Calculation and Optimization of the Two Stage Worm-Gear Reducers Speed Ratio

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1 Introduction

Worm gear drives are used in transmission systems because of their ability to achieve high reduction ratio with compact size (Rai and Barman 2019). Currently, there have been a number of studies in the world on worm gear optimization (Mogal and Wakchaure 2013; Padmanabhan et al. 2013; Godwin Raja et al. 2017; Alexandru 2010). The growing demand for compact, efficient, and reliable reducers forces the designer to use the optimal design approach.

Rai and Barman (2019) presented the power loss reduction of worm gear drive (objective function) using simulated annealing (SA), together with design variables (gear tooth number z_2, helix angle γ, and coefficient of friction) and constraints on bending stress, defection of worm and linear pressure. The obtained results showed that the performance of SA is better than that of other approaches used previously to minimize power loss. The reduction percentage in power loss using SA ranged from 78.02 to 22.98%. Mogal and Wakchaure (2013) used genetic algorithm (GA) to optimize the worm using design variables, including gear ratio, face width, and pitch circle diameters. The obtained results were 17.91% reduction in volume of worm and worm wheel, 20.34% reduction in center distance between worm and worm wheel, and 51.05% reduction in deflection of worm. Padmanabhan et al. (Padmanabhan et al. 2013) built an optimization model for worm gear drives in terms of design variables m, z_2, and p and design constraints on stress, using ant colony optimization (ACO) method and objective functions on capacity, mass, efficiency, and shaft distance, obtaining problem values with modulus between 4 and 8 mm. Godwin Raja

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R. K Agarwal (ed.), *Recent Advances in Manufacturing Engineering and Processes*, Lecture Notes in Mechanical Engineering,
https://doi.org/10.1007/978-981-19-6841-9_5

Ebenezer et al. (2017) used optimization algorithms including SA, FA, CS, and GA by MATLAB solver with design variables as gear ratio, face width, and pitch circle diameters. The result was a 42.06% reduction in worm drive mass. The design parameter values reduced by 13.04% in gear ratio, 35.53% in face width, and increased by 32.97% in pitch circle diameter of worm.

Some other studies also gave similar results on optimizing geometric parameter values (Alexandru 2010; Dudas 2005; Park et al. 2013; Su and Peng 2008; Chong et al. 2002), increasing efficiency values (Padmanabhan et al. 2013; Krol and Sokolov 2021), reducing power loss (Rai and Barman 2019; Patil et al. 2019), volume reduction (Mogal and Wakchaure 2013; Godwin Raja Ebenezer et al. 2017; Patil et al. 2019; Golabi et al. 2014), low noise (Chul Kim et al. 2020), yet with old and incomplete formulas. Optimizing the design of a worm drive is complex and time consuming because of the variety of objectives, design variables, constraints, empirical formulas, various graphs, and tables (Padmanabhan et al. 2013; Chandrasekaran et al. 2015; Elkholy and Falah 2015; Marjanovic et al. 2012).

At present, the speed ratio distribution in the two stage worm-gear reducer is mainly done according to the graph (Miltenović et al. 2017). Based on the preselected parameters and the total speed ratio u_h of the reducers, researchers look up the speed ratio u_1 of the fast drive in the graph, then calculate the speed ratio u_2 of the low drive according to the formula $u_h = u_1\ u_2$.

This method not only has low accuracy due to the need to look up information in the graph but is also inconvenient because of the necessity to carry lookup documents, and it is very difficult to program automatic calculations. In addition, studies only focus on the worm type in the below position, while the worm in the above position and two stage worm-gear reducers with bevel gear have not had enough research.

In this paper, the speed ratio distribution in a two stage worm-gear reducer with the worm in the above position, spur gears (straight and helical) and bevel gears is presented. The calculation is based on the constant condition of the torque on the worm shaft and the uniform contact strength condition of the drives in the reducer, satisfying the oil-immersed lubrication conditions and the minimum volume.

2 Fundamental of Gear and Worm Drives Calculations

For well-lubricated gear and worm drives, design calculations are required according to contact strength. To save manufacturing materials, when designing, engineers must ensure equal contact strength of worm drive and gear pairs. The problem of distribution of gear speed ratios of two stage worm-gear reducers to ensure that uniform contact strength and lubrication conditions with minimization volume is considered as an optimization problem and is stated as follows

$$X = \begin{Bmatrix} u_{12} \\ u_{34} \\ \ldots \\ u_{n-1,n} \end{Bmatrix} \tag{1}$$

Along with constraints:

- Equal contact strength between the drives

$$\sigma_{H12} \approx \sigma_{H34} \approx \ldots \approx \sigma_{H(n-1),n} \tag{2}$$

- Gear oil immersion lubrication conditions $1 \geq \frac{d_2}{d_4} \geq \frac{2}{3}$.
- The parameters have standard values: gear ratio, gear width, gear width factor, and diameter factor…
- Constraint volume: minimum volume $d_2 = d_4$.

