

Chapter 14

Role of Single Valued Linear Octagonal Neutrosophic Numbers in Multi-attribute Decision-Making Problems



S. Subasri, S. Arul Roselet Meryline, and Felbin C. Kennedy

14.1 Introduction

L. A. Zadeh in 1965 [14] set forth the exceptional concept of fuzzy sets, that have Broddingnagian applications in several fields of study. In 1986, a generalization of fuzzy sets was created by Atanassov [2], which is understood as Intuitionistic fuzzy sets (IFS). In IFS, in addition to one membership grade, there will additionally be another grade called non-membership grade that's hooked up to every part. To boot, there's a restriction that the total of those two grades at most be unity. A new theory was introduced by Smarandache [12] in 1999, called neutrosophic sets and logic. Uncertainty describes a lack of knowledge in one's knowledge but whereas ambiguity describes the ability to entertain more than in one interpretation. Thus, Neutrosophic set is used to deal with incomplete, indeterminate, and inconsistent information present in the real world. A neutrosophic set (NS) is employed to tackle uncertainty using the truth membership, indeterminacy membership, and falsity membership grades which are considered to be independent. The generalization of IFS is the neutrosophic sets since there is no restriction between the degree of truth, indeterminacy, falsity, and these degrees can individually vary within $]0^-, 1^+[$. Applications of Neutrosophic set can be found in the field of medicine, information technology, information system, decision support system, etc [8, 9].

From scientific or engineering purpose of reading, the neutrosophic set and set theory-based operators got to be specific. Hence, Wang et al. in 2005 [13], presented

S. Subasri (✉) · S. Arul Roselet Meryline · F. C. Kennedy
Stella Maris College, Cathedral Road, Chennai, India
e-mail: subasri30@gmail.com

S. Arul Roselet Meryline
e-mail: arulroselet@stellamariscollege.edu.in

F. C. Kennedy
e-mail: felbin@stellamariscollege.edu.in

an associate case of the neutrosophic set called single valued neutrosophic sets that were intended from the sensible purpose of reading wherein the degree of truth, indeterminateness, and falsity takes the value from the unit interval $[0, 1]$. To capture the imprecise information in the truth membership degree, indeterminate degree, falsity degree, Deli et al. [7] in 2016 introduced Single Valued Neutrosophic Numbers (SVNNs) which is a single valued neutrosophic subset of a real line that satisfies normality, convexity, and upper semicontinuity for truth degree, lower semicontinuity for indeterminate and falsity degree with bounded support. Several researchers studied different types of single valued neutrosophic numbers (to cite few [4, 11]). Various ranking methods were developed for SVNNs to make the optimal decision in real-life problems involving indeterminate data. Ranking using the value and ambiguity for the membership grades captures the ill-defined magnitude which underlies any fuzzy number and using the same ranking of the Single Valued Trapezoidal Neutrosophic number (SVTrNN) was studied by Biswas et al. [3]. In this paper, we introduce Single Valued Linear Octagonal Neutrosophic Numbers (SVLONNs) where the truth membership, indeterminacy membership, falsity membership functions are exhibited as Linear Octagonal Fuzzy Numbers. Ranking technique on SVLONNs plays a crucial role in higher cognitive process issues that involve indeterminate data in ordering and comparing the same.

The remaining paper is structured as follows: The definition of SVLONNs, $(\alpha_o, \beta_o, \gamma_o)$ -cuts of SVLONNs and arithmetic operations on SVLONNs are proposed in Sect. 14.2. Section 14.3 is dedicated to discussing the value and ambiguity indices of SVLONNs and a ranking technique of SVLONNs is introduced for defuzzification processes. In Sect. 14.4, we deal with the formulation of a Multi-Attribute Decision-Making Problem. In Sect. 14.5, a hypothetical problem is conferred for SVLONNs. In Sect. 14.6, we record closing remarks and some applications of the planned technique are put forth for future study.

14.2 Single Valued Linear Octagonal Neutrosophic Number

In this section, we define SVLONNs, cuts of SVLONNs, and arithmetic operations on SVLONNs.

Definition 14.1 A Single Valued Linear Octagonal Neutrosophic Number

(SVLONN) \tilde{A}_N denoted by $\langle p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8; q_1, q_2, q_3, q_4, q_5, q_6, q_7, q_8; r_1, r_2, r_3, r_4, r_5, r_6, r_7, r_8; k \rangle$ where $p_1 \leq p_2 \leq \dots \leq p_8$; $q_1 \leq q_2 \leq \dots \leq q_8$; $r_1 \leq r_2 \leq \dots \leq r_8$ are real numbers, its truth membership function $T_{\tilde{A}_N}$, indeterminacy membership function $I_{\tilde{A}_N}$, falsity membership function $F_{\tilde{A}_N}$ are defined as follows: For $0 < k < 1$,

$$T_{\tilde{A}_{\mathcal{N}}}(x) = \begin{cases} 0 & \text{for } x < p_1; \\ k \left(\frac{x - p_1}{p_2 - p_1} \right) & \text{for } p_1 \leq x \leq p_2; \\ k & \text{for } p_2 \leq x \leq p_3; \\ k + (1 - k) \left(\frac{x - p_3}{p_4 - p_3} \right) & \text{for } p_3 \leq x \leq p_4; \\ 1 & \text{for } p_4 \leq x \leq p_5; \\ k + (1 - k) \left(\frac{p_6 - x}{p_6 - p_5} \right) & \text{for } p_5 \leq x \leq p_6; \\ k & \text{for } p_6 \leq x \leq p_7; \\ k \left(\frac{p_8 - x}{p_8 - p_7} \right) & \text{for } p_7 \leq x \leq p_8; \\ 0 & \text{for } x > p_8; \end{cases}$$

$$I_{\tilde{A}_{\mathcal{N}}}(x) = \begin{cases} 1 & \text{for } x < q_1; \\ 1 + (1 - k) \left(\frac{q_1 - x}{q_2 - q_1} \right) & \text{for } q_1 \leq x \leq q_2; \\ k & \text{for } q_2 \leq x \leq q_3; \\ k + k \left(\frac{q_3 - x}{q_4 - q_3} \right) & \text{for } q_3 \leq x \leq q_4; \\ 0 & \text{for } q_4 \leq x \leq q_5; \\ k \left(\frac{x - q_5}{q_6 - q_5} \right) & \text{for } q_5 \leq x \leq q_6; \\ k & \text{for } q_6 \leq x \leq q_7; \\ k + (1 - k) \left(\frac{x - q_7}{q_8 - q_7} \right) & \text{for } q_7 \leq x \leq q_8; \\ 1 & \text{for } x > q_8; \end{cases}$$

$$F_{\tilde{A}_{\mathcal{N}}}(x) = \begin{cases} 1 & \text{for } x < r_1; \\ 1 + (1 - k) \left(\frac{r_1 - x}{r_2 - r_1} \right) & \text{for } r_1 \leq x \leq r_2; \\ k & \text{for } r_2 \leq x \leq r_3; \\ k + k \left(\frac{r_3 - x}{r_4 - r_3} \right) & \text{for } r_3 \leq x \leq r_4; \\ 0 & \text{for } r_4 \leq x \leq r_5; \\ k \left(\frac{x - r_5}{r_6 - r_5} \right) & \text{for } r_5 \leq x \leq r_6; \\ k & \text{for } r_6 \leq x \leq r_7; \\ k + (1 - k) \left(\frac{x - r_7}{r_8 - r_7} \right) & \text{for } r_7 \leq x \leq r_8; \\ 1 & \text{for } x > r_8; \end{cases}$$

Remark 14.1 The diagrammatic representation of SVLONNs for different values of k (Figs. 14.1, 14.2 and 14.3).

