Guidance Law Design Against Unknown Maneuvering Target Based on Two ESOs

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Abstract. With the development of high maneuvering missiles and hypersonic maneuvering unmanned aerial vehicles, accurately intercepting such high-speed moving targets typically involves knowing the target's precise velocity and acceleration. Obtaining the velocity and acceleration of the target in real time is difficult and expensive in practice. On the other hand, developing a guidance law for maneuvering target with unknown velocity and acceleration is still a work in progress. This paper proposes three types of sliding-mode guidance rules based on two extend state observers (ESOs) for intercepting maneuvering targets with unpredictable velocity and acceleration, where the only information required is the LOS angle and distance, but not the LOS angle velocity or the relative velocity orthogonal to the LOS. To estimate the unknown things associated to the objective, two ESOs are built. The auxiliary signal system has been devised to compensate for the saturation constraint of the guidance command. To test the effectiveness and robustness of the proposed guidance legislation, numerical simulations are run with a non-moving target and a maneuvering target.

Keywords: Unknown maneuvering targets · Extend State Observer (ESO) · Sliding mode · Saturation

1 Introduction

Because of its simple design and ease of implementation, proportional navigation guidance (PNG) is the most preferred terminal homing guidance approach in unmanned craft guidance [1, 2]. In Refs. [3–5], it was shown that PNG is the best guidance for non-maneuvering targets. PNG's performance, on the other hand, will deteriorate and even fail against maneuvering targets.

With the advent of high maneuvering missiles and hypersonic maneuvering unmanned aerial vehicles in recent years, intercepting such targets now necessitates the ability to intercept high speed moving targets with greater precision. To deal with maneuvering targets, different PNG laws have been proposed to improve guidance precision, such as augmented PNG (APNG) law [6–8] and optimal guidance law (OGL) [9, 10], which both require correct target acceleration information or prediction. In practice, however, obtaining the goal acceleration or an exact estimate is difficult. As a result, nonlinear robust control approaches such as nonlinear geometric method [11], sliding

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mode control method [12–14], and others have been employed to create guidance rules. For maneuvering targets, a robust geometric guidance technique is provided, in which the terms relating to target acceleration are treated as disturbance [11]. In Ref. [15], an all-aspect guidance law based on backstepping is proposed to achieve the interception without knowing the target's acceleration.

Because the target's maneuvering acceleration is unknown, the variable structure control (VSC) guidance law is designed with the line of sight (LOS) normal velocity as the sliding surface [16]. However, when the angle between the line of sight and the missile flight path is equal to 90°, the VSC guidance law is no longer effective. An adaptive nonsingular terminal sliding mode control (SMC) with an extended state observer (ESO) to estimate the uncertain term is provided to handle the unknown target acceleration [17, 18]. However, it requires knowing the relative velocities orthogonal to the LOS.

Furthermore, all of the above guidance approaches imply that the target is stationary or moving at a constant velocity that must be known. The problem of designing a guidance law for a target with uncertain velocity and acceleration remains unsolved. It is quite difficult to collect the target's velocity information without delay in actual applications. As a result, designing a guidance law using the target's unknown acceleration and velocity is more practicable. To deal with the unknown target, two ESOs are used in this study. ESO has been shown to be a powerful technique for dealing with unfamiliar terms [18–23]. Sliding mode control is also used to improve the robustness of the guidance law. Furthermore, this guidance strategy just needs to know the LOS angle and distance, but neither the LOS angle velocity or the relative velocity orthogonal to the LOS.

2 Problem Formulation

The two-dimensional engagement geometry is described in Fig. 1, where the missile M is trying to intercept a ma-neuvering target T. Assuming that the target and missile are point masses, the dynamic equations of missile and target are given as follows

$$\begin{aligned} \dot{x}_M &= V_M \cos\varphi_M,\ \dot{y}_M = V_M \sin\varphi_M \\ \dot{\varphi}_M &= \frac{a_M}{V_M} \end{aligned} \tag{1}$$

$$\begin{aligned} \dot{x}_T &= V_T \cos\varphi_T,\ \dot{y}_T = V_T \sin\varphi_T \\ \dot{\varphi}_T &= \frac{a_T}{V_T} \end{aligned} \tag{2}$$

where φ_M and φ_T are the flight path angle of the missile and the flight path angle of the target, V_M and a_M are the velocity and acceleration of the missile, and V_T and a_T are the velocity and acceleration of the target.

