



Consensus for Second-Order Nonlinear Multi-agent Systems with Position Constraints and Communication Delays

Gang Wang¹ and Chaoli Wang²(✉)

¹ Institute of Machine Intelligence, University of Shanghai for Science and Technology, Shanghai 200093, China

² Department of Control Science and Engineering, University of Shanghai for Science and Technology, Shanghai 200093, China
c1c1wang@126.com

Abstract. This paper details a new distributed consensus algorithm for second-order multi-agent systems with position constraints on general directed graphs. The developed algorithm has the attractive property of adapting to nonuniform communication delays and relies on minimal information from neighboring agents, namely their positions. We demonstrate that all agents reach consensus with the developed distributed algorithm without violating constraints while ensuring the boundedness of all closed-loop signals. Simulations are performed to illustrate and validate the algorithms.

Keywords: Distributed consensus · Multi-agent system · Nonlinear system · Adaptive control

1 Introduction

Distributed consensus in dynamical multi-agent systems involves driving certain variables of all agents to a common value [1, 2]. While great progress has been made in consensus methods for first-order integrators, there is still a lack of designing consensus algorithms for uncertain nonlinear dynamics that can endow real-life robotic teams with the expected autonomy.

For heterogeneous multi-agent systems with unknown inertias, a class of distributed consensus algorithms is presented through establishing connections between directed and undirected graphs in [3]. Moreover, for linear agents with matching uncertainties and second-order agents with mismatched uncertainties, a model reference adaptive consensus scheme is provided on general directed graphs in [4]. Subsequently, considering the unknown control directions, Nussbaum-based and nonlinear proportional-integral-based consensus control approaches are designed for first- and second-order agents in [5, 6]. However, these works focus on the canonical consensus control issue and ignore the actual constraints of the agent.

There are many robotics systems whose operating environment or safety specifications inherently limit their position to a certain range. By employing logarithmic barrier functions and projection-based operators, consensus in a continuous-time system with a state constraint is achieved in [7, 8]. Nevertheless, only linear multi-agent systems are considered in these aforementioned works. For nonlinear multi-agent systems, distributed controllers are developed to achieve asymptotic consensus and ensure that output constraints are satisfied in [9–11]. For uncertain multi-agent systems, all the above consensus results impose the assumption of zero communication delay, even though in most practical situations this assumption is violated. Recently, the communication delay in nonlinear systems has also been considered in solving the formation control problem of unmanned aerial vehicles [12], attitude synchronization of spacecraft formation [13], and the synchronization of Euler-Lagrange systems [14–16]. In [17], a proportional and delayed integral consensus error variable is proposed to recast the original consensus control problem into a regulation control problem and thereafter employed the standard backstepping technique to handle the consensus problem of higher-order nonlinear agents with communication delays.

This work investigates the consensus problem of second-order nonlinear multi-agent systems subject to communication delays and position constraints. The communication topology is assumed to contain a directed spanning tree and that all agents can only receive the delayed information of their neighbors. A new adaptive controller is proposed by designing continuously differentiable reference output and auxiliary error variables. We show that with the proposed distributed algorithm, the asymptotic consensus is achieved in the presence of nonuniform communication delays and the position constraints are satisfied at all times.

2 Preliminaries and Problem Statement

2.1 Interconnection Graph

A weighted directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ is used to describe the interconnection topology between the n agents, where the node set is $\mathcal{V} = \{1, \dots, n\}$, the edge set is $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$, and the weighted adjacency matrix $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{n \times n}$ associated with \mathcal{G} is defined by $a_{ij} > 0$ if $(j, i) \in \mathcal{E}$; in addition, $a_{ij} = 0$ if $(j, i) \notin \mathcal{E}$. Here, an edge $(i, j) \in \mathcal{E}$ indicates that node j has access to the information of node i but not vice versa, and node i is called a neighbor of node j . We say that a directed graph has a directed spanning tree if there exists at least one node such that the node has directed paths to all other nodes in \mathcal{G} . We define the Laplacian matrix $\mathcal{L} \in \mathbb{R}^{n \times n}$ corresponding to \mathcal{G} as $\mathcal{L} = \mathcal{D} - \mathcal{A}$, where $\mathcal{D} = \text{diag}\{d_1, \dots, d_n\}$ is the in-degree matrix with $d_i = \sum_{j=1}^n a_{ij}$.