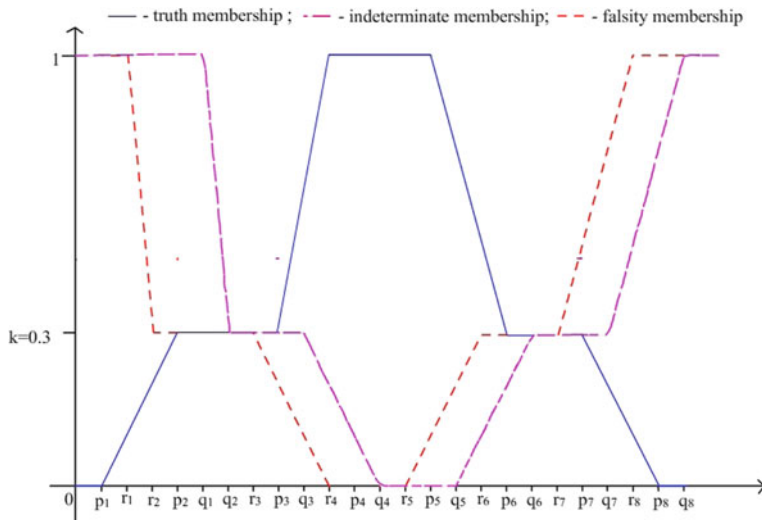


Fig. 14.1 SVLONN for k = 0.3

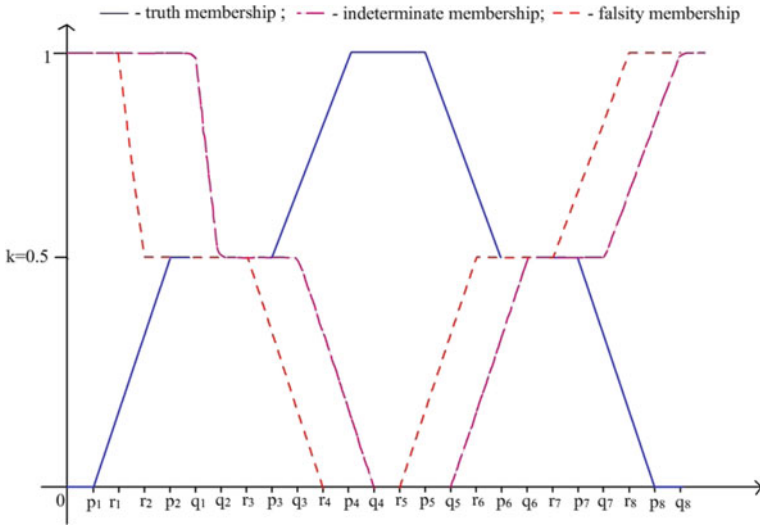


Fig. 14.2 SVLONN for $k = 0.5$

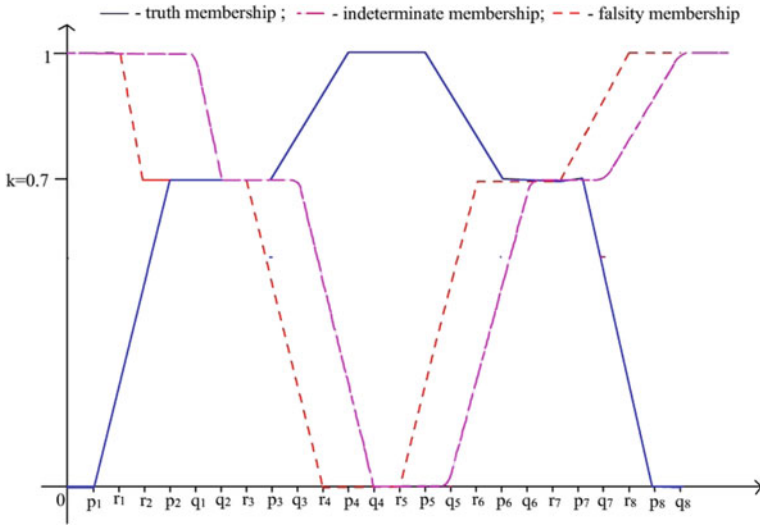


Fig. 14.3 SVLONN for $k = 0.7$

Remark 14.2 When $k = 0$ and $k = 1$, SVLONN reduces to the Single Valued Trapezoidal Neutrosophic number (SVTrNN) denoted $\langle p_3, p_4, p_5, p_6; q_1, q_2, q_7, q_8; r_1, r_2, r_7, r_8 \rangle$ and $\langle p_1, p_2, p_7, p_8; q_3, q_4, q_5, q_6; r_3, r_4, r_5, r_6 \rangle$ respectively (Figs. 14.4 and 14.5).

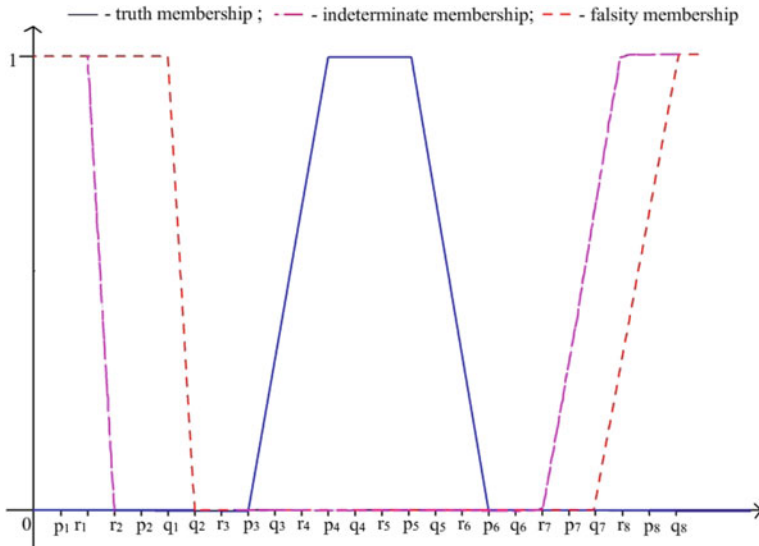


Fig. 14.4 SVLONN for $k = 0$

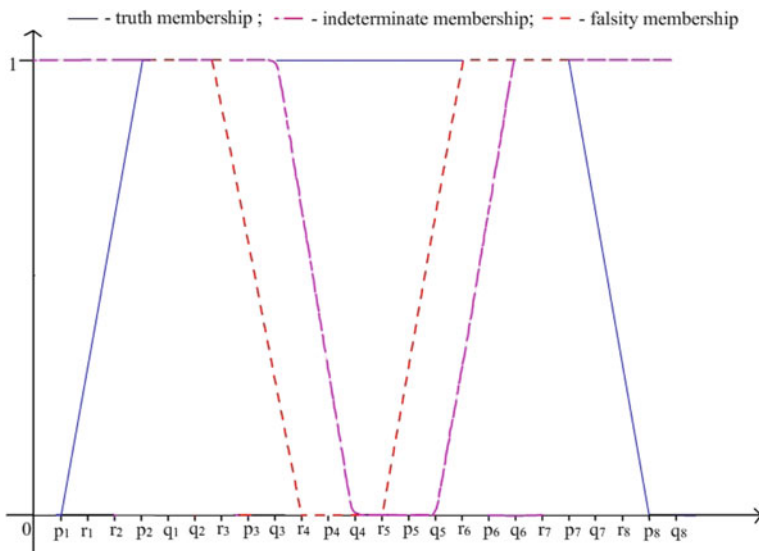


Fig. 14.5 SVLONN for $k = 1$

Definition 14.2 An $(\alpha_o, \beta_o, \gamma_o)$ -cut set of SVLONN \tilde{A}_N is given as

$$(\tilde{A}_N)_{\alpha_o, \beta_o, \gamma_o} = \{x : T_{\tilde{A}_N}(x) \geq \alpha_o, I_{\tilde{A}_N}(x) \leq \beta_o, F_{\tilde{A}_N}(x) \leq \gamma_o\} = ((\tilde{A}_N)_{\alpha_o}, (\tilde{A}_N)_{\beta_o}, (\tilde{A}_N)_{\gamma_o})$$

where

$$\begin{aligned}
 (\tilde{A}_N)_{\alpha_o} &= [(\tilde{A}_N)_{\alpha_o}^L, (\tilde{A}_N)_{\alpha_o}^R] = \begin{cases} [((\tilde{A}_N)_{\alpha_o}^L)_1, ((\tilde{A}_N)_{\alpha_o}^R)_1], & \text{for } \alpha_o \in [0, k] \\ [((\tilde{A}_N)_{\alpha_o}^L)_2, ((\tilde{A}_N)_{\alpha_o}^R)_2], & \text{for } \alpha_o \in (k, 1] \end{cases} \\
 (\tilde{A}_N)_{\beta_o} &= [(\tilde{A}_N)_{\beta_o}^L, (\tilde{A}_N)_{\beta_o}^R] = \begin{cases} [((\tilde{A}_N)_{\beta_o}^L)_1, ((\tilde{A}_N)_{\beta_o}^R)_1], & \text{for } \beta_o \in [0, k] \\ [((\tilde{A}_N)_{\beta_o}^L)_2, ((\tilde{A}_N)_{\beta_o}^R)_2], & \text{for } \beta_o \in (k, 1] \end{cases} \\
 (\tilde{A}_N)_{\gamma_o} &= [(\tilde{A}_N)_{\gamma_o}^L, (\tilde{A}_N)_{\gamma_o}^R] = \begin{cases} [((\tilde{A}_N)_{\gamma_o}^L)_1, ((\tilde{A}_N)_{\gamma_o}^R)_1], & \text{for } \gamma_o \in [0, k] \\ [((\tilde{A}_N)_{\gamma_o}^L)_2, ((\tilde{A}_N)_{\gamma_o}^R)_2], & \text{for } \gamma_o \in (k, 1] \end{cases}
 \end{aligned}$$