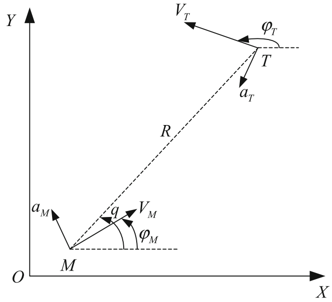

Fig. 1. Planar interception geometry

Combine Fig. 1 with Eq. (1) and Eq. (2), the dynamic of interception can be given by

$$\begin{aligned} \dot{R} &= -V_M \cos(q - \varphi_M) + V_T \cos(q - \varphi_T) \\ \dot{q} &= \frac{V_M \sin(q - \varphi_M)}{R} - \frac{V_T \sin(q - \varphi_T)}{R} \\ \dot{\varphi}_M &= \frac{a_M}{V_M} \\ \dot{\varphi}_T &= \frac{a_T}{V_T} \end{aligned} \tag{3}$$

where R and q represent the relative distance between the missile and the target and the line-of-sight (LOS) angle.

Assumption 1 both V_T and a_T can be treated as unknown term.

The reason of assumption 1 is that it is difficulty to obtain the accurate values of V_T and a_T without delay, and the sensors are affected by noise. Then the flight path angle of the target φ_T is also unknown. Therefore, $V_T \cos(q - \varphi_T)$ and $V_T \sin(q - \varphi_T)$ are unknown terms too, which means that $\dot{R}$ and $\dot{q}$ contain unknown terms. Therefore, $\dot{R}$ and $\dot{q}$ can not be directly used in the PNG law in this paper.

Assumption 2 both the relative distance R and the LOS angle q are available, and R doesn't equal zero.

It is reasonable for Assumption 2, because as long as $0 < R \leq r_0$ where r_0 is the sum of size of missile and target (see Ref. [1]), the hit-to-kill interception will be successful.

Assumption 3: V_T is less than V_M.

3 SMC Guidance Law with ESO

In order to design a guidance law for a homing missile against a high maneuvering target whose velocity and ac-celeration are unknown, SMC and ESO are applied. Meanwhile, the auxiliary signal is introduced to prevent the ef-fect of saturation.

3.1 ESO Design for Unknown Items

Obviously, the unknown terms $V_T \cos(q - \varphi_T)$ and $\frac{V_T \sin(q - \varphi_T)}{R}$ are differential and bounded, and their derivatives are also bounded in practical applications due to that the velocity and acceleration of target could not be infinite large. Define $V_{Tc} = V_T \cos(q - \varphi_T)$ and $V_{Ts} = -\frac{V_T \sin(q - \varphi_T)}{R}$, according to $0 < R \le r_0$, then there exists two positive constant C_a and C_V, which satisfy that $\left|V_{Tc}^{'}\right| \le C_V$ and $\left|V_{Ts}^{'}\right| \le C_a$.

As in Refs. [18–21], in order to handle the unknown terms in $\dot{R}$ and $\dot{q}$, two ESOs for $\dot{R}$ and $\dot{q}$ can be constructed as follows.

$$\begin{cases} E_{1,1} = Z_{1,1} - R \\ \dot{Z}_{1,1} = Z_{1,2} - \beta_1 g_{c1}(E_{1,1}) - V_M \cos(q - \varphi_M) \\ \dot{Z}_{1,2} = -\beta_2 g_{c2}(E_{1,1}) \end{cases} \tag{4}$$

$$\begin{cases} E_{2,1} = Z_{2,1} - q \\ \dot{Z}_{2,1} = Z_{2,2} - \beta_1 g_{c1}(E_{2,1}) + \dfrac{V_M \sin(q - \varphi_M)}{R} \\ \dot{Z}_{2,2} = -\beta_2 g_{c2}(E_{2,1}) \end{cases} \tag{5}$$

