

# Computer Aided Analysis of Involute Gear Tooth for Minimization of Bending Stress



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## 1 Introduction

The most efficient way of delivering power from the source to the driven part is through gear drives. They are very effective in transmitting torque when the centre distance between the driver and the driven members is very close. The designs of these gears for various industrial applications are over a century old. Many researchers have contributed to the development of materials, modified geometry, analyzed the gears for various applications, and have proposed developments over those of the existing ones in the past.

Cavdar et al. [1] devised a method for estimating the bending stress minimization of an involute spur gear. A computer programme was used to analyze the variation of bending stress and contact ratio dependent on the pressure angle on the driving side. Kapelevich [2] presented a method for boosting load capacity while lowering weight, vibration, and size, and his research shows that asymmetric tooth geometry allows for increased load capacity while reducing dimensions and weight for specific gear types. Litvin et al. [3] developed a redesigned geometry to allow them to more easily localize and stabilize the bearing contact while obtaining a more advantageous form of transmission errors. Shekhtman and Kapelevich [4] discussed how to reduce bending stress by adjusting the fillet radius. By optimizing the fillet profile, the maximum bending stress in the gear tooth root area is lowered by 10–30%. Direct gear design, introduced by Kapelevich and Kleiss [5], allows for the assessment of a wide range of parameters for all feasible gear combinations to discover the optimal solution for a given application. Direct gear design with an asymmetric tooth profile opens up new sources for development in gear drives with unidirectional load cycles,

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such as those present in many mechanical gearboxes. According to McNamara and Kapelevich [6], when compared to traditionally made gears, direct gear design leads to a 15–30% reduction in stress. This translates to greater load capacity (15–30%), smaller and lighter size and weight (10–20%), longer life, lower costs, increased dependability, lower noise and vibration, higher gear efficiency, and lower maintenance costs. Karpat et al. [7] presented innovative gear designs that are required due to rising performance demands such as long life, high endurance, high load capacity, quick speed, and low cost. Deng et al. [8] discussed the newly developed and modified gear models that could be used for mating analysis. According to the findings, the involute modification of the produced gear helped improve transmission performance. Senthil Kumar et al. [9] investigate the use of an asymmetric tooth edge that can be used to improve the bending capacity of symmetric involute gears constructed using a standard technique since the tipping development limits the load-bearing capacity at higher pressure angles. Masuyama and Miyazaki [10] presented the asymmetric tooth profile gear strength when the load-sharing ratio is taken into account and the performance of the gear is compared to the torque transmission capacity. The researchers looked at tooth pressure angles ranging from 20° to 45°. Vaghela and Prajapati [11] addressed how to lessen the bending stress of asymmetric involute spur gear teeth by optimizing the root shape. The Von Mises stress of the optimized root profile is investigated and compared to the regular asymmetric gear fillet profile. The root profile is optimized, and the Von Mises stress is lowered by 16.68%. The stress concentration of the improved asymmetric spur gear has been considerably reduced. According to Olguner and Filiz [12], asymmetric gears with a bigger pressure angle on the driving side contribute significantly to boosting load-bearing capacity and flow rate, as well as lowering flow rate volatility, which is detrimental to the gear pumps' dynamic behaviour. Malleth et al. [13] examined bending stresses and how they decreased with the increasing number of teeth and pressure angle on the drive side. When all gear teeth are subjected to the same load, the one with the most teeth will be stressed less. According to research, bending stress reduces when the pressure angle on the drive side increases. Prabhu Sekar [14] proposes that asymmetric teeth can be used to enhance gear efficiency by increasing contact force and load carrying capacity, as well as improving wear resistance. He analyzed the load shared by a tooth pair, frictional power losses, wear resistance, fillet and contact stresses, and respective mechanical efficiencies to conduct comparative performance research of symmetric and asymmetric spur gears.

An asymmetric gear tooth profile and modifying the root fillet radius are considered in this work to reduce bending stresses. The advantages of reducing the bending stress are increased life, a reduction in cost, and increased efficiency.