Computing the $(\tilde{A}_N)_{\alpha_o, \beta_o, \gamma_o}$ of \tilde{A}_N in Definition 14.1, we have

$$\begin{aligned}
 [((\tilde{A}_N)_{\alpha_o}^L)_1, ((\tilde{A}_N)_{\alpha_o}^R)_1] &= [p_1 + \frac{\alpha_o}{k}(p_2 - p_1), p_8 - \frac{\alpha_o}{k}(p_8 - p_7)] \\
 [((\tilde{A}_N)_{\alpha_o}^L)_2, ((\tilde{A}_N)_{\alpha_o}^R)_2] &= [p_3 + \frac{\alpha_o - k}{1 - k}(p_4 - p_3), p_6 - \frac{\alpha_o - k}{1 - k}(p_6 - p_5)] \\
 [((\tilde{A}_N)_{\beta_o}^L)_1, ((\tilde{A}_N)_{\beta_o}^R)_1] &= [q_3 - \frac{\beta_o - k}{k}(q_4 - q_3), q_5 + \frac{\beta_o}{k}(q_6 - q_5)] \\
 [((\tilde{A}_N)_{\beta_o}^L)_2, ((\tilde{A}_N)_{\beta_o}^R)_2] &= [q_1 - \frac{\beta_o - 1}{1 - k}(q_2 - q_1), q_7 + \frac{\beta_o - k}{1 - k}(q_8 - q_7)] \\
 [((\tilde{A}_N)_{\gamma_o}^L)_1, ((\tilde{A}_N)_{\gamma_o}^R)_1] &= [r_3 - \frac{\gamma_o - k}{k}(r_4 - r_3), r_5 + \frac{\gamma_o}{k}(r_6 - r_5)] \\
 [((\tilde{A}_N)_{\gamma_o}^L)_2, ((\tilde{A}_N)_{\gamma_o}^R)_2] &= [r_1 - \frac{\gamma_o - 1}{1 - k}(r_2 - r_1), r_7 + \frac{\gamma_o - k}{1 - k}(r_8 - r_7)]
 \end{aligned}$$

We introduce arithmetic operation on SVLONNs as follows:

Definition 14.3 Let $\tilde{A}_N = \langle p_1, p_2, \dots, p_8; q_1, q_2, \dots, q_8; r_1, r_2, \dots, r_8 \rangle$ and $\tilde{B}_N = \langle x_1, x_2, \dots, x_8; y_1, y_2, \dots, y_8; z_1, z_2, \dots, z_8 \rangle$ be two SVLONNs and let s be any real number, then addition and scalar multiplication by $(\alpha_o, \beta_o, \gamma_o)$ -cut approach are given by:

1. Addition:

$$\begin{aligned}
 (\tilde{A}_N)_{\alpha_o, \beta_o, \gamma_o} + (\tilde{B}_N)_{\alpha_o, \beta_o, \gamma_o} = \\
 [((\tilde{A}_N)_{\alpha_o}^L, (\tilde{A}_N)_{\alpha_o}^R) + ((\tilde{B}_N)_{\alpha_o}^L, (\tilde{B}_N)_{\alpha_o}^R); [(\tilde{A}_N)_{\beta_o}^L, (\tilde{A}_N)_{\beta_o}^R] + [(\tilde{B}_N)_{\beta_o}^L, (\tilde{B}_N)_{\beta_o}^R]; \\
 [(\tilde{A}_N)_{\gamma_o}^L, (\tilde{A}_N)_{\gamma_o}^R] + [(\tilde{B}_N)_{\gamma_o}^L, (\tilde{B}_N)_{\gamma_o}^R] \tag{14.1}
 \end{aligned}$$

where

$$\begin{aligned}
 [(\tilde{A}\mathcal{N})_{\alpha_0}^L, (\tilde{A}\mathcal{N})_{\alpha_0}^R] + [(\tilde{B}\mathcal{N})_{\alpha_0}^L, (\tilde{B}\mathcal{N})_{\alpha_0}^R] &= \begin{cases} [((\tilde{A}\mathcal{N})_{\alpha_0}^L)_1 + ((\tilde{B}\mathcal{N})_{\alpha_0}^L)_1, ((\tilde{A}\mathcal{N})_{\alpha_0}^R)_1 + ((\tilde{B}\mathcal{N})_{\alpha_0}^R)_1] & \text{for } \alpha_0 \in [0, k] \\ [((\tilde{A}\mathcal{N})_{\alpha_0}^L)_2 + ((\tilde{B}\mathcal{N})_{\alpha_0}^L)_2, ((\tilde{A}\mathcal{N})_{\alpha_0}^R)_2 + ((\tilde{B}\mathcal{N})_{\alpha_0}^R)_2] & \text{for } \alpha_0 \in (k, 1] \end{cases} \\
 &= \begin{cases} [p_1 + y_1 + \frac{\alpha_0}{k}(p_2 - p_1 + y_2 - y_1), p_8 + y_8 - \frac{\alpha_0}{k}(p_8 - p_7 + y_8 - y_7)] & \text{for } \alpha_0 \in [0, k] \\ [p_3 + y_3 + \frac{\alpha_0 - k}{1 - k}(p_4 - p_3 + y_4 - y_3), p_6 + y_6 - \frac{\alpha_0 - k}{1 - k}(p_6 - p_5 + y_6 - y_5)] & \text{for } \alpha_0 \in (k, 1] \end{cases} \\
 [(\tilde{A}\mathcal{N})_{\beta_0}^L, (\tilde{A}\mathcal{N})_{\beta_0}^R] + [(\tilde{B}\mathcal{N})_{\beta_0}^L, (\tilde{B}\mathcal{N})_{\beta_0}^R] &= \begin{cases} [((\tilde{A}\mathcal{N})_{\beta_0}^L)_1 + ((\tilde{B}\mathcal{N})_{\beta_0}^L)_1, ((\tilde{A}\mathcal{N})_{\beta_0}^R)_1 + ((\tilde{B}\mathcal{N})_{\beta_0}^R)_1] & \text{for } \beta_0 \in [0, k] \\ [((\tilde{A}\mathcal{N})_{\beta_0}^L)_2 + ((\tilde{B}\mathcal{N})_{\beta_0}^L)_2, ((\tilde{A}\mathcal{N})_{\beta_0}^R)_2 + ((\tilde{B}\mathcal{N})_{\beta_0}^R)_2] & \text{for } \beta_0 \in (k, 1] \end{cases} \\
 &= \begin{cases} [q_3 + z_3 - \frac{\beta_0 - k}{k}(q_4 - q_3 + z_4 - z_3), q_5 + z_5 + \frac{\beta_0}{k}(q_6 - q_5 + z_6 - z_5)] & \text{for } \beta_0 \in [0, k] \\ [q_1 + z_1 - \frac{\beta_0 - 1}{1 - k}(q_2 - q_1 + z_2 - z_1), q_7 + z_7 + \frac{\beta_0 - k}{1 - k}(q_8 - q_7 + z_8 - z_7)] & \text{for } \beta_0 \in (k, 1] \end{cases} \\
 [(\tilde{A}\mathcal{N})_{\gamma_0}^L, (\tilde{A}\mathcal{N})_{\gamma_0}^R] + [(\tilde{B}\mathcal{N})_{\gamma_0}^L, (\tilde{B}\mathcal{N})_{\gamma_0}^R] &= \begin{cases} [((\tilde{A}\mathcal{N})_{\gamma_0}^L)_1 + ((\tilde{B}\mathcal{N})_{\gamma_0}^L)_1, ((\tilde{A}\mathcal{N})_{\gamma_0}^R)_1 + ((\tilde{B}\mathcal{N})_{\gamma_0}^R)_1] & \text{for } \gamma_0 \in [0, k] \\ [((\tilde{A}\mathcal{N})_{\gamma_0}^L)_2 + ((\tilde{B}\mathcal{N})_{\gamma_0}^L)_2, ((\tilde{A}\mathcal{N})_{\gamma_0}^R)_2 + ((\tilde{B}\mathcal{N})_{\gamma_0}^R)_2] & \text{for } \gamma_0 \in (k, 1] \end{cases} \\
 &= \begin{cases} [r_3 + g_3 - \frac{\gamma_0 - k}{k}(r_4 - r_3 + g_4 - g_3), r_5 + g_5 + \frac{\gamma_0}{k}(r_6 - r_5 + g_6 - g_5)] & \text{for } \gamma_0 \in [0, k] \\ [r_1 + g_1 - \frac{\gamma_0 - 1}{1 - k}(r_2 - r_1 + g_2 - g_1), r_7 + g_7 + \frac{\gamma_0 - k}{1 - k}(r_8 - r_7 + g_8 - g_7)] & \text{for } \gamma_0 \in (k, 1] \end{cases}
 \end{aligned}$$

2. Scalar Multiplication:

$$s(\tilde{A}\mathcal{N})_{\alpha_0, \beta_0, \gamma_0} = \begin{cases} [(s\tilde{A}\mathcal{N})_{\alpha_0}^L, (s\tilde{A}\mathcal{N})_{\alpha_0}^R]; [(s\tilde{A}\mathcal{N})_{\beta_0}^L, (s\tilde{A}\mathcal{N})_{\beta_0}^R]; [(s\tilde{A}\mathcal{N})_{\gamma_0}^L, (s\tilde{A}\mathcal{N})_{\gamma_0}^R] & \text{for } s \geq 0 \\ [(s\tilde{A}\mathcal{N})_{\alpha_0}^R, (s\tilde{A}\mathcal{N})_{\alpha_0}^L]; [(s\tilde{A}\mathcal{N})_{\beta_0}^R, (s\tilde{A}\mathcal{N})_{\beta_0}^L]; [(s\tilde{A}\mathcal{N})_{\gamma_0}^R, (s\tilde{A}\mathcal{N})_{\gamma_0}^L] & \text{for } s < 0 \end{cases} \tag{14.2}$$

