Magnetohydrodynamic Mixed Convection Flow in a Vertical Channel Filled with Porous Media

Nidhi Singh and Manish K. Khandelwal

Abstract We report a linear instability mechanism of MHD mixed convection flow in a porous medium channel under a transverse magnetic field. The stability results are reported for an electrically conducting water-based electrolytes fluid. The governing equations are solved by a Chebyshev spectral collocation method. The linear disturbance equations formed a generalized eigenvalue problem. The results show that the basic flow contains the inflection point. The linear stability analysis shows that the growth of the disturbance reduces by increasing the strength of the magnetic field and decreasing the media permeability of the porous medium flow. The linear stability boundaries show that the relatively higher strength of the applied magnetic field stabilizes the flow, whereas an increase in the media permeability destabilizes the basic flow.

Keywords MHD flow · Mixed convection · Porous medium

1 Introduction

The phenomenon of mixed convection occurs due to thermal buoyancy force as well as external pressure gradient. The mixed convection flows through porous medium have been examined extensively due to wide applications in the electronic industry. The porous medium is an excellent candidate to enhance the heat in many heat transfer applications. The stability of mixed convection flow in vertical geometries has been the object of great interest in several applications namely heat exchangers, nuclear

N. Singh (✉) · M. K. Khandelwal
Department of Mathematics, Indira Gandhi National Tribal University, Amarkantak, MP 484887, India

M. K. Khandelwal
e-mail: khandelwal@igntu.ac.in; manish@iiitdm.ac.in

M. K. Khandelwal
Department of Mathematics, Indian Institute of Information Technology, Design and Manufacturing, Kancheepuram, Chennai 600127, India

R. K. Sharma et al. (eds.), *Frontiers in Industrial and Applied Mathematics*,
Springer Proceedings in Mathematics & Statistics 410,
https://doi.org/10.1007/978-981-19-7272-0_24

reactors, solar collectors, electronic equipment. The researchers have already studied the hydrodynamic instability properties of mixed convection flow in the vertical configurations [1–6] under different types of heating conditions. The flow of an electrically conducting fluid through a porous medium under an applied magnetic field has received considerable attention in many technological and laboratory flows. The study of magnetohydrodynamic (MHD) flow of electrically conducting fluid through a porous medium is essential in many MHD-related applications such as blankets (e.g., dual-coolant lead–lithium (DCLL) blanket) for thermonuclear reactors, MHD generators, crystal growth, and stirring of melts in the metallurgical industry, and electronic devices [7, 8]. The interaction of electrically conducting fluid with the magnetic field through inter-connected porous medium flow gives a significant heat enhancement in the above-mentioned applications.

The understanding of hydrodynamic stability analysis of MHD porous media flow is a fundamental interest in many applications. The present paper focuses on the stability analysis of parallel mixed convection flow in a linearly heated vertical porous medium channel with a transverse magnetic field. The flow instabilities may appear by many factors, particularly by a magnetic field, heat transfer, and medium permeability of the porous medium. The present study will provide the basic concept of the flow instability mechanism of an electrically conducting fluid through a porous medium in the presence of a magnetic field. There are some studies relevant to the present investigation in a vertical configuration. We summarize some important conclusions, which may support the present investigation.

The instability mechanism of non-magnetic fully developed mixed convection flow in a vertical channel filled with a porous medium is well established in the open literature using linear and weakly nonlinear stability analysis [9–15]. In these studies, two different types of state the local thermal equilibrium (LTE) state and local thermal non-equilibrium (LTNE) state are used for energy equations to examine the instability characteristics of porous medium flow. The effect of media permeability [9], the influence of Prandtl number [10], impact of different models [11], the impact of inter-phase heat transfer coefficient [12] for porous medium flow are discussed in terms of stability analysis. These porous medium parallel mixed convection channel flow studies show that flow is most unstable under two-dimensional. The kinetic energy balance mainly gives three different instability types: buoyant, shear, and thermal-shear (mixed) instability for assisted flow and Rayleigh–Taylor instability for buoyancy-opposed flow.

The MHD flow through porous media is investigated very little in the open literature. However, few important theoretical and experimental studies on porous medium flow in the presence of a magnetic field in different geometries are available in the literature [7, 16–22]. Wallace et al. [16] have investigated an experimental study for the flow of mercury in porous media (sandstone) under a magnetic field. They have shown that the flow rate of mercury through the porous media under the magnetic field does not change. Later, Rudraiah et al. [22] have performed a theoretical and numerical study of Hartmann flow to validate experimentally obtained results of Wallace et al. [16]. Using multiple scale expansions, Geindreau and Auriault [7] have examined the macroscopic description of seepage in porous media under magnetic field.

