Design of Pitch Angle Rate Control Law Based on L1 Adaptive Control Theory

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Abstract. Aiming at the problems of high-frequency oscillation easily caused by the traditional model-referenced adaptive control and the poor uncertainty and robustness of the traditional PID control model. This paper takes L1 adaptive control theory as the background and a scaled-down UAV in the laboratory as the object, first linearizes the UAV leveling to obtain the UAV longitudinal equations, then designs the pitch angle rate control law based on L1 adaptive theory, and designs a low-pass filter based on Liapunov's positive definite principle. Finally, perturbations and uncertainties are added to the model to verify the robustness and resistance to model uncertainties of the L1 adaptive control law, and compared with the traditional model-referenced adaptive and PID control methods, respectively. The results show that the designed L1 adaptive control system can still control the pitch angle rate well with the addition of disturbance and model uncertainty. And it ensures fast adaption with certain robustness and avoids the high-frequency oscillation problem brought by the traditional model-referenced adaption.

Keywords: Flight control · Pitch angle rate · L1 adaptive control · Longitudinal

Flight control system as the "brain" of the aircraft, its performance has an important impact on the stability and safety of the UAV. The current control methods for UAVs are mainly linear and nonlinear methods. The classical PID, LQR and other linear control methods are generally weak against model uncertainty. Nonlinear control methods such as dynamic inverse and backstepping generally require high model accuracy, and it is difficult to exclude the problem of insufficient system robustness caused by errors and external disturbances.

To solve the above problems, L1 adaptive control theory was proposed by Cao Chengyu and Naria Hovakimyan in 2006 [1]. It decouples fast adaptability and robustness, defines adaptive law with error input, introduces feedback loop into low-pass filter, and improves the transient performance of the system by increasing high adaptive gain. Therefore, it has been widely used in wing flutter suppression [2], path tracking [3], longitudinal control design [4], super maneuver control law [5], etc. At present, the research on lateral control of UAV mainly focuses on linear UAV control system [6, 7].

This paper focuses on the linear longitudinal control problem of UAV. In the paper, a laboratory type of scaled-down UAV is used as the research object, and the UAV pitch angle rate controller is designed based on the L1 adaptive theory, and compared

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with the MRAC method and PID control method under disturbance respectively, and the effectiveness and robustness of the L1 adaptive control algorithm are verified by simulation.

1 Construction of the Controlled Object Model

First of all, at the state point of velocity V = 30 m/s and altitude H = 2000 m, a certain type of UAV in the laboratory is leveled and linearized, and the mathematical model of the longitudinal motion of the aircraft is obtained as follows:

$$\dot{x} = Ax + bu \tag{1}$$

Selecting one set of state vectors, i.e., $x = \begin{bmatrix} \alpha \; q \end{bmatrix}^T$ and $u = [\delta_e]$, the collocation linearization results in a longitudinal linear model of the aircraft calculated as follows:

$$A = \begin{bmatrix} -6.1426 & 0.9599 \\ -74.0624 & -6.4824 \end{bmatrix}, b = \begin{bmatrix} -0.4454 \\ -123.5795 \end{bmatrix} \tag{2}$$

where, α denotes the angle of approach, q denotes the pitch rate, and δ_e denotes the elevator deflection. The simulation analysis of the longitudinal L1 adaptive longitudinal pitch angle rate flight control law design can be done based on the above linear model.

2 Design of Pitch Angle Rate Control Law Based on L1 Adaptive Control Theory

Pitch rate maintenance means that the aircraft system is given a pitch reference input q_{REF}, the system output can track well on the system input, and the control system works to maintain this attitude according to the pilot's demand.

2.1 Design of the State Predictor

The uncertain mathematical model of the aircraft system is constructed as follows.

$$\begin{cases} \dot{x}(t) = Ax(t) + b(wu(t) + \theta^T(t)x(t) + \delta(t)) \\ y(t) = c^T x(t) \;\; x(0) = x_0 \end{cases} \tag{3}$$

where, $x(t) \in R^{2\times 1}$ is the observable system state vector; $b, c \in R^{2\times 1}$ is the known constant vector; $A \in R^{2\times 2}$ is the known system matrix; $w \in R$ is the unknown input gain; $\theta_1(t) \in R^{2\times 1}$ is the unknown time-varying parameter vector; $\delta(t) \in R$ is the time-varying disturbance; $y(t) \in R$ is the system output; $u(t) \in R$ is the control signal.