## 2 Materials and Methods

The material used to make gears is determined by its strength and service conditions such as wear, noise, and so on. Metal or non-metallic materials can be used to make

the gears. Commercially available metallic gears with cut teeth are made of cast iron, steel, and bronze. Non-metallic materials including wood, rawhide, compressed paper, and synthetic polymers like nylon are used in gears to reduce noise.

Because of its outstanding wear qualities, excellent machinability, and ease of manufacturing intricate geometries using the casting method, cast iron is commonly used in the production of gears. Where a smooth motion is not required, cast iron gears with chopped teeth might be used. Plain carbon steel or alloy steel may be utilized for high-strength gears. To achieve a good balance of toughness and tooth hardness, steel gears are frequently heat-treated. Worm gears are commonly made of phosphorus bronze to decrease wear, which would be extreme with steel or cast iron.

## ***2.1 Geometric Modelling and Analysis of Gear Tooth***

The bending stresses of the gear tooth have been reduced in this study by either utilizing an asymmetric tooth or modifying the root fillet radius of the tooth. The next two sections go over these two options in detail.

### **2.1.1 To Minimize the Bending Stress in the Gear Tooth, the Asymmetric Gear Tooth is Considered as the First Alternative Design**

The following are the specifications for a single reduction spur gear.

- Gear ratio = 10:1,
- Centre to centre distance = 660 mm,
- No. of teeth on gear = 150,
- Module (m) = 8 mm,
- Pitch circle diameter = 1200 mm,
- Addendum = Module (m),
- Dedendum = 1.25 m,
- Tooth thickness = 1.5708 m,
- Minimum clearance = 0.25 m,
- Fillet radius = 0.4 m.

The values of the theoretical bending stresses and percentage reduction in bending stresses are calculated for various angles of pressure on the drive side and are given in Table 1.

#### **Input parameters for analysis of gear tooth using in a software package**

Thickness of gear = 40 mm, Force applied ( $W$ ) = 50 N.

Properties of material:  $E = 3e6$ ,  $\mu = 0.24$ .

**Table 1** Percentage reduction in bending stress

S. No.	Pressure angle ( $\phi$ ) in degrees	Stress ( $\sigma$ ) in kgf/mm <sup>2</sup>	% reduction in $\sigma_b$ (%)
1	20–20	1.091	–
2	20–25	0.975	10.5
3	20–30	0.874	20.18
4	20–35	0.814	25.32
5	20–40	0.756	31.19
6	20–45	0.629	42.29

Figure 1 shows the involute profile of the gear tooth for the selected pressure angles of 20–20°, 20–25°, 20–30°, 20–35°, 20–40° and 20–45° respectively.

For comparison, the bending stresses obtained from ANSYS software along with theoretical values are given in Table 2.

Figure 2 shows the distribution of bending stresses in the gear tooth with respective numerical values for the pressure angles of 20–20°, 20–25°, 20–30°, 20–35°, 20–40° and 20–45° respectively.

The bending stresses are found to be decreasing from 1.091 to 0.629 kgf/mm<sup>2</sup> as the pressure angle on the drive side increased from 20° to 45°. Further, to investigate the bending stresses in the tooth, a modified root fillet radius is also considered. This is discussed in the following section in detail.

