$\vec{e} = \frac{\vec{\boldsymbol{R}}_{lf}^n}{\left|\vec{\boldsymbol{R}}_{lf}^n\right|} = \begin{pmatrix} \frac{R_{lfx}}{L} & \frac{R_{lfy}}{L} & \frac{R_{lfz}}{L} \end{pmatrix}$$

Then the relative velocity between the leader and follower shells can be expressed as:

$$\hat{V}_{lf}^e = \hat{\boldsymbol{V}}_{lf}^n \cdot \vec{e} = \left(\hat{\boldsymbol{V}}_l^n - \hat{\boldsymbol{V}}_f^n\right)\vec{e} = \frac{V_{lfx}R_{lfx} + V_{lfy}R_{lfy} + V_{lfz}R_{lfz}}{\hat{L}} \tag{24}$$

Carry out Taylor series expansion of Eq. (23) at $\left(R_{lfx}\ R_{lfy}\ R_{lfz}\right)$ and $\left(V_{lfx}\ V_{lfy}\ V_{lfz}\right)$ respectively, and take the primary term error to obtain:

$$\begin{aligned}\hat{\boldsymbol{V}}_{lf}^e =& \boldsymbol{V}_{lf}^e + \frac{\partial \hat{\boldsymbol{V}}_{lf}^e}{\partial R_{lfx}} \cdot \delta R_{lfx} + \frac{\partial \hat{\boldsymbol{V}}_{lf}^e}{\partial R_{lfy}} \cdot \delta R_{lfy} + \frac{\partial \hat{\boldsymbol{V}}_{lf}^e}{\partial R_{lfz}} \cdot \delta R_{lfz} + \frac{\partial \hat{\boldsymbol{V}}_{lf}^e}{\partial V_{lfx}} \cdot \delta V_{lfx} \\ &+ \frac{\partial \hat{\boldsymbol{V}}_{lf}^e}{\partial V_{lfy}} \cdot \delta V_{lfy} + \frac{\partial \hat{\boldsymbol{V}}_{lf}^e}{\partial V_{lfz}} \cdot \delta V_{lfz}\end{aligned} \tag{25}$$

where:

$$\frac{\partial \hat{\boldsymbol{V}}_{lf}^e}{\partial R_{lfx}} = \frac{\left(R_{lfy}^2 + R_{lfz}^2\right)V_{lfx}}{\left(R_{lfx}^2 + R_{lfy}^2 + R_{lfz}^2\right)^{\frac{3}{2}}} - \frac{\left(V_{fly}R_{fly} + V_{flz}R_{flz}\right)R_{lfx}}{\left(R_{lfx}^2 + R_{lfy}^2 + R_{lfz}^2\right)^{\frac{3}{2}}}$$

$$\frac{\partial \hat{\boldsymbol{V}}_{lf}^{e}}{\partial R_{lfy}} = \frac{\left(R_{lfx}^2 + R_{lfz}^2\right)V_{lfy}}{\left(R_{lfx}^2 + R_{lfy}^2 + R_{lfz}^2\right)^{\frac{3}{2}}} - \frac{\left(V_{lfx}R_{lfx} + V_{lfz}R_{lfz}\right)\hat{R}_{lfy}}{\left(R_{lfx}^2 + R_{lfy}^2 + R_{lfz}^2\right)^{\frac{3}{2}}}$$

$$\frac{\partial \hat{\boldsymbol{V}}_{lf}^{e}}{\partial R_{lfz}} = \frac{\left(R_{lfx}^2 + R_{lfy}^2\right)V_{lfz}}{\left(R_{lfx}^2 + R_{lfy}^2 + R_{lfz}^2\right)^{\frac{3}{2}}} - \frac{\left(V_{lfx}R_{lfx} + V_{lfy}R_{lfy}\right)R_{lfz}}{\left(R_{lfx}^2 + R_{lfy}^2 + R_{lfz}^2\right)^{\frac{3}{2}}}$$

