

# Imperceptible Digital Image Watermarking Based on Discrete Wavelet Transform and Schur Decomposition



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**Abstract** The excessive use of multimedia content in the past decades led to threats of illegal copying and duplication of digital contents. This paper proposes one of the solutions to this problem based on an image watermarking technique using Discrete Wavelet Transform (DWT) and Schur decomposition. The Schur decomposition takes a lesser number of computations than singular value decomposition, which makes it suitable for the decomposition of the image. In the proposed algorithm, the watermark is embedded by manipulating the Schur coefficients of the LL subband of the cover image with the Schur vectors of the watermark. In the proposed work, the properties of both DWT and Schur decomposition are combined. The proposed technique performs better against, rotation, histogram equalization attacks, and various filtering attacks. The performance of the proposed technique is evaluated using PSNR and SSIM, which shows better visual imperceptibility against various attacks. The experimental result of the proposed scheme is also compared with other prevailing techniques, and the proposed method is found more effective among the other techniques.

**Keywords** Image watermarking · Schur decomposition · Wavelet · Imperceptibility · Histogram equalization

## 1 Introduction

The excessive use of multimedia content in the past decades led to threat of illegal copying and duplication of digital content. In the recent multimedia age, the sharing of multimedia content or information like images, videos, and data with another person is widespread. Current advances in transmission invention, distribution, and

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process have created new occasions for the unlawful modification, duplication, and spreading of digital content. In recent years, growth in digital transmission and within the cyber world has been incredibly fast, enhancing the requirement of confidentiality and authenticity of multimedia data. So digital watermarking techniques are used to solve the problem of modern digital content transmission. Digital image watermarking is defined as a scheme for embedding a watermark logo or image into the cover image using any key which will be extracted from watermarked image for copyright protection and authentication purpose [1]. In watermarking scheme, watermark can be visible or invisible. On the basis of information needed at the time of the extraction process, the watermarking techniques can be classified into three types: blind, semi-blind, and non-blind watermarking schemes [2]. In the blind types of techniques, there is no requirement of cover image only secret key is required, whereas in non-blind techniques, the cover image with the secret key both are required during extraction process. In semi-blind type of schemes, watermark bit sequence and secret key both are required [3]. Watermarking can be performed in spatial domain and transform domain. In spatial domain techniques, pixel values of cover image are modified to embed watermark, whereas in transform domain techniques, the cover image is transformed into the frequency domain and then watermark is embedded [4]. In hybrid image watermarking techniques, watermark information is embedded in both spatial domain and transform domain. The transform domain schemes have many advantages like better imperceptibility and robustness and higher payload capacity, whereas spatial domain techniques have advantages like lesser complexity and lesser cost. Wavelet-based watermarking schemes show wider scope as it is a time–frequency approach utilizes multi-resolution property [5].

Mohan et al. [6] developed a Schur-based technique that employs Schur triangular matrix to embed watermark. Liu et al. [7] suggested technique based on combination of Contourlet transform and Schur decomposition. Su et al. [8] investigated a technique based on Schur decomposition and uses dual color image watermark that employ correlation between Schur matrixes to embed the watermark. Ahmad [9] proposes a non-invertible technique in which cover image is converted into the blocks then subjected to Schur decomposition, and PN-sequences are generated to embed as watermark. Su et al. [10] suggested a primitive Schur decomposition based color image watermarking technique in which the watermark is embedded by modifying the relationship of two elements in the first column coefficient of the unitary matrix derived by Schur decomposition. This paper presents DWT and Schur decomposition-based watermarking scheme which provide better imperceptibility and robustness as well as it have lesser computational time.

This paper is arranged in following manner: In Sect. 2, the theory of DWT is explained, whereas in Sect. 3, the theory of Schur decomposition is presented, and in Sect. 4, proposed watermarking techniques are described. Section 5 contains the simulation result and finally conclusions are given in Sect. 6.

**Fig. 1** DWT decomposition of image

LL2	LH2	LH1
HL2	HH2	
HL1		HH1

### 2 Discrete Wavelet Transform (DWT)

DWT decomposed image into four subbands, in each decomposition level which provides multi-resolution property. DWT uses based on wavelets as basis function, whereas DFT and DCT have everlasting sinusoidal or complex exponentials. Wavelets are generated by dilation and translation of mother wavelet. The DWT is mainly based on subband filtering. First original image is transformed into the four different subbands like LL1, LH1, HL1, and HH1. LL1 contains lower frequency components so it has maximum energy of image [3]. High frequency components LH1, HL1, HH1 hold the detailed information. It can be further divided into the subband by applying the second level wavelet decomposition into the LL1 subband which further provides four subbands which are LL2, LH2, HL2, and HH2. The process can be repeated to decompose image to the nth level subbands to get suitable subbands for watermarking where  $n$  is the level of decomposition. Similarly by using Inverse DWT, original image can be recovered. DWT provides suitable subband for watermarking according to human visual system. So watermark can be embedded into HL or LH subbands as they are not visible to human visible system [11, 12]. The DWT has limitation of shift invariance and phase sensitivity (Fig. 1).