3 Problem Formulation

A multi-agent system with n agents is considered. The i th agent has the form

$$\begin{aligned} \dot{x}_i &= v_i + \theta_i^\top \varphi_{i,1}(x_i), \quad i = 1, \dots, n \\ \dot{v}_i &= u_i + \theta_i^\top \varphi_{i,2}(x_i, v_i) \end{aligned} \quad (1)$$

$x_i \in \mathbb{R}$, $v_i \in \mathbb{R}$, and $u_i \in \mathbb{R}$ denote position, velocity-like state, and control input of agent i , respectively. $\theta_i \in \mathbb{R}^{p_i}$ are uncertain dynamic parameters, and $\varphi_{i,1}, \varphi_{i,2} \in \mathbb{R}^{p_i}$ represent known vectors of smooth nonlinearities. The position x_i is required to satisfy $\underline{k} < x_i(t) < \bar{k}$ for all $t \geq 0$, where \underline{k} and \bar{k} represent the predefined position constraints.

The control objective is to present a new control algorithm for agents (1) that depends solely upon their states, position constraints, and the delayed neighboring agents' positions such that (i) consensus among the n agents can be reached asymptotically, i.e., $\lim_{t \rightarrow \infty} (x_i(t) - x_j(t)) = 0$, and (ii) the position constraint for each agent is not transgressed, i.e.,

$$\underline{k} < x_i(t) < \bar{k}, \forall t \geq 0 \tag{2}$$

for all $i, j = 1, \dots, n$.

4 Consensus Control with Communication Delays

4.1 Controller Design

The controller design and convergence analysis for the uncertain nonlinear multi-agent system (1) with nonuniform communication delays and position constraints is given in this section. Define $\gamma_i = \ln(x_i - \underline{k}) - \ln(\bar{k} - x_i)$. We construct a dynamic system for agent i to yield a reference output using the delayed information of the neighboring agents as:

$$\dot{z}_{i,1} = -\lambda_{i,1}(z_{i,1} - z_{i,2}) \tag{3}$$

$$\dot{z}_{i,2} = -\lambda_{i,2} \sum_{j=1}^n a_{ij}(z_{i,2} - \gamma_j(t - \tau_{ij})) \tag{4}$$

where $\tau_{ij} \geq 0$ denotes the communication delay from agent j to agent i for $(j, i) \in \mathcal{E}$, $\lambda_{i,p} > 0$ for $p = 1, \dots, m$, and $\gamma_j(t - \tau_{ij})$ is set to zero for all $0 \leq t \leq \tau_{ij}$.

Next, we consider the following auxiliary error for agent i :

$$e_{i,1} = (\gamma_i - z_{i,1}) + \int_0^t (\gamma_i(\sigma) - z_{i,1}(\sigma)) d\sigma \tag{5}$$

which, in view of (1) and (3), satisfies

$$\begin{aligned} \dot{e}_{i,1} &= \frac{(\bar{k} - \underline{k})}{(x_i - \underline{k})(\bar{k} - x_i)} (v_i + \theta_i^\top \varphi_{i,1}) + (\gamma_i - z_{i,1}) + \lambda_{i,1}(z_{i,1} - z_{i,2}) \\ &= \frac{(\bar{k} - \underline{k})}{(x_i - \underline{k})(\bar{k} - x_i)} (e_{i,2} + v_{i,d} + \theta_i^\top \varphi_{i,1}) + (\gamma_i - z_{i,1}) + \lambda_{i,1}(z_{i,1} - z_{i,2}) \end{aligned} \tag{6}$$

where $v_{i,d}$ represents the desired velocity and $e_{i,2} = v_i - v_{i,d}$. We select the desired velocity v_i as

$$v_{i,d} = -m_{i,1}e_{i,1} - \hat{\theta}_i^\top \varphi_{i,1} - \frac{(x_i - \underline{k})(\bar{k} - x_i)}{(\bar{k} - \underline{k})} ((\gamma_i - z_{i,1}) + \lambda_{i,1}(z_{i,1} - z_{i,2})) \tag{7}$$

where $m_{i,1} > 0$ and $\hat{\theta}_i$ is an estimator of the unknown parameter θ_i . We define a Lyapunov function candidate at this step as $V_{i,1} = \frac{1}{2}e_{i,1}^2 + \frac{1}{2}\tilde{\theta}_i^\top \Upsilon_i^{-1} \tilde{\theta}_i$ with $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$ being the parameter estimation error and $\Upsilon_i \in \mathbb{R}^{r_i \times r_i}$ a positive definite matrix. Clearly, we can verify from (6) and (7) that

