Consolidation of Layered Soils with Variable Compressibility

Deepak Kumar Singh , **Chitranshu Kumawat, and Siddharth Mehndiratta**

Abstract The distribution of consolidating load in any soil is related directly to the distribution of pore water pressure. Terzaghi's consolidation theory that is used to define this distribution is based on some certain linearized assumptions that may or may not be reasonable in practice. The linearized equation surely may have some mathematical benefits; however, they are of slight importance when working on non-homogeneous soils. In the following study, a numerical method is developed. A finite difference approach for computing 1D consolidation in analysing consolidation problem is presented. Most software applications utilize an explicit finite difference evaluation method. While using this approach, the solution space in the problem is discretized in time and space. The same problem may also be solved implicitly, so that the excess pore pressures at all nodes are solved together at the selected time. A nonlinear formulation, involving variations of compressibility, indicated in the results is deemed essential for the behaviour that is observed. Critical investigation may be done to investigate the nature of these variations.

Keywords Terzaghi's theory · Finite difference method · Consolidation

1 Introduction

Tiny solid particles form soil, and these particles are not bonded together, with the exception of the force of cohesion, internal friction, minor van der Waals forces and adsorbed double-layer water. Whenever a load is experienced by a soil mass, the elastic deformation undergone by the solid particles is quite small when compared

D. K. Singh (✉) · C. Kumawat · S. Mehndiratta
MNIT Jaipur, Jaipur, India
e-mail: 2020rce9526@mnit.ac.in

C. Kumawat
e-mail: 2020pct5254@mnit.ac.in

S. Mehndiratta
e-mail: siddharth.ce@mnit.ac.in

with the deformation that is due to variation in the relative position of distinct soil particles, and this following reduction in the soil mass' volume is due to reduction in volume of voids. If the voids are occupied by only air, rapid compression of soil takes place, since air can easily escape from the voids. When soil is saturated and its voids are occupied with water that is incompressible, any volume reduction or compression may only take place once water is ousted from the voids. This phenomenon occurs due to an extended period of static load, and the resulting expulsion of pore water is called consolidation. Also, we can conclude that consolidation is a natural phenomenon because it is due to the very long-term static load.

According to Terzaghi, 'every process involving a decrease in the water content of a saturated soil without the replacement of water by air is a process of consolidation'.

Xie et al. [1] presented a solution for 1D consolidation that was entirely explicit analytical. In their study, a double-layered soil with partly drained boundaries was taken. The results were indicative that the boundary drainage conditions and the soils' layered characteristics also impact the consolidation behaviour.

Conte and Troncone [2] analysed consolidation problem regarding time-dependent loading on thin clay. A modest solution to nonlinear 1D theory of consolidation that utilized certain analytical expressions in combination with Fourier series was forwarded.

Liu and Lei [3] presented a universal analytical solution that was explicit for one-dimensional consolidation of layered soil problems. An applicable computer code was written, and the results for some examples were conveyed. According to these results, a thorough study pertaining to one-dimensional consolidation behaviour of layered soil system was conducted.

2 Objectives

In the application of Terzaghi's theory, many significant shortcomings were pointed out in later researches arising due to the limitations of the assumptions in the theory, especially those involving soft clays, and, in his theory, the constant value of coefficient of consolidation is one of the major limitations. The objective of the present study is to obtain results between pore water pressure and time by varying different parameters.

3 Methodology

According to Terzaghi's theory, the value of C_v (coefficient of consolidation) is assumed constant. For the purpose of this study, a 1D nonlinear partial differentiation equation derived by Abbasi et al. [4] for estimation of consolidation characteristics of

clays is utilized. To approach towards the solution of the above-mentioned nonlinear equation, a finite difference methodology is selected.

