Modeling Deployment of Tape Spring Antennas and Its Effects on CubeSat Dynamics

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Abstract Tape spring mechanisms have been widely used to design deployable structures for satellites. In this paper, the dynamics of the deployment of VHF tape spring antennas from a CubeSat have been discussed. A set of nonlinear equations of motion representing the coupled dynamics of the deployment and the overall motion of the CubeSat were obtained using Kane's method. These were integrated numerically to obtain parameters of interest like the time taken for deployment, the trajectory of the antenna, and variation of the CubeSat angular velocities during the interval of deployment. These models have been used to analyze the design of Advitiy (1U CubeSat conceptualized by IIT Bombay's Student Satellite Team). Based on simulations, comments have been made on this design of the antenna deployment system, and few performance specifications have been laid down for the CubeSat.

Keywords Tape spring · Deployable antenna · Kane's method · CubeSat dynamics

1 Introduction

Over the last decade, rapid growth has been seen in the CubeSat market. Due to the miniaturization of technology, it is possible to send numerous small satellites or CubeSats rather than one "mega" satellite. Several deployable members like antennas and solar panels are typically incorporated in CubeSats, allowing a smaller volume during the launch. These members are deployed after reaching the desired orbit. The design of such mechanisms is a complicated task. Investigations to develop passive and active control for primitive spacecraft, which were typically rigid, have been carried out. The same analysis cannot be directly extended to modern spacecraft, the primary reason being the inclusion of several flexible members and deployable structures [5].

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R.S. Priyadarshini and T. Sundararajan (eds.), *Advances in Small Satellite Technologies*, Lecture Notes in Mechanical Engineering, https://doi.org/10.1007/978-981-19-7474-8_15

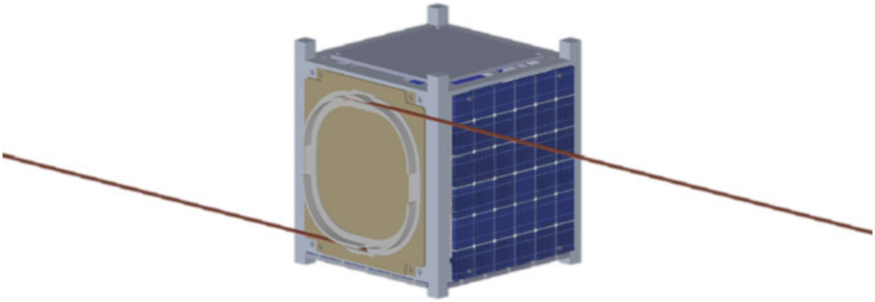

Fig. 1 Advitiy

This paper analyzes the antenna deployment system for CubeSats being developed by the Student Satellite Team, IIT Bombay.[1] The dynamics of the deployment are of interest as it influences several mission parameters. It is essential to know at what angular rates of the satellite can the antennas be deployed. Time taken for the deployment to complete is of interest. Based on the analysis, some remarks have been made on the design of other subsystems.

2 Design of the Antenna Deployment System for CubeSat Application

Student Satellite Team, IIT Bombay, has been developing an antenna deployment system for CubeSat applications [6]. This system comprises two tape spring antennas wrapped around a circular ring supported by a PCB. Advitiy, the second student satellite of IIT Bombay, was being designed with this antenna deployment system [4]. In Fig. 1, Advitiy, along with the antenna deployment system, is shown. Advitiy, a 1U CubeSat, was being designed for a Low Earth Orbit (LEO) mission. The amateur band 144–146 MHz was selected for communication. The monopole or dipole antennas for this band cannot be accommodated inside 1U volume constraints. To meet this, an antenna deployment system was designed. An antenna deployment system, capable of deploying dipole antennas of 51 cm length, was designed and prototyped by the team.

[1] Student Satellite Program, IIT Bombay, is an initiative taken up by the students to build a centre of excellence in space science and technology at IIT Bombay. The first satellite, Pratham, under this program, was launched onboard the PSLV-C35 on 26th September 2016. Information about current projects can be found here: https://www.aero.iitb.ac.in/satlab/.