Because the system matrix does not satisfy the Hurwitz matrix condition, the control law $u(t) \in R$ of the controller can be divided into two parts, the linear state feedback control law $u_1(t)$ and the adaptive law $u_2(t)$ as follows.

$$u(t) = u_1(t) + u_2(t) \tag{4}$$

where, $u_1(t) = -K^T x(t)$, $u_2(s) = kD(s)\bar{r}(s)$.

2.2 Design of the Adaptation Lows

From Eqs. (1) and (2), the model of the controlled object at the state point of velocity V = 30 m/s, height H = 2000 m can be described as:

$$\begin{bmatrix} \dot{\alpha} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} -6.1426 & 0.9599 \\ -74.0624 & -6.4824 \end{bmatrix} \begin{bmatrix} \alpha \\ q \end{bmatrix} + \begin{bmatrix} -0.4454 \\ -123.5795 \end{bmatrix} [\delta_e] \tag{5}$$

First check the controllability of the system and whether A satisfies the Hurwitz condition, and then configure the poles of the closed-loop system according to the desired system stability and dynamic quality so that $A_m = A - bK^T$ satisfies the Hurwitz condition.

Since $rank([b, Ab]) = 2$, the system is controllable by L1 adaptive theory, and thus the poles of the matrix $A_m = A - bK^T$ can be arbitrarily configured. The LQR method is used to determine the feedback gain and to configure the poles of $A_m = A - bK^T$ in a reasonable way. Letting Q = diag(50,300) and R = 90, the calculation is able to obtain K as follows.

$$K^T = [0.6250 - 1.7382] \tag{6}$$

At this time, the matrix A_{m} satisfies the Hurwitz matrix.

Since the goal of system control is to enable output $y(t)$ to track a given bounded segmented continuous input signal $r(t) \in R$ by selecting the control signal $u(t)$ of the state feedback controller, the state observer model is as follows.

$$\begin{cases} \dot{x}_m(t) = A_m x_m(t) + b k_g r(t) \\ y_m(t) = c^T x_m(t) \; x_m(0) = x_0 \end{cases} \tag{7}$$

where the role of k_g is to ensure that $y_m(t)$ can track the upper step reference control input with zero static error.

From Eqs. (5), (6) and (7), calculate the value of k_g as follows.

$$k_g = -\frac{1}{c^T A_m^{-1} b} = -1.7180 \tag{8}$$

2.3 Design of the Low-Pass-Filter

In the case where the system control gain is unknown, the low-pass filter cannot be applied directly to the control signal. To solve this problem, the control signal is transformed into a form containing a filter. Using the L1 adaptive controller to generate the control signal, the control input can be described as:

$$u_2(s) = kD(s)\overline{r}(s) \tag{9}$$

where $\overline{r}(s)$ is the Rasch transform of $\overline{r}(t)$, $u_2(s)$ is the Rasch transform of $u_2(t)$, $k > 0$ is the feedback gain and $D(s)$ is a strictly canonical transfer function.

In order to make the selected filter strictly canonical and stable, the filter can be described as:

$$C(s) = \frac{kwD(s)}{1 + kwD(s)} \quad \forall w \in \Omega \tag{10}$$

Here $D(s) = 1/s$ is chosen, then $C(s)$ is a first order filter and satisfies:

$$C(s) = \frac{kw}{s + kw} \quad \forall w \in \Omega \tag{11}$$

Further determination of the parameters in the low-pass filter, using the L1 gain stabilization requisite.

$$\lambda = \|G(s)\|_{L_1} L < 1 \tag{12}$$

$L = 20$ can be calculated. Let $\omega_c = k\omega$, Fig. 1 shows the relationship curve between λ and ω_c:

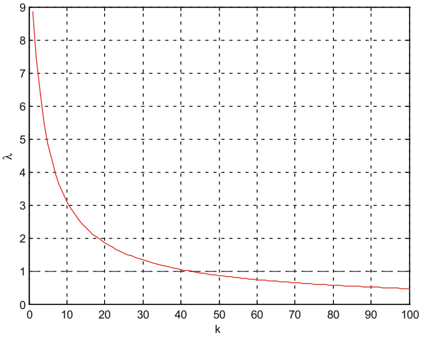

Fig. 1. The relationship curve between λ and ω_c.

