Joint TDOA and FDOA Estimation Based on Keystone Transform and Chirp-Z Transform

Xiaoyang Chen[1,2], Lede Qiu[1,2](✉), Shuai Li[1,2], Ming Li[1,2], and Yihao Song[1,2]

[1] Institute of Telecommunication and Navigation Satellites, China Academy of Space Technology, Beijing 100094, China
Qld_satcom@sina.com

[2] Innovation Center of Satellite Communication System, CNSA, Beijing 100094, China

Abstract. The traditional time difference of arrival (TDOA) and frequency difference of arrival (FDOA) estimation method suffers from the huge computational complexity and the estimate stepwise caused by the discrete parameter space. Moreover, the estimation performance deteriorates severely when the target moves rapidly. Therefore, a high accuracy method for joint TDOA and FDOA estimation based on the keystone transform and chirp-z transform is proposed. Firstly, the received signal data is divided into two-dimensional array of Fast Time (FT) and Slow Time (ST). And then the keystone transform is used to compensate the time delay migration due to the movement of target. Based on the compensated data, the coarse estimation of two parameters can be obtained by the utilization of the two-dimensional fast Fourier transform. In the following, the two-dimensional chirp-z transform is applied to reduce the search step of the region around the coarse estimation, so that the fine estimation of two parameters can be obtained. Finally, the quadratic function fitting is used to get the accurate estimation. Numerical simulations demonstrate the superiority of this method.

Keywords: TDOA · FDOA · Keystone transform · Chirp-z transform

1 Introduction

In the field of location of non-cooperative emitters, TDOA and FDOA are two important parameters. Accurate parameter estimation can significantly improve the localization accuracy [1]. For joint estimation of TDOA and FDOA, the traditional cross ambiguity function (CAF) is a maximum likelihood method [2]. However, CAF method needs many computing resources, so it is impossible for real-time computation. In order to abate computational complexity, a new coherent processing approach was extended to the field of estimation of TDOA and FDOA [3–5], which has been widely used in the maneuvering target detection [6], and the synthetic aperture radar (SAR) [7, 8]. The method divides the received signal into two-dimensional array of FT and ST, the Fast Time denotes a small segment of signal and the Slow Time denotes the whole signal length. In each Fast Time segment, the signal is viewed to have a fixed TDOA and a fixed FDOA. Meanwhile, the TDOA and FDOA change along the Slow Time. The focus

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of existing papers is to compensate the time delay migration (TM) and the Doppler shift migration (DM), such as the keystone transform (KT) [3, 6, 7], the second-order keystone transform (SKT) [4, 5], the Lv's distribution (LVD) [5, 6, 8] and so on. However, the discrete parameter space of TDOA and FDOA leads the estimate stepwise, whereas the true value as continuous so that the quantitative error cannot be avoided.

This paper proposes a high-accuracy joint TDOA and FDOA estimation method. The received signal is divided into two-dimensional array of FT and ST. The keystone transform is used to compensate the TM, and two-dimensional fast Fourier transform (2D-FFT) is utilized to get the coarse estimation. And based on the coarse estimation, the two-dimensional chirp-z transform (2D-CZT) is used to reduce the search step and get the fine estimation. Then, the quadratic function fitting is applied to get the accurate estimation.

2 Signal Model

Assuming that the noises, the variation of the amplitude and phase are ignored, the two received signals $s_1(t)$, $s_2(t)$ are [3]

$$\begin{aligned} s_1(t) &= a(t) \\ s_2(t) &= a(t + D(t))e^{j2\pi f_c D(t)} \end{aligned} \tag{1}$$

where f_c denotes the carrier frequency. And $D(t)$ denotes time delay. It should be noted that $D(t)$ is time-varying due to the relative motion between receiving machine and emitter, which can be expanded as Taylor series

$$D(t) = \tau_0 + vt + \varphi(t) \tag{2}$$

where $\varphi(t)$ is the higher-order terms, v is the generalized velocity, τ_0 is the time delay when $t = 0$.

Then the signal can be divided into two-dimensional array of FT and ST [3–5]. The TDOA and FDOA are assumed to be constant in a Fast Time segment t, but change along the Slow Time axis t_m. Let $D(t) = D(t_m)$, the signal model can be expressed in two-dimensional form, which is denoted as

$$\begin{aligned} s_1(t, t_m) &= a(t) \\ s_2(t, t_m) &= a(t + \tau_0 + vt_m)e^{j2\pi f_c(\tau_0 + vt_m)} \end{aligned} \tag{3}$$

3 The Proposed Method

After the signal partition, there are three steps about the proposed method. In the first step, KT is used to correct the linear time delay migration. Then based on the compensated date, 2D-FFT is utilized to obtain the coarse estimation. The second step is to reduce the search step by 2D-CZT, so that the fine estimation can be obtained from the finer parameter space grids. The third step is to apply quadratic function fitting to get the accurate estimation.