In addition, depending on the goal of the problem, there may be many different constraints.

2.1 *Speed Ratio Distribution in Two Stage Worm-Spur Gear Reducer*

The scheme of the two stage worm-spur gear reducer is shown in Fig. 1: 1–2 is worm drive, and 3–4 is spur gear pair.

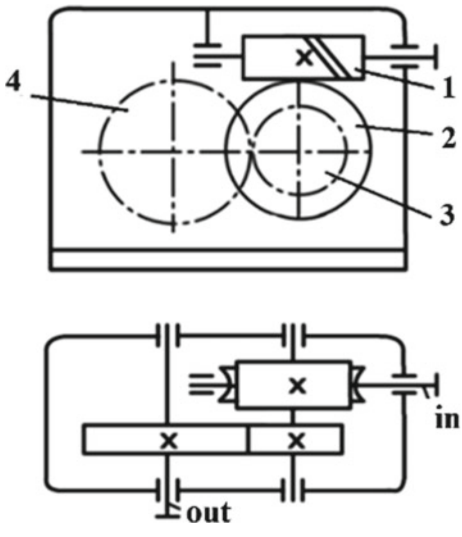


Fig. 1 Kinematic scheme of two stage worm-spur gear reducer

For Worm Drive 1–2. According to Nguyen (2020), the center distance of the worm drive is determined by the formula

$$a_w = \left(1 + \frac{q}{z_2}\right)\sqrt[3]{\left(\frac{5400}{[\sigma_H]}\right)^2 \frac{T_2 K_H}{(q/z_2)}} \quad (3)$$

where q is the diameter factor; z_2 is the number of worm gear teeth; a_ω is the center distance of worm drive; T_2 is the torque on the worm gear shaft [Nm]; K_H is the contact calculated factor; $[\sigma_H]$ is the allowable contact stress of the worm gear wheel.

From (3), it is possible to determine the allowable torque on the worm gear shaft

$$[T_2] = \frac{1}{5400^2}\left(\frac{a_{12}}{1 + q/z_2}\right)^3 \cdot \frac{(q/z_2)[\sigma_{H2}]^2}{K_{H2}} \quad (4)$$

Substituting $a_{12} = \frac{1}{2}d_2(1 + q/z_2)$ into formula (4), we have

$$[T_2] = \frac{d_2^3}{8 \cdot 5400^2 \cdot u_{12} \cdot \tan\gamma} \frac{[\sigma_{H2}]^2}{K_{H2}} \quad (5)$$

where $d_2 = mz_2$ is the pitch circle diameter and the worm wheel; u_{12} is the speed ratio of worm drive, and γ is lead angle

$$\tan\gamma = \frac{z_1}{q} = \frac{z_2}{u_{12} \cdot q} \quad \text{or} \quad \frac{q}{z_2} = \frac{1}{u_{12} \cdot \tan\gamma} \quad (6)$$

For Second Stage of Spur Gear Pair 3–4. For well-lubricated gear pairs, we calculate by contact strength (Nguyen 2020; ISO 6336-2:2006), and we have

$$\sigma_{H3} = Z_M Z_H Z_\varepsilon \sqrt{\frac{2T_3 \cdot 10^3 \cdot K_H(u_{34} \pm 1)}{u_{34} b_{\omega 34} d_1^2}} \le [\sigma_{H3}] \quad (7)$$

where Z_M is the elasticity factor that takes into account gear material properties (modulus of elasticity and Poisson's ratio); Z_H is the factor taking into account the shape of the contact surface; Z_ε is the factor taking into account the effect of total contact length (contact ratio factor); T_3 is the torque on the driving shaft; $b_{\omega 34}$ is the width of driving gear 3; d_3 is the pitch diameter of the driving gear 3; and u_{34} is the speed ratio of worm drive.