In contrast to instability properties of porous medium flow under magnetic field, the instability mechanism for viscous-medium flow in a vertical channel under applied magnetic field is discussed rigorously. For example, a survey of existing published literature: related to natural convection [23–25], in connection with forced convection [26–28], and in connection with mixed convection [29–32] focuses on the instability mechanism of parallel channel flow under magnetic field. In these studies, the magnetic field, in general, stabilizes the basic flow. However, in the mixed convection flow, thermal buoyancy force destabilizes the MHD flow.

The above literature review shows that the hydrodynamic stability characteristics of mixed convection MHD flow in a vertical channel filled with a porous medium is not considered yet in our best knowledge. Therefore, we aim to discuss the stability properties of mixed convection MHD porous medium flow using linear stability analysis in a vertical channel. The present study will provide a new research development in many MHD applications.

2 Governing Equations

We consider an incompressible MHD flow in a long vertical channel filled with a porous medium. The width of the channel is *2L*. A uniform transverse magnetic field of strength $\boldsymbol{B_0}$ is applied perpendicularly to the direction of the flow, as shown in Fig. 1. Buoyancy force and an external pressure gradient drive the flow under a uniform magnetic field. A linearly varying temperature is considered on the walls as $T_w = T_0 + Cz$ where T_0 denotes the reference temperature and C is a positive constant. The schematic of flow configuration is displayed in Fig. 1. Thermo-physical properties of the fluid are assumed constant except density in the buoyancy force term. The Boussinesq approximation is used for the density variation. In the present study, we have adopted volume-averaged Navier–Stokes (VANS) equations for transporting porous medium [33] to analyze the flow instability. The non-dimensional governing equation of the present problem is given by

$$\nabla.\boldsymbol{V} = 0 \tag{1}$$

$$\frac{1}{\varepsilon}\frac{\partial \boldsymbol{V}}{\partial t} + \frac{1}{\varepsilon^2}(\boldsymbol{V}.\nabla)\boldsymbol{V} + F\boldsymbol{V}|\boldsymbol{V}| = -\nabla p + \frac{\lambda}{Re}\nabla^2\boldsymbol{V} - \frac{1}{DaRe}\boldsymbol{V} + \frac{Ra}{Re}\theta\overrightarrow{e_z} + \frac{Ha^2}{Re}(\boldsymbol{j}\times\overrightarrow{e_x}) \tag{2}$$

$$\sigma\frac{\partial\theta}{\partial t} + V.\nabla\theta = \frac{1}{RePr}\left(\nabla^2\theta - w\right) \tag{3}$$

$$\boldsymbol{j} = -\nabla\phi + (\boldsymbol{V}\times\overrightarrow{e_x}) \tag{4}$$

$$\nabla.\boldsymbol{j} = 0 \tag{5}$$

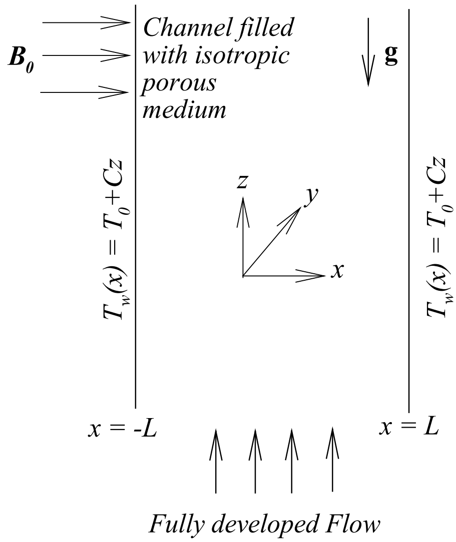

Fig. 1 Physical problem and coordinate system

where $\vec{e_x}$, $\vec{e_z}$, σ, and ε are the unit vector in the x-direction and z-direction, the ratio of the volumetric heat capacities of the fluid and medium and porosity of the medium, respectively. Following non-dimensional quantities are used to non-dimensionalized the above governing equations

$$(x, y, z) = \frac{(x^*, y^*, z^*)}{L}, \boldsymbol{V} = \frac{\boldsymbol{V^*}}{\overline{\overline{W_0}}}, P = \frac{p^*}{\rho_0 \overline{W_0}^2}, t = \frac{t^* \overline{W_0}}{L}, \theta = \frac{T - T_W}{CLRePr},$$

