Design of the CNC Router Structure for Machining Wood Materials Using Reliability-Based Design Optimization Method

Huu Loc Nguyen and Van Thuy Tran

1 Introduction

The uncertainties of the structural system (i.e., loads, materials, dimensions, models, etc.) in deterministic design optimization are taken into account in an indirect and subjective method, based on partial safety factors determined in design ranges. Therefore, the level of reliability can be reduced when using deterministic optimal solutions (Beck and Santana Gomes 2012; Thuy and Nguyen 2018). The objective of a RBDO method is to optimize structures, and make sure that a minimum reliability level is upholded by the optimum structure.

The optimization algorithms of the RBDO method use the reliability methods to evaluate the probability constraints, and the objective functions are utilized to prescribe the reliability (Chiralaksanakul and Mahadevan 2005; Royset et al. 2001). In RBDO methods, the commonly used design parameters are the mean of random system parameters and solve a mathematical nonlinear programming problem to determine the optimized cost according to the prescribed probabilistic constraints. Therefore, the solution from RBDO supplies not only improves the design quality but also increases the reliability of the design, reduces the production cost and improves the model more beautiful (Youn and Choi 2004a; Tu et al. 2001).

There are two different methods used to evaluate the RBDO incorporating probabilistic constraints, the performance measure approach (PMA) and the reliability index approach (RIA) (Youn et al. 2003; Tu et al. 1999; Cheng et al. 2018). According

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to the RIA method, the probability constraint is defined as reliability. Nevertheless, RIA does not converge for some problems or converges slowly. To avoid this shortcoming, PMA is recommended by solving an inverse problem for the first-order reliability method (FORM) (Youn and Choi 2004b; Enevoldsen and Sørensen 1994; Padmanabhan et al. 2006).

For reliability analysis, FORM has proven to be effective and widely used. Therefore, to evaluate the objective functions and the number of probabilistic constraints, the previous RBDO methods used FORM. Over the years, the structural and engineering disciplines have adopted. Several first-order RBDO methods have been shown to be more effective than previous RBDO methods (Loc et al. 2019).

2 Theoretical Background

RBDO problem in system parameter design is introduced as follows (Nguyen 2015; Wang and Ma 2017):

$$\text{Find } \mathbf{X} = \begin{Bmatrix} X_1 \\ X_2 \\ . \\ . \\ . \\ X_n \end{Bmatrix} \tag{1}$$

To minimize $f(\mathbf{X})$,

Subject to:

$$P(g_j(\mathbf{X}, \mathbf{p}) \geq 0)) \geq R_j^*; \quad j = 1, 2, \ldots n_g$$
$$h_k(\mathbf{X}, m_p) \leq 0; \quad k = 1, 2, \ldots n_h$$
$$X_i^1 \leq X_i \leq X_i^n; \quad i = 1, 2, \ldots n.$$

where X_i is the design variable (deterministic or random), $\mathbf{X}$ is the design variable vectors (including deterministic and random), $\mathbf{p}$ are random parameter vectors (mean value is $m_\mathbf{p}$), $g_j(\mathbf{X}, \mathbf{p})$ is the limit state function, $f(\mathbf{X})$ is the objective function, n are the numbers of design variables, n_g are the numbers of probability constraints, $R_j{}^*$ are the desired reliability and $X_i{}^1$, $X_i{}^n$ are the upper and upper limits of the design variable.

The inverse reliability analysis method is used to transform the RBDO problem into a deterministic optimization problem. To solve this problem, it is first necessary to determine the limit state function value g^F correspond with the desired reliability $R_j{}^*$. The steps are as follows (Fig. 2).

Find the value of limit state function g^F correspond with a failure probability F:

$$F = P\left(g(X) < g^F\right) \tag{2}$$

The new limit state function is obtained after estimation by an approximation method:

$$g'(X) = g(X) - g^F \tag{3}$$

Assume the most probable point (MPP) of a function $P\left(g'(X) < 0\right) = P\left(g(X) < g^F\right)$ is $u*$, according to the FORM, if the probability of failure F is known in advance, the index of reliability β is determined as follows:

$$\beta = \left|\Phi^{-1}(F)\right| \tag{4}$$

In Fig. 1, the most probable point $u*$ is the point of contact of a circle of radius g^F and the function $g(X) - g^F = 0$. In other words, it is the point of contact that determines the shortest distance from the limit state function surface $g(X) - g^F = 0$ to the origin O in U space.