From (7), the allowable torque on the driving gear can be determined by

$$[T_3] = \frac{u_{34} b_{\omega 34} d_3^2 [\sigma_{H3}]}{2 \times 10^3 \cdot (Z_M Z_H Z_\varepsilon)^2 K_{H3}(u_{34} \pm 1)} \quad (8)$$

where $b_{w34} = \psi_{bd34} d_3$ is width of spur gear; ψ_{bd34}, ψ_{ba34} is a width ratio; and ψ_{ba34} is taken from the sequence of standard numbers: 0.1, 0.125, 0.16, 0.2, 0.25, 0.315, 0.4, 0.5, and 0.63 depending on the wheel position relative to the bearings $\psi_{bd34} = \frac{1}{2}\psi_{ba34}(u_{34} \pm 1)$.

From (8) and replacing $d_4 = u_{34} \cdot d_3$, we have

$$[T_3] = \frac{\psi_{ba34} d_4^3 [\sigma_{H3}]^2}{2K_d^3 K_{H3} u_{34}^2} \quad (9)$$

where $K_d = \sqrt[3]{2 \times 10^3 \cdot (Z_M Z_H Z_\varepsilon)^2}$, since the gear material is steel, then calculated value $K_d = 756$ for straight and $K_d = 680$ for helical spur gears.

To ensure the conditions of equal contact stress, from (5), (9), with $u_{12} = \frac{u_h}{u_{34}}$, u_h is the speed ratio of the two stage worm-gear reducer, we infer

$$u_{34}^3 = 4 \cdot 5400^2 \cdot \tan\gamma \frac{d_4^3}{d_2^3} \frac{\psi_{ba34}}{K_d^3} \frac{[\sigma_{H3}]^2}{[\sigma_{H2}]^2} \frac{K_{H2}}{K_{H3}} \cdot u_h \quad (10)$$

The obtained formula depends on the following conditions:

1. Worm gear—gear material;
2. The width ratio of the gear drive, type of the spur gear: straight or helical;
3. The relationship between d_2 and d_4: $d_2 = d_4$ or $1 \geq \frac{d_2}{d_4} \geq \frac{2}{3}$

To ensure oil-immersed lubrication conditions, we have the formula

$$1 \leq u_{34}^3 = 4 \cdot 5400^2 \cdot \tan\gamma \frac{d_4^3}{d_2^3} \frac{\psi_{ba34}}{K_d^3} \frac{[\sigma_{H3}]^2}{[\sigma_{H2}]^2} \frac{K_{H2}}{K_{H3}} \cdot u_h \leq \left(\frac{3}{2}\right)^3 \quad (11)$$

2.2 *Speed Ratio Distribution in Two Stage Worm-Bevel Gear Reducer*

The scheme of the two stage worm-bevel gear reducer is shown in Fig. 2 (1–2 is worm drive, and 3–4 is bevel gear drive). The contact stress for bevel gears is determined

$$\sigma_{H3} = 2 \cdot Z_M Z_H Z_\varepsilon \sqrt{\frac{T_3 \cdot 10^3 \cdot K_{H\beta3}.u_{34}^2}{0.85 \cdot (1 - 0.5 \cdot \psi_{be})^2 \cdot \psi_{be} \cdot d_4^3}} \leq [\sigma_{H3}] \quad (12)$$

From here, we deduce the formula to determine the allowable torque value

$$[T_3] = \frac{d_4^3}{(2 \cdot Z_M Z_H Z_\varepsilon)^2 \cdot 10^3} \frac{0.85 \cdot (1 - 0.5\psi_{be})^2 \cdot \psi_{be} \cdot [\sigma_{H3}]^2}{K_{H\beta3} \cdot u_{34}^2} \quad (13)$$

Or shorten to

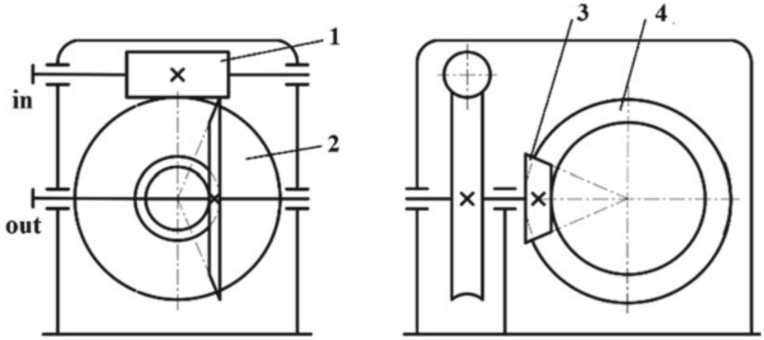


Fig. 2 Scheme of two stage worm-bevel gear reduce

$$[T_3] = \frac{d_4^3}{2 \cdot K_d^3} \cdot \frac{0.85 \cdot (1 - 0.5\psi_{be})^2 \cdot \psi_{be} \cdot [\sigma_{H3}]^2}{K_{H\beta 3} \cdot u_{34}^2} \quad (14)$$

with $K_d = \sqrt[3]{2 \cdot 10^3 \cdot (Z_M Z_H Z_\varepsilon)^2}$.