$$\phi = \frac{\varnothing}{L\boldsymbol{B}_0 \overline{\overline{W_0}}}, \boldsymbol{j} = \frac{\boldsymbol{j}^*}{\sigma_1 \boldsymbol{B}_0 \overline{\overline{W_0}}} \tag{6}$$

where $\boldsymbol{V}$, P, t, θ, $\boldsymbol{j}$, ϕ are the dimensionless velocity vector, pressure, time, temperature, current density, and electrical potential, respectively. The following non-dimensional parameters are appeared in the present problem: Rayleigh number ($Ra = gCL^4\beta_T/\nu\alpha$), Reynolds number ($Re = \overline{W_0}L/\nu$), Prandtl number ($Pr = \nu/\alpha$), Darcy number ($Da = K/L^2$), Forchheimer number ($F = C_F L/|K|^{1/2}$), viscosity ratio ($\lambda = \overline{\mu}/\mu_f$), interaction parameter ($N = \sigma_1 L \boldsymbol{B}_0{}^2/\rho_0 \overline{W_0}$), and Hartmann number ($Ha = \sqrt{NRe}$). Furthermore, $\overline{W_0}$ is average base velocity, ρ_0 is reference fluid density, α is the thermal diffusivity, β_T is the thermal expansion coefficient, ν is the kinematic viscosity, g acceleration due to gravity, K the permeability of the porous medium, C_F is form drag coefficient, $\overline{\mu}$ is coefficient of effective viscosity, μ_f is the fluid viscosity, and σ_1 is the electrical conductivity. Note that the value of σ and λ is taken 1 for the present investigation.

2.1 Basic Flow

To investigate the instability mechanism of the mixed convective flow, first we derive the basic flow that is steady state, unidirectional, and fully developed. Using these conditions, the governing Eqs. (1)–(5) reduces into the following ordinary differential equations

$$\lambda \frac{d^2 W_0}{dx^2} - \frac{1}{Da} W_0 - ReF|W_0|W_0 - Ha^2 W_0 + Ra\Theta_0 = Re\frac{dP_0}{dz} \tag{7}$$

$$\frac{d^2\Theta_0}{dx^2} = W_0 \tag{8}$$

The boundary conditions for the basic flow at the channel walls are

$$W_0 = \Theta_0 = 0 \text{ at } x = \pm 1 \tag{9}$$

where W_0, Θ_0, and P_0 are the basic state velocity, temperature, and pressure, respectively.

2.2 Linear Stability Analysis

The classical normal mode analysis [34] is considered to examine the linear stability analysis of the above MHD mixed convection basic flow. The infinitesimal disturbance is imposed on the basic flow. Thus the velocity, temperature, and pressure field can be written as

$$(u, v, w, \theta, P) = \left(u', v', W_0(x) + w', \Theta_0(x) + \theta', P_0(z) + p'\right) \tag{10}$$

In the above equation, primed quantities denote infinitesimal disturbance to the corresponding field variable. The infinitesimal disturbance can be written in the form of traveling waves [34]

$$X'(x, y, z, t) = \widehat{X}(x)e^{i(\alpha z + \beta y - \alpha c t)} \tag{11}$$

where X' denotes field variables, α and β are wavenumbers in the z and y directions, respectively. $c = c_r + ic_i$ represents the complex wave speed. The behavior of disturbance (growth/decay) depends upon the sign of c_i. The flow is unstable, neutrally stable, or stable accordingly as $c_i > 0$, $c_i = 0$, or $c_i < 0$, respectively. The linear disturbance equations for above basic flow are given as

$$i\alpha\widehat{w} + i\beta\widehat{v} + \frac{d\widehat{u}}{dx} = 0 \tag{12}$$

$$-\frac{1}{\varepsilon^2}i\alpha W_0\widehat{u} - \frac{d\widehat{p}}{dx} + \frac{\lambda}{Re}\left(\frac{d^2\widehat{u}}{dx^2} - \left(\alpha^2+\beta^2\right)\widehat{u}\right) - \frac{\widehat{u}}{DaRe} - F|W_0|\widehat{u} = -\frac{1}{\epsilon}i\alpha c\widehat{u} \tag{13}$$

$$-\frac{1}{\varepsilon^2}i\alpha W_0\widehat{v} - i\beta\widehat{p} + \frac{\lambda}{Re}\left(\frac{d^2\widehat{v}}{dx^2} - \left(\alpha^2+\beta^2\right)\widehat{v}\right) - \frac{\widehat{v}}{DaRe} - F|W_0|\widehat{v} - N(i\alpha\widehat{\phi} + \widehat{v}) = -\frac{1}{\varepsilon}i\alpha c\widehat{v} \tag{14}$$