At the MPP $u*$:

$$u^{*F} = -\left\|u^{*F}\right\|a^* = -\beta a^* \tag{5}$$

In the kth loop:

$$u^{k+1} = -\beta a^k\{u\} \tag{6}$$

where

$$a^k = \frac{\nabla g\left(u^k\right)}{\left\|\nabla g\left(u^k\right)\right\|} \tag{7}$$

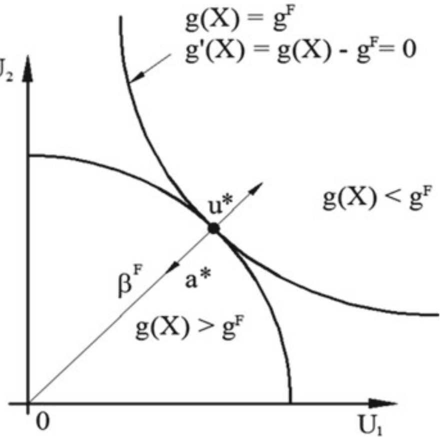

Fig. 1 Find MPP by inverse reliability analysis

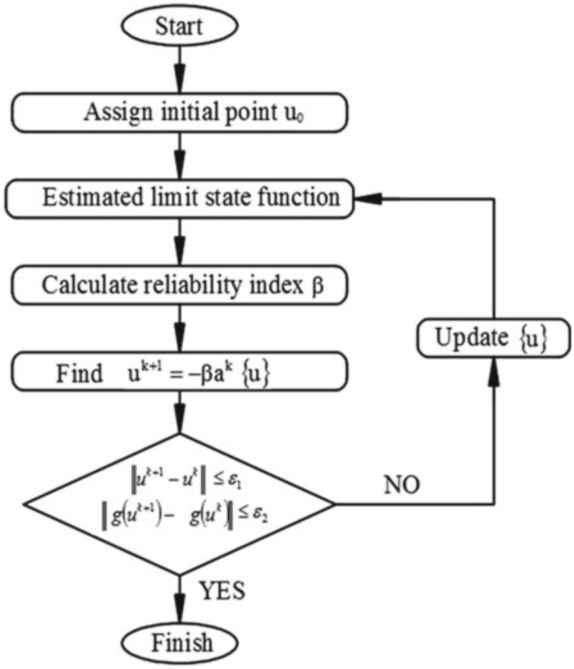

Fig. 2 Algorithm to find MPP according to the inverse reliability analysis method

Convergence conditions are used to finish the loop:

$$\left\|u^{k+1} - u^k\right\| \le \varepsilon_1 v\grave{a} \left\|g\left(u^{k+1}\right) - g\left(u^k\right)\right\| \le \varepsilon_2 \tag{8}$$

After determining the MPP $u*$, the value of the function g^F is determined as follows:

$$g^F = g\left(u^*\right) \tag{9}$$

In summary, the algorithm to find the MPP according to the inverse reliability analysis method is presented as Fig. 2.

After determining g^F, the RBDO problem turns into a deterministic optimization problem and the first constraint of the RBDO problem (1) become $g_j(\mathbf{X}, \mathbf{p}) \ge g^F$.

3 Numerical Example

RBDO approach is used to optimize the design of the CNC router structure as shown in Fig. 3. Supposing that the CNC router structure is composed of four elements linked together such as the head (1), crossbeam (2), column (3) and the base (4). To simplify the calculations, it is accepted that the head, crossbeam and column are the