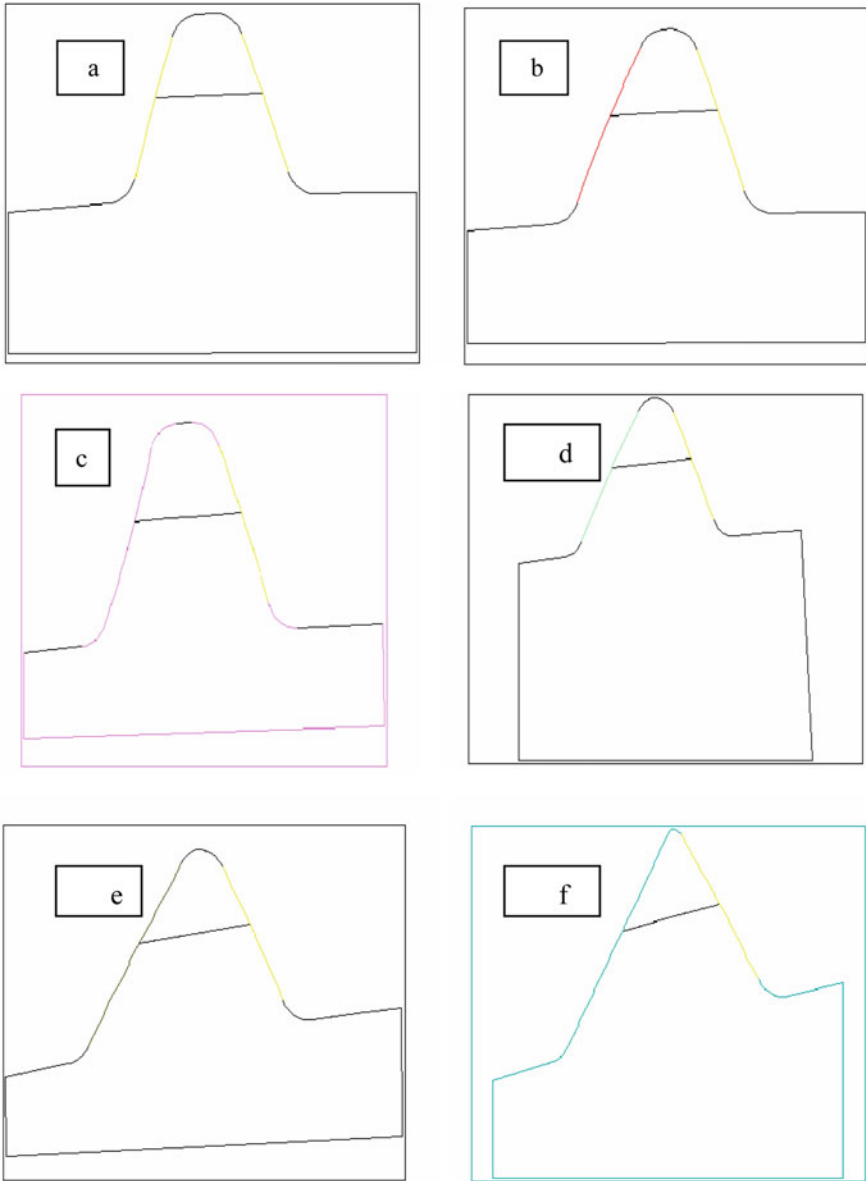
### 2.1.2 Minimization of Bending Stress in the Gear Tooth by Modifying the Root Fillet

It is proposed to study the effects of the bending stresses concerning modified fillet radii. Keeping all parameters of alternative 1 except the fillet radii, an attempt is made to explore the possibility of obtaining better bending stresses by a suitable curve fitting method. (Best curve fit method).

The concept is given below:

Bending stress minimization is achieved by designing a fillet profile that has a low bending stress concentration and meets specific requirements (e.g., manufacturability). This problem can be solved in a variety of ways. They are based on a curve-fitting technique in which the trochoid fillet profile, which is common in rack or mating gear generating methods, is substituted by an ellipsis, parabola, chain line, or other curves to decrease bending stress.

The trace of the mated gear tooth is the initial fillet profile. This profile is the top-level boundary that limits the optimization search area to avoid interfering with the mating gear. The start and last fillet positions on the form diameter circle cannot be changed throughout the optimization process. In the random search approach, the fillet nodes (save the first and last) are relocated along the beams that pass between the fillet centre and the initial fillet profile nodes. The fillet's centre corresponds to the centre of the best-fitting circular. For each new fillet point combination, the bending



**Fig. 1** Involute profile of gear teeth when pressure angles on coast side and drive side are **a** 20–20° **b** 20–25° **c** 20–30° **d** 20–35° **e** 20–40° **f** 20–45° respectively