$$-\frac{1}{\epsilon^2}i\alpha W_0\widehat{w} - i\alpha\widehat{p} + \frac{\lambda}{Re}\left(\frac{d^2\widehat{w}}{dx^2} - \left(\alpha^2+\beta^2\right)\widehat{w}\right) - \frac{1}{\epsilon^2}\frac{dW_0}{dx}\widehat{u} - \frac{\widehat{w}}{DaRe} - 2F|W_0|\widehat{w} + N\left(i\beta\widehat{\phi} - \widehat{w}\right) + \frac{Ra}{Re}\widehat{\theta} = -\frac{1}{\varepsilon}i\alpha c\widehat{w} \tag{15}$$

$$-i\alpha W_0\widehat{\theta} + \frac{1}{RePr}\left(\frac{d^2\widehat{\theta}}{dx^2} - \left(\alpha^2+\beta^2\right)\widehat{\theta} - \widehat{w}\right) - \frac{d\Theta_0}{dx}\widehat{u} = -\sigma i\alpha c\widehat{\theta} \tag{16}$$

$$\left[\frac{d^2\widehat{\phi}}{dx^2} - \left(\alpha^2+\beta^2\right)\widehat{\phi}\right] - i\beta\widehat{w} + i\alpha\widehat{v} = 0 \tag{17}$$

Finally, Eqs. (12)–(17) constitute a generalized eigenvalue problem.

2.3 Numerical Method

In order to determine the numerical solution of Eqs. (7) and (8) (basic flow equations) and (12)–(17) (linear disturbance equations) along with boundary conditions, a high accurate spectral collocation method has been used. The Chebyshev polynomial is used as a basis set for spectral collocation method. The equations have been discretized along the x-direction at Gauss–Lobatto points. These are the extreme points of an M-degree Chebyshev polynomial and given by

$$x_j = cos\left(\frac{\pi j}{M}\right),\ j = 0, 1, 2, \ldots, M$$

The main emphasis in spectral collocation method is to construct differential operator which is given by

Table 1 Comparison between published and present results under special case $Ha = 0$, $\mathrm{Da} = 10^{12}$, $\boldsymbol{\varepsilon} = 1$

Re	*Pr*	Published result [1]		Present	
		Ra_c	α_c	Ra_c	α_c
100	0.7	41.65	0.875	41.646	0.873
1000	0.7	30.26	1.355	30.263	1.355
100	7.0	15.73	0.24	15.738	0.237
1000	7.0	15.60	0.024	15.605	0.024
100	100	8.61	0.108	8.614	0.108
1000	100	8.6	0.011	8.597	0.011

$$D_{jk}^{(1)} = \begin{cases} \frac{c_j(-1)^{k+j}}{c_k\left(x_j - x_k\right)}, & j \neq k \\ \frac{x_j}{2\left(1 - x_j^2\right)}, & 1 \leq j = k \leq N-1 \\ \frac{2N^2+1}{6}, & j = k = 0 \\ -\frac{2N^2+1}{6}, & j = k = N \end{cases}$$

The other higher order derivative can be obtained from lower order derivative by differentiating them. In this process, the differentiation operator takes the role of the derivative. Using spectral method discretization scheme, equations are transformed into a generalized eigenvalue problem of the form

$$AX = cBX \tag{18}$$

where X represents the eigenvector of the field variable, c is an eigenvalue. The square matrices A and B represent the coefficients of linear disturbance equations. The details of the considered numerical method with implementation procedure can be seen in the reference [32]. The eigenvalues and eigenvectors of the eigenvalue problem (18) are calculated by MATLAB software. The obtained results are compared with published results [1] under some special cases. The results calculated by our numerical code are in good match with the published one (Table 1).

3 Results and Discussion

In the present section, we discuss the results of mixed convection flow of electrically conducting fluid in a vertical channel filled with porous medium. The present problem is governed by six non-dimensional parameters, namely Darcy number (*Da*), Reynolds number (*Re*), Prandtl number (*Pr*), Rayleigh number (*Ra*), Hartmann number (*Ha*), and Forchheimer number (*F*). The main emphasis is considered on basic flow, disturbance growth rate profiles, and stability boundaries under a weak to a moderate value of the magnetic field. The stability results are determined for electrically conducting water-based electrolytes fluid, whose Prandtl number (Pr) is 7.01 [24]. The analysis is considered for high permeable porous medium flow. Therefore, the porosity of the medium is taken at 0.9. The Reynolds number is fixed at $Re = 1000$ for the present investigation. The value of F is calculated in terms of C_F and Da, i.e., $F = C_F/\sqrt{Da}$, where C_F is fixed at 0.006.