### 3 Schur Decomposition

The Schur decomposition [9, 13, 14] provides Eigen values of pixel block easily. It can be defined for any matrix  $M$  as

$$M = U \times A \times U^T \tag{1}$$

where  $M \in R^{n \times n}$  and  $A$  is upper triangular matrix and  $U$  is a unitary matrix [8, 9].

The Schur decomposition provides two matrices unitary and upper triangular matrix. The Schur decomposition can be applied on square matrices. The Schur decomposition is based on theory of unitary transformation, and it is intermediate

step of singular value decomposition (SVD) due to which it has lesser the computation time than SVD. The Schur decomposition diagonal matrices are same for  $A$  and  $A^T$  as the Eigen value of both are same. The Schur vectors are invariants to scaling. The Schur decomposition is mainly based on unitary transformation which makes so it stable. These properties make the Schur decomposition suitable for its application in image decomposition and in image watermarking as well. The watermarking information can be embedding into both the matrices either unitary matrix or upper triangular matrix but generally upper triangular matrix is preferred.

## 4 Proposed Watermarking Technique

The proposed watermarking algorithm is discussed in this section. In this work, a cover image as ( $I$ ) of size  $M \times N$  and binary watermark image as ( $I_w$ ) of size  $m \times n$  is considered.

### 4.1 Embedding Algorithm

The watermark is embedded based on procedure whose is shown in Fig. 2:

- i. Perform DWT on cover image to decompose image into four subbands LL1, LH1, HL1, and HH1.
- ii. The Schur decomposition is obtained by applying the Schur transform into the LL1 subband.

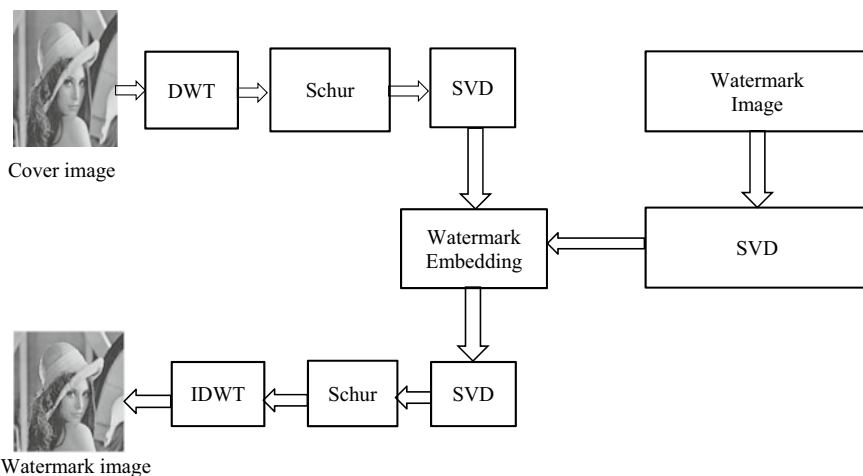


Fig. 2 Block diagram representation of proposed schemes

- iii. The upper triangular matrix of LL subband of cover image is further subjected to singular values decomposition (SVD) which gives three matrices.
- iv. The watermark image is also subjected to SVD to obtain the singular values (SV) matrices.
- v. The watermark is embedded by manipulating the SVs of cover image to SVs of watermark image

$$I_2(i, j) = I_1(i, j) + kI_w(i, j) \quad (2)$$

where  $k$  is gain factor.

- vi. Inverse process of SVD and Schur transform is applied to recover the modified LL subband.
- vii. The watermarked image is obtained by applying inverse DWT to the modified LL subband.

## 4.2 Extraction Process

The extraction procedure of binary watermark image is as following steps

- i. Apply DWT to obtain watermark image which results to four subband of watermarked image.
- ii. Perform Schur transform to LL subband to obtain the Schur vector matrix.
- iii. In next step, SVD is applied into the upper triangular matrix of Schur decomposition.
- iv. Extract transformed watermark image coefficients from watermarked image by applying