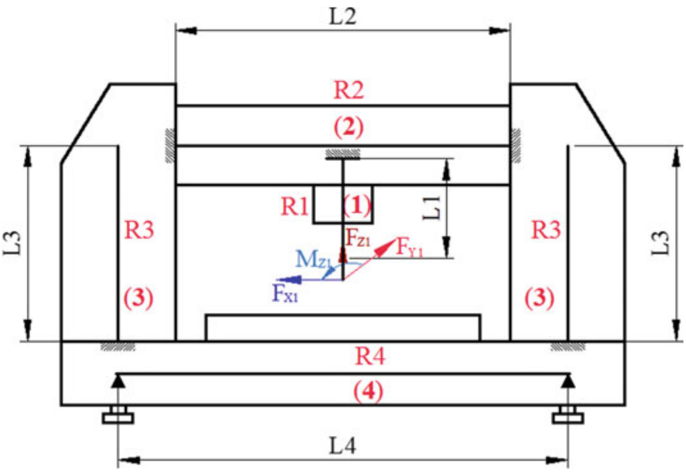

Fig. 3 Model of a CNC router structure

cantilever beam correspond with the length L_1, L_2, L_3 and the base L_4 which is fixed by 2 supports.

The applied forces F_{X1}, F_{Y1}, F_{Z1} and M_{Z1} are put into the computational model. During machining, force components F_{X1}, F_{Y1}, F_{Z1} and torque M_{Z1} from the cutting tool are applied to the spindle of the machine body structure (Nguyen and Tran 2022).

If we consider the planar problem, the element (1) is subjected to bending and compression. The compressive stress in the Z direction is usually small compared to the bending stress, so it can be ignored. The planar problem with element (1) is then only subjected to bending and torsion. Elements (2), (3) and (4) are similarly considered. Therefore, the structural design problem of the machine body is to solve the problems for elements (1), (2), (3) and (4), respectively, which are considered as bars and beams. Then, combining the solutions obtained together, we have the preliminary dimensions of the body. We separate elements (1), (2) and (3) from the model, convert the model to s simple form and set the force conditions (cutting force is constant) as shown in Fig. 4.

Preliminary selection of body structure geometry dimensions: The basis for preliminary selection of the geometric dimensions of the body structure is as follows: The CNC router on the market were analyzed, and it is found that the working area of the medium CNC router according to X is less than 1000 mm, Y less than 2000 mm and Z less than 100 mm. In addition, wood workpiece in sheet form is common, so the thickness in the Z direction is usually less than 100 mm. On that basis, the dimensions of the machine structure are preliminary selected as follows: $L_1 = 250$ mm, $L_2 = 1000$ mm and $L_3 = 350$ mm.

For the preliminary design of the CNC router body structure, the desired reliability $R^* = 0.999$ is selected and the RBDO method is used. Two design variables that represent the height V_i and width H_i are determined and given in Table 1.

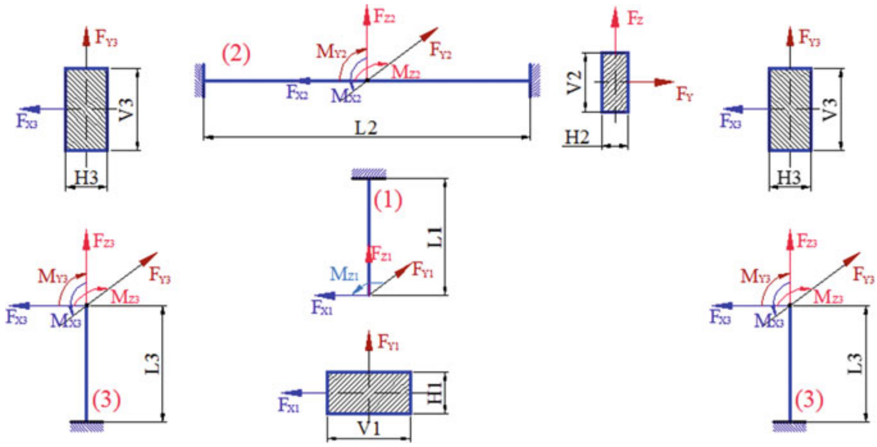

Fig. 4 Simple model of elements (1), (2) and (3)

Table 1 Dimensions of the upper and lower limits of the geometry parameters

Elements	Parameters	Basic values, mm	Lower limits V_{imin}, H_{imin}, mm	Upper limits V_{imax}, H_{imax}, mm
Head 1	V_1	200	150	250
	H_1	25	20	30
Crossbeam 2	V_2	275	225	325
	H_2	25	20	30
Column 3	V_3	350	300	400
	H_3	35	30	40

Material for all elements is steel C45, yield strength $\sigma_y = 360$ MPa and standard deviation $S_y = 6$ MPa. Coefficient of variation of forces and moment is 0.1. Mechanical properties of steel materials C45 are in Table 2.