3 Dynamics of the CubeSat Over the Interval of Deployment

Tape springs are straight, thin-walled strips with a curved cross-section [10]. A carpenter's tape is a tape spring. Due to its curvatures, it exhibits unique behavior. It is stored as a coil but can be extended into a stiff tape that can be easily folded. Except for the region of the fold, the rest of the tape spring is mostly straight and undeformed. Tape springs store elastic strain energy during folding and, in principle, would deploy freely into the straight unstrained configuration once the restraining forces have been released. Tape spring can be folded in two ways, depending upon the sense of curvatures. Several deployable structures that make use of tape springs have been analyzed [2, 9, 11]. Different modeling approaches have been proposed, ranging from a theoretical model [10] to develop a FE-based model [3, 7]. In this paper, a suitable extension is proposed to the theoretical model proposed by Seffen [10].

A model to study the dynamics of uncoiling of a tape spring mounted on a spool and its experimental verification is presented by Seffen [10]. This analysis is applicable when the radius of curvature of the tape spring and tape spool is nearly the same. The tape spring is modeled as a rigid body of varying length, which uncoils from the circular base. This modeling approach assumes the conservation of energy.

The antenna deployment system consists of a disk around which the antennas are wrapped. The CubeSat can be assumed to consist of a rigid cube with two flexible tape spring antennas wrapped around a disk mounted on one of the faces. Figure 2 shows the schematic and parameters used to denote the unwrapping of the antenna from the deployment system. The x and y axes are parallel with the face containing the antenna deployment system, and the z-axis is normal to the face with the antenna deployment system, pointing towards the reader. The x-axis is parallel to the deployed antenna and points towards the right side. It was assumed that no external torques and forces are acting on the system during the deployment. It is expected that the angular momentum of the system remains conserved. The system gains kinetic energy at the expense of the potential energy stored in the coiled tape spring.

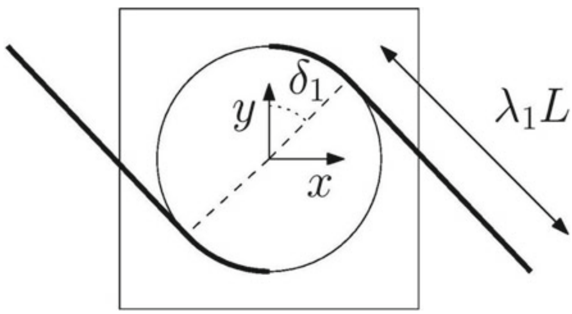

Fig. 2 Schematic of the model

Table 1 Results of simulation on disk

Test ID	Parameters				Results	
	No. of antenna	Thickness (mm)	Direction of folding	Total energy (J)	Deployment time (sec)	Angular velocity (rad/sec)
FT01	1	0.2	Opposite	0.4115	0.177	−10.76
FT02	1	0.1	Opposite	0.0514	0.364	−2.86
FT03	1	0.1	Equal	0.0277	0.496	−2.10
FT04	1	0.2	Equal	0.2216	0.242	−7.92
FT05	2	0.2	Opposite	0.8231	0.170	−19.53
FT06	2	0.1	Opposite	0.1029	0.355	−5.40
FT07	2	0.1	Equal	0.0554	0.484	−3.96
FT08	2	0.2	Equal	0.4432	0.232	−14.35

Before developing the full model for the CubeSat, a more straightforward problem was analyzed - a disk with one or two antennas in a plane, where translation in two axes and rotation about one axis were permitted.[2] Kane's method was used to obtain the dynamics equations. This set of nonlinear differential equations was then integrated numerically. A set of simulations were performed to understand the effect of the number of antennas, thickness, initial energy, and the sense of folding for the tape springs on the final angular velocity attained and the time taken for the deployment. This was later extended for the CubeSat by simulating a cube with one or two antennas, where translations in three axes and rotations about three axes were permitted [8]. Values assigned for various parameters were obtained from the design of Advitiy.

4 Results

4.1 Simulations on Disk

Different test cases, their parameters, and the results have been tabulated in Table 1. For all these simulations, initially, the disk is at rest, and 0.99 fraction of the antenna is wrapped around the disk. The numerical integration is performed over two intervals. Timestep of 0.00001 was used from 0 s to 0.001 s, and a timestep of 0.001 was used from 0.001 s for the rest of the period of analysis. The mass of the disk is the same as that of the satellite, and the moment of inertia for the disk is the same as that for the satellite, about the z-axis.

[2] Refer Appendix—Derivation of dynamics of disk with 1 coiled antenna.