On the basis of Terzaghi's fundamental equation and in view of the point that every variation in pore pressure equals a variation in the effective stress, we have

$$\frac{\partial^2 u}{\partial z^2} = \frac{\partial^2 \sigma'}{\partial z^2} \tag{1}$$

For homogeneous soils:

$$\frac{\partial k}{\partial z} = 0 \tag{2}$$

When the applied total stress is constant, the equation for continuity for 1D consolidation is given as follows:

$$\left(\frac{1}{1 + e_0}\right)\frac{\partial e}{\partial t} = -\frac{k}{\gamma_w}\frac{\partial^2 \sigma'}{\partial z^2} \tag{3}$$

For soft soil/clays:

$$e = a - C_c \operatorname{Log} \sigma' \tag{4}$$

e and k relationship:

$$e = b + M \operatorname{Log}(k) \tag{5}$$

where

'a' and 'b' are intercepts of slope line.
A = void ratio of unit effective stress.
b = void ratio of unit coefficient of permeability, ($k = 1$).

On combining both equations:

$$k = 10^{(a-b)/M}(\sigma')^{-C_c/M} \tag{6}$$

Differentiate Eq. 4 with respect to t:

$$\frac{\partial e}{\partial t} = \frac{-2.3 C_c}{\sigma'}\frac{\partial \sigma'}{\partial t} \tag{7}$$

Substituting Eqs. 6 and 7 in Eq. 3.

$$\frac{\partial \sigma'}{\partial t} = \frac{2.3(1 + e_0)}{C_c \gamma_w} 10^{(a-b)/M}(\sigma')^{\left(1 - \frac{C_c}{M}\right)}\frac{\partial^2 \sigma'}{\partial z^2} \tag{8}$$

Assuming

$$\alpha = 1 - \frac{C_c}{M} \tag{9}$$

and

$$C_n = \frac{2.3(1 + e_0)}{C_c \gamma_w} 10^{(a-b)/M} \tag{10}$$

So

$$\frac{\partial \sigma'}{\partial t} = C_n (\sigma')^\alpha \frac{\partial^2 \sigma'}{\partial z^2} \tag{11}$$

Assuming linear relationship for $e - Log(\sigma')$ and $e - Log(k)$, Equation 11 can be reworked in terms of the excess pore water pressure, (u) seeing that $\sigma' = \sigma_t - u$

$$\frac{\partial u}{\partial t} = C_n (\sigma_t - u)^\alpha \frac{\partial^2 u}{\partial z^2} \tag{12}$$

$$C_n (\sigma_t - u)^\alpha = C_v \tag{13}$$

According to Eq. 12, the coefficient of consolidation, C_v, fluctuates during consolidation when the surplus pore water pressure, u, varies. It is clearly observed here that coefficient of consolidation is associated nonlinearly with effective stress. α is a dimensionless factor that relies upon compressibility and permeability characteristics (from Eq. 9 (M and C_c) of the soil, and C_n, a coefficient, can be ascertained using Eq. 10 and is dependent on permeability and compressibility characteristics (M, b, C_c, a), initial void ratio (e) and the unit weight of water (γ_w). In the present study, the coefficients C_n and α will be labelled as basic coefficient of consolidation and nonlinearity factor, respectively. For special cases, i.e. $\alpha = 0$ (or $C_c/M = 1$), C_v will remain constant, equivalent to C_n, and Eq. 12 will diminish to Terzaghi's theory.

$$\frac{\partial u}{\partial t} = C_v \frac{\partial^2 u}{\partial z^2} \tag{14}$$

Table 1 Characteristics of samples

Sample number	Coefficient of consolidation C_v (m^2/sec)	Applied pressure (kPa)	α
1	8.44×10^{-7}	15	0.34
2	1.25×10^{-5}	60	-0.14

4 Single-Layered Profile: Parametric Study

4.1 Problem Statement

The problem undertaken in the paper involves weightless soil (depth $= 12$ cm) where water table is at the ground surface. Two soil samples having different coefficient of consolidation and applied pressure are considered (Table 1). At time zero, excessive pore water pressure is equivalent to applied pressure at all nodes except at the drainage boundaries. It is found that successive application of finite difference equation soon leads to absurd values of pore water pressures (is divergent) if $\beta = C_v \Delta t / (\Delta z^2)$ exceeds 0.5. In the present study, β is forced to remain less than 0.5. The accuracy of the solution is improved by using a small spacing of node points in the z direction.

4.2 Effect of Parameter A

In the analysis, applied pressure is taken as 100 kPa and initial coefficient of consolidation is considered as 8.44×10^{-7} m^2/sec.