The processing material is Goldden oak wood, whose scientific name is Homalium Caryophyllaceum Benth with the density of 1000 kg/m^3, durability at 70 MPa, hardness at 7.7 HB and humidity of 15%. The components of the applied force are defined rely on a cutting force in a woodworking process. Components of the applied force are presented in Table 3 (Wang and Ma 2017).

According to the strength criteria, the limit state function is written as follows:

Table 2 Mechanical properties of steel material C45

C	Si	Mn	S	P	Cr
0.42–0.5	0.15–0.35	0.50–0.80	≤ 0.025	≤ 0.025	0.2–0.4

Table 3 Applied forces

Elements	Force F_{Zi}, N	Force F_{Xi}, N	Force F_{Yi}, N	Moment M_{Xi}, Nmm	Moment M_{Yi}, Nmm	Torque M_{Zi}, Nmm	Length L_i, mm
Head 1	215	390	390	0	0	1170	250
Crossbeam 2	215	390	390	97,500	97,500	1170	1000
Column 3	185	120	196	533	48,843	16,607	350

$$g\left(\sigma_{\text{iy}}, \sigma_{\text{ieq}}\right) = \sigma_y - \sqrt{\left(\frac{F_{Zi}}{H_i V_i} + 6F_{Xi}L_i\left(\frac{1}{H_i V_i^2} + \frac{1}{V_i H_i^2}\right)\right)^2 + 4\left(\frac{M_{Zi}}{3.07V_i H_i^2}\right)^2} \tag{10}$$

According to the stiffness criteria, the limit state function is written as follows:

$$g(u_{i\,\text{lim}}, u_{i\,\text{max}}) = u_{i\,\text{lim}} - u_{i\,\text{max}} = u_{i\,\text{lim}} - \sqrt{\left(\frac{\text{F}_{\text{Xi}}L_i^3}{3\text{EI}_{\text{Yi}}}\right)^2 + \left(\frac{\text{F}_{\text{Yi}}L_i^3}{3\text{EI}_{\text{Xi}}}\right)^2} \tag{11}$$

The RDBO problem for the elements is presented as follows:
Design variable: V_i, H_i.
Objective function: $m_i = \rho L_i V_i H_i \quad \rightarrow$ min.
Subject to:

$$\begin{cases} V_{i\,\text{min}} \le V_i \le V_{i\,\text{max}} \\ H_{i\,\text{min}} \le H_i \le H_{i\,\text{max}} \\ P\left(g\left(\sigma_{iy}, \sigma_{ieq}\right) > 0\right) = P\left(\left(\sigma_{iy} - \sigma_{ieq}\right) > 0\right) \ge R_i^* \\ P(g(u_{i\,\text{lim}}, u_{i\,\text{max}}) > 0) = P((u_{\text{lim}} - u_{i\,\text{max}}) > 0) \ge R_i^* \end{cases} \tag{12}$$

where limit displacement $u_{\text{ilim}} = 0.035$ mm and desired reliability $R_i^* = 0.999$.

The inverse reliability analysis method is utilized to shift the RBDO problem into the deterministic optimization problem. After four loops, the function g^F at the MPP has the following value in Table 4.

Thus, the RBDO problem is transformed into the deterministic optimization problem as follows:

Design variable: V_i, H_i.