Table 2 Results of simulations on cube

Test ID	Parameters				Results	
	No. of antenna	Thickness (mm)	Direction of folding	Total energy (J)	Deployment time (sec)	Angular velocity (rad/sec)
FT09	1	0.2	Opposite	0.4115	0.177	{3.98, 1.08, – 11.59}
FT10	2	0.2	Opposite	0.8236	0.173	{4.04, 1.65, – 20.13}

4.2 Simulations on Cube

Different test cases, their parameters, and the results have been tabulated in Table 2. For all these simulations, initially, the cube is at rest, and 0.99 fraction of the antenna is wrapped around. The numerical integration is performed over two intervals. Timestep of 0.00002 was used from 0 to 0.001 s, and a timestep of 0.001 for FT09 and 0.002 for FT10 was used from 0.001 s for the rest of the period of analysis.

For the simulations performed, the velocity of the center of mass and the angular momentum were monitored. It was observed that the velocity of the center of mass remains nearly zero, and the angular momentum is conserved. These slight variations have been attributed to numerical errors, but they are within engineering approximations and can be neglected. Since the system begins from rest, the total energy is equal to the potential energy stored due to the tape spring coiling. This potential energy depends on the strain energy per unit area and the coiled area of the tape. The opposite sense of folding is responsible for a higher value of the strain energy, while the equal sense of folding is responsible for a lower value. A thicker antenna, when coiled, stores more strain energy than a thinner antenna.

The plots for test case FT10 are shown. Figure 3 denotes the plot of the fraction of deployment for both the antennas over time. Both the antennas take nearly the same amount of time for deployment. Figure 4 shows the angular velocity of the cube. ω_1, ω_2, and ω_3 denote angular rates about the 3 axes, where ω_3 is about the z-axis, about which the antennas are wrapped around. Hence, maximum change is observed about the z-axis. The non-zero cross-terms in the moment of the inertia matrix are responsible for angular rates about the other two axes.

5 Conclusions

In this paper, a model for coupled dynamics of the deployment of the tape spring antennas mounted on disk and cube were derived, and simulations were performed. The following qualitative remarks can be made from the analysis:

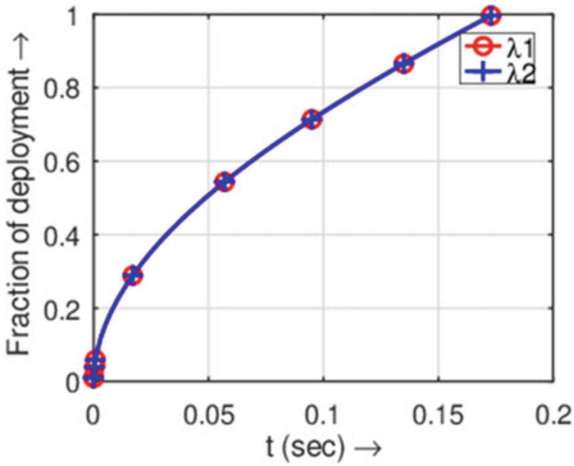

Fig. 3 Fraction of deployment

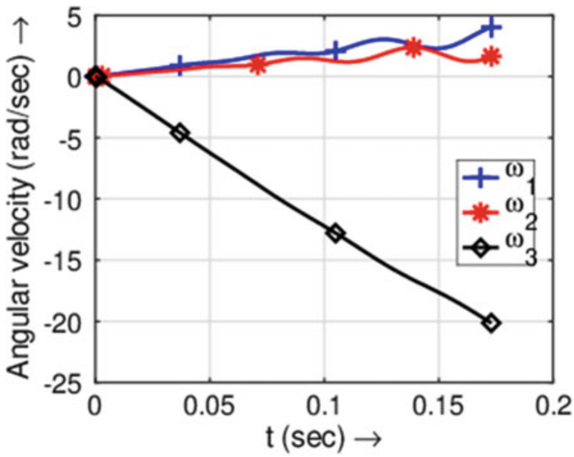

Fig. 4 Angular rates of CubeSat

- The maximum change in angular velocity is observed along the axis about which the antennas are wrapped around.
- A thicker antenna with an opposite sense of bending produces the maximum angular velocity, while a thinner antenna with an equal sense of bending produces the least angular velocity.
- For the same thickness and direction of folding, the system with one antenna and two antennas have nearly the same deployment time, with the system with two antennas requiring slightly less time than the system with one antenna.