Figure 1 illustrates the consequence of α on U-t curve profile. Regarding negative α values, the U-t curve is located under the curve projected by Terzaghi's solution, i.e. (constant C_v, or $\alpha = 0$). Indicating that values of α that are negative, the consolidation will be at a slower rate than expected with respect to Terzaghi's solution and consolidation gets slower when the value of α decreases. Alternatively, for positive α values, consolidation is quicker than anticipated by Terzaghi's solution, and with increase in the value of α, the rate of consolidation increases.

As presented earlier that during consolidation, the coefficient of consolidation varies as effective stress (or pore pressure) varies in both spatial and temporal directions. Figures 2 and 3 display distinctive deviations of C_v with time and depth, respectively. The effect that α has on C_v is depicted in Fig. 3. This figure displays that the coefficient of consolidation, C_v, has increasing and decreasing tendencies with respect to time for positive and negative α values, respectively. The permeability of soil declines at a rate faster when likened to decrease in compressibility when α is larger than 0. Thus, as the consolidation advances, an increase in the coefficient of consolidation can be observed. Alternatively, when the value α is below 0, during consolidation, the coefficient of consolidation reduces. It can also be observed that
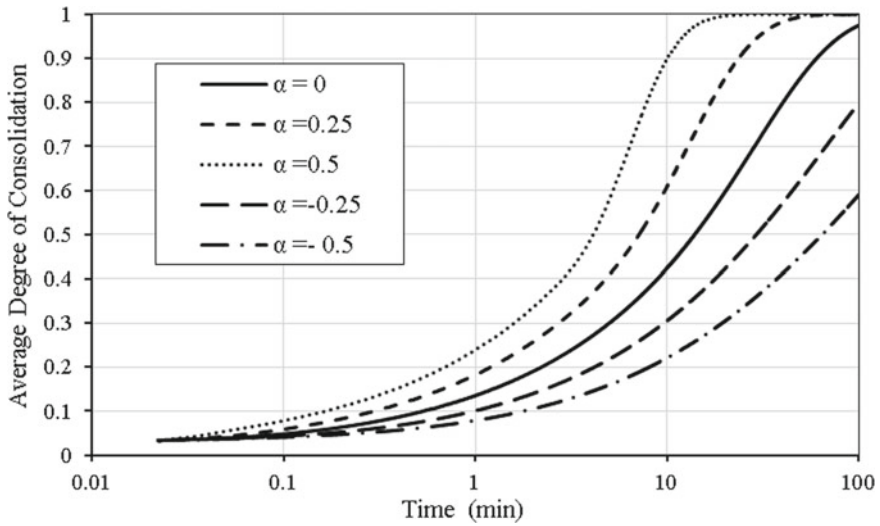
Fig. 1 Average degree of consolidation versus time curve for different value of α

the coefficient of consolidation is unchanged, and finally when α is equal to zero, the outcomes are identical to Terzaghi's solution.
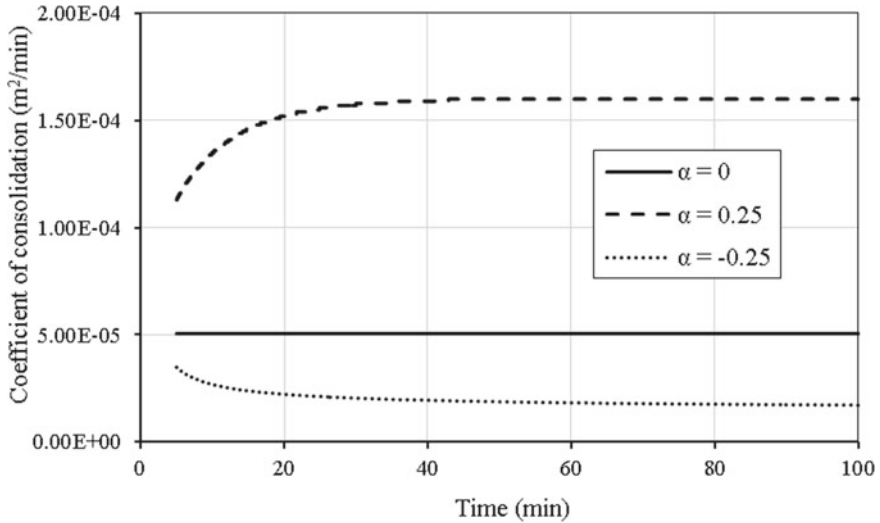