Table 4 Calculation results

The limit state function g_i^F	Head 1	Crossbeam 2	Column 3
$g_{i\sigma}^F = g_{i\sigma}(u^*)$	0.0327	0.0362	0.0298
$g_{iS}^F = g_{iS}(u^*)$	-7.65×10^{-4}	-8.21×10^{-4}	-6.82×10^{-4}

Objective function: $m_i = \rho L_i V_i H_i \quad \rightarrow$ min.
Subject to:

$$\begin{cases} V_{i\,\min} \leq V_i \leq V_{i\,\max} \\ H_{i\,\min} \leq H_i \leq H_{i\,\max} \\ g(\sigma_{iy}, \sigma_{i\mathrm{eq}}) = \sigma_{iy} - \sigma_{i\mathrm{eq}} \geq g_{i\sigma}^F \\ g(u_{i\,\lim}, u_{i\,\max}) = u_{i\,\lim} - u_{i\,\max} \geq g_{iS}^F \end{cases} \quad (13)$$

The genetic algorithm method in Tool Optimization of MATLAB is utilized to solve the design optimization problem. The results achieved after optimizing of the machine head, horizontal beam and vertical column of the CNC router are presented in Table 5.

Equivalent stresses $\boldsymbol{\sigma}_{\mathrm{eq1}}$, $\boldsymbol{\sigma}_{\mathrm{eq2}}$, $\boldsymbol{\sigma}_{\mathrm{eq3}}$ and displacement $u_{1\max}$, $u_{1\max}$, $u_{1\max}$, respectively, of machine head, crossbeam and vertical column of CNC router are all smaller than the limit strength value $\boldsymbol{\sigma}_y$ and displacement $u_{\lim}$.

The MCS method is used to analyze the reliability of the machine structure after optimal design. The reliability analysis results according to the MCS method are compared with the desired reliability of the body structure. Simulation results with sample number $N = 5.10^6$ are presented in Figs. 5 and 6.

Table 5 Optimal design results

Elements	Mass m_i, kg	Height H_i, mm	Width V_i, mm	Stress σ_{eqi}, MPa	Maximum displacement u_{imax}, mm
Head 1	13.326	29	230	3.56	0.0199
Crossbeam 2	34.67	28	300	4.22	0.0184
Column 3	33.66	35	353	1.40	0.0195

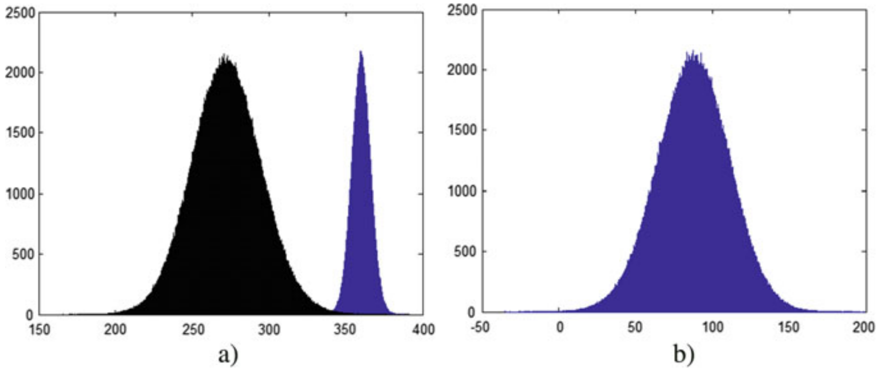

Fig. 5 Monte Carlo simulation according to the strength criteria: **a** yield strength $\boldsymbol{\sigma}_y$ and stress $\boldsymbol{\sigma}_{\max}$ and **b** limit state function $g(\mathbf{X})$

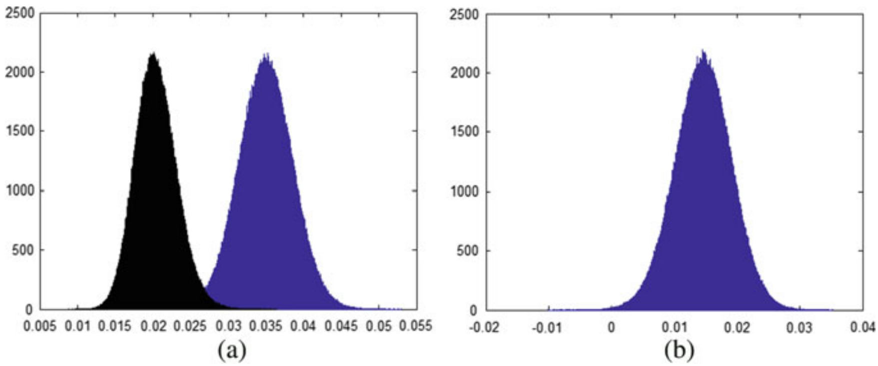

Fig. 6 Monte Carlo simulation according to the stiffness criteria: **a** displacement $u_{\max}$, $u_{\lim}$ and **b** limit state function g(**X**)