with

$$g_{c1}(E_{i,1}) = E_{i,1},$$
$$g_{c2}(E_{i,1}) = \begin{cases} \left|E_{i,1}\right|^{\alpha} \mathrm{sign}(E_{i,1}), \left|E_{i,1}\right| > \sigma \\ E_{i,1} \big/ \sigma^{1-\alpha}, \textit{otherwise} \end{cases} \quad 0 < \alpha < 1, i = 1, 2$$

where $E_{i,1}$ is the estimation error of the ESO, $Z_{i,1}$ and $Z_{i,2}$ are the observer output, $\beta_1 > 0$, $\beta_2 > 0$ are the observer gains. It has been proved [18–21] that for appropriate values of $\beta_1, \beta_2, \alpha, \sigma$, the observer output $Z_{1,1}$ and $Z_{2,1}$ can approaches to R and q respectively, meanwhile, $Z_{1,2}$ and $Z_{2,2}$ can approaches to V_{Tc} and V_{Ts} respectively. As well, $\dot{Z}_{1,1}$ and $\dot{Z}_{2,1}$ can approaches to $\dot{R}$ and $\dot{q}$ respectively. It had been proved that $E_{1,2} = Z_{1,2} - V_{Tc}$ and $E_{2,2} = Z_{2,2} - V_{Ts}$ are bounded [20]. In other words, we have that $\left|E_{1,2}\right| \le \frac{\beta_1}{\beta_2}\sigma^{1-\alpha} C_V$ and $\left|E_{2,2}\right| \le \frac{\beta_1}{\beta_2}\sigma^{1-\alpha} C_a$ [20], where C_V and C_a are positive constants need not to know the exact value.

3.2 SMC Guidance Law Design Procedure

As we all known, $V_r = -V_M \cos(q - \varphi_M) + V_T \cos(q - \varphi_T)$ denotes the velocity along LOS. Then it is necessary to keep $V_r < 0$ for a direct hit [14]. Also, it is well known that a direct hit can be achieved when $R = 0$ or $\theta_M = q - \varphi_M = 0$, where $\theta_M = 0$ mean that the velocity of missile is always in the LOS.

Then, the guidance task can be realized by stabilizing the following sliding mode surface with the missile acceleration

$$\begin{aligned} & S_1 = R + c_0 \dot{R} = 0, \\ \textit{or} \quad & S_2 = \theta_M = 0, \\ \textit{or} \quad & S_3 = \theta_M + c_1 R = 0 \end{aligned} \tag{6}$$

where c_0 and c_1 are positive designed constant parameters. Note that the units of variable θ_M and variable R are not the same, so the units of variable θ_M and variable R need to be consistent when selecting parameter c_1. Meanwhile, in order to hit the target, it is necessary to ensure that the missile's speed is along the LOS line firstly, that is to say it is necessary to ensure that the speed of convergence of θ_M is much faster than the speed of convergence of R. Therefore, we can choose $c_1 = \frac{\theta_M(0)}{100\times R(0)}$, where $\theta_M(0)$ and $R(0)$ are the initial values of variable θ_M and variable R, respectively.

Next, this article focuses on the analysis of the guide law when choosing the third sliding mode surface. The analysis of the other sliding mode surfaces is similar to this.

Derivate the sliding mode surface S_3, we can obtain

$$\begin{aligned}\dot{S}_3 = \frac{V_M \sin(q - \varphi_M)}{R} + V_{Ts} - \frac{a_M}{V_M} \\ + c_1\left[-V_M \cos(q - \varphi_M) + V_{Tc}\right]\end{aligned} \tag{7}$$

Obviously, the Eq. (7) includes the unknown terms V_{Tc} and V_{Ts}, which can be estimated by ESO. Then we can design guidance law as follows

$$a_M = V_M \left[\begin{array}{l} c_1\left(-V_M \cos(q - \varphi_M) + Z_{1,2}\right) \\ + \dfrac{V_M \sin(q - \varphi_M)}{R} + Z_{2,2} - \tau S_3 - \delta |S_3|^{\gamma} \operatorname{sgn}(S_3) \end{array}\right] \tag{8}$$

where $Z_{1,2}$ and $Z_{2,2}$ are the states of the ESO Eqs. (4) and (5), τ, δ are positive designed parameters, and $\gamma \in (0, 1)$ is also positive designed parameter.