Fig. 2 Coefficient of consolidation (C_v in m^2/min) versus time factor curve for various values of α
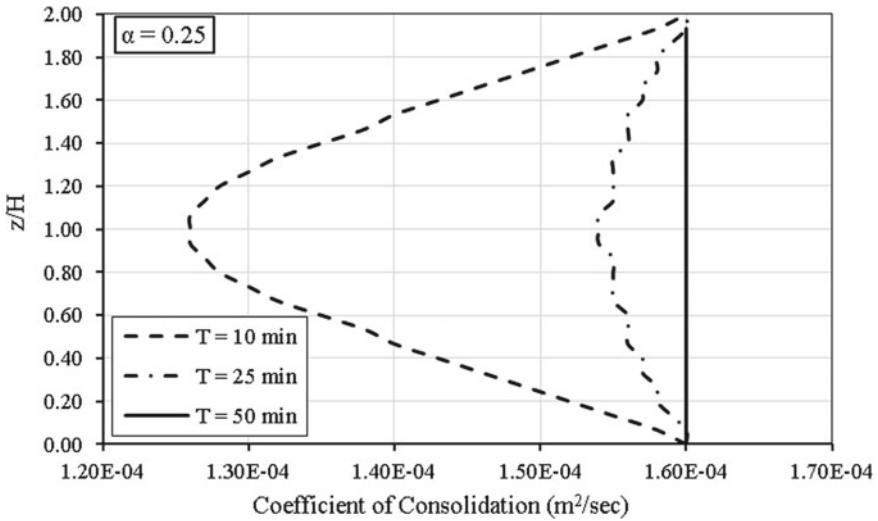
Fig. 3 Average degree of consolidation versus time factor curve for differing values of Δz

4.3 Effect of Applied Pressure

Figure 4 displays the outcome of applied pressure on the U–t curve shape ($\alpha = 0.25$). It can be clearly observed that increasing applied pressure increases the consolidation rate. The surcharge pressures have a great influence on consolidation characteristics.
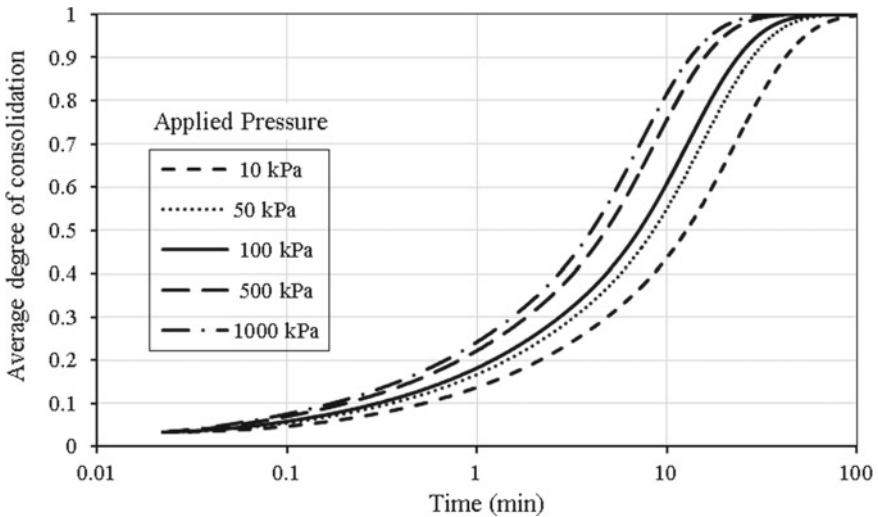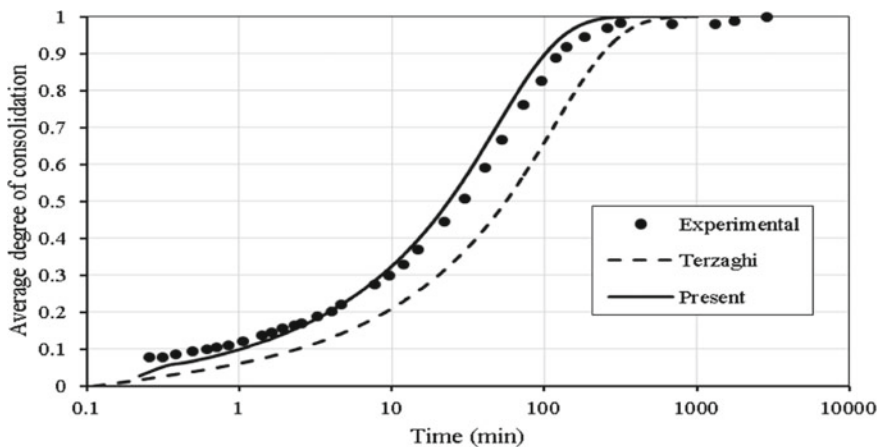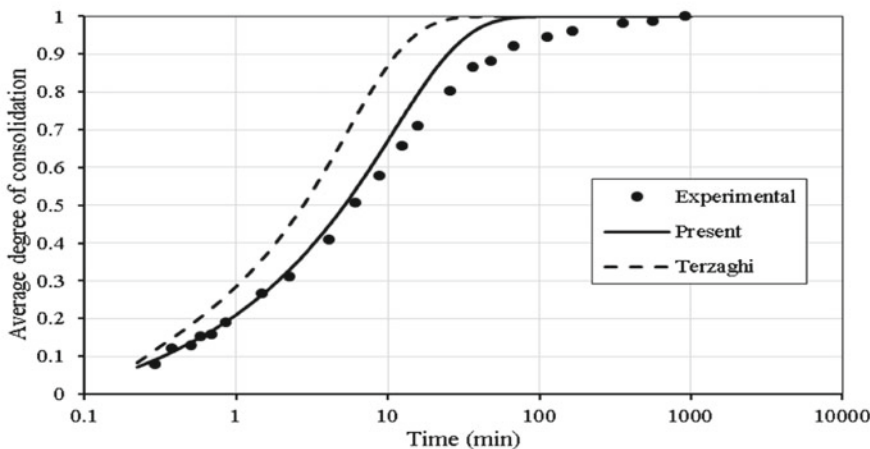