where, for $s \geq 0$

$$\begin{aligned}
 [(s\tilde{A}\mathcal{N})_{\alpha_0}^L, (s\tilde{A}\mathcal{N})_{\alpha_0}^R] &= \begin{cases} [(s(\tilde{A}\mathcal{N})_{\alpha_0}^L)_1, (s(\tilde{A}\mathcal{N})_{\alpha_0}^R)_1] & \alpha_0 \in [0, k] \\ [(s(\tilde{A}\mathcal{N})_{\alpha_0}^L)_2, (s(\tilde{A}\mathcal{N})_{\alpha_0}^R)_2] & \alpha_0 \in (k, 1] \end{cases} \\
 &= \begin{cases} [sp_1 + \frac{\alpha_0}{k}(sp_2 - sp_1), sp_8 - \frac{\alpha_0}{k}(sp_8 - sp_7)] & \text{for } \alpha_0 \in [0, k] \\ [sp_3 + \frac{\alpha_0 - k}{1 - k}(sp_4 - sp_3), sp_6 - \frac{\alpha_0 - k}{1 - k}(sp_6 - sp_5)] & \text{for } \alpha_0 \in (k, 1] \end{cases} \\
 [(s\tilde{A}\mathcal{N})_{\beta_0}^R, (s\tilde{A}\mathcal{N})_{\beta_0}^L] &= \begin{cases} [(s(\tilde{A}\mathcal{N})_{\beta_0}^R)_1, (s(\tilde{A}\mathcal{N})_{\beta_0}^L)_1] & \beta_0 \in [0, k] \\ [(s(\tilde{A}\mathcal{N})_{\beta_0}^R)_2, (s(\tilde{A}\mathcal{N})_{\beta_0}^L)_2] & \beta_0 \in (k, 1] \end{cases} \\
 &= \begin{cases} [sq_3 - \frac{\beta_0 - k}{k}(sq_4 - sq_3), sq_5 + \frac{\beta_0}{k}(sq_6 - sq_5)] & \text{for } \beta_0 \in [0, k] \\ [sq_1 - \frac{\beta_0 - 1}{1 - k}(sq_2 - sq_1), sq_7 + \frac{\beta_0 - k}{1 - k}(sq_8 - sq_7)] & \text{for } \beta_0 \in (k, 1] \end{cases} \\
 [(s\tilde{A}\mathcal{N})_{\gamma_0}^L, (s\tilde{A}\mathcal{N})_{\gamma_0}^R] &= \begin{cases} [(s(\tilde{A}\mathcal{N})_{\gamma_0}^L)_1, (s(\tilde{A}\mathcal{N})_{\gamma_0}^R)_1] & \gamma_0 \in [0, k] \\ [(s(\tilde{A}\mathcal{N})_{\gamma_0}^L)_2, (s(\tilde{A}\mathcal{N})_{\gamma_0}^R)_2] & \gamma_0 \in (k, 1] \end{cases} \\
 &= \begin{cases} [sr_3 - \frac{\gamma_0 - k}{k}(sr_4 - sr_3), sr_5 + \frac{\gamma_0}{k}(sr_6 - sr_5)] & \text{for } \gamma_0 \in [0, k] \\ [sr_1 - \frac{\gamma_0 - 1}{1 - k}(sr_2 - sr_1), sr_7 + \frac{\gamma_0 - k}{1 - k}(sr_8 - sr_7)] & \text{for } \gamma_0 \in (k, 1] \end{cases}
 \end{aligned}$$

and for $s < 0$

$$\begin{aligned}
 [(s\tilde{A}\mathcal{N})_{\alpha_0}^R, (s\tilde{A}\mathcal{N})_{\alpha_0}^L] &= \begin{cases} [(s(\tilde{A}\mathcal{N})_{\alpha_0}^R)_1, (s(\tilde{A}\mathcal{N})_{\alpha_0}^L)_1] & \alpha_0 \in [0, k] \\ [(s(\tilde{A}\mathcal{N})_{\alpha_0}^R)_2, (s(\tilde{A}\mathcal{N})_{\alpha_0}^L)_2] & \alpha_0 \in (k, 1] \end{cases} \\
 &= \begin{cases} [sp_8 - \frac{\alpha_0}{k}(sp_8 - sp_7), sp_1 + \frac{\alpha_0}{k}(sp_2 - sp_1)] & \text{for } \alpha_0 \in [0, k] \\ [sp_6 - \frac{\alpha_0 - k}{1 - k}(sp_6 - sp_5), sp_3 + \frac{\alpha_0 - k}{1 - k}(sp_4 - sp_3)] & \text{for } \alpha_0 \in (k, 1] \end{cases}
 \end{aligned}$$

$$\begin{aligned}
 [(s\tilde{A}_{\mathcal{N}})^R_{\beta_o}, (s\tilde{A}_{\mathcal{N}})^L_{\beta_o}] &= \begin{cases} [(s(\tilde{A}_{\mathcal{N}})^R_{\beta_o})_1, (s(\tilde{A}_{\mathcal{N}})^L_{\beta_o})_1] & \beta_o \in [0, k] \\ [(s(\tilde{A}_{\mathcal{N}})^R_{\beta_o})_2, (s(\tilde{A}_{\mathcal{N}})^L_{\beta_o})_2] & \beta_o \in (k, 1] \end{cases} \\
 &= \begin{cases} [sq_5 + \frac{\beta_o}{k}(sq_6 - sq_5), sq_3 - \frac{\beta_o - k}{k}(sq_4 - sq_3)] & \text{for } \beta_o \in [0, k] \\ [sq_7 + \frac{\beta_o - k}{1 - k}(sq_8 - sq_7), sq_1 - \frac{\beta_o - 1}{1 - k}(sq_2 - sq_1)] & \text{for } \beta_o \in (k, 1] \end{cases} \\
 [(s\tilde{A}_{\mathcal{N}})^R_{\gamma_o}, (s\tilde{A}_{\mathcal{N}})^L_{\gamma_o}] &= \begin{cases} [(s(\tilde{A}_{\mathcal{N}})^R_{\gamma_o})_1, (s(\tilde{A}_{\mathcal{N}})^L_{\gamma_o})_1] & \gamma_o \in [0, k] \\ [(s(\tilde{A}_{\mathcal{N}})^R_{\gamma_o})_2, (s(\tilde{A}_{\mathcal{N}})^L_{\gamma_o})_2] & \gamma_o \in (k, 1] \end{cases} \\
 &= \begin{cases} [sr_5 + \frac{\gamma_o}{k}(sr_6 - sr_5), sr_3 - \frac{\gamma_o - k}{k}(sr_4 - sr_3)] & \text{for } \gamma_o \in [0, k] \\ [sr_7 + \frac{\gamma_o - k}{1 - k}(sr_8 - sr_7), sr_1 - \frac{\gamma_o - 1}{1 - k}(sr_2 - sr_1)] & \text{for } \gamma_o \in (k, 1] \end{cases}
 \end{aligned}$$

Definition 14.4 Let $\tilde{A}_{\mathcal{N}} = \langle p_1, p_2, \dots, p_8; q_1, q_2, \dots, q_8; r_1, r_2, \dots, r_8 \rangle$ and

$\tilde{B}_{\mathcal{N}} = \langle x_1, x_2, \dots, x_8; y_1, y_2, \dots, y_8; z_1, z_2, \dots, z_8 \rangle$ be two SVLONNs and let s be any real number, then the coordinatewise addition and scalar multiplication are defined as follows:

$$1. \tilde{A}_{\mathcal{N}} \oplus \tilde{B}_{\mathcal{N}} = \langle p_1 + x_1, \dots, p_8 + x_8; q_1 + y_1, \dots, q_8 + y_8; r_1 + z_1, \dots, r_8 + z_8 \rangle \tag{14.3}$$

$$2. s\tilde{A}_{\mathcal{N}} = \begin{cases} \langle sp_1, \dots, sp_8; sq_1, \dots, sq_8; sr_1, \dots, sr_8 \rangle & \text{for } s \geq 0 \\ \langle sp_8, \dots, sp_1; sq_8, \dots, sq_1; sr_8, \dots, sr_1 \rangle & \text{for } s < 0 \end{cases} \tag{14.4}$$

Theorem 14.1 The $(\alpha_o, \beta_o, \gamma_o)$ -cut approach and coordinate approach of the addition and scalar multiplication of SVLONNs yields the same SVLONN.

Proof It is enough to show that

$$(\tilde{A}_{\mathcal{N}})_{\alpha_o, \beta_o, \gamma_o} + (\tilde{B}_{\mathcal{N}})_{\alpha_o, \beta_o, \gamma_o} = (\tilde{A}_{\mathcal{N}} + \tilde{B}_{\mathcal{N}})_{\alpha_o, \beta_o, \gamma_o} \text{ and } s(\tilde{A}_{\mathcal{N}})_{\alpha_o, \beta_o, \gamma_o} = (s\tilde{A}_{\mathcal{N}})_{\alpha_o, \beta_o, \gamma_o}.$$

From Definition 14.3 and Definition 14.4, we observe that

RHS of equation (14.1) = $(\tilde{A}_{\mathcal{N}} + \tilde{B}_{\mathcal{N}})_{\alpha_o, \beta_o, \gamma_o}$ and RHS of equation (14.2) = $(s\tilde{A}_{\mathcal{N}})_{\alpha_o, \beta_o, \gamma_o}$ respectively. □

14.3 Value and Ambiguity Index-Based Ranking for SVLONNs

In this section, we define Value and Ambiguity of SVLONNs. And by using the same, we define value and ambiguity index of SVLONNs.

