Activity Time Variations and Its Influence on Realization of Different Critical Paths in a PERT Network: An Empirical Study Using Simulations

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Abstract. Despite the simple and extensively used program evaluation and review technique (PERT), a project management tool for estimating project duration poses many limitations. PERT only considers the overly estimated critical path as the lifeline of the project and neglects the variance of other paths, which later affect the project's timely completion. This paper uses a simulation-based approach to illustrate that a project network contains multiple critical paths that depend upon the network's variability. A real construction project scenario augmented with five different cases. The outcome shows many instances of other critical paths along with the dominant critical path in simulation.

Keywords: Scheduling · PERT · Project delays · Stochastic network analysis · Project scheduling · Project management

1 Introduction

Unlike the manufacturing industry, the construction industry suffers project delays evident in literature [1]. In India, as per the report published by the Ministry of Statistics and Programme Implementation [2] on 343 construction projects valuing more than $135 million, cost overruns are averaged around 12.49%, along with time overrun ranging from 1 month to 61 months [3]. Similarly, for those 1701 construction projects valued at more than $20 million, the cost overrun is about 19% and an average time overrun of around 39 months [2]. This suggests the need to better understand bottlenecks and lacuna in the current construction project planning and control approaches. Robust planning facilitates better execution and helps stakeholders to achieve timely project completion.

The critical path method (CPM) and the program evaluation and review technique (PERT) are commonly used for planning and monitoring construction projects for the last few decades. The CPM approach is used when the duration of activities is deterministic,

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while PERT tackles variability in activity durations during project planning. Despite both methods focuses on the critical path and its continuous monitoring, but timely completion of projects continues to be an enigma. Thus, the impact of variability in the activity durations on the critical path needs to be researched further.

Published research has highlighted multiple drawbacks of the PERT and suggested certain improvements to increase its planning efficiency. Still, all that research follows the notion of a single critical path (mostly using the expected activity duration estimate) as the critical element of the project network. This work presents an empirical simulation-based study that indicates the influence of variability in the activity durations vis-a-vis the notion of a single critical path. Variability in activity durations is modelled using appropriate probability distributions. Random variates of the activity durations are sampled using the probability distributions to achieve large number of multiple instantiations of the project network. These instances are solved to identify the critical path and estimate the existence of other critical paths and their relation to the expected critical path. We found that there are potentially many instances when the other paths become critical with respect to the expected critical path. Hence, focusing on the expected critical path while ignoring the dominance of other possible critical paths could hinder the timely completion of the project.

2 Modifications to PERT

PERT, which is considered as robust project planning and control method, fails to prove in construction projects as most projects struggle to achieve timely completion. Farnum & Stanton, 1987 [4] questioned the efficacy of PERT, especially for activity times following beta distribution. Golenko-Ginzburg suggested a correction factor for calculating the mean and variance of each activity [5, 6]. Simulation studies by Badiru [7, 8] demonstrated that the beta distribution is not always effective.

Researchers mainly used three approaches viz., (i) modifying the probability distribution function (PDF), (ii) stochastic simulation, and (iii) using optimization algorithms to enhance PERT results. Mohan et al. proposed using lognormal distribution instead of beta distribution that required only two inputs viz., (i) most likely times with (ii) either optimistic or pessimistic times, which works better in many cases [9]. Similarly, Hajdu and Bokor (2014) conducted another simulation study used uniform and triangular distributions and compared with the efficiency of beta distribution [10, 11]. They showed that even $\pm 10\%$ deviations can cause significant difference for a large project. Similarly, other researches using compound Poisson [12], Weibull [13], Beta rectangular [14], tilted beta [15], compared with beta distribution to explain its shortcoming.