Fig. 4 Variation of average degree of consolidation with time at different applied pressure

5 Validation

The experimental results of Abbasi et al. [4] and from Terzaghi's theory are compared with the results that were achieved through numerical solution using nonlinear equations. It is observed from Fig. 5 that the results achieved using nonlinear theory are similar to the results obtained experimentally when compared with the results achieved by Terzaghi's linear theory. Additionally, for the soil samples 1 having positive values of α (Fig. 5a), consolidation occurs quicker than anticipated by Terzaghi's theory. For samples 2 and 3 that comprise α values that are negative (Fig. 5b), consolidation occurs at a rate slower than anticipated by Terzaghi's theory.

(a) Sample 1

(b) Sample 2

Fig. 5 Change in average degree of consolidation with time

6 Double-Layered Profile

Laplace equation for two-dimensional seepages is written as follows:

$$k_x \frac{\partial^2 h}{\partial x^2} + k_z \frac{\partial^2 h}{\partial x^2} = 0 \qquad (15)$$

In the case of flow through the boundary of a homogenous soil layer to another, the equation is modified (Das [5]).

Referring to Fig. 6, flow area is situated half in soil 1 and has a permeability coefficient k_1 and the other part in soil 2 having a permeability coefficient, k_2, we say that:

$$k_{\text{avg}} = \frac{1}{2}(k_1 + k_2) \qquad (16)$$

If soil 1 and soil 2 are swapped, the replaced (soil 1) evidently has a hydraulic head of h_4' instead of h_4.
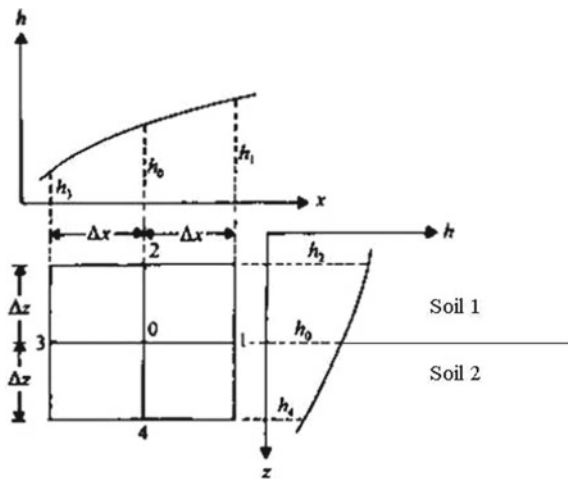
Having equal velocity,