$$\begin{aligned} \dot{V}_{i,1} &= e_{i,1} \dot{e}_{i,1} + \tilde{\theta}_i^\top \Upsilon_i^{-1} (-\dot{\hat{\theta}}_i) \\ &= \frac{-m_{i,1}(\bar{k} - \underline{k})e_{i,1}^2}{(x_i - \underline{k})(\bar{k} - x_i)} + \tilde{\theta}_i^\top \Upsilon_i^{-1} (\Upsilon_i \frac{(\bar{k} - \underline{k})e_{i,1}\varphi_{i,1}}{(x_i - \underline{k})(\bar{k} - x_i)} - \dot{\hat{\theta}}_i) \\ &\quad + \frac{(\bar{k} - \underline{k})}{(x_i - \underline{k})(\bar{k} - x_i)} e_{i,1}e_{i,2}. \end{aligned} \tag{8}$$

By virtue of (3) and (7), the dynamics of $e_{i,2}$ satisfy

$$\begin{aligned} \dot{e}_{i,2} &= u_i + \theta_i^\top \varphi_{i,2} - \dot{v}_{i,d} \\ &= u_i + \theta_i^\top \bar{\varphi}_{i,2} - \beta_i \end{aligned} \tag{9}$$

with $\bar{\varphi}_{i,2} = \varphi_{i,2} - \frac{\partial v_{i,d}}{\partial x_i} \varphi_{i,1}$ and $\beta_i = \frac{\partial v_{i,d}}{\partial x_i} v_i + \frac{\partial v_{i,d}}{\partial \hat{\theta}_i} \dot{\hat{\theta}}_i + \frac{\partial v_{i,d}}{\partial z_{i,1}} \dot{z}_{i,1} + \frac{\partial v_{i,d}}{\partial z_{i,2}} \dot{z}_{i,2}$ involving only known variables. Choose the Lyapunov function as

$$V_{i,2} = V_{i,1} + \frac{1}{2}e_{i,2}^2.$$

Differentiating it with respect to (9) and noting (8) lead to

$$\begin{aligned} \dot{V}_{i,2} &= -m_{i,1} \frac{(\bar{k} - \underline{k})e_{i,1}^2}{(x_i - \underline{k})(\bar{k} - x_i)} + \tilde{\theta}_i^\top \Upsilon_i^{-1} (\Upsilon_i \frac{(\bar{k} - \underline{k})\varphi_{i,1}e_{i,1}}{(x_i - \underline{k})(\bar{k} - x_i)} - \dot{\hat{\theta}}_i) \\ &\quad + \frac{(\bar{k} - \underline{k})}{(x_i - \underline{k})(\bar{k} - x_i)} e_{i,1}e_{i,2} + e_{i,2}(u_i + \theta_i^\top \bar{\varphi}_{i,2} - \beta_i). \end{aligned} \tag{10}$$

The control input is thus defined as

$$u_i = -m_{i,2}e_{i,2} - \hat{\theta}_i^\top \bar{\varphi}_{i,2} - \frac{(\bar{k} - \underline{k})e_{i,1}}{(x_i - \underline{k})(\bar{k} - x_i)} + \beta_i \tag{11}$$

with $m_{i,2} > 0$. One can show that

$$\begin{aligned} \dot{V}_{i,2} &= -m_{i,1} \frac{(\bar{k} - \underline{k})e_{i,1}^2}{(x_i - \underline{k})(\bar{k} - x_i)} - m_{i,2}e_{i,2}^2 \\ &\quad + \tilde{\theta}_i^\top \Upsilon_i^{-1} (\Upsilon_i \frac{(\bar{k} - \underline{k})\varphi_{i,1}e_{i,1}}{(x_i - \underline{k})(\bar{k} - x_i)} + \Upsilon_i \bar{\varphi}_{i,2}e_{i,2} - \dot{\hat{\theta}}_i). \end{aligned} \tag{12}$$

Selecting the adaptive law for $\hat{\theta}_i$ as

$$\dot{\hat{\theta}}_i = \Upsilon_i \left(\frac{(\bar{k} - \underline{k})\varphi_{i,1}e_{i,1}}{(x_i - \underline{k})(\bar{k} - x_i)} + \bar{\varphi}_{i,2}e_{i,2} \right) \tag{13}$$

we have the following results.