As the bandwidth value ω_c of $C(s)$ gradually increases, the value of λ decreases accordingly, and $\omega_c = 90$ can be chosen in the simulation.

2.4 L1 Control Structure and Parameter Setting

The L1 adaptive pitch angle rate control structure consisting of Eqs. (1), (4), (7) and (11) is shown in Fig. 2.

Further set the range of unknown parameters:

$$w \in \Omega = [w_l, w_u], \ \theta(t) \in \Theta, \ |\delta(t)| \le \Delta, \ \left\|\dot{\theta}\right\| \le d_\theta, \ \left\|\dot{\delta}\right\| \le d_\delta \tag{13}$$

where Θ is a given convex set when $t \ge 0$; $0 < w_l < w_u < \infty$ and d_θ, d_δ is a known boundary. So the convex set of set uncertainty parameters is chosen to be $\Omega = [0, 1, 2]$, $\Theta = [-10, 10]$, $\Delta = 10$.

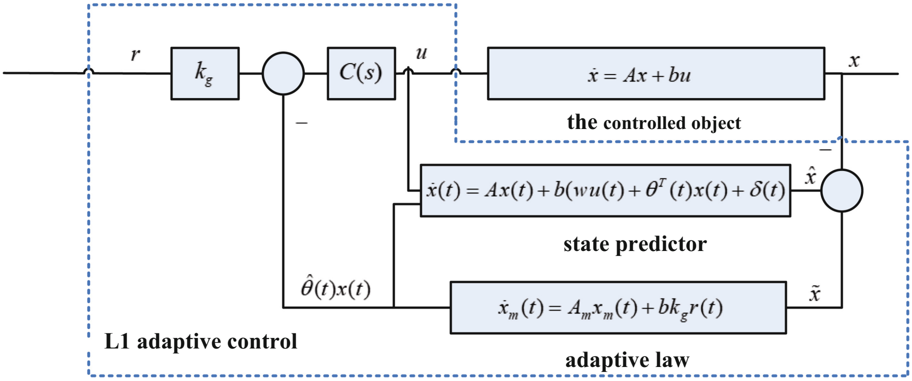

Fig. 2. L1 adaptive pitch angle rate control structure

As the L1 adaptive control effect is to be verified and compared with other control methods. Set the unknown input gain $w = 1$ for the aircraft uncertainty mathematical model of Eq. (2), and the corresponding uncertainty parameter $\theta(t)$ is:

$$\theta(t) = [0.1\sin(\pi t + {}^{\pi}/_{2})\ \ 0.3\sin(0.1\pi t)]^T \tag{14}$$

The time-varying perturbation $\delta(t)$ is:

$$\delta(t) = 0.1\sin(\pi/2t) \tag{15}$$

3 Simulation Comparison

Taking a UAV as the research object, the L1 adaptive control structure established above is used to control the UAV. The initial state of UAV is set as:

$$\begin{aligned} &V = 30\ \text{m/s},\ \alpha = 0.024\,rad,\ \beta = 0,\ p = q = r = 0 \\ &\phi = 0,\ \theta = 0.024\,rad,\ \psi = 0,\ x = 0,\ y = 0,\ z = 2000\ \text{m} \\ &\delta_e = 0.049\ rad,\ \delta_a = \delta_r = 0 \end{aligned}$$

Further collocation linearization yields the longitudinal state matrix of the aircraft as follows: $A = \begin{bmatrix} -6.1426 & 0.9599 \\ -74.0624 & -6.4824 \end{bmatrix}$, $b = \begin{bmatrix} -0.4454 \\ -123.5795 \end{bmatrix}$.

The L1 longitudinal control model is simulated using Matlab/Simulink, and the following scenarios are analyzed.

3.1 Comparison of the Effect of L1 Adaptive Control in the Case of System Without and with Disturbance Factors

The controlled object is the object defined by Eq. (1), the adaptive law takes $\Gamma = 100000$, the input pitch angle rate signal is selected as a square wave signal with $5/57.3$. If the system has disturbance, the selection of the object uncertainty parameter $\theta(t)$ and external time-varying disturbance $\delta(t)$ are shown in Eqs. (14) and (15), respectively; otherwise, both $\theta(t)$ and $\delta(t)$ are made to be 0.