$$k_1 \cdot \frac{h_4' - h_0}{\Delta z} = k_2 \cdot \frac{h_4 - h_0}{\Delta z} \qquad (17)$$

or

$$h_4' = \frac{k_2}{k_1} \cdot (h_4 - h_0) + h_0 \qquad (18)$$

Therefore, from Eq. 15

Fig. 6 Hydraulic heads for flow in a region (Das [5])

$$\frac{k_1 + k_2}{2} \frac{h_1 + h_3 - 2h_0}{(\Delta x)^2} + k_1 \frac{h_2 + h'_4 - 2h_0}{(\Delta z)^2} = 0 \tag{19}$$

Taking $\Delta x = \Delta z$ and substituting the value of h'_4 in above equation,

$$h_0 = \frac{1}{4} \left(h_1 + \frac{2k_1}{k_1 + k_2} h_2 + h_3 + \frac{2k_2}{k_1 + k_2} h_4 \right) \tag{20}$$

6.1 Finite Difference Formulation 1D Consolidation for Layered Soil

Every time it may not be feasible to formulate a sealed-form result for consolidation in layered soils. Many a time, numerous variables are involved like the thickness of layers, different coefficients of permeability and differing coefficient of consolidation. Hence, numerical solutions are said to be a prudent method. If evaluation of surplus pore water pressure at the boundary of two dissimilar types (i.e. varying values of C_v) of clayey soils is involved, finite difference equations are altered to some degree. According to Terzaghi's consolidation theory:

$$\frac{k}{c_v} \frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial z^2} \tag{21}$$

where

$$k \frac{\partial^2 h}{\partial z^2} = k_1 \left[\frac{h_1 + h'_4 - 2h_0}{(\Delta z)^2} \right]$$
$$= \frac{k_1}{(\Delta z)^2} \left[h_2 + \frac{k_2}{k_1} (h_4 - h_0) + h_0 - 2h_0 \right]$$

On dividing and multiplying above equation with 2 and taking $(k_1 + k_2)$ common, then

$$k \frac{\partial^2 u}{\partial z^2} = \frac{1}{2} \left(\frac{k_1 + k_2}{(\Delta z)^2} \right) \left[\frac{2k_1}{k_1 + k_2} u_{1,t} + \frac{2k_2}{k_1 + k_2} u_{3,t} - 2u_{0,t} \right] \tag{22}$$

For the boundary element, average change in volume is

$$\frac{k}{C_v} \frac{\partial u}{\partial t} = \frac{1}{2} \left[\frac{k_1}{C_{v1}} + \frac{k_2}{C_{v2}} \right] \frac{1}{\Delta t} \left[u_{0,t+\Delta t} - u_{0,t} \right] \tag{23}$$

Therefore, from Eqs. 21, 22 and 23

$$\therefore u_{0,t+\Delta t} = \frac{\Delta t}{\Delta z^2} \frac{(k_1 + k_2)}{\left[\frac{k_1}{C_{v1}} + \frac{k_2}{C_{v2}}\right]} \left[\frac{2k_1}{k_1 + k_2} u_{1,t} + \frac{2k_2}{k_1 + k_2} u_{3,t} - 2u_{0,t}\right] + u_{0,t} \quad (24)$$

From the above equation, at the interface of two dissimilar types of soil at any $(t + \Delta t)$ time, we can calculate pore water pressure.