$$\frac{\partial \hat{\boldsymbol{V}}_{lf}^{e}}{\partial V_{lfx}} = \frac{R_{lfx}}{\sqrt{R_{lfx}^2 + R_{lfy}^2 + R_{lfz}^2}}$$

$$\frac{\partial \hat{\boldsymbol{V}}_{lf}^{e}}{\partial V_{lfy}} = \frac{R_{lfy}}{\sqrt{R_{lfx}^2 + R_{lfy}^2 + R_{lfz}^2}}$$

$$\frac{\partial \hat{\boldsymbol{V}}_{lf}^{e}}{\partial V_{lfz}} = \frac{R_{lfz}}{\sqrt{R_{lfx}^2 + R_{lfy}^2 + R_{lfz}^2}}$$

So, we can immediately write

$$M_r = \begin{bmatrix} \frac{\partial \hat{\boldsymbol{V}}_{lf}^{e}}{\partial R_{lfx}} & \frac{\partial \hat{\boldsymbol{V}}_{lf}^{e}}{\partial R_{lfy}} & \frac{\partial \hat{\boldsymbol{V}}_{lf}^{e}}{\partial R_{lfz}} \end{bmatrix} \quad M_v = \begin{bmatrix} \frac{\partial \hat{\boldsymbol{V}}_{lf}^{e}}{\partial V_{lfx}} & \frac{\partial \hat{\boldsymbol{V}}_{lf}^{e}}{\partial V_{lfy}} & \frac{\partial \hat{\boldsymbol{V}}_{lf}^{e}}{\partial V_{lfz}} \end{bmatrix}$$

The radar seeker line can measure the velocity of the leading projectile relative to the subordinate projectile in the line of sight, it can be seen as follow

$$\tilde{\boldsymbol{V}}_{lf}^{e} = \boldsymbol{V}_{lf}^{e} + \varepsilon_v \tag{26}$$

Then subtract Eq. (26) from Eq. (25) to obtain:

$$\begin{aligned} Z_v &= \hat{\boldsymbol{V}}_{lf}^{e} - \tilde{\boldsymbol{V}}_{lf}^{e} \\ &= \frac{\partial \hat{\boldsymbol{V}}_{lf}^{e}}{\partial R_{lfx}} \cdot \delta R_{lfx} + \frac{\partial \hat{\boldsymbol{V}}_{lf}^{e}}{\partial R_{lfy}} \cdot \delta R_{lfy} + \frac{\partial \hat{\boldsymbol{V}}_{lf}^{e}}{\partial R_{lfz}} \cdot \delta R_{lfz} + \frac{\partial \hat{\boldsymbol{V}}_{lf}^{e}}{\partial V_{lfx}} \cdot \delta V_{lfx} \\ &+ \frac{\partial \hat{\boldsymbol{V}}_{lf}^{e}}{\partial V_{lfy}} \cdot \delta V_{lfy} + \frac{\partial \hat{\boldsymbol{V}}_{lf}^{e}}{\partial V_{lfz}} \cdot \delta V_{lfz} - \varepsilon_v \end{aligned}$$

$$H_v = \begin{bmatrix} \mathbf{0}_{1\times3} & \mathbf{0}_{1\times3} & \mathbf{0}_{1\times3} & M_r & M_v & \mathbf{0}_{1\times3} \end{bmatrix}$$

5 Simulation Analysis

The state variables of cooperative navigation are 18 dimensions in total, and the measurement equation has 5 dimensions, which includes data link ranging measurement dimension, radar position measurement dimension and radar speed measurement dimension. Kalman filter is used for processing in a sequential manner.