Definition 14.5 Let $\tilde{A}_{\mathcal{N}}$ be a SVLONN, then the value of the truth $[V_T(\tilde{A}_{\mathcal{N}})]$, indeterminacy $[V_I(\tilde{A}_{\mathcal{N}})]$ and falsity $[V_F(\tilde{A}_{\mathcal{N}})]$ membership grade of $\tilde{A}_{\mathcal{N}}$ are respectively defined using the weighting function $f(\alpha_o)$, $g(\beta_o)$ and $h(\gamma_o)$ as

$$V_T(\tilde{A}_N) = \int_0^k [((\tilde{A}_N)_{\alpha_o}^L)_1 + ((\tilde{A}_N)_{\alpha_o}^R)_1]f(\alpha_o)d\alpha_o + \int_k^1 [((\tilde{A}_N)_{\alpha_o}^L)_2 + ((\tilde{A}_N)_{\alpha_o}^R)_2]f(\alpha_o)d\alpha_o \tag{14.5}$$

$$V_I(\tilde{A}_N) = \int_0^k [((\tilde{A}_N)_{\beta_o}^L)_1 + ((\tilde{A}_N)_{\beta_o}^R)_1]g(\beta_o)d\beta_o + \int_k^1 [((\tilde{A}_N)_{\beta_o}^L)_2 + ((\tilde{A}_N)_{\beta_o}^R)_2]g(\beta_o)d\beta_o \tag{14.6}$$

$$V_F(\tilde{A}_N) = \int_0^k [((\tilde{A}_N)_{\gamma_o}^L)_1 + ((\tilde{A}_N)_{\gamma_o}^R)_1]h(\gamma_o)d\gamma_o + \int_k^1 [((\tilde{A}_N)_{\gamma_o}^L)_2 + ((\tilde{A}_N)_{\gamma_o}^R)_2]h(\gamma_o)d\gamma_o \tag{14.7}$$

Remark 14.3 1. The decision maker can set the Weighting functions according to the nature of the problems in real situations.

2. For example the choice of the functions $f(\alpha_o) = \alpha_o$, $g(\beta_o) = 1 - \beta_o$ and $h(\gamma_o) = 1 - \gamma_o$ where $\alpha_o, \beta_o, \gamma_o \in [0, 1]$, give rise to different weights to elements in different α_o -cut, β_o -cut, γ_o -cut sets and make less the contribution of the lower α_o -cut sets, reduce the contribution of higher β_o -cut, γ_o -cut sets. These are acceptable as the cut sets arising from values of $T_{\tilde{A}_N}(x)$, $I_{\tilde{A}_N}(x)$, and $F_{\tilde{A}_N}(x)$ deals with a considerable amount of uncertainty.
3. $V_T(\tilde{A}_N)$, $V_I(\tilde{A}_N)$, and $V_F(\tilde{A}_N)$ synthetically reflects the information on every membership degree, indeterminacy degree, and falsity degree respectively.
4. $V_T(\tilde{A}_N)$, $V_I(\tilde{A}_N)$, and $V_F(\tilde{A}_N)$ may be considered as a central value that represents the membership function, indeterminacy function, and falsity function.

Substituting $f(\alpha_o) = \alpha_o$, $g(\beta_o) = 1 - \beta_o$ and $h(\gamma_o) = 1 - \gamma_o$ in Eqs. (14.5), (14.6) and (14.7) respectively, we compute the values of truth membership function, indeterminacy membership function, and falsity membership function using MathCad 14 and are given by

$$V_T(\tilde{A}_N) = \frac{k^2}{6} (p_1 + 2p_2 + 2p_7 + p_8) + \frac{k-1}{6} (2kp_3 + kp_4 + kp_5 + 2kp_6 + p_3 + 2p_4 + 2p_5 + p_6)$$

$$V_I(\tilde{A}_N) = \frac{k}{2} (q_3 + q_4 + q_5 + q_6) - \frac{k^2}{6} (2q_3 + q_4 + q_5 + 2q_6) + \frac{(k-1)^2}{6} (q_1 + 2q_2 + 2q_7 + q_8)$$

$$V_F(\tilde{A}_N) = \frac{k}{2} (r_3 + r_4 + r_5 + r_6) - \frac{k^2}{6} (2r_3 + r_4 + r_5 + 2r_6) + \frac{(k-1)^2}{6} (r_1 + 2r_2 + 2r_7 + r_8)$$

Definition 14.6 Let \tilde{A}_N be a SVLONN. Then the ambiguity of the truth $[A_T(\tilde{A}_N)]$, indeterminacy $[A_I(\tilde{A}_N)]$ and falsity $[A_F(\tilde{A}_N)]$ membership grade of \tilde{A}_N are respectively defined using the functions $f(\alpha_o)$, $g(\beta_o)$ and $h(\gamma_o)$ as

$$A_T(\tilde{A}_N) = \int_0^k [((\tilde{A}_N)_{\alpha_o}^R)_1 - ((\tilde{A}_N)_{\alpha_o}^L)_1]f(\alpha_o)d\alpha_o + \int_k^1 [((\tilde{A}_N)_{\alpha_o}^R)_2 - ((\tilde{A}_N)_{\alpha_o}^L)_2]f(\alpha_o)d\alpha_o \tag{14.8}$$

$$A_I(\tilde{A}_N) = \int_0^k [((\tilde{A}_N)_{\beta_o}^R)_1 - ((\tilde{A}_N)_{\beta_o}^L)_1]g(\beta_o)d\beta_o + \int_k^1 [((\tilde{A}_N)_{\beta_o}^R)_2 - ((\tilde{A}_N)_{\beta_o}^L)_2]g(\beta_o)d\beta_o \tag{14.9}$$

$$A_F(\tilde{A}_N) = \int_0^k [((\tilde{A}_N)_{\gamma_o}^R)_1 - ((\tilde{A}_N)_{\gamma_o}^L)_1]h(\gamma_o)d\gamma_o + \int_k^1 [((\tilde{A}_N)_{\gamma_o}^R)_2 - ((\tilde{A}_N)_{\gamma_o}^L)_2]h(\gamma_o)d\gamma_o \tag{14.10}$$

Remark 14.4 In the above definition, we observe that $((\tilde{A}_{\mathcal{N}})^R_{\alpha_o})_j - ((\tilde{A}_{\mathcal{N}})^L_{\alpha_o})_j$, $((\tilde{A}_{\mathcal{N}})^R_{\beta_o})_j - ((\tilde{A}_{\mathcal{N}})^L_{\beta_o})_j$, $((\tilde{A}_{\mathcal{N}})^R_{\gamma_o})_j - ((\tilde{A}_{\mathcal{N}})^L_{\gamma_o})_j$ for $j=1,2$ express the length of the intervals of $(\tilde{A}_{\mathcal{N}})_{\alpha_o}$, $(\tilde{A}_{\mathcal{N}})_{\beta_o}$, $(\tilde{A}_{\mathcal{N}})_{\gamma_o}$ respectively. Thus $A_T(\tilde{A}_{\mathcal{N}})$, $A_I(\tilde{A}_{\mathcal{N}})$, $A_F(\tilde{A}_{\mathcal{N}})$ can be viewed as the global spreads of the truth membership function, indeterminacy membership function, and falsity membership function respectively. The vagueness of $\tilde{A}_{\mathcal{N}}$ is determined using the ambiguity of these three functions.

We derive the ambiguity of truth membership function, indeterminacy membership function, and falsity membership function using MathCad 14 for $f(\alpha_o) = \alpha_o$, $g(\beta_o) = 1 - \beta_o$ and $h(\gamma_o) = 1 - \gamma_o$ in Eqs. (14.8), (14.9) and (14.10) respectively yield:

$$A_T(\tilde{A}_{\mathcal{N}}) = \frac{k-1}{6}(2kp_3 + kp_4 - kp_5 - 2kp_6 + p_3 + 2p_4 - 2p_5 - p_6) + \frac{k^2}{6}(2p_7 - 2p_2 + p_8 - p_1)$$

$$A_I(\tilde{A}_{\mathcal{N}}) = \frac{k}{2}(q_5 - q_4 - q_3 + q_6) - \frac{(k-1)^2}{6}(q_1 + 2q_2 - 2q_7 - q_8) + \frac{(k)^2}{6}(2q_3 + q_4 - q_5 - q_6)$$

$$A_F(\tilde{A}_{\mathcal{N}}) = \frac{k}{2}(r_5 - r_4 - r_3 + r_6) - \frac{(k-1)^2}{6}(r_1 + 2r_2 - 2r_7 - r_8) + \frac{(k)^2}{6}(2r_3 + r_4 - r_5 - r_6)$$

By using the value and ambiguity of truth membership function, indeterminacy membership function, falsity membership function computed in Remarks 14.3, 14.4, the value and ambiguity index of SVLONNs are defined as follows:

Definition 14.7 For a SVLONN $\tilde{A}_{\mathcal{N}}$, the value index and ambiguity index are given by

$$V_{\psi, \eta, \zeta} = \psi V_T + \eta V_I + \zeta V_F \tag{14.11}$$

$$A_{\psi, \eta, \zeta} = \psi A_T + \eta A_I + \zeta A_F \tag{14.12}$$

where the co-efficient ψ, η, ζ appearing in Eqs. (14.11) and (14.12) expresses respectively the preference value of the decision maker such that $\psi + \eta + \zeta = 1$.