Most research [16] continue to focus on the three PERT time estimates of activities along with the expected time estimates. Also, most work focus on the conventional critical path obtained using expected times for project management, which fails to ensure timely project completion in most cases. Typically, PERT only focus on variance of the critical path and ignores variance of all other paths, thereby resulting in an unrealistic estimate of probability of completion. For example, if the expected mean of the critical path is equal to the X, the probability of completion of any project at is 50% (Refer Eq. 1). In this research, we propose a simulation-based approach that empirically illustrates that even with a small amount of variability introduced in the project activity times, multiple paths can become critical. Then, many random instances of the project network are simulated to estimate the share of dominance of such critical paths.

$$Z = \frac{X - \mu}{\sigma} \quad (1)$$

where Z is the standard normal variate, μ is the mean, and σ is the standard deviation of the critical path.

3 Research Methodology

The research methodology flowchart is shown in Fig. 1. To illustrate the hypothesis, we used an example problem from (Pinedo, 2000) as shown in Fig. 2, where activity durations are taken in days. The project network has 14 activities with a balanced precedence relationship. The activity time estimates used in the example problem was scaled to 10 times so that realistic construction project timelines can be mimicked. After scaling up the deterministic time (t) of the activities, variability is introduced by using five different uniform distributions, viz., (i) UNIF ± 10% (V1), (ii) ± 20% (V2), (iii) ± 30% (V3), (iv) ± 40% (V4), and (v) ± 50% (V5) which is the range of t_{min} and t_{max}, as shown in Table 1. Then, for each variability instance, random project durations are sampled to achieve 10,000 random instances of the project network, for the same precedence constraints. Each instance is solved to identify the critical path and associated maximum completion time (C_{max}). Finally, the dominance of each critical path is quantified by estimating the proportion of times within the 10,000 simulation instances the particular path became the critical path.

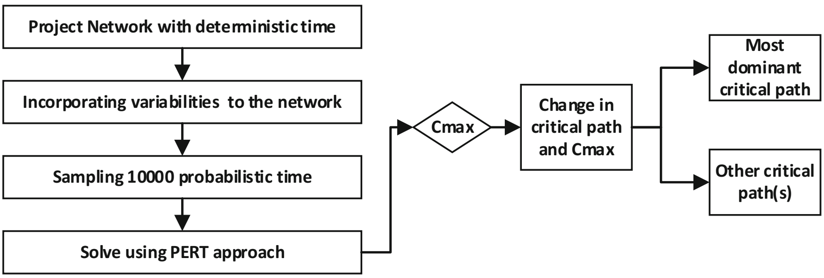

Fig. 1. Proposed research methodology

Now using these three-point estimates, the expected time ($\overline{x}$), and standard deviation (σ_x) for each activity is calculated to find the maximum completion time of critical path (C_{max}) of the network using PERT approach. The path duration, path variance is also calculated for each path. With the increase in variability, it was observed that other paths also started to become critical in addition to the dominant critical path. A MATLAB code is developed for simulating the 10,000 random instances of the project network for each variability setting.

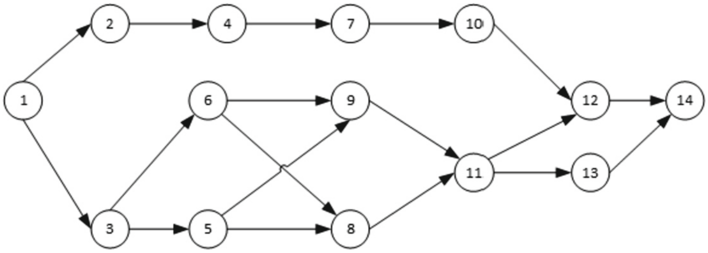

Fig. 2. Project network used in the study

Table 1. Variability incorporation in deterministic time using the uniform distribution

A	t	10t	V1		V2		V3		V4		V5	
			±10%		±20%		±30%		±40%		±50%	
1	5	50	45	55	40	60	35	65	30	70	25	75
2	6	60	54	66	48	72	42	78	36	84	30	90
3	9	90	81	99	72	108	63	117	54	126	45	135
4	12	120	108	132	96	144	84	156	72	168	60	180
5	7	70	63	77	56	84	49	91	42	98	35	105
6	12	120	108	132	96	144	84	156	72	168	60	180
7	10	100	90	110	80	120	70	130	60	140	50	150
8	6	60	54	66	48	72	42	78	36	84	30	90
9	10	100	90	110	80	120	70	130	60	140	50	150
10	9	90	81	99	72	108	63	117	54	126	45	135
11	7	70	63	77	56	84	49	91	42	98	35	105
12	8	80	72	88	64	96	56	104	48	112	40	120
13	7	70	63	77	56	84	49	91	42	98	35	105
14	5	50	45	55	40	60	35	65	30	70	25	75