The results of the reliability analysis of the MCS method: according to the strength criteria $R_{\mathrm{MCS}\sigma} = 0.99977$ and stiffness criteria $R_{\mathrm{MCSu}} = 0.99973$. Because $\mathrm{R}_{\mathrm{MCS}\sigma}$ and R_{MCSu} are more than $R^* = 0.999$, the body structure after optimizing the design according to RDBO method is satisfactory.

After the complete design, CAD model has the form as shown in Fig. 7a, an analysis and a test are conducted according to strength and stiffness on the CAE system as shown in Fig. 7b, c. The results of the test in accordance with the stiffness and strength criteria is satisfactory.

After the complete design, CAD model has the form as shown in Fig. 7a, an analysis and a test are conducted in accordance with the stiffness and strength on the CAE system as shown in Fig. 7b, c. The results of the test in accordance with the stiffness and strength criteria are satisfactory.

4 Conclusions

This paper applies the RBDO approach to analyze and design CNC router structure. With desired reliability R*, the inverse reliability analysis method is utilized to transform the RBDO problem into a deterministic optimization problem.

Genetic Algorithm method in Tool Optimization of MATLAB is utilized to solve the design optimization problem. The results obtained after design optimization include equivalent stresses $\boldsymbol{\sigma}_{\mathrm{eq1}}$, $\boldsymbol{\sigma}_{\mathrm{eq2}}$, $\boldsymbol{\sigma}_{\mathrm{eq3}}$ and displacement $u_{1\max}$, $u_{1\max}$, $u_{1\max}$, respectively, of machine head, crossbeam and vertical column of CNC router are all smaller than limit strength value $\boldsymbol{\sigma}_y$ and displacement $u_{\lim}$.

MCS is used to analyze the machine structure reliability after an optimal design according to strength and stiffness criteria. The reliability analysis results according to the MCS method are compared with the desired reliability R^* of the body structure.

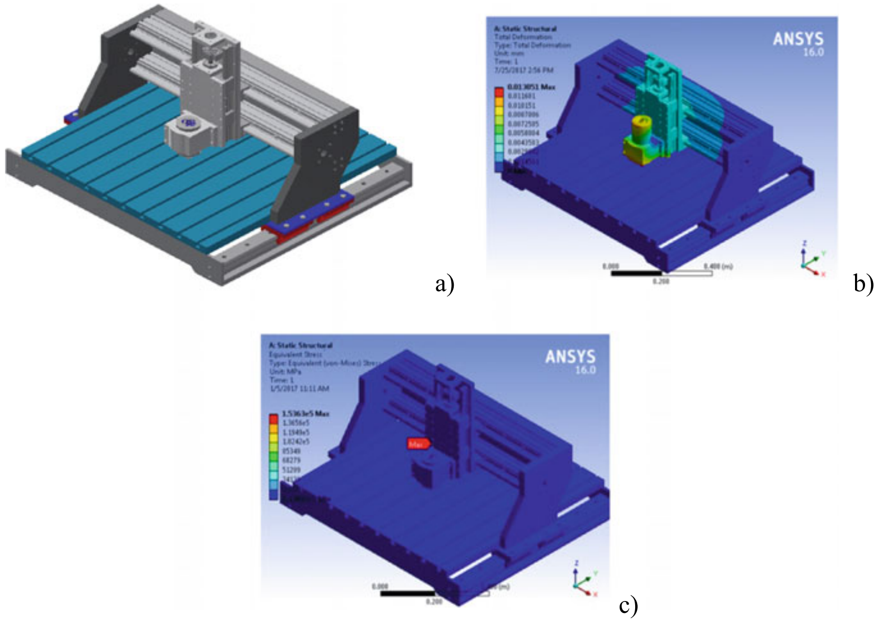

Fig. 7 CAD and CAE model

The obtained findings highlighted that body structure after optimizing the design according to the RDBO method is satisfactory.

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