5.1 Construction of Simulation Environment

The cooperative navigation algorithm is verified by software simulation. The simulation environment structure is shown in Fig. 3. First the simulation trajectory of the pilot and follower is setted, and IMU inertial device errors are generated. The navigator carries out inertial navigation calculation independently, the pilot and follower transmit data through a high-speed data link capable of ranging. The follower uses its own seeker to measure the distance, angle and line of sight speed of the pilot. Data from follower participate in the collaborative navigation filtering to obtain the estimation of follower error and relative error, so as to realize the collaborative navigation filtering calculation.

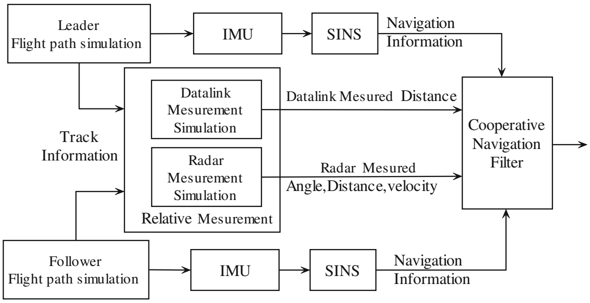

Fig. 3. Simulation platform structure diagram

5.2 Simulation Conditions

In the simulation, the gyro zero drift is set to 1°/h, the acceleration zero offset is 0.5 mg, and the speed measurement error is set to <5 m/s(1σ), rang measurement error <10 m(1σ). The relative attitude deviation between the pilot and the follower, and the relative attitude deviation angle of roll, heading and pitch are $(20, 20, 20)'$. The flight path of the pilot and follower in the solidified geographical system is shown in Fig. 4. The initial transverse spacing is about 500 m, the total simulation time is 200 s, and the flight distance is about 250 km.

The filtering estimation of cooperative navigation is carried out under the flight trajectory of the follower to obtain the estimation of the relative position, relative speed and relative attitude deviation of the follower relative to the pilot, as shown in Figs. 5, 6 and 7.

It can be seen from the figure that the relative position estimation error of the follower relative to the pilot is less than 20 m. The relative velocity estimation error is less than 5 m/s, with the cooperation of missile maneuver, the relative attitude estimation error is less than $8'$.