Note that, the acceleration of missile is usually constrained due the overload in practical applications. Therefore, the practical acceleration of missile with saturation constraint can be described as follows

$$a_M = sat(a_{M0}) = \begin{cases} a_{M0}, & |a_{M0}| \le a_{M\max} \\ a_{M\max} \operatorname{sgn}(a_{M0}), & |a_{M0}| > a_{M\max} \end{cases} \tag{9}$$

where $a_{M\max}$ is the maximum acceleration of missile and a_{M0} is the designed acceleration of missile without consideration of saturation.

In order to compensate the effect of saturation, the auxiliary signal system can be constructed as

$$\dot{\lambda} = -\tau\lambda + \frac{\Delta a_M}{V_M} \tag{10}$$

where $\Delta a_M = a_{M0} - a_M$, and τ is defined in Eq. (8).

Obviously, the auxiliary signal system is bounded input bounded output stable (BIBO). When $|a_{M0}| \le a_{M\max}$, the auxiliary signal λ will converge to zero.

Then the sliding mode surface S_3 in Eq. (6) can be compensated as followings:

$$\overline{S}_3 = S_3 - \lambda \tag{11}$$

Derivate the compensated sliding mode surface $\overline{S}_3$, we can obtain

$$\begin{aligned}\dot{\overline{S}}_3 &= \frac{V_M \sin(q-\varphi_M)}{R} + V_{Ts} - \frac{a_M}{V_M} \\ &+c_1\left[-V_M \cos(q-\varphi_M) + V_{Tc}\right] + \tau\lambda - \frac{\Delta a_M}{V_M} \\ &= \frac{V_M \sin(q-\varphi_M)}{R} + V_{Ts} \\ &+c_1\left[-V_M \cos(q-\varphi_M) + V_{Tc}\right] + \tau\lambda - \frac{a_{M0}}{V_M}\end{aligned} \tag{12}$$

Therefore, we can design guidance law as follows

$$\begin{aligned} a_M &= sat(a_{M0}) \\ a_{M0} &= V_M \begin{bmatrix} c_1\left(-V_M \cos(q-\varphi_M) + Z_{1,2}\right) \\ +\dfrac{V_M \sin(q-\varphi_M)}{R} + Z_{2,2} - \tau S_3 - \delta\left|\overline{S}_3\right|^{\gamma} \operatorname{sgn}\left(\overline{S}_3\right)\end{bmatrix}\end{aligned} \tag{13}$$

Remark 1 when the sliding mode surface is selected as S_1, the derivation of the unknown term $V_T \cos(q-\varphi_T)$ exists in the derivation of S_1 due to the target is maneuvering. Then the guidance law is

$$a_{M0} = \begin{cases} \dfrac{1}{c_0 \sin(q-\varphi_M)}\begin{bmatrix} c_0 \dot{Z}_{1,2} - V_M \cos(q-\varphi_M) \\ +Z_{2,2} + \tau S_1 + \delta\left|\overline{S}_1\right|^{\gamma} \operatorname{sgn}\left(\overline{S}_1\right)\end{bmatrix}, & |q-\varphi_M| > \varepsilon \\ 0, \quad |q-\varphi_M| < \varepsilon \end{cases} \tag{14}$$

where $\dot{Z}_{1,2}$ is the estimation of the derivation of $V_T \cos(q-\varphi_T)$, and ε is a small positive designed parameter, $\overline{S}_1 = S_1 - \lambda$.

Remark 2 when the sliding mode surface is selected as S_2, then the guidance law is

$$a_{M0} = V_M\left[\frac{V_M \sin(q-\varphi_M)}{R} - Z_{2,2} - \tau S_2 - \delta\left|\overline{S}_2\right|^{\gamma} \operatorname{sgn}\left(\overline{S}_2\right)\right] \tag{15}$$

where $\overline{S}_2 = S_2 - \lambda$ is the compensated sliding mode.