**Table 2** Comparison of theoretical and ANSYS software calculated stresses

S. No.	Pressure angle ( $\emptyset$ ) in degrees	Theoretical stress, $\sigma_{\text{Bending}}$ (kgf/mm <sup>2</sup> )	Stress has shown by ANSYS (kgf/mm <sup>2</sup> )
1	20–20	1.290	1.090
2	20–25	1.117	0.975
3	20–30	1.027	0.875
4	20–35	0.904	0.814
5	20–40	0.762	0.756
6	20–45	0.638	0.629

stresses are calculated, and the figures and results for each of the adjusted fillet radii are as follows:

Table 3 shows the bending stresses at different root fillet shapes of 20° involute symmetric teeth.

Figures 3, 4, 5 and 6 show the involute profile of the gear tooth for the designed fillet, circular root fillet, and elliptical root fillets, respectively.

Figures 7, 8, 9 and 10 show the distribution of bending stresses in the gear tooth for different root fillet shapes.

The best-fit curves at root are tried to study the trend of the bending stresses right from the designed fillet radius to an elliptical<sup>2</sup> shape. The obtained stresses and the percentage reduction in bending stresses are given in Table 3.

### 3 Results and Discussions

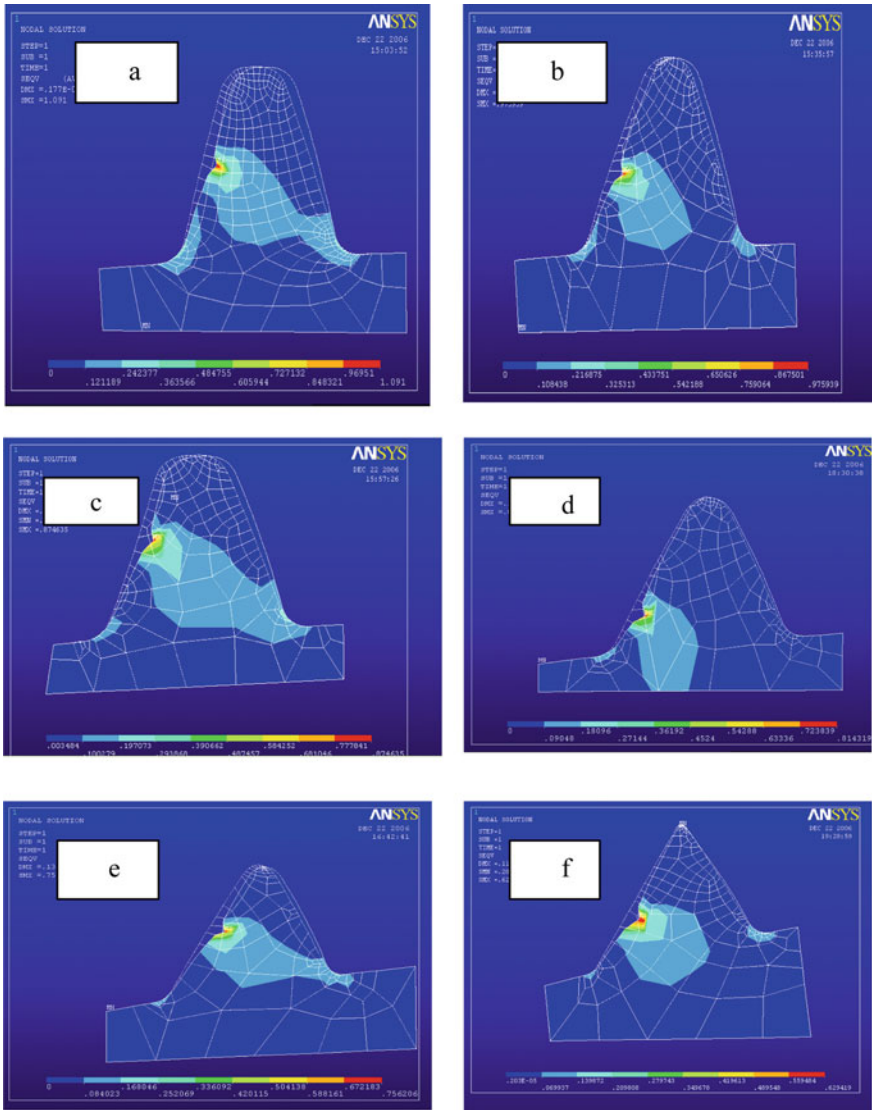
The goal of this study was to reduce the bending stresses in the gear wheel teeth by considering the asymmetric tooth and modifying the root fillet radius. The values for the above two cases are given in detail in the above section. The discussions about these results are given in the following sections.