First, we examine the basic flow results to examine the impact of magnetic field and media permeability of the MHD porous medium flow. Figure 2 shows a variation of the basic velocity under different magnetic field strengths for $Da = 10^{-2}$. The basic velocity profiles contain the point of inflection near the channel walls. It is observed that point of inflection in the velocity profile smooth out slowly on increasing the value of magnetic field parameter Ha, and the velocity profile becomes flattened. The maximum velocity occurs near the channel walls. The impact of the media permeability in terms of the Darcy number under magnetic field on basic velocity is investigated in Fig. 3. The high permeable flow provokes a clear point of inflection in the velocity profile. The velocity profile is relatively more flattened under a low permeability case (see for $Da = 10^{-2}$). We have also examined the impact of thermal buoyancy force under magnetic field on basic velocity and temperature in Fig. 4a–b. It is observed that increasing the thermal buoyancy force in terms of Rayleigh number invites the point of inflection in the basic velocity profile. The magnitude of the basic velocity near channel walls increases on increasing the strength of the thermal buoyancy force. The increase in the value of Ra also results in increase in the magnitude of basic temperature (see Fig. 4b).

The above basic flow analysis indicates that the basic velocity profiles contain the inflection point, which could increase the flow's instability. Based on this analysis, we can predict that the instability of the flow decreases on increasing the value of Hartmann number, i.e., applied magnetic field has a tendency to stabilize the flow. However, increased media permeability acts in the reverse way of the magnetic field. We have examined the linear stability properties to confirm the observations of basic flow.

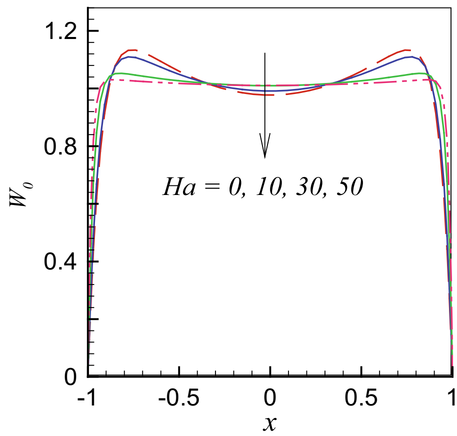

Fig. 2 Basic velocity profile for $Da = 10^{-2}$, $Ra = 150$, and different values of Ha

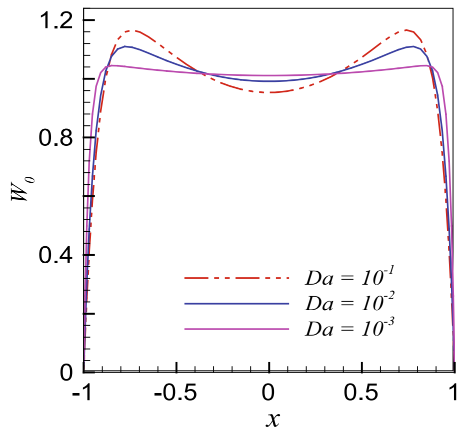

Fig. 3 Basic velocity profile for $Ha = 10$, $Ra = 150$, and different values of Da

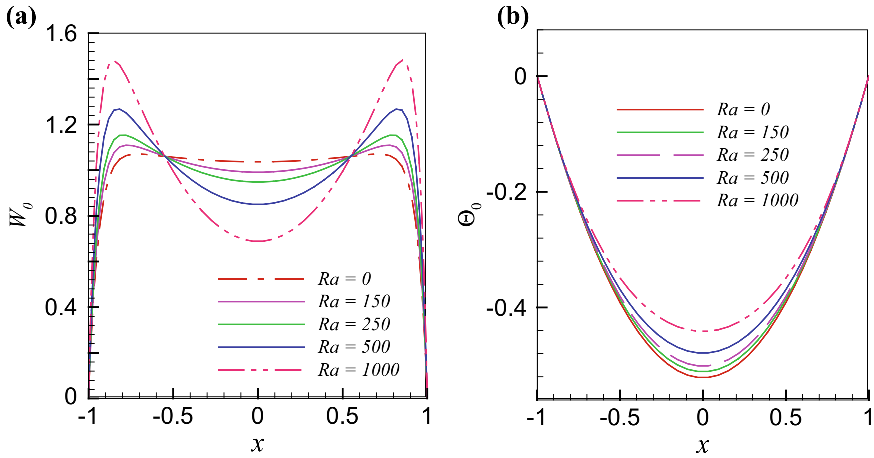

Fig. 4 **a** Basic velocity profile. **b** Basic temperature profile for $Ha = 10$, $Da = 10^{-2}$, and different values of Ra