Following are some considerations which shall influence the design of other subsystems:

- The initial angular velocity of Advitiy, while in orbit, may be non-zero. Depending on the direction of the unfolding of the antennas, this can either increase or decrease the angular velocity. Hence, the onboard logic should check the angular rates and factor in the direction of the unfolding of antennas. Based on this, an appropriate instant should be decided to issue the command for deployment.
- From the model, the anticipated angular rates with which Advitiy spins after deployment can be evaluated. The attitude determination and control subsystem should be able to control Advitiy under these conditions. Thus, a set of requirements must be generated by performing a more exhaustive set of simulations on the model.

Acknowledgements We want to thank the Student Satellite Team for their inputs, which made this research work possible, and the Department of Mechanical Engineering for their support.

Appendix—Derivation of Dynamics of Disk with 1 Coiled Antenna

The system with a uniform disk and one tape spring antenna wrapped around its periphery is considered. The schematic is shown in Fig. 5. The disk has radius r, mass m_d, and moment of inertia about its center of mass I_d. The tape spring has length L, mass per unit length ρ, the radius of curvature R, and thickness t. The inertial frame is defined using N. The frame attached to the disk, located at its center of mass, is denoted by A. At $\psi = 0$, the frames N and A are parallel to each other. x and y denote the position of the center of mass of the disk in N frame. Let λ_1 denote the fraction of length of tape spring, which has been unwrapped. Let δ_1 denote the angle subtended at the center of the disk by the portion of tape spring coiled, at a given instant. The subscript has been added to identify the antenna. There are two parts of the tape spring, a coiled section, and an unwrapped section. A generic point on the coiled section is denoted by **xP11Aa** and on the unwrapped section by **xP12Aa**. ζ_1 is the parameter, which is dependent on δ_1. Kane's method has been used to derive these equations [1, 8]. **xANn** denotes the position vector of the center of mass of disk with respect to frame N, expressed in unit vectors of N frame. Similar notations have been used throughout.

$$\mathbf{vANa} = \{u_1,\ u_2, 0\}^T \tag{1}$$

$$\mathbf{\&ANa} = \{0, 0, u_3\}^T \tag{2}$$

u_1, u_2, u_3 are generalized speeds.

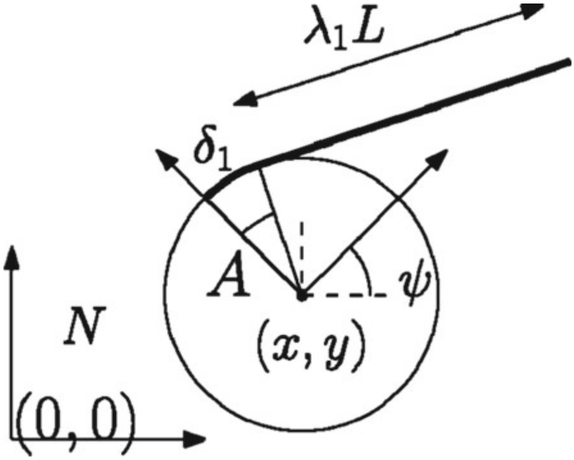

Fig. 5 Schematic for 1 antenna uncoiling from disk

$$L(1 - \lambda_1) = r\delta_1 \tag{3}$$

$$\mathbf{xP11Aa} = \{r\ \sin(\zeta_1 L/r),\ r\ \cos(\zeta_1 L/r),\ 0\}^T \tag{4}$$

$$\mathbf{xP12Aa} = \{r\ \sin(\delta_1) + (L\ \zeta_1 - r\ \delta_1)\ \cos(\delta_1),\ r\cos(\delta_1) \\ -(L\ \zeta_1 - r\ \delta_1)\ \sin\ (\delta_1),\ 0\}^T \tag{5}$$

Velocities are written as follows. $\mathrm{d}\delta_1/\mathrm{d}t$ is set as u_4, the 4th generalized speed, in addition to the 3, defined before.

$$\mathbf{vP11Na} = \mathbf{vANa} + \mathrm{d}\mathbf{xP11Aa}/\mathrm{d}t + \mathbf{\&ANa} \times \mathbf{xP11Aa} \tag{6}$$

$$\mathbf{vP12Na} = \mathbf{vANa} + \mathrm{d}\mathbf{xP12Aa}/\mathrm{d}t + \mathbf{\&ANa} \times \mathbf{xP12Aa} \tag{7}$$

Using these, partial velocities (i =4 terms each. p indicates that these are partial velocities)-$\mathbf{vANap_i}$, $\mathbf{vP11Nap_i}$, $\mathbf{vP12Nap_i}$, and partial angular velocities $\boldsymbol{\omega}\mathbf{ANap_i}$ are evaluated by taking derivatives with generalized speeds. Accelerations are evaluated as follows.