Remark 14.5 For $\psi \in [0, \frac{1}{3}]$ and $\eta + \zeta \in [\frac{1}{3}, 1]$, a decision maker makes a pessimistic decision in an uncertain environment. On the other hand, the decision maker may desire to make an optimistic decision in an uncertain environment for $\psi \in [\frac{1}{3}, 1]$ and $\eta + \zeta \in [0, \frac{1}{3}]$. Also, if a decision maker chooses $\psi = \eta = \zeta = \frac{1}{3}$, then there is an equal importance to truth, indeterminacy, and falsity. Therefore, the value index and ambiguity index reflect the attitude of the decision maker for SVLONN.

Theorem 14.2 Let $\tilde{A}_{\mathcal{N}_1}$ and $\tilde{A}_{\mathcal{N}_2}$ be two SVLONNs. Then for $\psi, \eta, \zeta \in [0, 1]$ and $\phi \in \mathbb{R}$,

$$V_{\psi, \eta, \zeta}(\tilde{A}_{\mathcal{N}_1} + \tilde{A}_{\mathcal{N}_2}) = V_{\psi, \eta, \zeta}(\tilde{A}_{\mathcal{N}_1}) + V_{\psi, \eta, \zeta}(\tilde{A}_{\mathcal{N}_2}) \tag{14.13}$$

$$V_{\psi, \eta, \zeta}(\phi \tilde{A}_{\mathcal{N}_1}) = \phi V_{\psi, \eta, \zeta}(\tilde{A}_{\mathcal{N}_1}) \tag{14.14}$$

Proof We prove this theorem by using the Definitions 14.4 and 14.7.

$$\begin{aligned}
 V_{\psi, \eta, \zeta}(\tilde{A}_{\mathcal{N}_1} + \tilde{A}_{\mathcal{N}_2}) &= \psi V_T(\tilde{A}_{\mathcal{N}_1} + \tilde{A}_{\mathcal{N}_2}) + \eta V_I(\tilde{A}_{\mathcal{N}_1} + \tilde{A}_{\mathcal{N}_2}) + \zeta V_F(\tilde{A}_{\mathcal{N}_1} + \tilde{A}_{\mathcal{N}_2}) \\
 &= \frac{\psi}{6} \left[\frac{k^2}{6} ((p_1 + x_1) + 2(p_2 + x_2) + 2(p_7 + x_7) + (p_8 + x_8)) \right. \\
 &\quad + \frac{k-1}{6} (2k(p_3 + x_3) + k(p_4 + x_4)) + k(p_5 + x_5) \\
 &\quad + 2k(p_6 + x_6) + (p_3 + x_3) + 2(p_4 + x_4) + 2(p_5 + x_5) + (p_6 + x_6) \Big] \\
 &\quad + \frac{\eta}{6} \left[\frac{k}{2} ((q_3 + y_3) + (q_4 + y_4) + (q_5 + y_5) + (q_6 + y_6)) \right. \\
 &\quad - \frac{k^2}{6} (2(q_3 + y_3) + (q_4 + y_4) + (q_5 + y_5) \\
 &\quad + 2(q_6 + y_6)) + \frac{(k-1)^2}{6} \\
 &\quad \times ((q_1 + y_1) + 2(q_2 + y_2) + 2(q_7 + y_7) + (q_8 + y_8)) \Big] \\
 &\quad + \frac{\zeta}{6} \left[\frac{k}{2} ((r_3 + z_3) + (r_4 + z_4) + (r_5 + z_5) + (r_6 + z_6)) \right. \\
 &\quad - \frac{k^2}{6} (2(r_3 + z_3) + (r_4 + z_4) + (r_5 + z_5) + 2(r_6 + z_6)) \\
 &\quad + \frac{(k-1)^2}{6} ((r_1 + z_1) + 2(r_2 + z_2) + 2(r_7 + z_7) \\
 &\quad + (r_8 + z_8)) \Big] - \frac{k^2}{6} (2(r_3 + z_3) + (r_4 + z_4) \\
 &= V_{\psi, \eta, \zeta}(\tilde{A}_{\mathcal{N}_1}) + V_{\psi, \eta, \zeta}(\tilde{A}_{\mathcal{N}_2})
 \end{aligned}$$

and

$$\begin{aligned}
 V_{\psi, \eta, \zeta}(\phi \tilde{A}_{\mathcal{N}_1}) &= \psi V_T(\phi \tilde{A}_{\mathcal{N}_1}) + \eta V_I(\phi \tilde{A}_{\mathcal{N}_1}) + \zeta V_F(\phi \tilde{A}_{\mathcal{N}_1}) \\
 &= \frac{\psi}{6} \left[\frac{k^2}{6} (\phi p_1 + 2\phi p_2 + 2\phi p_7 + \phi p_8) + \frac{k-1}{6} (2k\phi p_3 + k\phi p_4 + k\phi p_5 + 2k\phi p_6 + \phi p_3 + 2\phi p_4 + 2\phi p_5 \right. \\
 &\quad + \phi p_6) + \frac{\eta}{6} \left[\frac{k}{2} (\phi q_3 + \phi q_4 + \phi q_5 + \phi q_6) - \frac{k^2}{6} (2\phi q_3 + \phi q_4 + \phi q_5 + 2\phi q_6) + \frac{(k-1)^2}{6} (\phi q_1 + 2\phi q_2 \right. \\
 &\quad + 2\phi q_7 + \phi q_8) + \frac{\zeta}{6} \left[\frac{k}{2} (\phi r_3 + \phi r_4 + \phi r_5 + \phi r_6) - \frac{k^2}{6} (2\phi r_3 + \phi r_4 + \phi r_5 + 2\phi r_6) \right. \\
 &\quad \left. + \frac{(k-1)^2}{6} (\phi r_1 + 2\phi r_2 + 2\phi r_7 + \phi r_8) \right] \\
 &= \phi V_{\psi, \eta, \zeta}(\tilde{A}_{\mathcal{N}_1})
 \end{aligned}$$

which completes the proof. □

Theorem 14.3 *Let $\tilde{A}_{\mathcal{N}_1}$ and $\tilde{A}_{\mathcal{N}_2}$ be two SVLONNs. Then for $\psi, \eta, \zeta \in [0, 1]$ and $\phi \in \mathbb{R}$,*

$$A_{\psi, \eta, \zeta}(\tilde{A}_{\mathcal{N}_1} + \tilde{A}_{\mathcal{N}_2}) = A_{\psi, \eta, \zeta}(\tilde{A}_{\mathcal{N}_1}) + V_{\psi, \eta, \zeta}(\tilde{A}_{\mathcal{N}_2}) \tag{14.15}$$

$$A_{\psi, \eta, \zeta}(\phi \tilde{A}_{\mathcal{N}_1}) = \phi A_{\psi, \eta, \zeta}(\tilde{A}_{\mathcal{N}_1}) \tag{14.16}$$

Proof We prove this theorem by using the Definitions 14.4 and 14.7. The proof of this theorem is as that of theorem (14.2). \square

Definition 14.8 We compare two SVLONNs $\tilde{A}_{\mathcal{N}_1}$ and $\tilde{A}_{\mathcal{N}_2}$ by using value and ambiguity indices as follows:

1. If $\tilde{A}_{\mathcal{N}_1} \leq \tilde{A}_{\mathcal{N}_2}$ then $V_{\psi, \eta, \zeta}(\tilde{A}_{\mathcal{N}_1}) \leq V_{\psi, \eta, \zeta}(\tilde{A}_{\mathcal{N}_2})$.
2. If $\tilde{A}_{\mathcal{N}_1} \geq \tilde{A}_{\mathcal{N}_2}$ then $V_{\psi, \eta, \zeta}(\tilde{A}_{\mathcal{N}_1}) \geq V_{\psi, \eta, \zeta}(\tilde{A}_{\mathcal{N}_2})$.
3. If $V_{\psi, \eta, \zeta}(\tilde{A}_{\mathcal{N}_1}) = V_{\psi, \eta, \zeta}(\tilde{A}_{\mathcal{N}_2})$, then by using ambiguity index we compare them in the following ways:
 - a. If $A_{\psi, \eta, \zeta}(\tilde{A}_{\mathcal{N}_1}) \geq A_{\psi, \eta, \zeta}(\tilde{A}_{\mathcal{N}_2})$ then $\tilde{A}_{\mathcal{N}_1} \geq \tilde{A}_{\mathcal{N}_2}$.
 - b. If $A_{\psi, \eta, \zeta}(\tilde{A}_{\mathcal{N}_1}) \leq A_{\psi, \eta, \zeta}(\tilde{A}_{\mathcal{N}_2})$ then $\tilde{A}_{\mathcal{N}_1} \leq \tilde{A}_{\mathcal{N}_2}$.
 - c. If $A_{\psi, \eta, \zeta}(\tilde{A}_{\mathcal{N}_1}) = A_{\psi, \eta, \zeta}(\tilde{A}_{\mathcal{N}_2})$ then $\tilde{A}_{\mathcal{N}_1} \approx \tilde{A}_{\mathcal{N}_2}$.