We have tested several numerical tests for different parameter sets to know the least stable mode of linear stability. It is found that MHD porous medium flow is least stable under two-dimensional mode. Therefore, we have examined present linear stability results for spanwise wavenumber $\beta = 0$. The disturbance growth of the most unstable mode is one of the important features in the instability of the flow. To know the instability behavior of mixed convection MHD flow of electrically conducting water-based electrolytes fluid, we plot the disturbance growth rate contours in (Ha, α)-plane for $Da = 10^{-2}$ and $Da = 10^{-3}$. The positive (negative) value of the growth rate indicates the unstable (stable) nature. The disturbance growth rate contours show

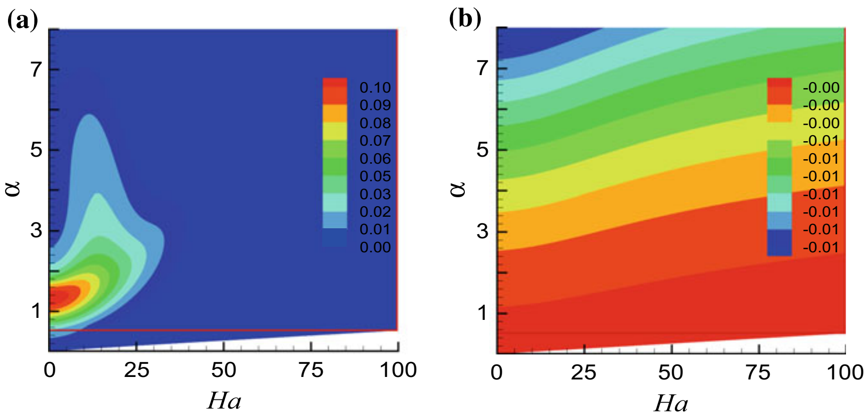

Fig. 5 Contour of the growth rate of the most unstable mode in (Ha, α)-plane at $Ra = 1000$ **a** $Da = 10^{-2}$ and **b** $Da = 10^{-3}$

that growth of the disturbance reduces on enhancing the strength of the magnetic field, i.e., flow instability reduces on enhancing the value of Hartmann number. Figure 5a shows an unstable zone for weak magnetic field strength, but for relatively high magnetic field strength, there is no unstable zone. We have also observed the decrease in the media permeability gives a more stable flow. Figure 5b shows a complete stable flow for the same parameters as Fig. 5a. The analysis shows that the instability of the flow grows by increasing the media permeability of the porous medium. The qualitative behavior of the growth rate analysis helps to examine the linear instability boundaries. The linear stability results are calculated in terms critical value of the Rayleigh number.

The instability boundaries for three different values of Darcy number ($Da = 10^{-1}$, 10^{-2}, and 10^{-3}) in (Ha, Ra_c)-plane is plotted in Fig. 6 to understand the influence of magnetic field on instability boundaries. The critical value of the Rayleigh number increases on increasing the strength of the magnetic field. For the higher value of Ha, the generated Lorentz force in the flow stabilizes the basic flow. On the other hand, increasing the Darcy number's value reduces the critical value of Rayleigh number, i.e., the flow stability reduces by increasing the media permeability of the porous medium flow.

To gain further insight about the characteristics of the instability mechanism, we have plotted eigenfunctions of a disturbance at a critical level for different values of magnetic field parameter, i.e., Hartmann number in Fig. 7 for $Da = 10^{-2}$. The disturbance fluctuation reduces by increasing the value of Ha. The magnitude of temperature disturbance is larger in comparison to the velocity disturbance eigenfunction for all considered values of the Ha.