4 Result and Discussion

The five cases of variability of the same network, V1, V2…. V5, were used to test the proposed hypothesis. All possible nine paths of the project network are enumerated in Table 2. The precedence is kept constant throughout the experiment, which helps to analyse the effect of variability in project completion and critical path.

The study clearly indicates that the project network can have different critical paths, depending upon the sampled activity durations, which are in turn influenced by the variance associated with the duration of activities. Figure 3 compares the critical paths and their frequencies observed throughout the simulations. The horizontal axis of each graph has the nine possible paths, and the vertical axis indicates the number of simulations out of 10,000 the specific path became critical path. Even when a small amount of variability of 10% in activity times (V1) is introduced to the project network; two more paths (P1 and P9) in addition to P5 in the project network shows up critical 18.56% and 0.44% of time respectively, as illustrated in Fig. 3a.

Table 2. Shows the path in the project network.

S.No.	Paths in the network	Path name
1	1-2-4-7-10-12-14	P1
2	1-3-5-8-11-12-14	P2
3	1-3-6-8-11-12-14	P3
4	1-3-5-9-11-12-14	P4
5	1-3-6-9-11-12-14	P5
6	1-3-5-8-11-13-14	P6
7	1-3-6-8-11-13-14	P7
8	1-3-5-9-11-13-14	P8
9	1-3-6-9-11-13-14	P9

Considering all sub-figures in Fig. 3, P5 is the dominant critical path by around 81% and 58.63% for low variability instances, i.e., at 10% (V1) and 20% (V2) respectively. Also, two more paths, viz., P1, and P9 were also critical paths for the remaining proportions with P1 being more dominant among the two paths. The notable decrease in the dominance of the critical path P5 was observed when variabilities V2 and V3 are introduced, which is usual in the construction business. At the same time, the P1 and P9 frequencies increased significantly as shown in Fig. 3b and Fig. 3c. The frequencies of P1 and P5 become almost similar at 39% when the network is simulated with 40% variability (V4) as illustrated in Fig. 3d. Also, P3, P7, and P8 also became critical paths along with P1, P5, and P9 in a few simulated instances. The frequencies of P3, P7, and P8 further increased by 1.02%, 0.43%, and 0.38% respectively with 50% variability introduced in activity durations (V5) and the most dominant critical path switched from P5 to P1 (Fig. 3e). These simulation results indicate that when the construction project activities have higher variability in their associated activity durations, the expected critical path may not be the ideal baseline for project scheduling. Since the critical path(s) are heavily dependent on the activity duration, the path may get changed during the execution as activity times changes due to various unforeseen reasons; considering only one critical path as the baseline for project planning reduces the robustness of the approach. This indicates the need of a more robust baseline for project planning that also incorporates the probability of completion concept.