6.2 Problem Statement

The diagram shown in Fig. 7 depicts one-dimensional, nonlinear consolidation of double-layered clayey soil. In the same figure, h_i = thickness of the clay layer i (i = 1, 2); $H = h_1 + h_2$, the total thickness of two clay layers; $q(t)$ = load applied on the top surface of the soil that is uniformly distributed. In the present study, h_1 and h_2 are considered as 4 m. The amount of the instant load over the top surface is equal to 100 kPa. Material properties used in the analysis are reported in Table 2.
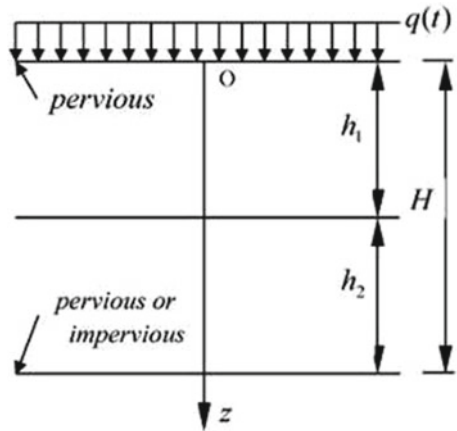

Fig. 7 Double-layered soil system

Table 2 Material properties

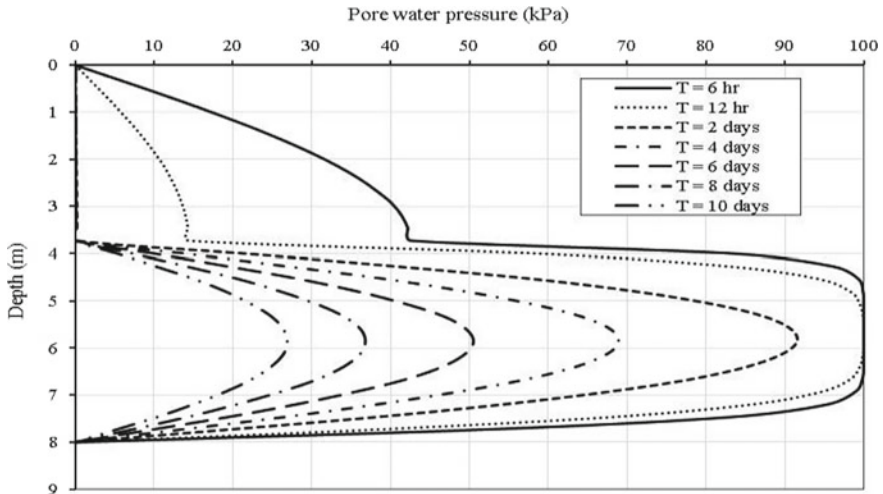| Layer	Depth (m)	Coefficient of permeability, k (m/sec)	Coefficient of consolidation, C_v (m²/sec)	α
1	4.0	0.28×10^{-6}	7.25×10^{-5}	0.1
2	4.0	0.20×10^{-9}	8.44×10^{-9}	0.34

Fig. 8 Variation of pore water pressure with depth at various time

6.3 Variation of Pore Water Pressure and Coefficient of Consolidation C_v Temporally and Spatially

Figure 7 shows the variation of PWP with depth at various time. Layer 1 is sand (as its permeability is high) and Layer 2 is clay (due to low permeability in comparison with above layer). Top surface of Layer 1 is pervious and bottom surface of Layer 2 is pervious so it is the case of double drainage. From Fig. 8, it is clearly seen that Layer 1 pore water pressure dissipates rapidly due to high value of coefficient of permeability, but Layer 2 will take time to dissipate pore water pressure. It is observed from Fig. 8 that after 2 days Layer 1 is completely drained and pore water pressure along the depth is zero, but for Layer 2 as it is clay it takes large time to drain completely.

Figure 9 shows that C_v in Layer 1 is constant along the depth at various time, but at the interface of two soils (transition from Layer 1 to Layer 2), there is a sudden increase in the value of coefficient of consolidation (C_v). This is because at the interface of two soils, water from the Layer 2 at the top moves towards Layer 2, and when water reaches to a particular depth, it simply drains out from the top of Layer 1. In the Layer 2, pore water pressure is maximum at centre and decreases with time. As proved earlier, the coefficient of consolidation is inversely proportional to the pore water pressure; hence, C_v is minimum at the center, and it increases with time.