3.1 Coarse Estimation

Define the mixing product as

$$s(f, t_m) = FFT_t^*[s_1(t, t_m)] \odot FFT_t[s_2(t, t_m)] = |A(f)|^2 e^{j2\pi f_c(\tau_0 + vt_m)} e^{j2\pi f(\tau_0 + vt_m)} \quad (4)$$

where $A(f)$ is the expression of $a(t)$ in frequency domain, and f is the FT frequency. $FFT_t[\cdot]$ represents the FFT operation in FT domain. $\odot$ denotes the Hadamard product. $*$ is the complex conjugation. The expression of (4) in FT time domain is

$$s(t, t_m) = r_{aa}(t + \tau_0 + vt_m)e^{j2\pi f_c(\tau_0 + vt_m)} \quad (5)$$

where $r_{aa}(t) = E(a(x + t)a(x))$ is the time domain expression of $|A(f)|^2$, and the autocorrelation function of $a(t)$ in physical meaning. The focus position of $r_{aa}(t)$ in FT time domain is the time-varying TDOA. And the phase term reflects a single-frequency signal along the ST axis, whose frequency is FDOA.

Utilize the keystone transform to clear off the linking between f and t_m. And the KT is defined as

$$f_c t_n = (f + f_c)t_m \quad (6)$$

where t_n is the time with respect to ST after the KT, so (4) can be changed as

$$s(f, t_n) = |A(f)|^2 e^{j2\pi(f_c + f)\tau_0} e^{j2\pi f_c v t_n} \quad (7)$$

Performing the generalized 2D-FFT on (7) along the FT axis and the ST axis, we have

$$s(t, f_n) = r_{aa}(t + \tau_0)\delta(f_n + f_c v) \quad (8)$$

where f_n is the frequency with respect to ST. It is obvious from (8) that τ_0 and $f_c v$ can be obtained, which correspond to the initial TDOA and FDOA respectively.

3.2 Fine Estimation

As shown in above, the calculation of TDOA and FDOA is carried out by FFT, so the "Picket Fence Effect" degrades the accuracy of the final result. Note that the resolution of FT time domain is $\Delta\tau = 1/f_s$, where f_s is the sampling frequency. And the resolution of ST frequency domain is $\Delta f = 1/T$, where T is the total integral time. To improve the resolution, sampling rate and the length of signal should be increased, which will greatly increase the cost. So the 2D-CZT is applied to magnify the region of interest around the peak, which comes from the result of the coarse estimation. And the fine estimation can be gotten from the new peak position.

The 2D-CZT of (7) is defined as

$$S(t, f_n) = czt[s(f, t_n)] = W_1^{t^2/2} W_2^{f_n^2/2} \sum_{f=-f_s/2}^{f_s/2} \sum_{t_n=0}^{T} g(f, t_n) W_1^{-(t-f)^2/2} W_2^{-(f_n - t_n)^2/2} \quad (9)$$

where $S(t, f_n)$ is the result of 2D-CZT of $s(f, t_n)$ and the expression of $s(t, f_n)$ with smaller search step. And t, f are time and frequency with respect to FT. t_n, f_n are time and frequency with respect to ST. $g(f, t_n) = s(f, t_n)A_1^{-f}A_2^{-t_n}W_1^{f^2/2}W_2^{t_n^2/2}$ and the terms A_1, A_2, W_1, W_2 are complex exponential scalars, whose expressions are $W_1 = \exp[-j2\pi(T_2 - T_1)/(PT')]$, $A_1 = \exp[j2\pi T_1/T']$, $W_2 = \exp[-j2\pi(F_2 - F_1)/(Qf_{s,m})]$, $A_2 = \exp[j2\pi F_1/f_{s,m}]$, where T' denotes the length of a Fast Time segment. T_2, T_1 are two endpoints of zoom interval of TDOA. $f_{s,m}$ denotes the sampling frequency in Slow Time axis. F_2, F_1 are two endpoints of zoom interval of FDOA. P, Q is the interval length with respect to TDOA and FDOA.

3.3 Quadratic Function Fitting

The quadratic function is used for interpolation to get the accurate estimation. Based on the fine estimation, two slices can be gotten which along the two axes of the array respectively. Select the largest value and the adjacent two points in a slice $[(x_{k-1}, y_{k-1}), (x_k, y_k), (x_{k+1}, y_{k+1})]$, so that the real peak can be accurately located which is halfway between (x_k, y_k) and the other point. The expression of the quadratic function is defined as

$$a_1x^2 + a_2x + a_3 = y \tag{10}$$

where a_1, a_2, a_3 are the coefficient of the quadratic function. So real peak position is $x_{\max} = -a_2/(2a_1)$, which is the accurate estimation.