$$\mathbf{aANa} = \mathrm{d}\mathbf{vANA}/\mathrm{d}t + \mathbf{\&ANa} \times \mathbf{vANa} \tag{8}$$

$$\mathbf{aANa} = \mathrm{d}\mathbf{\&ANa}/\mathrm{d}t \tag{9}$$

$$\mathbf{aP11Na} = \mathrm{d}\mathbf{vP11Na}/\mathrm{d}t + \mathbf{\&ANa} \times \mathbf{vP11Na} \tag{10}$$

$$\mathbf{aP12Na} = \mathrm{d}\mathbf{vP12Na}/\mathrm{d}t + \mathbf{\&ANa} \times \mathbf{vP12Na} \tag{11}$$

The potential energy for the coiled section of the tape spring is written as PE = $\mu R^2 \alpha \delta_1$ where μ denotes strain energy per unit area, α angle of the embrace of the cross-section. Generalized inertia forces, $F^*{}_i$, and generalized active force, F_i, are written

$$F_i^* = -\Big[m_d \mathbf{aANa} \cdot \mathbf{vANap_i} + I_d \mathbf{aANa} \cdot \mathbf{\&ANap_i} + \int_0^{r\,\delta 1\,/L} \rho L(\mathbf{aP11Na} \cdot \mathbf{vP11Nap_i})\mathrm{d}\,\zeta_1 + \int_{r\,\delta 1\,/L}^{1} \rho L(\mathbf{aP12Na} \cdot \mathbf{vP12Nap_i})d\,\zeta_1\Big] \tag{12}$$

$$F_4 = -\mathrm{dPE}/\mathrm{d}\delta_1 \tag{13}$$

Other generalized active force terms are zero. Using these, equations of motion are obtained.

$$\begin{aligned}
&-(L\rho + m_d)\mathrm{d}u_1/\mathrm{d}t \\
&-0.5\rho\big[-2r^2\sin(\delta_1) - 2r(L - r\,\delta_1)\cos(\delta_1) + (L - r\,\delta_1)^2\sin(\delta_1)\big]\mathrm{d}u_3/\mathrm{d}t \\
&+0.5\rho\big[(L - r\,\delta_1)^2\sin(\delta_1)\big]\,\mathrm{d}u_4/dt = -(L\rho + m_d)u_2u_3 \\
&+2r^2u_3^2(1 - \cos(\delta_1)) + 2r(L - r\,\delta_1)\big(u_4^2 - u_3^2\big)\sin(\delta_1) \\
&+(L - r\,\delta_1)^2(u_4 - u_3)^2\cos(\delta_1)
\end{aligned} \tag{14}$$

$$\begin{aligned}
&-(L\rho + m_d)\mathrm{d}u_2/\mathrm{d}t \\
&-0.5\rho\big[2r^2(1 - \cos(\delta_1)) + 2r(L - r\,\delta_1)\sin(\delta_1) + (L - r\,\delta_1)^2\cos(\delta_1)\big]\mathrm{d}u_3/\mathrm{d}t \\
&+0.5\rho\big[(L - r\,\delta_1)^2\cos(\delta_1)\big]\,\mathrm{d}u_4/\mathrm{d}t = (L\rho + m_d)u_1u_3 - 2r^2u_3^2\sin(\delta_1) \\
&+2r(L - r\,\delta_1)\big(u_4^2 - u_3^2\big)\cos(\delta_1) + (L - r\,\delta_1)^2(u_4 - u_3)^2\sin(\delta_1)
\end{aligned} \tag{15}$$

$$\begin{aligned}
&0.5\rho\big[-2r^2\sin(\delta_1) - 2r(L - r\,\delta_1)\cos(\delta_1) + (L - r\,\delta_1)^2\sin(\delta_1)\big]\mathrm{d}u_1/\mathrm{d}t \\
&+0.5\rho\big[2r^2(1 - \cos(\delta_1)) + 2r(L - r\,\delta_1)\sin(\delta_1) + (L - r\,\delta_1)^2\cos(\delta_1)\big]\mathrm{d}u_2/\mathrm{d}t \\
&+\big[I_d + \rho L r^2 + (\rho/3)(L - r\,\delta_1)^3\big]\mathrm{d}u_3/\mathrm{d}t \\
&-(\rho/3)(L - r\,\delta_1)^3\mathrm{d}u_4/\mathrm{d}t \\
&\quad = -r^2u_1u_3\rho(1 - \cos(\delta_1)) - r\rho(L - r\,\delta_1)^2(u_4 - u_3)u_4 \\
&-r\rho u_3(L - r\,\delta_1)[u_2\cos(\delta_1) + u_1\sin(\delta_1)] \\
&-0.5\rho u_3(L - r\,\delta_1)^2[u_1\cos(\delta_1) - u_2\sin(\delta_1)]
\end{aligned} \tag{16}$$