The conditions of equal contact stress $[T_3] = [T_2]$; from (5) and (14), we deduce

$$\frac{d_4^3}{2 \cdot K_d^3} \cdot \frac{0.85 \cdot (1 - 0.5 \cdot \psi_{be})^2 \cdot \psi_{be} \cdot [\sigma_{H3}]^2}{K_{H\beta 3} \cdot u_{34}^2} = \frac{d_2^3}{8 \cdot 5400^2 \cdot u_{12} \cdot \tan\gamma} \cdot \frac{[\sigma_{H2}]^2}{K_{H2}} \quad (15)$$

Replacing $u_{12} = \frac{u_h}{u_{34}}$ and after simplifying, we get

$$u_{34}^3 = \left(\frac{d_4}{d_2 \cdot K_d}\right)^3 \cdot 4 \cdot 5400^2 \cdot \tan\gamma \cdot 0.85(1 - 0.5 \cdot \psi_{be})^2 \psi_{be} \frac{K_{H2}}{K_{H\beta 3}} \frac{[\sigma_{H3}]^2}{[\sigma_{H2}]^2} u_h \quad (16)$$

To ensure oil-immersed lubrication conditions, we choose $1 \geq \frac{d_2}{d_4} \geq \frac{2}{3}$, and from there deduce the formula

$$1 \leq u_{34}^3 = \left(\frac{d_4}{d_2}\right)^3 \cdot 4 \cdot 5400^2 \cdot \tan\gamma \cdot 0.85 \cdot \frac{K_{H2}}{K_{H\beta 3}} \frac{(1 - 0.5\psi_{be})^2 \cdot \psi_{be}}{K_d^3} \frac{[\sigma_{H3}]^2}{[\sigma_{H2}]^2} u_h \leq \left(\frac{3}{2}\right)^3 \quad (17)$$

To ensure the minimum volume, $d_2 = d_4$, we should choose the smallest value u_{34} in the range of the above gear ratio values.

3 Calculation Results and Applications

3.1 Speed Ratio Distribution in Two Stage Worm-Spur Gear Reducer

For worm drive, we choose the lead angle $\gamma = 10°(< 30°)$ and the material for making worm gear: tinless bronze and brass are used with intermediate slip velocities (<5 m/s). This bronze has high mechanical strength, but low score resistance, so it is used together with quenched (>45 HRC) ground and polished worms (Grote and Antonsson 2009).

$$[\sigma_{H2}] = (276 \div 300) - 25v_s \approx 200 \text{ MPa}$$

Calculated factors $K_{H2} = K_v \cdot K_\beta = 1.1$ (Nguyen 2020), where K_v is internal gearing dynamics caused by manufacturing errors (accuracy grade 7, slip velocities $v_s = 3 - 7.5$ m/s), and $K_\beta \approx 1$ is the factor that takes into account the unevenness of the load distribution in the contact area due to distortion of the worm and wheel shaft.

For spur gear pairs, material selection for gear train with driving gear being made of C45 steel, normalization or refining with the hardness of 300HB, $\sigma_{0H\,\mathrm{lim}} = 2 \cdot HB + 70$= 670 MPa, we select $[\sigma_{H3}] = 603$ MPa.

For design, we select $K_{H3} = K_{H\beta} = 1.01$ depending on factor ψ_{bd} and hardness (Grote and Antonsson 2009). The width ratio taken from the sequence of standard numbers depending on the wheel position relative to the bearings: for symmetrical arrangement, we select $\psi_{ba34} = 0.315$; $\psi_{ba34} = 0.4$; and $\psi_{ba34} = 0.5$.

The calculation results and the combination of selecting the gear ratio u_{34} of the second stage are according to 2 standard ranges (series 1 is the priority series in bold), and the smallest value should be selected to ensure the smallest volume and weight as shown in Table 1.

Analysis of the results on Minitab software obtained the dependence of the minimum gear ratio of the gear pair u_{34} and the gear pair ratio of the two speed reducer u_h in the form of the quadratic regression equations with testing for lack of fit (R-square = 99.7%) presented in Table 2.