14.4 Formulation of a Multi-attribute Decision-Making Problem

In this section, we consider an Multi-Attribute Decision Making (MADM) problem where the attributes are given by SVLONNs. Let us assume that for an MADM problem, $U = \{U_1, U_2, \dots, U_t\}$ be the set of t alternatives, and $E = \{E_1, E_2, \dots, E_v\}$ be the set of v attributes and the weight vector provided by the decision maker for the attributes be $W = (w_1, w_2, \dots, w_v)^T$, where $w_j \in [0, 1]$, $\sum_{j=1}^v w_j = 1$ and w_j is the degree of importance for the attribute E_j . Therefore, we express the alternatives U_i over the attributes E_j by SVLONN $d_{ij} = \langle p_{ij}^1, p_{ij}^2, p_{ij}^3, p_{ij}^4, p_{ij}^5, p_{ij}^6, p_{ij}^7, p_{ij}^8; q_{ij}^1, q_{ij}^2, q_{ij}^3, q_{ij}^4, q_{ij}^5, q_{ij}^6, q_{ij}^7, q_{ij}^8; r_{ij}^1, r_{ij}^2, r_{ij}^3, r_{ij}^4, r_{ij}^5, r_{ij}^6, r_{ij}^7, r_{ij}^8 \rangle$ where $p_{ij}^k, q_{ij}^k, r_{ij}^k \in \mathbb{R}$ for $i = 1, 2, 3, \dots, t; j = 1, 2, 3, \dots, v$ and $k = 1, 2, 3, \dots, 8$ and the neutrosophic decision matrix $D = (\tilde{d}_{ij})_{m \times n}$, where

$$\begin{array}{c}
 \hline
 \hline
 \begin{array}{cccc}
 E_1 & E_2 & \dots & E_n
 \end{array} \\
 \hline
 (\tilde{d}_{ij})_{m \times n} = \begin{array}{cccc}
 U_1 & \tilde{d}_{11} & \tilde{d}_{12} & \dots & \tilde{d}_{1v} \\
 U_2 & \tilde{d}_{21} & \tilde{d}_{22} & \dots & \tilde{d}_{2v} \\
 \vdots & \vdots & \vdots & \vdots & \vdots \\
 U_t & \tilde{d}_{t1} & \tilde{d}_{t2} & \dots & \tilde{d}_{tv}
 \end{array} \\
 \hline
 \hline
 \end{array}$$

for $i = 1, 2, \dots, t$ and $j = 1, 2, \dots, v$. Here we apply Value and ambiguity indices of SVLONNs to solve MADM problem.

1. Normalization of SVLONN based on decision matrix

In order to eliminate the effect of different physical dimensions during the process of final decision-making, the decision matrix $(\tilde{d}_{ij})_{t \times v}$ is normalized into $(\tilde{r}_{ij})_{t \times v}$ by using the linear normalization technique.

2. Aggregation of the weighted values of alternatives

The aggregated weighted values of the alternatives $U_i (i = 1, 2, \dots, t)$ is determined by

$$\tilde{S}_i = \sum_{j=1}^v w_j \tilde{r}_{ij} \tag{14.17}$$

respectively. Here the aggregated weighted values $\tilde{S}_i (i = 1, 2, \dots, t)$ are defined using SVLONNs.

3. Ranking of all alternatives

Ranking of all alternatives of all alternatives is determined by using the value and ambiguity indices of SVLONN using \tilde{S}_i .

14.5 Illustration of MADM Problem

Consider the following situation experienced by a Principal of a College in selecting a candidate for the post of Assistant Professor in the college. Suppose three candidates say U_1, U_2, U_3 has been shortlisted after a written test to be appointed as an assistant professor based on the criteria such as educational qualification (E_1), past experience (E_2), and research publications (E_3). The assessment of the recruiter for each candidate based on the information corresponding to the attributes are expressed as SVLONNs using the linguistic terms:

Extremely Low-(0.0, 0.0, 0.0, 0.0, 0.0, 0.05, 0.1, 0.15;0.3);

Low-(0.05, 0.1, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4;0.3);

Neither low nor medium-(0.2, 0.25, 0.3, 0.35, 0.4, 0.45, 0.5, 0.55;0.3);

Medium-(0.35, 0.40, 0.45, 0.5, 0.55, 0.6, 0.65, 0.7;0.3);

Neither medium nor high-(0.5, 0.55, 0.6, 0.65, 0.7, 0.75, 0.8, 0.85;0.3);

High-(0.6, 0.65, 0.7, 0.75, 0.8, 0.85, 0.9, 0.95;0.3);

Extremely High-(0.75, 0.8, 0.85, 0.9, 0.92, 0.96, 0.98, 1.0;0.3). The assigned weight vectors of three attributes are $w = \{0.31, 0.34, 0.35\}$. The complete perception of the Principal about the individual is modeled as SVLONNs and the decision matrix is given by Table 14.1 where E_i represents the attributes, U_1, U_2, U_3 are alternatives and \tilde{d}_{ij} are SVLONNs.

Table 14.1 SVLONNs-based decision matrix

| | | | |
|-----------------------------------|-------|------------------|------------------|
| | E_1 | E_2 | E_3 |
| $(\tilde{d}_{ij})_{3 \times 3} =$ | U_1 | \tilde{d}_{11} | \tilde{d}_{12} |
| | U_2 | \tilde{d}_{21} | \tilde{d}_{22} |
| | U_3 | \tilde{d}_{31} | \tilde{d}_{33} |

Based on the individual U_1 's educational qualification, we have

$$\tilde{d}_{11} = \langle \text{extremely high; medium; extremely low} \rangle \\ = (0.75, 0.80, 0.85, 0.90, 0.92, 0.96, 0.98, 1.00; 0.35, 0.40, 0.45, 0.50, 0.55; 0.60, 0.65, 0.70; 0.00, 0.00, 0.00, 0.05, 0.1, 0.15, 0.20; 0.3)$$

similarly for U_1 's past experience, we have

$$\tilde{d}_{12} = \langle \text{medium; low; extremely low} \rangle \\ = (0.35, 0.40, 0.45, 0.50, 0.55, 0.60, 0.65, 0.70; 0.05, 0.10, 0.15, 0.20, 0.25, 0.30, 0.35, 0.40; 0.00, 0.00, 0.00, 0.05, 0.10, 0.15, 0.20; 0.3)$$

and for U_1 's research publications, we have

$$\tilde{d}_{13} = \langle \text{neither low nor medium; low; neither medium nor high} \rangle \\ = (0.20, 0.25, 0.30, 0.35, 0.40, 0.45, 0.50, 0.55; 0.05, 0.10, 0.15, 0.20, 0.25, 0.30, 0.35, 0.40; 0.50, 0.55, 0.60, 0.65, 0.70, 0.75, 0.80, 0.85; 0.3)$$

Based on the individual U_2 's educational qualification, we have

$$\tilde{d}_{21} = \langle \text{neither low nor medium; high; neither medium nor high} \rangle \\ = (0.20, 0.25, 0.30, 0.35, 0.40, 0.45, 0.50, 0.55; 0.60, 0.65, 0.70, 0.75, 0.80, 0.85, 0.90, 0.95; 0.50, 0.55, 0.60, 0.65, 0.70, 0.75, 0.80, 0.85; 0.3)$$

for U_2 's past experience, we have

$$\tilde{d}_{22} = \langle \text{high; medium; extremely low} \rangle \\ = (0.60, 0.65, 0.70, 0.75, 0.80, 0.85, 0.90, 0.95; 0.35, 0.40, 0.45, 0.50, 0.55, 0.60, 0.65, 0.70; 0.00, 0.00, 0.00, 0.05, 0.10, 0.15, 0.20; 0.3)$$

and for U_2 's research publications, we have

$$\tilde{d}_{23} = \langle \text{high; medium; extremely low} \rangle \\ = (0.60, 0.65, 0.70, 0.75, 0.80, 0.85, 0.90, 0.95; 0.35, 0.40, 0.45, 0.50, 0.55, 0.60, 0.65, 0.70; 0.00, 0.00, 0.00, 0.05, 0.10, 0.15, 0.20; 0.3)$$

Based on the individual U_3 's educational qualification, we have

$$\tilde{d}_{31} = \langle \text{extremely low; neither low nor medium; high} \rangle \\ = (0.00, 0.00, 0.00, 0.00, 0.05, 0.10, 0.15, 0.20; 0.20, 0.25, 0.30, 0.35, 0.40, 0.45, 0.50, 0.55; 0.60, 0.65, 0.70, 0.75, 0.80, 0.85, 0.90, 0.95; 0.3)$$

for U_3 's past experience, we have

$$\tilde{d}_{32} = \langle \text{neither low nor medium; high; extremely high} \rangle \\ = (0.20, 0.25, 0.30, 0.35, 0.40, 0.45, 0.50, 0.55; 0.60, 0.65, 0.70, 0.75, 0.80, 0.85, 0.90, 0.95; 0.75, 0.80, 0.85, 0.90, 0.92, 0.96, 0.98, 1.00; 0.3)$$

and for U_3 's research publications, we have

$$\tilde{d}_{33} = \langle \text{extremely high; neither low nor medium; low} \rangle \\ = (0.75, 0.80, 0.85, 0.90, 0.92, 0.96, 0.98, 1.00; 0.20, 0.25, 0.30, 0.35, 0.40, 0.45, 0.50, 0.55; 0.05, 0.10, 0.05, 0.20, 0.25, 0.30, 0.35, 0.40; 0.3)$$

We rank the alternatives U_1, U_2, U_3 by examining the value index and the ambiguity index of each alternative for different values of $\psi, \eta, \zeta \in [0, 1]$ as tabulated in Table 14.2 (using MathCad 14).