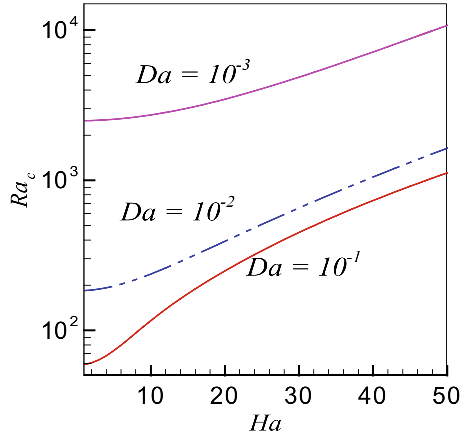

Fig. 6 Variation of critical *Ra* as a function of *Ha* for different values of *Da*

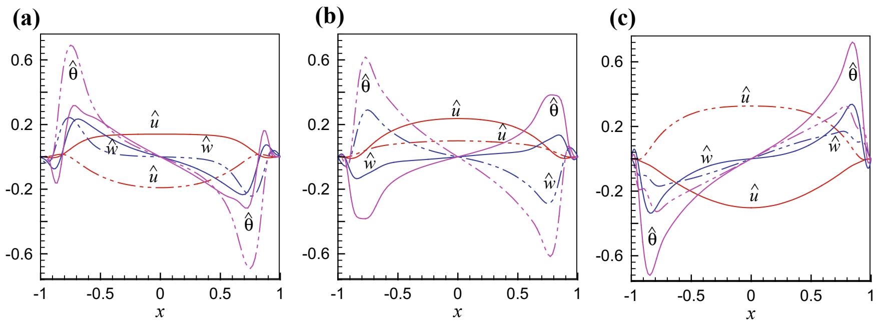

Fig. 7 Eigenfunctions ("—" real part, "–..–" imaginary part) of u (red), w (blue), and θ (purple) on linear stability critical point at $Da = 10^{-2}$ for **a** $Ha = 10$, **b** $Ha = 20$, and **c** $Ha = 50$

4 Conclusion

In this paper, we have studied the instability mechanism of MHD mixed convection flow in a vertical channel filled with the porous medium through a linear stability analysis. The present study results are examined for electrically conducting water-based electrolytes fluid at a fixed value of $Re = 1000$. A high permeable porous medium flow situation is considered. In the present study, we have examined basic flow characteristics, growth rate, and linear stability boundaries under a wide range of magnetic fields. The basic flow and linear disturbance equations are solved by Chebyshev spectral collocation method. The considered flow is least stable under two-dimensional disturbance. The enhancement in the media permeability and thermal buoyancy force could give rise to the inflection point in the velocity profile. The relatively strong magnetic field makes the velocity profile flatten. The growth rate profile shows that the stability domain enlarges by increasing the strength of the magnetic field and decreasing the media permeability. The linear stability conforms to the applied magnetic field stabilizes porous medium flow, whereas increase in the media permeability destabilizes the basic flow. These results of the present study may serve as a piece of fruitful information in many porous medium MHD applications.