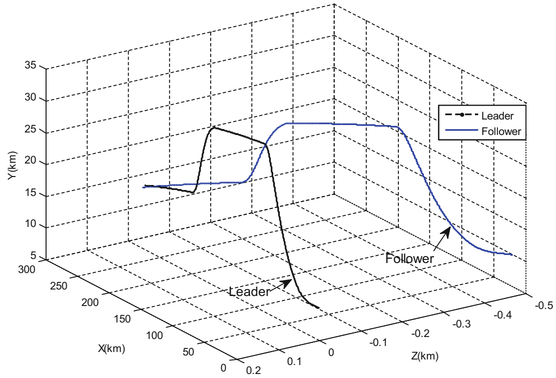

Fig. 4. Trajectory of Missile

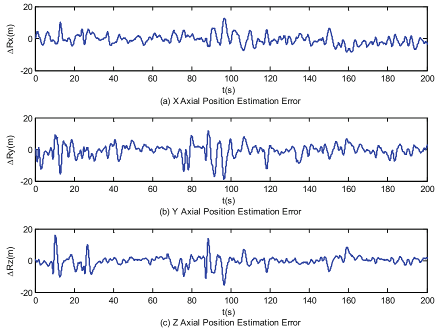

Fig. 5. Relative position estimation error

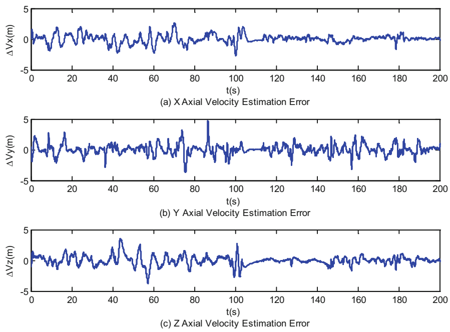

Fig. 6. Relative velocity estimation error

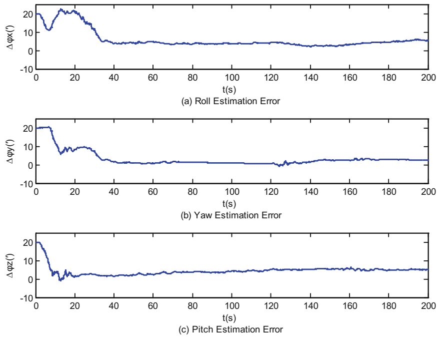

Fig. 7. Relative attitude estimation error

6 Summary

Cooperative detection, cooperative guidance and cooperative attack will be the necessary capabilities of future air-borne missiles to adapt to the new situation of future multi platform cooperative air combat. The basis of realizing multi missile cooperation is the real-time acquisition of relative information between multiple missiles. This paper presents a new method to obtain the relative information of Multiple Missiles Based on satellite navigation. This method can solve the problem of obtaining the relative velocity, relative position, especially the relative attitude deviation between air-borne missiles; it lays a foundation for the realization of multi missile cooperative operation of air-borne missiles.

References

1. Jun, W., Tonglin, R., Jin, L., et al.: The networking cooperative guidance combat pattern for air-borne missile. Aero Weaponry **5**, 32–35 (2015)
2. Xushengli, C.F., et al.: Application of multi missile cooperative technology in the development of air-defense missile. Electron. Opt. Control **2017**(2), 55–59
3. Hong, W., Jun, W., Wen bo, L., et al.: Multi-missile cooperative attacking strategy of new type of air-borne missile. Aero Weaponry **2011**(2), 7–11
4. Jianbo, Z., Shuxing, Y.: Review of multi-missile cooperative guidance[J]. Acta Aeronautica ET Astronautica Sinica **38**(1), 17–29 (2017)
5. Jipeng, D., Dong, T.: An Algorithm of Target Tracking Based on Two Missiles Cooperative Location Information[J]. Aero Weaponry **3**, 3–7 (2014)
6. Junhong, S., Song Shenmin, X., Shengli.: A Cooperative Guidance Law for Multiple Missiles to Intercept Maneuvering Target[J]. Journal of Astronautics **37**(12), 1432–1440 (2016)
7. Woods, J.O., Christian, J.A.: Lidar-based relative navigation with respect to non-cooperative objects[J]. Acta Astro- nautica **126**, 298–311 (2016)
8. Scaramuzza, D., Achtelik, M.C., Doitsidis, L., et al.: Vision-controlled micro flying robots: from system design to autonomous navigation and mapping in GPS-denied environments[J]. IEEE Robot. Autom. Mag. **21**(3), 26–40 (2014)
9. Fosbury, A.M.: Relative navigation of air vehicles. J. Guid. Control Dyn. **31**(4), 824–833 (2008)

10. Wu, T.-H., Flewelling, B., Leve, F., Lee, T.: Spacecraft relative attitude formation tracking on SO(3) based on line-of-sight measurements. In: American Control Conference (ACC)Washington, DC, USA, June 17–19, 2013
11. Jun, X., Xiong Zhi, Y., Yongjun, et al.: Close relative navigation algorithm for unmanned aerial vehicle aided by UWB relative measurement. J. Chin. Inert. Technol. **26**(3), 346–351 (2018)
12. Yandong, W., Yang Hongyu, F., Haifeng, et al.: Pseudorange differential GPS relative navigation without base-station for formation flight. J. Chin. Inert. Technol. **6**, 763–768 (2014)
13. Gross, J.N., Gu, Y., Rhudy, M.B.: Robust UAV relative navigation with DGPS, INS, and peer-to-peer radio ranging. IEEE Trans. Autom. Sci. Eng. **12**(3), 935–944 (2015)
14. Xiusen, W., Hongjin, Z., Shangyue, Z.: Relative navigation between vessels based on GPS single difference. J. Chin. Inert. Technol. **20**(4), 464–467 (2012)