3.3 Stability Analysis

Substituting Eq. (13) into Eq. (12), we can obtain the follows

$$\dot{\overline{S}}_3 = -\tau\overline{S}_3 - \delta\left|\overline{S}_3\right|^{\gamma} \operatorname{sgn}\left(\overline{S}_3\right) - E_{2,2} - c_1 E_{1,2} \tag{16}$$

Choose lyapunov function as $V = \frac{1}{2}\overline{S}_3^2$, then we can obtain the follows

$$\begin{aligned}\dot{V} &= -\tau\overline{S}_3^2 - \delta\left|\overline{S}_3\right|^{\gamma+1} - E_{2,2}\overline{S}_3 - c_1 E_{1,2}\overline{S}_3 \\ &\le -\tau\overline{S}_3^2 - \delta\left|\overline{S}_3\right|^{\gamma+1} + \left|\overline{S}_3\right|\left(\left|E_{2,2}\right| + \left|E_{1,2}\right|\right)\end{aligned} \tag{17}$$

Then $\left|E_{2,2}\right| + \left|E_{1,2}\right|$ can be viewed as the disturbance input of the closed loop system. When $\left\|\overline{S}_3\right\| \geq \chi\left(\left|E_{2,2}\right| + \left|E_{1,2}\right|\right)$, with $\chi\left(\left|E_{2,2}\right| + \left|E_{1,2}\right|\right) = \frac{1}{\tau}\left(\left|E_{2,2}\right| + \left|E_{1,2}\right|\right)$, the Eq. (17) is smaller than zero. Therefore, the closed loop system is input to state stable (ISS). Base on input to state stability (see [19, 20]), it is easy to conclude that the estimation error $\left|E_{2,2}\right| + \left|E_{1,2}\right|$ by ESO will affect that whether the dynamic of the compensated sliding mode converge to $\overline{S}_3 = 0$.That is to say the dynamic of the compensated sliding mode will be restricted into the neighborhood of $\overline{S}_3 = 0$ as

$$\lim_{t\to\infty} \overline{S}_3 \in \left(\left|\overline{S}_3\right| \leq \frac{1}{\tau}\left(\left|E_{2,2}\right| + \left|E_{1,2}\right|\right)\right) \tag{18}$$

Fortunately, this neighborhood can be reduced to any small size by selecting the ESO parameters β_1, β_2, α, σ and the controller parameter τ. Furthermore, τ, δ, γ determines the speed of convergence and the final error. According to Eq. (17), the bigger the parameters τ and δ are, the faster speed converges to zero, and the smaller approaching error is. From the above analysis, a_{M0} is bounded, therefore, Δa_M is bounded too, then we can conclude that λ is bounded. When $|a_{M0}| \leq a_{M\max}$, we can obtain that $\lim\limits_{t\to\infty} S_3 \in \left(|S_3| \leq \frac{1}{\tau}\left(\left|E_{2,2}\right| + \left|E_{1,2}\right|\right)\right)$.

Remark 3 From the above analysis, it is clear that S_3 will not converge to zero due to the estimation error of ESO. It implies that S_3 can only converge into a neighborhood of the desired sliding mode surface $S_3 = 0$ and remains within it.

4 Simulation Results

Numerical simulations are given to investigate the effectiveness of the proposed guidance law (13). It is assumed that the maximum acceleration value of the missile is $a_{M\max} = 50\,\mathrm{m}/\mathrm{s}^2$. The initial positions of the missile and the target are $x_M(0) = -20\,\mathrm{km}$, $y_M(0) = -20\,\mathrm{km}$ and $x_T(0) = 0\,\mathrm{m}$, $y_T(0) = 0\,\mathrm{m}$, respectively. The initial flight path angles of the missile and the target are $\varphi_M(0) = \frac{\pi}{2}$ rad and $\varphi_T(0) = \frac{\pi}{3}$ rad, respectively. The velocity of the missile is chosen as $V_M = 800\,\mathrm{m/s}$.