The effect of L1 adaptive control in the case of system without and with disturbance is compared as follows (The left figure shows the system without disturbance).

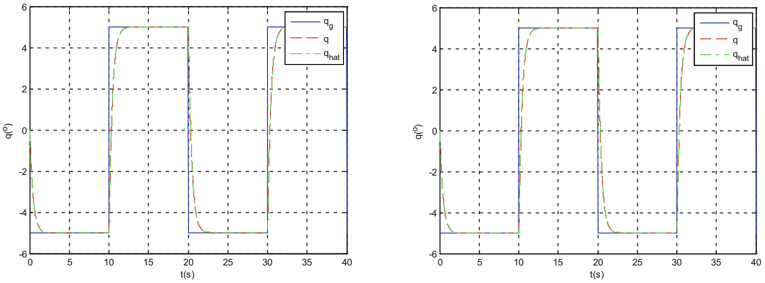

Fig. 3. Comparison of the pitch rate tracking effect of L1 adaptive control in the case of system without and with disturbance

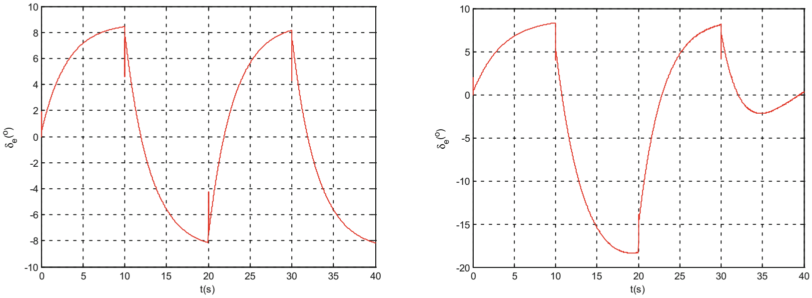

Fig. 4. Comparison of the rudder response effect of L1 adaptive control in the case of system without and with disturbance

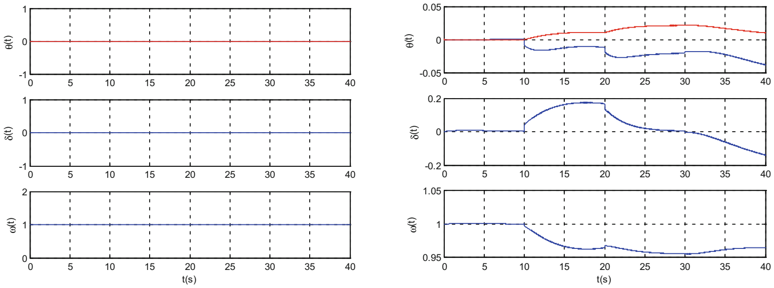

Fig. 5. Comparison of parameter estimation results of L1 adaptive control in the case of system without and with disturbance

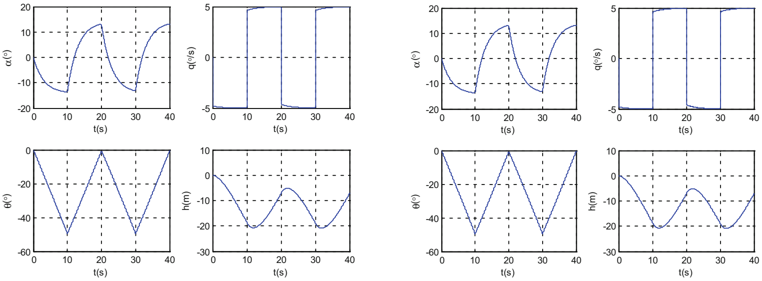

Fig. 6. Comparison of aircraft output response results of L1 adaptive control in the case of system without and with disturbance

From Fig. 3, it can be seen that the system with L1 adaptive control method can ensure good robustness, system output response and rudder adaptive deflection in both the system without and with disturbance, and the system output response can track the reference input well with zero static difference and can achieve fast adaption. Figure 4 shows that the rudder deflections of the elevator are in the controllable range and consistent and bounded when the system without and with disturbance. The rudder deflection is higher when the disturbance is added to the system than when it is not, to counteract the effect of the disturbance. Figure 5 shows that all parameter estimation results are consistently bounded and are guaranteed to vary within the set parameters. Figure 6 shows that all the aircraft output response results are stable and reliable, and all are within the acceptable range.