While considering the asymmetric teeth to explore the possibility of reducing the bending stresses, the theoretical values of 20° pressure angle symmetric teeth are compared with asymmetric teeth of different pressure angles on the drive side, and the percentage reduction in bending stresses is also given in Table 1.

From the above table, it is found that as the pressure angle varied from 20–20° to 20–45°, the reduction in bending stresses was found to be 42.29%.

Table 2 shows the comparison of theoretical and ANSYS software calculated values. The decreasing trend of the theoretical values of bending stresses due to the use of asymmetric teeth is further confirmed by using ANSYS software.

While considering the modification of the root fillet radius according to the best-fit curve technique, the resulting bending stresses for various alternative best-fit curves are given in Table 3. From the table, it is found that as the root fillet profile is modified



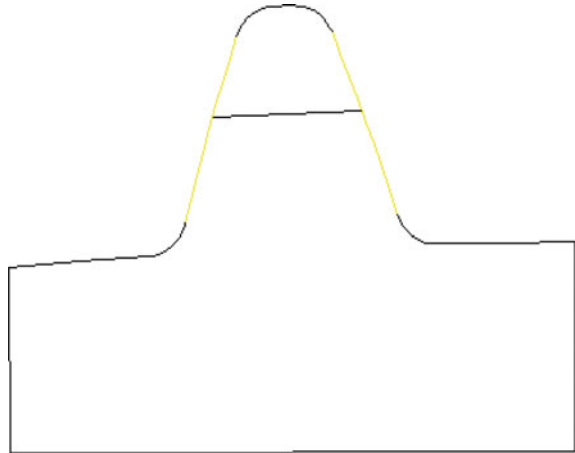
**Fig. 2** Analysis of Involute profile of gear teeth when pressure angles on coast side and drive side a 20–20° b 20–25° c 20–30° d 20–35° e 20–40° f 20–45° respectively

from the design fillet to an elliptical shape, the reduction in bending stresses is found to be 48.51%.

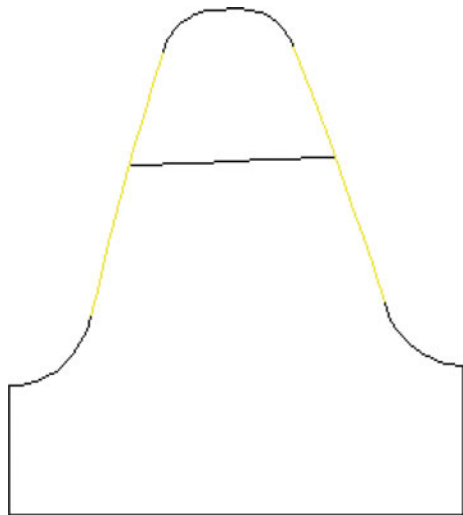
**Table 3** Bending stresses at different root fillet shapes of 20° involute symmetric teeth

S. No.	Root fillet shape	Obtained stress in kgf/mm <sup>2</sup>	% reduction in bending stress
1	Designed fillet	1.091	- -
2	Circular root	0.7287	33.20
3	Elliptical <sup>1</sup>	0.6728	38.33
4	Elliptical <sup>2</sup>	0.5617	48.51