We draw a graph showing the relationship between speed ratio u_h of two stage worm-spur gear reducer and speed ratio u_{34} of the second stage to ensure equal strength, lubrication and minimum volume with respective width ratio in Fig. 3.

Table 1 Calculation results of the distribution of minimum gear speed ratio u_{34}

uh	Spur gear						Bevel gear
	$\psi_{ba34} = 0.315$		$\psi_{ba34} = 0.4$		$\psi_{ba34} = 0.5$		
	Straight	Helical	Straight	Helical	Straight	Helical	
22.4	1.49	1.66	1.62	1.80	1.74	1.94	1.23
25	1.55	1.72	1.68	1.86	1.81	2.01	1.28
28	1.61	1.79	1.74	1.94	1.88	2.09	1.33
31.5	1.67	1.86	1.81	2.01	1.95	2.17	1.38
35.5	1.74	1.93	1.88	2.10	2.03	2.26	1.44
40	1.81	2.01	1.96	2.18	2.11	2.35	1.50
45	1.88	2.09	2.04	2.27	2.20	2.44	1.56
50	1.95	2.17	2.11	2.35	2.28	2.53	1.61
56	2.03	2.25	2.19	2.44	2.36	2.63	1.68
63	2.11	2.34	2.28	2.54	2.46	2.73	1.74
71	2.19	2.44	2.37	2.64	2.56	2.84	1.81
80	2.28	2.54	2.47	2.75	2.66	2.96	1.89
90	2.37	2.64	2.57	2.86	2.77	3.08	1.96
100	2.46	2.73	2.66	2.96	2.87	3.19	2.03
110	2.54	2.82	2.75	3.05	2.96	3.29	2.10
125	2.65	2.94	2.87	3.19	3.09	3.43	2.19
140	2.75	3.06	2.98	3.31	3.21	3.57	2.27
160	2.87	3.20	3.11	3.46	3.35	3.73	2.38
180	2.99	3.32	3.24	3.60	3.49	3.88	2.47
200	3.10	3.44	3.35	3.73	3.61	4.02	2.56

Table 2 Regression equations determines the minimum gear ratio

Two stage reducer	Factor ψ_{ba}	Regression equations	R-square (%)
Worm-straight spur gear	0.315	$u_{34} = 1.205 + 0.01598u - 0.000034u^2$	99.7
	0.4	$u_{34} = 1.304 + 0.01731u - 0.000036u^2$	99.7
	0.5	$u_{34} = 1.406 + 0.01866u - 0.000039u^2$	99.7
Worm-helical spur gear	0.315	$u_{34} = 1.337 + 0.01781u - 0.000038u^2$	99.7
	0.4	$u_{34} = 1.452 + 0.01923u - 0.000040u^2$	99.7
	0.5	$u_{34} = 1.567 + 0.02062u - 0.000043u^2$	99.7
Worm-bevel gear	0.285	$u_{34} = 0.9954 + 0.01323u - 0.000028u^2$	99.7