Theorem 1. Consider the nonlinear multi-agent system (1) with communication delays and position constraints, and suppose that \mathcal{G} contains a spanning tree and that the delays are time-invariant and finite. The distributed controller (11) with the dynamic system (3) and the parameter estimate (13) guarantees that all agents can reach consensus, i.e., $\lim_{t \rightarrow \infty} (x_i(t) - x_j(t)) = 0$ for all $i, j = 1, \dots, n$ and that the position constraint is not transgressed, i.e., $-\underline{k} < x_i(t) < \bar{k}$ for all $t \geq 0$.

Proof. Substituting the adaptive control law (13) into (12) results in $\dot{V}_{i,2} = -m_{i,1} \frac{(\bar{k}-k)e_{i,1}^2}{(x_i-k)(k-x_i)} - m_{i,2}e_{i,2}^2$. This, together with the definition of $V_{i,2}$, leads to $e_{i,1}, e_{i,2} \in \mathbb{L}_2 \cap \mathbb{L}_\infty$ and $\tilde{\theta}_i \in \mathbb{L}_\infty$. Noting that θ_i is a constant vector, one can further conclude that $\hat{\theta}_i \in \mathbb{L}_\infty$. Since $e_{i,1}$ is a linear stable differential operator acting on $\int_0^t (\gamma_i(\sigma) - z_{i,1}(\sigma))d\sigma$, we have $(\gamma_i - z_{i,1}), \int_0^t (\gamma_i(\sigma) - z_{i,1}(\sigma))d\sigma \in \mathbb{L}_\infty$.

Using (3), it can be deduced that

$$\begin{aligned} \dot{z}_{i,2} = & -\lambda_{i,2} \sum_{j=1}^n a_{ij}(z_{i,2} - z_{j,1}(t - \tau_{ij})) \\ & + \lambda_{i,2} \sum_{j=1}^n a_{ij}(\gamma_j(t - \tau_{ij}) - z_{j,1}(t - \tau_{ij})). \end{aligned} \tag{14}$$

Applying the Laplace transformation to (3) and (14) results in

$$\begin{aligned} sZ_{i,1}(s) = & -\lambda_{i,1}(Z_{i,1}(s) - Z_{i,2}(s)) + z_{i,1}(0) \\ sZ_{i,2}(s) = & -\lambda_{i,2} \sum_{j=1}^n a_{ij}(Z_{i,2}(s) - e^{-\tau_{ij}s}Z_{j,1}(s)) \\ & + z_{i,2}(0) + \lambda_{i,2} \sum_{j=1}^n a_{ij}Z_{ij}^*(s) \end{aligned} \tag{15}$$

$Z_{i,1}(s) = \mathfrak{L}[z_{i,1}(t)]$, $Z_{i,2}(s) = \mathfrak{L}[z_{i,2}(t)]$, and $Z_{ij,0}^*(s) = \mathfrak{L}[\gamma_j(t - \tau_{ij}) - z_{j,1}(t - \tau_{ij})]$ denote the Laplace transform of $z_{i,1}(t)$, $z_{i,2}(t)$, and $(\gamma_j(t - \tau_{ij}) - z_{j,1}(t - \tau_{ij}))$, respectively, and s is the Laplace variable. Define the column vector $z = [z_1^\top, z_2^\top]^\top$ with $z_1 = [z_{1,1}, \dots, z_{n,1}]^\top$ and $z_2 = [z_{1,2}, \dots, z_{n,2}]^\top$, $Z(s) = \mathfrak{L}[z(t)]$, and the matrix

$$\mathcal{Q} = \begin{bmatrix} \Lambda_1 & -\Lambda_1 \\ -\Lambda_2\mathcal{A} & \Lambda_2\mathcal{D} \end{bmatrix} \tag{16}$$

with $\Lambda_1 = \text{diag}\{\lambda_{1,1}, \dots, \lambda_{n,1}\}$, $\Lambda_2 = \text{diag}\{\lambda_{1,2}, \dots, \lambda_{n,2}\}$, \mathcal{D} and \mathcal{A} being, respectively, the in-degree and adjacency matrices of \mathcal{G} . Consequently, (15) can be reformulated with the following compact form:

$$sZ(s) = z(0) - (\mathcal{M} - \mathcal{T})Z(s) + \mathcal{B}(s) \tag{17}$$

where $\mathcal{M} = \text{diag}\{q_1, \dots, q_{2n}\}$ with q_i being the i th diagonal element of matrix \mathcal{Q} , $\mathcal{B}(s) = [0_{2(n-1)}^\top, 1_n^\top ((\Lambda_2 \mathcal{A}) \odot Z_0^*(s))^\top]^\top$, $\mathcal{T} = (\mathcal{M} - \mathcal{Q}) \odot \begin{bmatrix} 1_{n \times n} & 1_{n \times n} \\ \mathcal{W} & 1_{n \times n} \end{bmatrix} \in \mathbb{R}^{2n \times 2n}$, $\tau_{ij} = 0$ for each $(j, i) \notin \mathcal{E}$, and

$$Z_0^*(s) = \begin{bmatrix} 0 & Z_{12,0}^*(s) & \dots & Z_{1n,0}^*(s) \\ Z_{21,0}^*(s) & 0 & \dots & Z_{2n,0}^*(s) \\ \vdots & \vdots & \ddots & \vdots \\ Z_{n1,0}^*(s) & Z_{n2,0}^*(s) & \dots & 0 \end{bmatrix} \in \mathbb{R}^{n \times n}$$

$$\mathcal{W} = \begin{bmatrix} 0 & e^{-\tau_{12}s} & \dots & e^{-\tau_{1n}s} \\ e^{-\tau_{21}s} & 0 & \dots & e^{-\tau_{2n}s} \\ \vdots & \vdots & \ddots & \vdots \\ e^{-\tau_{(n-1)1}s} & e^{-\tau_{(n-1)2}s} & \dots & e^{-\tau_{(n-1)n}s} \\ e^{-\tau_{n1}s} & e^{-\tau_{n2}s} & \dots & 0 \end{bmatrix} \in \mathbb{R}^{n \times n}.$$

Defining the transfer function matrix $G(s) = (sI_{2n} + \mathcal{M} - \mathcal{T})^{-1}$, one can deduce from (17) that

$$Z(s) = G(s)(z(0) + \mathcal{B}(s)). \tag{18}$$

Performing elementary row operations, we have $\text{rank}(\mathcal{Q}) = 2n - 1$. Hence, we can conclude from [2] that these $2n$ agents must interact on an interconnection graph containing a spanning tree, which, together with [15], yields that all the poles of $G(s)$ excluding the simple zero pole are in the open left half plane. Recalling $\int_0^t (y_i(\gamma) - z_{i,1}(\gamma))d\gamma \in \mathbb{L}_\infty$ and applying [14, Prop. 1] to (18) lead to $z \in \mathbb{L}_\infty$. Keeping in mind the dynamics of z given in (3), we know that $\gamma_i, \dot{z}_{i,1}, \dot{z}_{i,2} \in \mathbb{L}_\infty$. Thus, there exists a constant $\gamma_i^* > 0$ such that $|\gamma_i(t)| \leq \gamma_i^*, \forall t \in [0, t_f]$. Applying the antilog of γ_i , we obtain that

$$\underline{k} < \underline{k}_i \leq x_i(t) \leq \bar{k}_i < \bar{k}, \forall t \in [0, t_f] \tag{19}$$

where $\underline{k}_i = \frac{\bar{k}e^{-\gamma_i^*} + k}{e^{-\gamma_i^*} + 1}$ and $\bar{k}_i = \frac{\bar{k}e^{\gamma_i^*} + k}{e^{\gamma_i^*} + 1}$. It then follows from (7) and $\hat{\theta}_i \in \mathbb{L}_\infty$ that $v_{i,d} \in \mathbb{L}_\infty$, which together with the boundedness of $e_{i,2}$ ensures $v_i \in \mathbb{L}_\infty$. One can finally come to a conclusion that $u_i \in \mathbb{L}_\infty$.