3.2 Comparison of the Effect of L1 Adaptive Control with Other Control Methods in the Case of System with Disturbance Factors

In order to fully illustrate the superiority of the L1 adaptive control method, the L1 adaptive algorithm is compared with the conventional model-referenced adaptive (MRAC) and PID in the case of the disturbance is added to the system.

The controlled object still selects the object defined in formula (1), the object uncertainty parameter $\theta(t)$ and external time-varying disturbance $\delta(t)$ are shown in Eqs. (14) and (15), the adaptive law is $\Gamma = 100000$ and the given input of the system is $q_g = 5/57.3$, The results of the comparison of L1 adaptive control method with MRAC control and PID control respectively are as follows.

Figure 6 shows that after adding the disturbance, the pitch angle rate output of the system with L1 adaptive control method and MRAC method can track the input command signal well without overshoot and the control effect is excellent, while the PID method has been unable to control the controlled object and the pitch angle rate output of the control system cannot track the upper reference input.

Figure 7 shows that the system with MRAC control method showed obvious high-frequency oscillation of the rudder surface, and the control effect was poor. Comparing MRAC method and PID method, the rudder output of the system with L1 adaptive control method has slow rudder response without spikes, smaller maximum peak of rudder deflection, and higher rudder efficiency (Fig. 8).

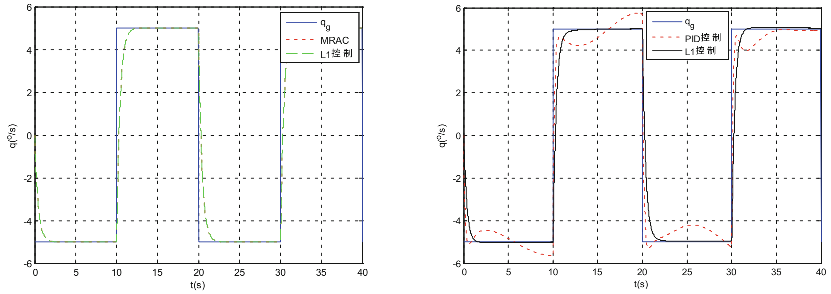

Fig. 7. Comparison of pitch rate control of the L1 adaptive control method with the traditional model-referenced adaptive (MRAC) method and PID method

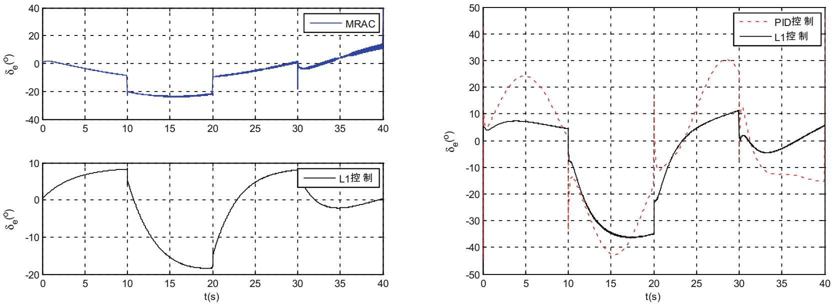

Fig. 8. Comparison of the rudder output effect of the L1 adaptive control method with the traditional model-referenced adaptive (MRAC) method and PID method

Thus, the L1 adaptive control method is superior to the MRAC method and the PID method on balance.

4 Conclusion

In the paper, a pitch angle rate controller based on L1 adaptive theory is designed with a laboratory type of scaled-down UAV as the research object, and the comparison of different disturbance conditions and different methods is carried out. From the simulation results, we can see that the L1 adaptive control method can still control the pitch angle rate well and the rudder adaptive deflection is stable and effective, which can reduce the influence caused by the disturbance well under the condition of adding model uncertainty and disturbance again. In contrast, after the disturbance is added, the traditional MRAC control method shows high frequency oscillation of the rudder surface, and the PID control can no longer control the pitch rate effectively under the disturbance addition. The effectiveness and superiority of the designed L1 adaptive theory algorithm are verified.

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