**Fig. 3** Involute profile of symmetric gear tooth with designed fillet

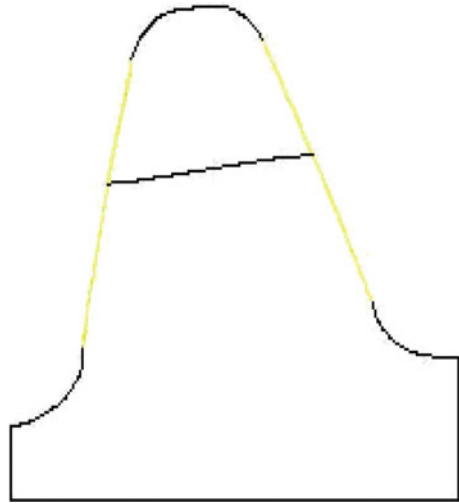


**Fig. 4** Involute profile of symmetric gear tooth with circular fillet

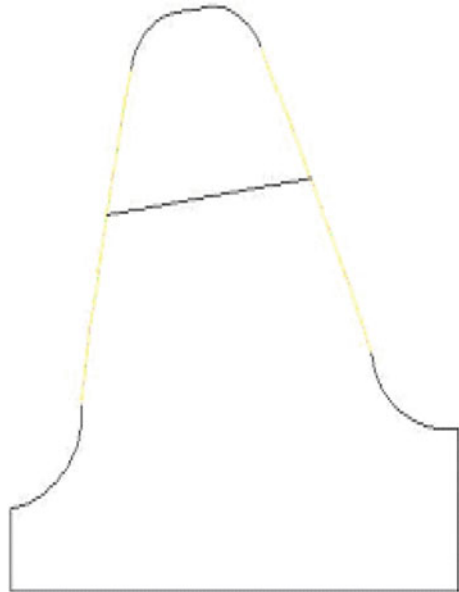




**Fig. 5** Involute profile of symmetric gear tooth with elliptical<sup>1</sup> root fillet



**Fig. 6** Involute profile of symmetric gear tooth with elliptical<sup>2</sup> root fillet



## 4 Conclusions

In the first suggested consideration, in which the asymmetric teeth are considered with an increased pressure angle on the drive side from 20–20° to 20–45°, it is noted that the bending stresses in the gear teeth are going to be reduced to a maximum value of 42.29%. As the pressure angle on the drive side is further increased beyond