$$\begin{aligned}&0.5\rho(L-r\ \delta_1)^2[\mathrm{d}u_2/\mathrm{d}t\ \ \cos(\delta_1)+\mathrm{d}u_1/\mathrm{d}t\ \ \sin(\delta_1)]\\&-(\rho/3)(L-r\ \delta_1)^3[\mathrm{d}u_4/\mathrm{d}t-\mathrm{d}u_3/\mathrm{d}t]\\&=-0.5\rho u_3(L-r\ \delta_1)^2[u_1\cos(\delta_1)-u_2\sin(\delta_1)]\\&-0.5r\rho(L-r\ \delta_1)^2\left(u_4^2-u_3^2\right)+\mu R^2\alpha\end{aligned} \tag{17}$$

Note that these equations of motion (14–17) are functions of the generalized speeds, their first derivatives, and only one generalized coordinate δ_1. These are solved simultaneously with the following four Eqs. (18–21) to obtain the trajectory of the system in terms of the generalized coordinates of the system.

$$\mathrm{d}x/\mathrm{d}t=u_1\cos(\psi)-u_2\sin(\psi) \tag{18}$$

$$\mathrm{d}y/\mathrm{d}t=u_2\cos(\psi)+u_1\sin(\psi) \tag{19}$$

$$\mathrm{d}\psi/\mathrm{d}t=u_3 \tag{20}$$

$$\mathrm{d}\delta_1/\mathrm{d}t=u_4 \tag{21}$$

The example considered by Seffen [10] restricts the translation of the disk. The set of equations can be reduced by setting $u_1=u_2=0$ and $\mathrm{d}u_1/\mathrm{d}t=\mathrm{d}u_2/\mathrm{d}t=0$ to verify these against the equations presented by Seffen [10].

This approach of deriving dynamics has been extended for a disk with two antennas and a cube with one or two antennas [8]. Use of symbolic manipulation software is highly recommended to obtain and solve these equations.

References

1. Banerjee AK (2016) Flexible multibody dynamics: efficient formulations and applications. John Wiley & Sons
2. Dewalque F, Collette JP, Brüls O (2016) Mechanical behaviour of tape springs used in the deployment of reflectors around a solar panel. Acta Astronaut 123:271–282
3. Guinot F, Bourgeois S, Cochelin B, Blanchard L (2012) A planar rod model with flexible thin-walled cross-sections. Application to the folding of tape springs. Int J Solids Struct 49(1):73–86
4. Kothari H, Lohiya P, Zubair A, Singh S (2020) Satellite structure of Advitiy (second student satellite of IIT Bombay). In: Advances in small satellite technologies. Springer, Singapore, pp 535–544
5. Modi VJ (1974) Attitude dynamics of satellites with flexible appendages-a brief review. J Spacecr Rocket 11(11):743–751
6. Patki A, Jagdale K, Munjal M (2020) Design approach to antenna deployment system for nano-satellite applications. In: Advances in small satellite technologies. Springer, Singapore, pp 523–533

7. Picault E, Marone-Hitz P, Bourgeois S, Cochelin B, Guinot F (2014) A planar rod model with flexible cross-section for the folding and the dynamic deployment of tape springs: improvements and comparisons with experiments. Int J Solids Struct 51(18):3226–3238
8. Ranade AR (2020) Dynamics modeling of deployment of tape spring antennas and its effects on CubeSat dynamics. Master's Thesis, Indian Institute of Technology Bombay
9. Seffen KA, Pellegrino S (1997) Deployment of a rigid panel by tape-springs
10. Seffen KA, Pellegrino S (1999) Deployment dynamics of tape springs. Proc Royal Soc London Ser A Math Phys Eng Sci 455(1983):1003–1048
11. Walker SJ, Aglietti G (2004) Study of the dynamics of three dimensional tape spring folds. AIAA J 42(4):850–856