In the following, the asymptotic consensus between agents is demonstrated. From (6) and (9), one has $\dot{e}_{i,1}, \dot{e}_{i,2} \in \mathbb{L}_\infty$ for $i = 1, \dots, n$. Combining this with $e_{i,1}, e_{i,2} \in \mathbb{L}_\infty \cap \mathbb{L}_2$, Barbalat's lemma can be used to show that $\lim_{t \rightarrow \infty} e_{i,1}(t) = 0$ and $\lim_{t \rightarrow \infty} e_{i,2}(t) = 0$. By (5), we further have $\lim_{t \rightarrow \infty} \int_0^t (\gamma_i(\sigma) - z_{i,1}(\sigma))d\sigma = 0$ and $\lim_{t \rightarrow \infty} (\gamma_i(t) - z_{i,1}(t)) = 0$. Performing the final value theorem of Laplace transform to $Z_{ij,0}^*(s)$ in (15) yields $\lim_{s \rightarrow 0} Z_{ij,0}^*(s) = 0$. Notably, [18] shows that there exists a constant vector $\rho \in \mathbb{R}^{2n}$ such that $\lim_{s \rightarrow 0} sG(s) = 1_{2n}\rho^\top$. For system (18), we have

$$\lim_{t \rightarrow \infty} z(t) = 1_{2n}\rho^\top z(0)$$

which indicates that $\lim_{t \rightarrow \infty} (z_{i,1}(t) - z_{j,1}(t)) = 0$ and $\lim_{t \rightarrow \infty} (x_i(t) - x_j(t)) = 0$ for all $i, j = 1, \dots, n$.

5 Simulation Study

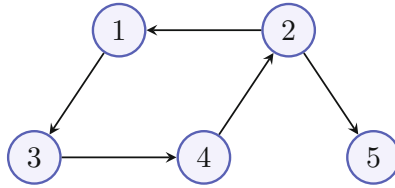


Fig. 1. Directed interaction topology.

To validate the proposed theoretical result, we consider a group of five nonlinear agents

$$\begin{aligned} \dot{x}_i &= v_i + \theta_i^\top \varphi_{i,1}(x_i) \\ \dot{v}_i &= u_i + \theta_i^\top \varphi_{i,2}(x_i, v_i) \end{aligned}$$

where $\varphi_{i,1} = \sin(x_{i,1})$, $\varphi_{i,2} = x_i \cos(v_i)$ for $i = 1, \dots, 5$. The position constraints for each agent are set as $-2 < x_{i,1}(t) < 2$ for all $t \geq 0$, i.e., $\bar{k} = 2$ and $\underline{k} = -2$. The initial positions chosen for our simulation scenario are $x_1(0) = 1.8$, $x_2(0) = 0.8$, $x_3(0) = 0.4$, $x_4(0) = 0$, and $x_5(0) = -1.2$, and the initial velocity-like states are set to $v_i(0) = 0$. The objective is to ensure the stability of the closed-loop system and asymptotic consensus with the position constraint.

The communication topology for these five agents is described Fig. 1 with communication delays $\tau_{ij} = 0.1$ s. The performance of the proposed algorithm is illustrated in Figs. 2, 3, and 4. The profiles of the agent positions together with the constraints are shown in Fig. 2, in which it can be observed that all agents achieve constrained consensus under the switching topology. Figure 3 shows the evolution of states v_i . The boundedness of the parameter estimation can be seen in Fig. 4. Clearly, x_i converge to the same value, as shown by the theoretical analysis.

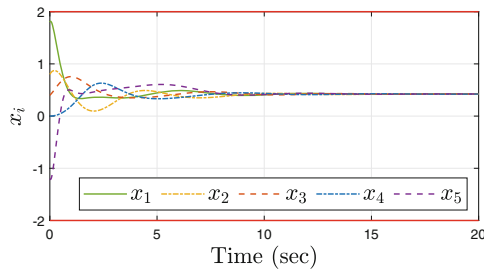


Fig. 2. Profiles of the agent position x_i together with its constraints.

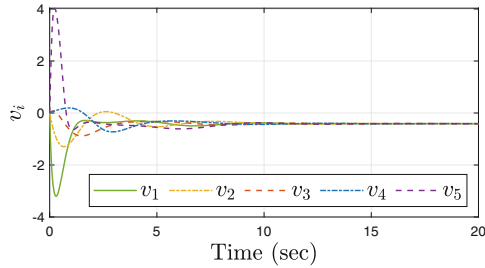


Fig. 3. Profiles of the agent velocity-like state v_i .

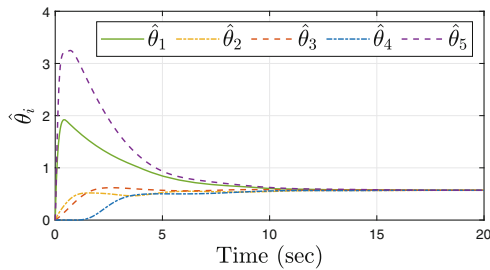


Fig. 4. Profiles of the parameter estimation $\hat{\theta}_i$.