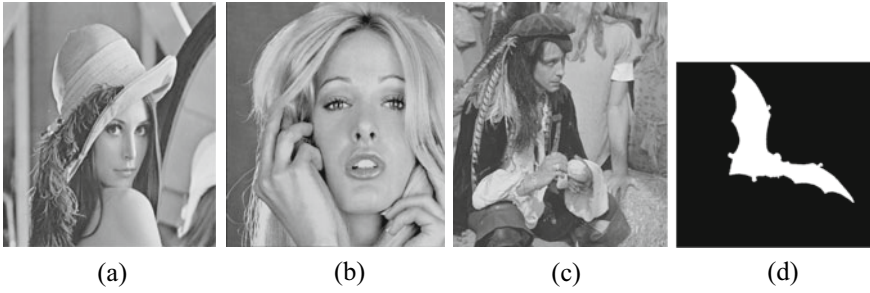
$$I_w(i, j) = (I_2(i, j) - I_1(i, j))/k \quad (3)$$

- v. Apply SVD to extract the watermark image.

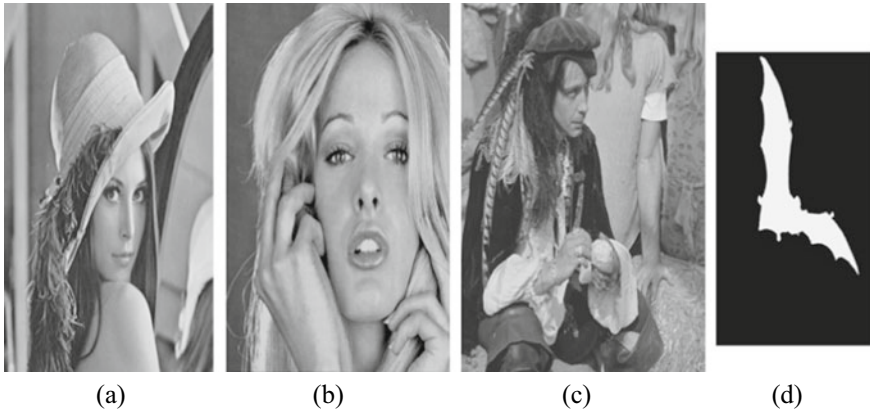
## 5 Simulation Result

The effectiveness of proposed scheme is tested using various performance parameters for three images which are lena, girl, and pirates of as cover image of size  $512 \times 512$  and binary watermark of size  $256 \times 256$  is taken as watermark image shown in Fig. 3. Watermarked image obtained after applying the proposed algorithm is shown in Fig. 4.

The performance of the proposed algorithm is evaluated by two parameters which are as follows.



**Fig. 3** Cover image **a** lena, **b** girl, **c** pirates, and **d** binary watermark



**Fig. 4** Watermarked images **a** lena, **b** pirates, **c** girl, and **d** extracted binary watermark

### 5.1 Peak Signal to Noise Ratio (PSNR)

It is the parameter to evaluate the superiority of the watermarked image. It is ratio of peak value of signal power to noise power. PSNR used for evaluation of watermarking algorithm which is defined as:

$$\text{PSNR} = 10 * \log \frac{255^2}{\text{MSE}} \quad (4)$$

where mean square error (MSE) for gray image is given by:

$$\text{MSE} = \frac{1}{M \times N} \sum_{x=1}^M \sum_{y=1}^N [I(x, y) - W(x, y)]^2 \quad (5)$$

## 5.2 Structural Similarity Index Measures (SSIM)

SSIM defines the structural similarity between the cover image and watermarked image. SSIM nearer to 1 shows that structure of cover image and watermarked image are same. SSIM provides knowledge about the imperceptibility of image which is an important characteristic of watermarking algorithm.

$$SSIM(I, W_c) = \frac{\sum_{x=0}^M \sum_{y=0}^N [C(x, y) W_c(x, y)]}{\sum_{x=0}^M \sum_{y=0}^N |C(x, y)|^2} \quad (6)$$

The SSIM and PSNR evaluated after applying various types of attacks.

**Noise attack:** The noise affected watermarked image is obtained by applying salt and pepper type noise with variance 0.01.

**Histogram equalization:** Histogram equalization modifies the intensity value of image which used to improve the contrast of the image.

**Filtering attack:** Various types of filtering attacks like Gaussian, median, and average filtering are performed.

**Geometrical attack:** There exist various types of geometrical attacks like rotation, translation, etc. Performance of proposed technique is tested against rotation attack by rotation of watermarked image.