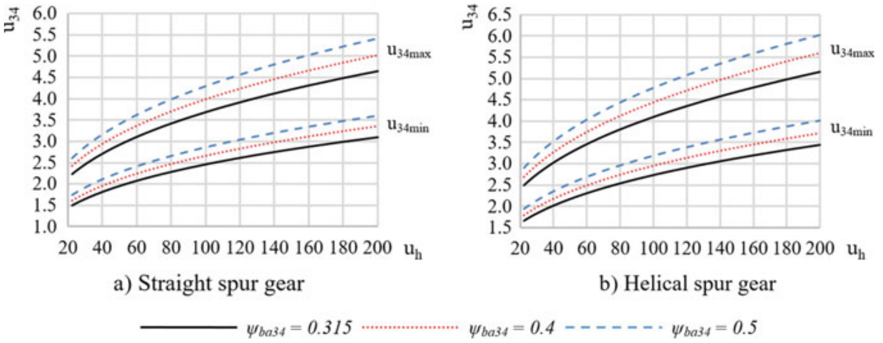


Fig. 3 Speed ratio distribution of two stage worm-spur gear reducer

3.2 Speed Ratio Distribution in Two Stage Worm-Bevel Gear Reducer

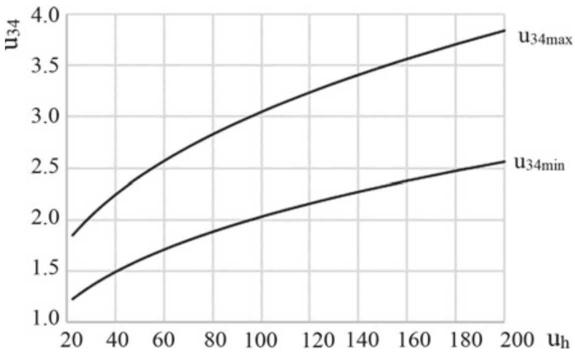
We select worm drive the same as two stage worm-spur gear reducer. For bevel gear, material selection for gear train with driving gear being made of C45 steel, normalization or refining with the hardness of 300 HB: $\sigma_{0H\,\lim} = 2HB + 70$ =670 MPa, we select $[\sigma_{H3}] = 603$ MPa. Preliminary selection of factor $K_{H3} = K_{H\beta} = 1.07$.

The calculation results and the combination of selecting the bevel gear ratio u_{34} of the second stage are according to standard ranges (series 1 is the priority series), and the smallest value should be selected to ensure the minimum volume and weight as shown in the last column of Table 1.

The results of the quadratic regression equations showing the dependence of minimum gear ratio u_{34} on u_h are presented in Table 2.

We construct a graph showing the relationship between speed ratio u_h of two stage worm-bevel gear reducer and speed gear ratio u_{34} of the second stage to ensure equal strength, lubrication and minimum volume in Fig. 4

Fig. 4 Speed ratio distribution of two stage worm-bevel gear reducer



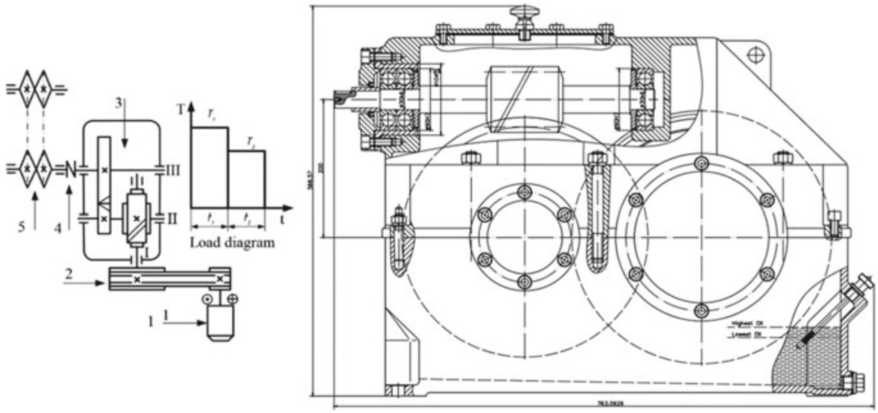


Fig. 5 Kinematic scheme and front view drawing of two stage worm-spur gear

In this section, the above research results are applied to select the gear ratio and to design the transmission systems for loads chain system with the kinematic scheme shown in Fig. 5.

The loads chain system includes: 1—motor; 2—V belt drive; 3—double stage worm-helical gear reducer; 4—coupling; 5—loads chain sprocket; and 6—loads chain. Input data for design: force on chain $F = 23{,}000$ [N]; chain velocity $v = 0.2$ [m/s]; pith diameter of loads chain sprocket $d = 315$ [mm].

According to the power 4.825 [kW], with the common speed ratio of the two stage reducer $u_r = 50$, we choose the gear ratio of the gear pair according to the priority set of value $u_{34} = 2.5$. To optimize the product volume of the reducer, we should choose $d_2 \approx d_4$, illustrated in drawing Fig. 5.

4 Conclusion

This paper has established an analytical formula for the distribution of speed ratios in a two stage worm-gear reducer with either spur gear or bevel gear. The research addresses the combination of three goals, which are to ensure the conditions of equal contact stress and conditions of oil-immersed lubrication as well as the minimum volume. Formulas in Table 2 not only allow quick and accurate determination of speed ratios u_{12}, u_{34} according to the total speed ratio u_h, but also facilitate programming to design automatically. The paper has constructed a series of speed ratios and graphs in the two stage worm-gear reducer (Figs. 3 and 4) to look up the gear ratios corresponding to each gear width ratio value. In addition, we can also extend this calculation method to apply to another scheme of two stage and triple stage worm-gear reducers.

This paper results can be used to design and calculate the distribution of speed ratios in two stage worm-gear reducers, as a scientific basis for the distribution of speed ratios for other types of reducers and used as reference when researching, as well as teaching and learning.

Acknowledgements We acknowledge Ho Chi Minh City University of Technology (HCMUT), VNU-HCM for supporting this study.

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