Table 1. Computational complexity comparison

The method	The computational complexity
The proposed method	$O(11MN \log_2 N + 25MN)$
KTM	$O(5MN \log_2 N + 9MN)$
CAF	$o(2LN_tN_f)$

4 Computational Complexity Analysis

This part discusses the computational load and compares method in this paper with the CAF [2] and the method based on keystone transform (KTM) [3]. Denote the length of FT and ST as N, M respectively. Denote the search length of TDOA and FDOA as N_t, N_f and the length of signal as L in CAF. The computational complexity comparison shows in the Table 1.

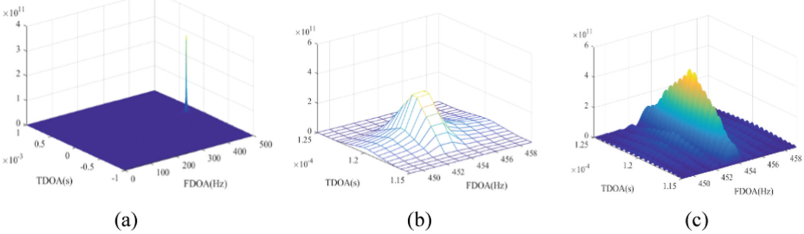

Fig. 1. The simulation result: (a) panorama of the coarse estimation; (b) the detail of the spectral peak of the coarse estimation; (c) the result of 2D-CZT, the display region is the same as the (b)

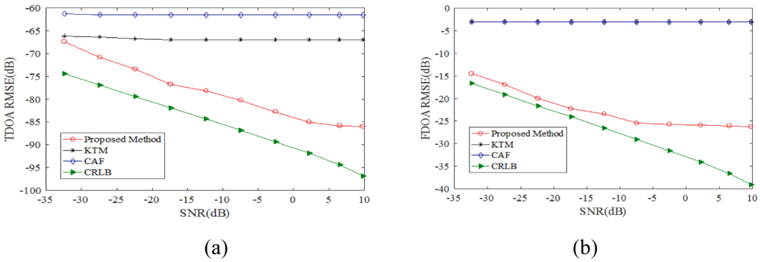

Fig. 2. (a) RMSEs of TDOA versus SNR; (b) RMSEs of FDOA versus SNR

5 Numerical Simulations

This part uses simulations to demonstrate the enforcement of method in this paper and compare it with conventional method. The signal modulation mode is the binary phase shift keying (BPSK). The bandwidth is $B_s = 600\,\text{kHz}$ and the carrier frequency is $f_c = 300\,\text{MHz}$.The sampling frequency is $f_s = 2\,\text{MHz}$, the total time is $T = 1\,\text{s}$. And the length of ST is $M = 1000$, the length of FT is $N = 2000$. The initial true value is $TDOA = 1.19799945 \times 10^{-4}\,\text{s}$ and $FDOA = 453.506919\,\text{Hz}$.

The results of the coarse estimation and the fine estimation with a relatively high SNR are shown in Fig. 1. It is found that the coarse estimation can form a peak at ($TDOA = 1.2 \times 10^{-4}\,\text{s}$, $FDOA = 454\,\text{Hz}$), whereas the low resolution limits the accuracy. And the fine estimation can overcome the disadvantage, the detail of the peak can be completely observed by the 2D-CZT, which magnifies the resolution of the region around the peak by 100 times. And the new peak position after the 2D-CZT is ($TDOA = 1.1980 \times 10^{-4}\,\text{s}$, $FDOA = 453.51\,\text{Hz}$), so the quantization error is decreased.

The accuracy of the CAF [2], the KTM [3] and the method in the paper are presented in Fig. 2. And 100 Monte Carlo trials are calculated for each SNR values. Note that the SNR r here is the equivalent input SNR of two received signals. And it can be expressed as $r = 2r_1r_2/(r_1 + r_2 + 1)$, where r_i denotes the input SNR of $s_i(t)$, $i = 1, 2$. And the RMSE is the evaluation norm of the estimation accuracy. As shown in the Fig. 2, due to the existence of the time delay migration, the error of the CAF is the highest. Compared with CAF, KTM has better estimation performance due to compensation time delay migration. Based on the compensated data, the other two methods are inferior to the proposed method, this is because that the search step of the proposed method is smaller and the quadratic function fitting is applied. So the proposed method can efficaciously enhance the accuracy of TDOA and FDOA estimation.

6 Conclusion

Based on KT and 2D-CZT, a high-accuracy estimation method for TDOA and FDOA has been investigated. The proposed method firstly divides the received signal into two-dimensional array of FT and ST. And the KT is used to compensate the linear time delay migration so that the coarse estimation of two parameters can be obtained by 2D-FFT. Sequentially, the 2D-CZT is used to reduce the search step based on the coarse estimation, and the fine estimation can be acquired. Then the accurate estimation can be found by the quadratic function fitting. The results of numerical experiments prove the superiority of the proposed method, and the accuracy can approach the Cramer-Rao lower bounds (CRLB).

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