References

1. Chen, Y.C., Chung, J.N.: The linear stability of mixed convection in a vertical channel flow. J. Fluid Mech. **325**, 29–51 (1996)
2. Chen, Y.C., Chung, J.N.: Stability of mixed convection in a differentially heated vertical channel. J. Heat Transf. **120**, 127–132 (1998)
3. Chen, Y.C., Chung, J.N.: A direct numerical simulation of k and h-type flow transition phenomenon in a heated vertical channel. Phys. Fluids **14**, 3327–3346 (2002)
4. Khandelwal, M.K., Bera, P.: Weakly nonlinear stability analysis of non-isothermal Poiseuille flow in a vertical channel. Phys. Fluids **27**, 064103-1-24 (2015)
5. Su, Y.C., Chung, J.N.: Linear stability analysis of mixed convection flow in a vertical pipe. J. Fluid Mech. **422**, 141–166 (2000)
6. Yao, L.S., Rogers, B.B.: The linear stability of mixed convection in a vertical annulus. J. Fluid Mech. **201**, 279–298 (1989)
7. Geindreau, C., Auriault, J.L.: Magnetohydrodynamic flows in porous media. J. Fluid Mech. **466**, 343–363 (2002)
8. Mc Whirter, J., Crawford, M., Klein, D., Sanders, T.: Modal for inertialess magnetohydrodynamic flow in packed beds. Fus. Technol. **34**, 187–197 (1998)
9. Bera, P., Khalili, A.: Stability of mixed convection in an anisotropic porous channel. Phys. Fluids **14**, 1617–1630 (2002)
10. Bera, P., Khalili, A.: Influence of Prandtl number on stability of mixed convective flow in a vertical channel filled with a porous medium. Phys. Fluids **18**, 124103 (2006)
11. Kumar, J., Bera, P., Khalili, A.: Influence of inertia and drag terms on the stability of mixed convection in a vertical porous-medium channel. Int. J. Heat Mass Transf. **53**, 5261–5271 (2010)
12. Bera, P., Khandelwal, M.K.: A thermal non-equilibrium perspective on instability mechanism of nonisothermal poiseuille flow in a vertical porous medium channel. Int. J. Ther. Sci. **105**, 159–173 (2016)
13. Khandelwal, M.K., Bera, P.: A thermal non equilibrium perspective on mixed convection in a vertical channel, Int. J. Ther. Sci. **56**, 23–34 (2012)
14. Sharma, A.K., Khandelwal, M.K., Bera, P.: Finite amplitude analysis of non-isothermal parallel flow in a vertical channel filled with a highly permeable porous medium. J. Fluid Mech. **857**, 469–507 (2018)
15. Khandelwal, M.K., Sharma, A.K., Bera, P.: Instability of mixed convection in a differentially heated channel filled with porous medium: a finite amplitude analysis. Phys. Fluids **33**, 024109 (2021)
16. Wallace, W.E., Pierce, C.I., Swayer, W.: Experiments on the flow of mercury in porous media in a transverse magnetic field. Technical Report, TN 23, U7, No. 7259. US Bureau of Mines
17. Raptis, A., Perdikis, C.: Magnetohydrodynamics effects on mass transfer flow through porous medium. Astrophys. Space Sci. **113**, 53–58 (1985)
18. Ram, G., Mishra, R.S.: Unsteady flow through magnetohydrodynamic porous media. Indian J. Pure Appl. Maths **8**, 637–647 (1977)
19. Tawil, M.A.E., Kamel, M.H.: MHD flow under stochastic porous media. Energy Conserv. Manag. **35**, 991–997 (1994)

20. Yih, K.A.: The effect of uniform suction/blowing on heat transfer of magnetohydrodynamic Hiemenz flow through porous media. Acta Mech. **130**, 147–158 (1998)
21. Prescott, P.J., Incropera, F.P.: Magnetically damped convection during solidification of a binary metal alloy. J. Heat Transf. **115**, 302–310 (1993)
22. Rudriah, N., Ramaiah, B.K., Rajasekhar, B.M.: Hartmann flow over a permeable bed. Intl J. Eng. Sci. **13**, 1–24 (1975)
23. Takashima, M.: The stability of natural convection in a vertical layer of electrically conducting fluid in the presence of a transverse magnetic field. Fluid Dyn. Res. **14**, 121–134 (1994)
24. Hudoba, A., Molokov, S.: Linear stability of buoyant convective flow in a vertical channel with internal heat sources and a transverse magnetic field. Phys. Fluids **28**(114103), 1–19 (2016)
25. Kolyshkin, A.A.: On the stability convection generated by internal heat sources in a magnetic field. Can. J. Phys. **66**, 990–993 (1988)
26. Stuart, J.T.: On the stability of viscous flow between parallel planes in the presence of co-planner magnetic field. Proc. Royal Soc. Lond. A **221**,189-XX (1954)
27. Lock, R.C.: The stability of the flow of an electrically conducting fluid between parallel plane under transverse magnetic field. Proc. R. Soc. A **233**, 105–125 (1955)
28. Hunt, J.C.R.: On the stability of parallel flows with parallel magnetic fields. Proc. R. Soc. A **293**, 342–358 (1966)
29. Satake, S., Sone, K., Furumi, K., Kunugi, T.: Direct numerical simulation of turbulent mixed convection in a vertical channel in a wall normal magnetic field. Fusion Eng. Des. **87**, 798–802 (2012)
30. Saleh, H., Hashim, I.: Flow reversal of fully-developed mixed MHD convection in vertical channels. Chin. Phys. Lett. **27**, 024401–024403 (2010)
31. Shankar, B.M., Kumar, J., Shivakumara, I.S.: Magnetohydrodynamic instability of mixed convection in a differentially heated vertical channel. Eur. Phys. J. Plus **134**(53), 1–12 (2019)
32. Singh, N., Khandelwal, M.K., Yu, P.: Instability of mixed convection flow in a differentially heated channel under a magnetic field with internal heating. Phys. Fluids **33**, 094102 (2021)
33. Nield, D.A., Bejan, A.: Convection in Porous Media, 4th edn. Springer, New York (2013)
34. Drazin, P.G., Reid, W.H.: Hydrodynamic Stability. Cambridge University Press, Cambridge (2004)