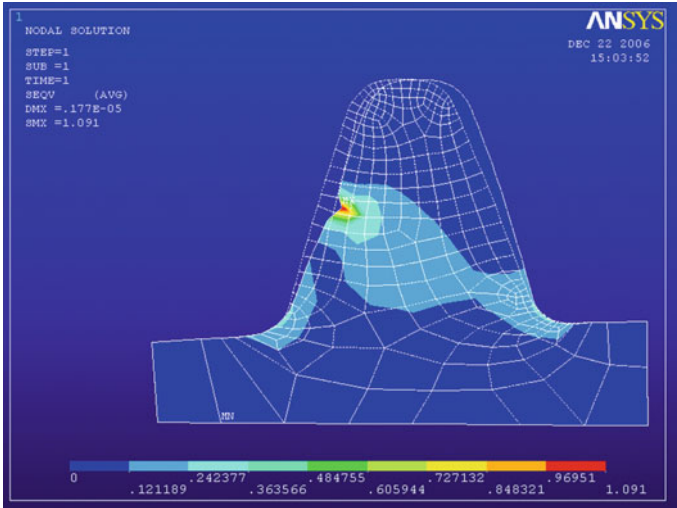


Fig. 7 Analysis of Involute profile of symmetric gear tooth with designed fillet

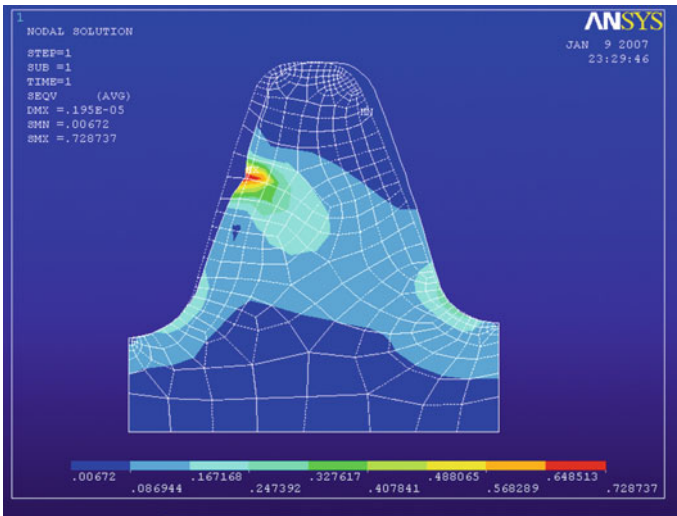


Fig. 8 Analysis of Involute profile of symmetric gear tooth with circular fillet

45°, the top land of the tooth is going to narrow and finally result in a single line because of the intersection of the two involute profiles on either side of the tooth.

In the second consideration, it is observed that the bending stresses are reduced by up to 48.51% for the modified fillet radius employing an elliptical profile.

From the above two, it is found that modifying the root fillet radius is the better option to employ in the manufacture of gears for reduced bending stresses. Longer

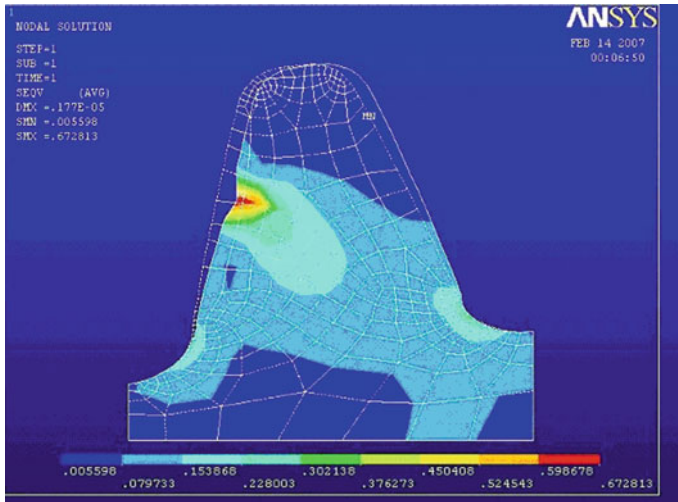


Fig. 9 Analysis of Involute profile of symmetric gear tooth with elliptical<sup>1</sup> root fillet

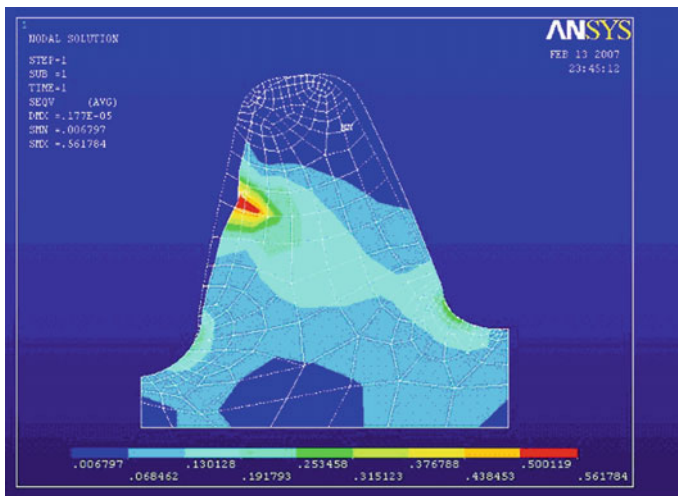


Fig. 10 Analysis of Involute profile of symmetric gear tooth with elliptical<sup>2</sup> root fillet

life, reduction in size and weight, application with a higher load, reduction of noise and vibration, cost reduction, and increased efficiency are all benefits of bending stress reduction.

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