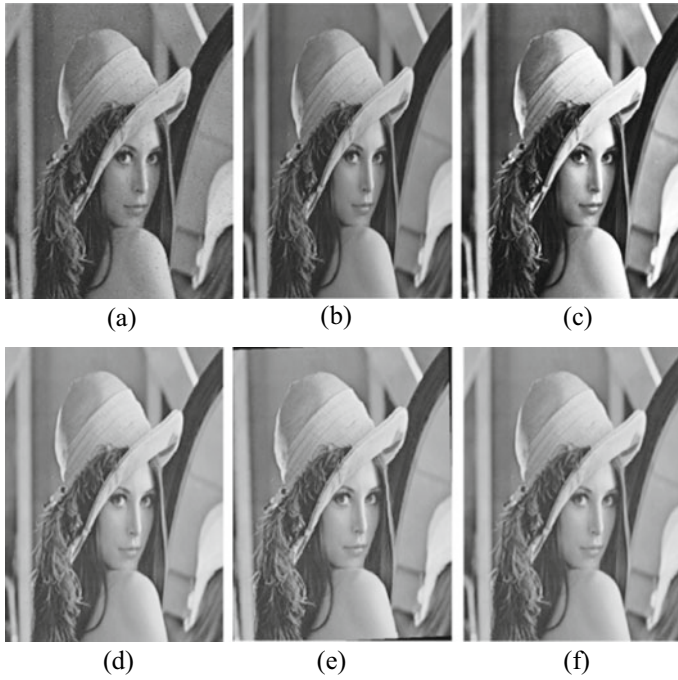
In Fig. 5, watermarked image obtained after various attacks are shown. In Fig. 6, PSNR value is plotted against gain  $k$ . Various values of parameter obtained after applying various attacks are given in Table 1.

It is clearly visible from Fig. 6, there exists an inverse relationship between the value of gain factor ( $k$ ) and PSNR value. It shows the imperceptibility of watermarking scheme decreases with the increase value of  $k$ . In proposed scheme, PSNR value is greater than 30 dB for all value of  $k$ , which is more than the acceptable value of PSNR for any watermarking schemes.

Figure 7 shows the imperceptibility of proposed scheme which is better as compared to other method based on Schur decomposition proposed in [15–17].

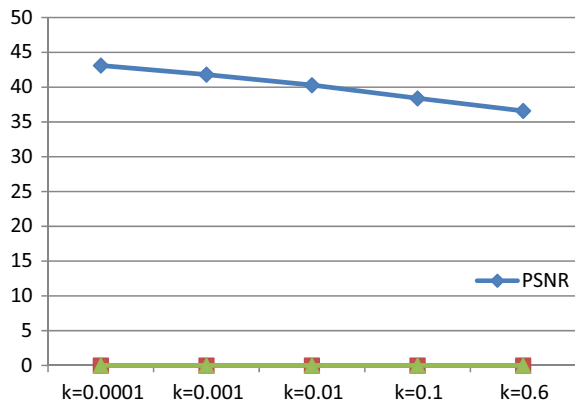
## 6 Conclusion

Image watermarking scheme which is based on DWT and Schur decomposition is proposed in this paper. Binary watermark is embedded in cover image, after applying wavelet transform in LL subband. The proposed scheme shows good visual imperceptibility against various attacks. The proposed scheme shows very good PSNR and SSIM values. This algorithm can be further modified using RDWT as it has shift invariance property. Furthermore, the value of gain factor  $k$  can be optimized.



**Fig. 5** Watermarked image after various attacks: **a** salt and pepper noise (0.01), **b** Gaussian noise (0.01), **c** histogram attack equalization, **d** median filter ( $3 \times 3$ ), **e** average filtering, and **f** rotation ( $2^\circ$ )

**Fig. 6** PSNR value on various values of k (gain)

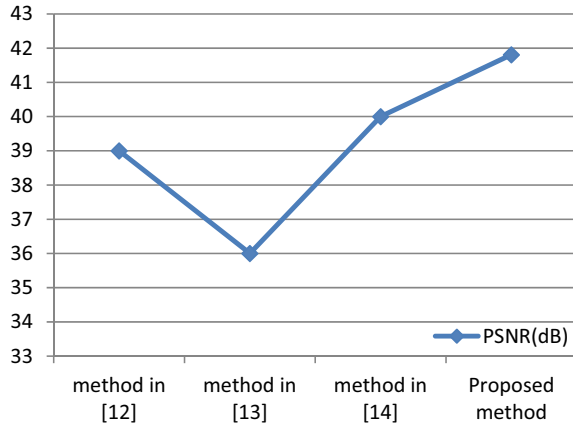




**Table 1** PSNR a value and structure similarity index measure (SSIM) of various watermarked image after applying attacks

Attack	PSNR(in dB)	SSIM
Salt and pepper noise	28.06	0.8122
Median filtering (3 × 3)	30.99	0.9934
Gaussian filtering (3 × 3)	29.597	0.8602
Average filtering	30.57	0.9412
Histogram attack equalization	25.64	0.8546

**Fig. 7** Comparison of PSNR values of proposed scheme with Schur-based algorithm presented in [15–17]



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