Table 14.2 Ranking results for alternatives

| Alternative | Value of ψ, η, ζ | Value index | Ambiguity index | Ranking order |
|-------------|---------------------------------------------------------------|-------------|-----------------|-------------------|
| U_1 | $\psi = 0.10, \eta = 0.40, \zeta = 0.50$ | 0.30 | 0.11 | $U_3 > U_2 > U_1$ |
| U_2 | | 0.38 | 0.10 | |
| U_3 | | 0.48 | 0.09 | |
| U_1 | $\psi = \frac{1}{3}, \eta = \frac{1}{3}, \zeta = \frac{1}{3}$ | 0.10 | 0.09 | $U_3 > U_2 > U_1$ |
| U_2 | | 0.15 | 0.09 | |
| U_3 | | 0.25 | 0.08 | |
| U_1 | $\psi = 0.70, \eta = 0.20, \zeta = 0.10$ | -0.22 | 0.07 | $U_3 > U_2 > U_1$ |
| U_2 | | -0.21 | 0.07 | |
| U_3 | | -0.10 | 0.06 | |

For this choice of values of $\psi, \eta, \zeta \in [0, 1]$, the ranking of alternatives are obtained as follows: $U_3 > U_2 > U_1$.

Remark 14.6 When the problem is carried for the value $k = 0$, the ranking is obtained as follows (Table 14.3):

Table 14.3 Ranking results for alternatives

| Alternatives | Value of ψ, η, ζ | Value Index | Ambiguity index | Ranking order |
|--------------|---------------------------------------------------------------|-------------|-----------------|-------------------|
| U_1 | $\psi = 0.10, \eta = 0.40, \zeta = 0.50$ | 0.29 | 0.17 | $U_3 > U_2 > U_1$ |
| U_2 | | 0.37 | 0.17 | |
| U_3 | | 0.47 | 0.12 | |
| U_1 | $\psi = \frac{1}{3}, \eta = \frac{1}{3}, \zeta = \frac{1}{3}$ | 0.06 | 0.22 | $U_3 > U_2 > U_1$ |
| U_2 | | 0.11 | 0.25 | |
| U_3 | | 0.23 | 0.10 | |
| U_1 | $\psi = 0.70, \eta = 0.20, \zeta = 0.10$ | -0.28 | 0.30 | $U_3 > U_2 > U_1$ |
| U_2 | | -0.29 | 0.37 | |
| U_3 | | -0.15 | 0.06 | |

Remark 14.7 For a different choice of weighting functions, $f(\alpha_o) = 1 - \alpha_o$, $g(\beta_o) = \beta_o$ and $h(\gamma_o) = \gamma_o$ we have the problem worked out along line and the outcome is tabulated in Table 14.4.

Table 14.4 Ranking results for alternatives

| Alternatives | Value of ψ, η, ζ | Value index | Ambiguity index | Ranking order |
|--------------|---------------------------------------------------------------|-------------|-----------------|-------------------|
| U_1 | $\psi = 0.10, \eta = 0.40, \zeta = 0.50$ | 0.39 | 0.16 | $U_3 > U_2 > U_1$ |
| U_2 | | 0.49 | 0.15 | |
| U_3 | | 0.55 | 0.14 | |
| U_1 | $\psi = \frac{1}{3}, \eta = \frac{1}{3}, \zeta = \frac{1}{3}$ | 0.45 | 0.14 | $U_2 > U_3 > U_1$ |
| U_2 | | 0.54 | 0.14 | |
| U_3 | | 0.52 | 0.12 | |
| U_1 | $\psi = 0.70, \eta = 0.20, \zeta = 0.10$ | 0.53 | 0.12 | $U_2 > U_1 > U_3$ |
| U_2 | | 0.61 | 0.12 | |
| U_3 | | 0.48 | 0.16 | |

Remark 14.8 When the problem is carried for the value $k = 0$, the ranking is obtained as follows:

Comparing Tables 14.2 and 14.4 we note that the variations in α_o, β_o and γ_o affect the ranking system. So depending on the importance given to the various criteria considered there will be variation in the ranking also. Thus the choice of the candidate will differ from recruiter to recruiter. This is one recruiter’s perception for two patterns.

Table 14.5 Ranking results for alternatives

| Alternative | Value of ψ, η, ζ | Value index | Ambiguity index | Ranking order |
|-------------|---------------------------------------------------------------|-------------|-----------------|-------------------|
| U_1 | $\psi = 0.10, \eta = 0.40, \zeta = 0.50$ | 0.38 | 0.15 | $U_3 > U_2 > U_1$ |
| U_2 | | 0.48 | 0.14 | |
| U_3 | | 0.54 | 0.13 | |
| U_1 | $\psi = \frac{1}{3}, \eta = \frac{1}{3}, \zeta = \frac{1}{3}$ | 0.44 | 0.13 | $U_2 > U_3 > U_1$ |
| U_2 | | 0.53 | 0.12 | |
| U_3 | | 0.51 | 0.11 | |
| U_1 | $\psi = 0.70, \eta = 0.20, \zeta = 0.10$ | 0.52 | 0.09 | $U_2 > U_1 > U_3$ |
| U_2 | | 0.61 | 0.09 | |
| U_3 | | 0.47 | 0.07 | |

From Tables 14.2, 14.3, 14.4, 14.5 we observe that for better ranking, SVLONNs are used.

14.6 Conclusion

In this paper, we introduced and studied the idea of SVLONN. Value index and Ambiguity index of SVLONNs are discussed. With the help of the same, a ranking

method for SVLONNs is developed and applied to a MADM problem. Depending on the need of the person making choices with respect to $\alpha_o, \beta_o, \gamma_o, \psi, \eta, \zeta$ there will be variation in the output. Based on the need one can choose truth value, indeterminacy value, falsity value, coefficients of value, and ambiguity indices which cause variation in the ranking. In a similar type of setup in any other field, this model can be used (to cite a few medical diagnosis, pattern recognition, personal selection). Further, value index and ambiguity index can be used in transportation problem.

Acknowledgements We thank DST (FIST 2006) MATCAD 14 which is used for computational purpose.

References

1. Abdel Aal, S.I., Mahmoud, M.A., Ellatif, A., Hassan, M.M.: Two ranking methods of single valued triangular neutrosophic numbers to rank and evaluate information system quality. *Neutrosophic Sets Syst.* **19**, 132–141 (2018)
2. Atanassov, K.: Intuitionistic fuzzy sets. *Fuzzy Sets Syst.* **20**(1), 87–96 (1986)
3. Biswas, P., Pramanik, S., Giri, B.C.: Value and ambiguity index based ranking method of single valued trapezoidal neutrosophic numbers and its application to multi-attribute decision making. *Neutrosophic Sets Syst.* **12**, 127–138 (2016)
4. Chakraborty, A., Mondal, S.P., Ahmadian, A., Senu, N., Alam, S., Salahshour, S.: Different forms of triangular neutrosophic numbers, de-neutrosophication techniques, and their applications. *Symmetry* **10**(8), 327 (2018)
5. Delgado, M., Vila, M., Voxman, W.: A fuzziness measure for fuzzy numbers: applications. *Fuzzy Sets Syst.* **94**(2), 205–216 (1998)
6. Delgado, M., Vila, M., Voxman, W.: On a canonical representation of fuzzy numbers. *Fuzzy Sets Syst.* **93**(1), 125–135 (1998)
7. Deli, I., Şubaş, Y.: A ranking method of single valued neutrosophic numbers and its applications to multi-attribute decision making problems. *Int. J. Mach. Learn. Cybern.* **8**(4), 1309–1322 (2016)
8. El-Hefnawy, N., Metwally, M.A., Ahmed, Z.M., El-henawy, I.M.: A review on the applications of neutrosophic sets. *J. Comput. Theor. Nanosci.* **13**(1), 936–944 (2016)
9. Koundal, D., Gupta, S., Singh, S.: Applications of neutrosophic sets in medical image denoising and segmentation. *New Trends Neutrosophic Theory Appl.* 257–275 (2016)
10. Malini, S.U., Kennedy, F.C.: An approach for solving fuzzy transportation problem using octagonal fuzzy numbers. *Appl. Math. Sci.* **7**, 2661–2673 (2013)
11. SahayaSudha, A., Vijayalakshmi, K.R.: Multi-attribute decision making method using value and ambiguity in single valued hexagonal neutrosophic numbers. *Int. J. Sci. Res. Math. Stat. Sci.* **6**(3), 84–90 (2019)
12. Smarandache, F.: *A Unifying Field in Logics. Neutrosophy: Neutrosophic Probability, Set and Logic.* Rehoboth: American Research Press. Published. Press (1999)
13. Wang, H., Smarandache, F., Zhang, Y., Sunderraman, R.: Single valued neutrosophic sets. *Tech. Sci. Appl. Math.* **10**, 10–14 (2012)
14. Zadeh, L.A.: Fuzzy sets. *Inf. Control* **8**, 338–353 (1965)