Figure 10 shows that in Layer 1 (Depth $= 2.67$ m), C_v is constant at any time, but for Layer 2 (Depth $= 5.34$ m) there is increment in C_v with increment in the time of consolidation. This is because the pore water pressure decreases with increase in time. Thus, C_v increases.
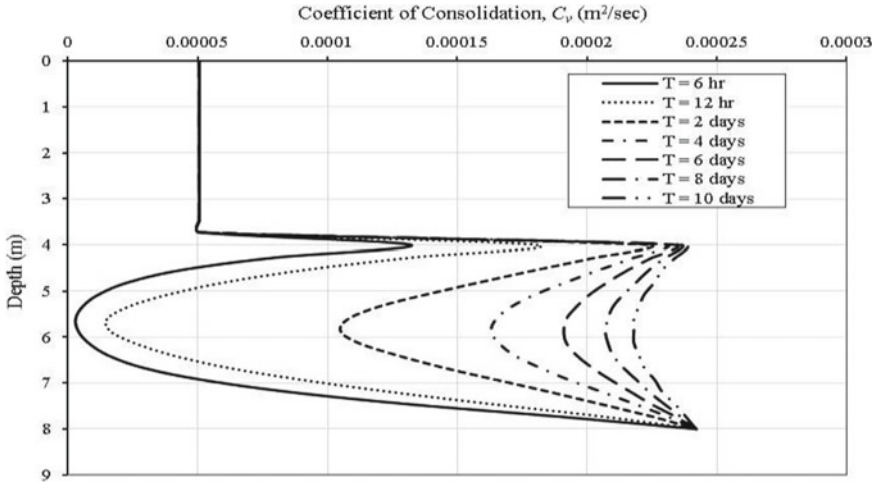
Fig. 9 Change in of coefficient of consolidation (C_v) with depth at different time period
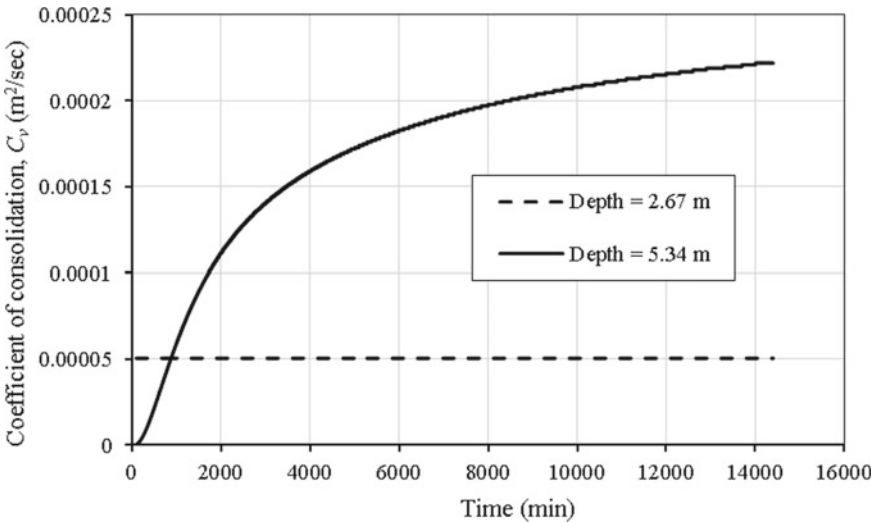

Fig. 10 Variation of coefficient of consolidation with time at various depths

7 Conclusion

The nonlinear theory specified by Abbasi et al. [4] for consolidation of soil while permitting for differences in coefficient of consolidation with time and space has been utilized in the present study. A finite difference approach that follows an iterative

technique to arrive at the result for the developed nonlinear principal equation was utilized. According to the results attained, the subsequent inferences are drawn:

(a) At different times, all the consolidation characteristics of soil, i.e. degree of consolidation, pore pressure, coefficient of consolidation and time factor, can be simply evaluated at desired depth (and the average over depths) while using the numerical model.

(b) For negative α values, consolidation is slower when compared with Terzaghi's solution, and for positive α values, consolidation is faster than anticipated from Terzaghi's solution.

(c) As C_v increases, pore water pressure along the depth decreases.

(d) As the value of C_v increases, time required for consolidation decreases because as we know from mathematical equation of C_v, it is directly related to coefficient of permeability (k).

(e) For the selected relative C_v values, the reduction in the thickness of the lower layer (clay) has greater effect than decrease in the thickness of the upper layer (sand).

(f) As the applied pressure increases, value of coefficient of consolidation increases at various depths.

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