The parameters for ESO are selected as $\beta_1 = 50$,$\beta_2 = 300$,$\alpha = 0.25$ and $\sigma = 0.15$. And the initial states of ESOs are set to $Z_{i,1}(0) = Z_{i,2}(0) = 0$, $i = 1, 2$. The parameters for the proposed guidance law are chosen as $\tau = 10$, $\delta = 1$ and $\gamma = 0.2$. The initial auxiliary signal is $\lambda(0) = 0$. The parameters for sliding mode are chosen as $c_0 = 0.1$ and $c_1 = \frac{\theta_M(0)}{100\times R(0)}$. And ε is selected as $\varepsilon = 0.01$, where it needs to note that the value of ε determines the stability of the slide mode S_1.

For comparison, the following PNG law is also considered.

$$a_M = NV_M\dot{q} = NV_M\left(\frac{V_M\sin(q-\varphi_M)}{R} + Z_{2,2}\right)$$

where $N = 3$ and $Z_{2,2}$ is defined above.

Define $a_M^{sum} = \int_0^{t_I} |a_M|dt$ as the total control effort of the required guidance command, where t_I is the interception time representing the smallest miss distance.

Case 1: It is assumed that the target is stationary. That is to say $V_T = 0\,\mathrm{m/s}$ and $a_T = 0\,\mathrm{m^2/s}$. The simulation time is set to 40 s and the simulation step is set to 0.0001 s. The engagement trajectory is shown in Fig. 2, and the relative distance is displayed in Fig. 3.

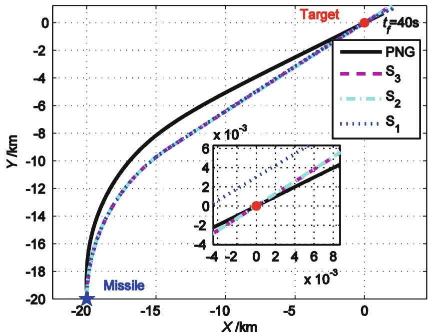

Fig. 2. Flight trajectories

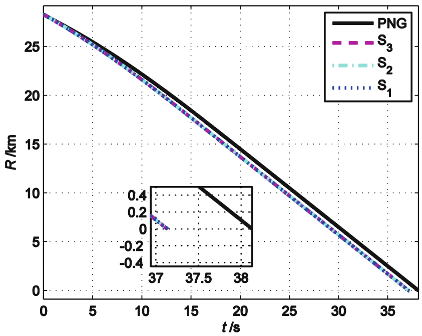

Fig. 3. Relative range

Table 1. Performance results of guidance laws for case 1

Methods	Miss distance (m)	Interception time (s)	Total control effort
PNG	0.0771	38.1296	9757110
S3	0.0402	37.1326	8087205
S2	0.0599	37.1325	10953274
S1	0.8956	37.1326	8027486

The performances at the interception time arerepresented in Table 1. By the simulation, the actual miss distances and corresponding interception times for four methods are respectively 0.0771 m and 38.1296 s, 0.0402 m and 37.1326 s, 0.0599 m and 37.1325 s, 0.8956 m and 37.1326 s. It can be clearly seen that when the target is stationary, the effect of the four guidance methods is similar. But the interception time of the three sliding mode guidance methods is shorter than that of PNG. Moreover, the engagement trajectories of the three sliding mode guidance methods are almost identical. The guidance command for the four methods is given in Fig. 4. As shown in Fig. 4 and Table 1, when the slide mode surface is selected as S_3, the proposed guidance law uses less guidance command than the other three guidance laws.

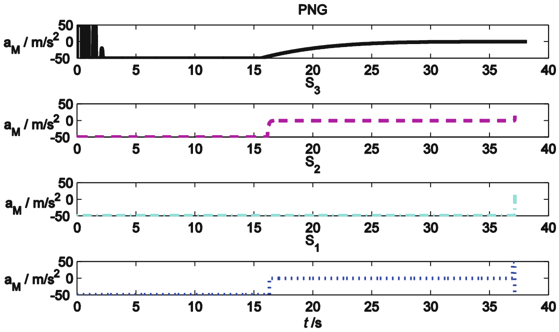

Fig. 4 Guidance command.