6 Conclusion

A distributed consensus framework was presented to deal with the consensus problem of multi-agent systems with position constraints. Only a delayed output measurement of the neighbors is required for second-order agents without having to consider the uncertain dynamics of neighbors such that a consensus can be reached through a structurally simple solution. Simulation results clarified and verified our theoretical findings.

References

1. Olfati-Saber, R., Murray, R.M.: Consensus problems in networks of agents with switching topology and time-delays. *IEEE Trans. Autom. Control* **49**(9), 1520–1533 (2004)
2. Ren, W., Beard, R.W.: *Distributed Consensus in Multi-vehicle Cooperative Control*. Springer, New York (2008). <https://doi.org/10.1007/978-1-84800-015-5>
3. Mei, J., Ren, W., Chen, J.: Distributed consensus of second-order multi-agent systems with heterogeneous unknown inertias and control gains under a directed graph. *IEEE Trans. Autom. Control* **61**(8), 2019–2034 (2016)
4. Mei, J., Ren, W., Song, Y.: A unified framework for adaptive leaderless consensus of uncertain multiagent systems under directed graphs. *IEEE Trans. Autom. Control* **66**(12), 6179–6186 (2021)

5. Chen, W., Li, X., Ren, W., Wen, C.: Adaptive consensus of multi-agent systems with unknown identical control directions based on a novel Nussbaum-type function. *IEEE Trans. Autom. Control* **59**(7), 1887–1892 (2014)
6. Wang, Q., Psillakis, H.E., Sun, C.: Cooperative control of multiple agents with unknown high-frequency gain signs under unbalanced and switching topologies. *IEEE Trans. Autom. Control* **64**(6), 2495–2501 (2019)
7. Lee, U., Mesbahi, M.: Constrained consensus via logarithmic barrier functions. In: *Proceedings of the 50th IEEE Conference on Decision and Control and European Control Conference, Orlando*, pp. 3608–3613. IEEE (2011)
8. Zhou, Z., Wang, X.: Constrained consensus in continuous-time multiagent systems under weighted graph. *IEEE Trans. Autom. Control* **63**(6), 1776–1783 (2018)
9. Fan, B., Yang, Q., Sarangapani, J., Sun, Y.: Output-constrained control of non-affine multi-agent systems with partially unknown control directions. *IEEE Trans. Autom. Control* **64**(9), 3936–3942 (2019)
10. Wang, G., Wang, C., Cai, X.: Consensus control of output-constrained multiagent systems with unknown control directions under a directed graph. *Int. J. Robust Nonl. Control* **30**(5), 1802–1818 (2020)
11. Wang, G., Wang, C.: Constrained consensus in nonlinear multiagent systems under switching topologies. *IEEE Trans. Circuits Syst. II Exp. Briefs.* **69**(6), 2857–2861 (2022)
12. Abdessameud, A., Tayebi, A.: Formation control of VTOL unmanned aerial vehicles with communication delays. *Automatica* **47**(11), 2383–2394 (2011)
13. Min, H., Wang, S., Sun, F., Gao, Z., Zhang, J.: Decentralized adaptive attitude synchronization of spacecraft formation. *Syst. Control Lett.* **61**(1), 238–246 (2012)
14. Wang, H.: Consensus of networked mechanical systems with communication delays: a unified framework. *IEEE Trans. Autom. Control* **59**(6), 1571–1576 (2014)
15. Nuño, E., Ortega, R., Basañez, L., Hill, D.: Synchronization of networks of non-identical Euler-Lagrange systems with uncertain parameters and communication delays. *IEEE Trans. Autom. Control* **56**(4), 935–941 (2011)
16. Wang, H.: Differential-cascade framework for consensus of networked Lagrangian systems. *Automatica* **112**, 108620 (2020)
17. Psillakis, H.E.: Adaptive NN cooperative control of unknown nonlinear multiagent systems with communication delays. *IEEE Trans. Syst. Man Cybern. Syst.* **51**(9), 5311–5321 (2021)
18. Nuño, E., Ortega, R., Basañez, L., Hill, D.: Trajectory tracking and consensus of networks of Euler-Lagrange systems. In: *Proceedings of the IFAC World Congress, Milan*, pp. 938–943 (2011)