Case 2: It is assumed that the target is maneuvering. The velocity and acceleration of target is respectively set to $V_T = 450\,\text{m/s}$ and $a_T = 100\sin(t)\,\text{m}^2/\text{s}$. The simulation time is set to 100 s and the simulation step is set to 0.001s. The engagement trajectory is shown in Fig. 5, and the relative distance is displayed in Fig. 6.

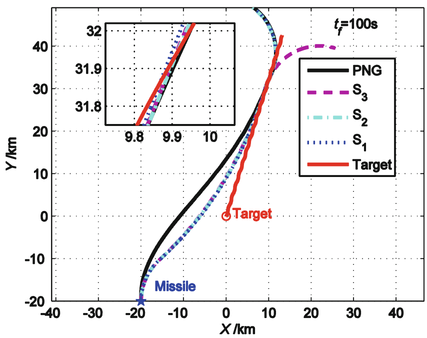

Fig. 5. Flight trajectories

The performances at the interception time are represented in Table 2. By the simulation, the actual miss distances and corresponding interception times for four methods are respectively 28.9759 m and 77.2706 s, 0.2217 m and 77.2920 s, 7.4823 m and 77.3372 s, 6.1301 m and 77.3396 s. It can be clearly seen that when the target is maneuvering, the effect of the three sliding mode guidance methods are better than that of PNG. The

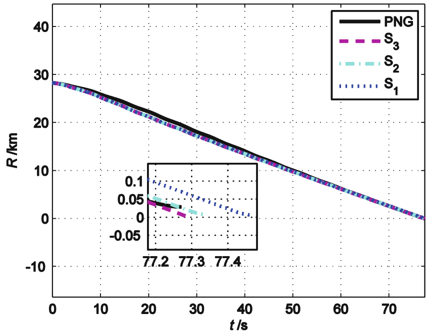

Fig. 6. Relative range

Table 2. Performance results of guidance laws for case 2

Methods	Miss distance (m)	Interception time (s)	Total control effort
PNG	28.9759	77.2706	13215847
S3	0.2217	77.2920	13873439
S2	7.4823	77.3372	13931688
S1	6.1301	77.3396	9626835

guidance command for the four methods is given in Fig. 7. As shown in Fig. 7 and Table 2, when the sliding surface is designed as S3, although the guidance command is not the least, the miss distance is the least compared to the other three methods.

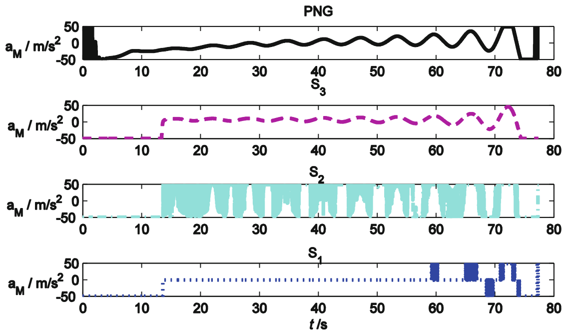

Fig. 7. Guidance command.

From the simulations in case 1 and case 2, it could be concluded that the proposed three slide mode guidance methods has better performance than PNG, especially when the slide mode surface is selected as S_3. The reason is that it can not only ensure that the velocity of the missile points to the target, but also ensure that the distance between the missile and the target is small, as the sliding surface S_3 converging to 0.

5 Conclusions

In this study, the problem of guidance law design against non-maneuvering and maneuvering targets with unknown velocity and acceleration is solved by designing three different sliding mode guidance methods. Compared to the traditional PNG method, the proposed guidance methods can ensures intercepting a maneuvering target. In addition, the proposed approach is novel in that the guidance law considers the uncertainties of target velocity as well as target acceleration. Simulation results, the stability and performance analysis show that the proposed schemes are effective against non-maneuvering and maneuvering targets.

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