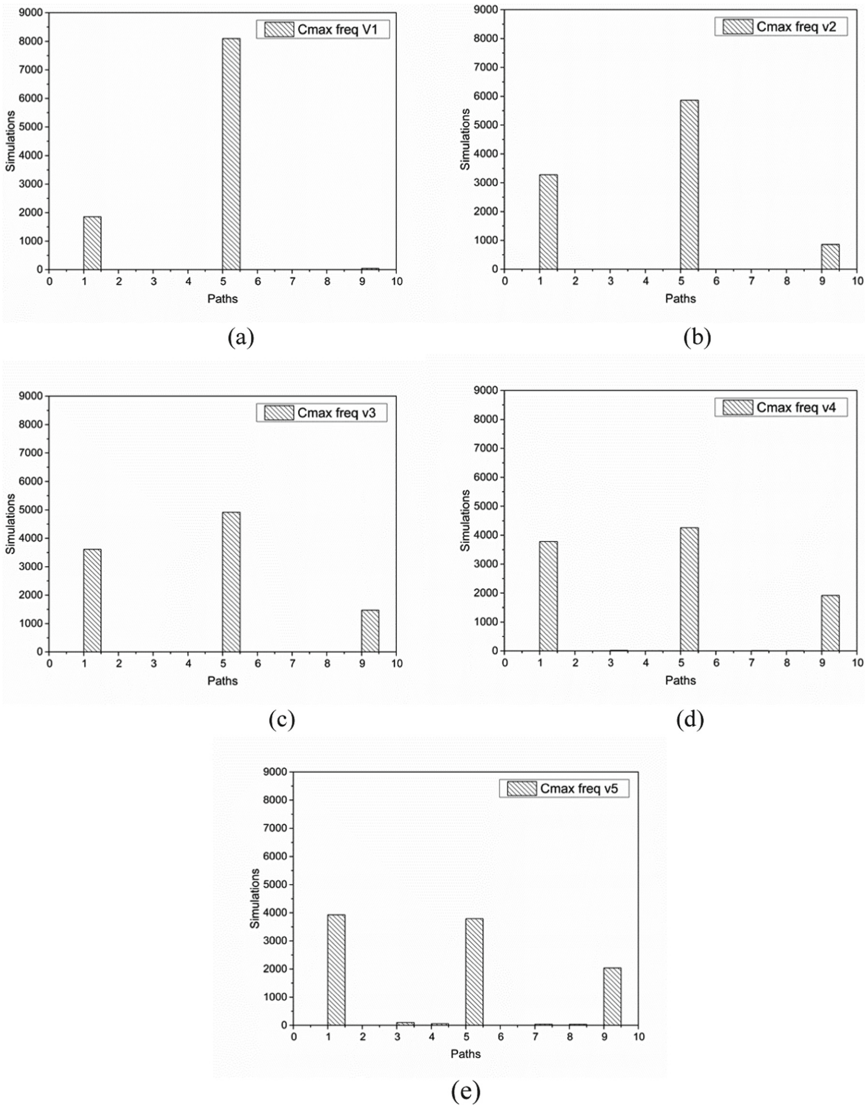

Fig. 3. Changes in the critical path when subjected to different variability

5 Conclusion

This study aimed to address the lacuna associated with the notion of a single critical path approach based on the expected time values in PERT, which is traditionally used for planning and control of construction projects. Through the literature review, rationale and supportive evidence were provided on the PERT's significant weakness and demonstrate the applicability of the present approach on project networks using an empirical simulation study. The variability in the activity durations can significantly influence the dominance of critical paths in a project network is demonstrated through simulation experiments. In the case of the dominant critical path and other critical paths, the PERT only accounts for variability of the critical path but neglect the variance of the other paths, which may get critical if variability increases due to improper monitoring and resources allocation. Finally, the approach numerically evaluated 10,000 instances of all five cases of variability to show its efficacy in scenarios like scheduled networks of real construction projects. The results indicate that with a slight to moderate increase in the variability in the activity durations of the network, the concept of dominant critical path based on expected activity duration becomes invalid.

References

1. Al-Momani, A.H.: Construction delay: a quantitative analysis. Int. J. Proj. Manag. **18**(1), 51–9 (2000).http://www.sciencedirect.com/science/article/B6V9V-40Y4JMT-7/2/903a5c429a333f7d212c1ade748aeebb
2. MoSPI G of I. FLASH REPORT ON (Rs . 150 crore and above) Government of India Ministry of Statistics and Programme Implementation Infrastructure and Project Monitoring Division (2019)
3. MoSPI G of I. 82 nd REPORT ON MEGA PROJECTS (Rs . 1000 Crore and above) March 2016 Government of India Ministry of Statistics and Programme Implementation Infrastructure and Project Monitoring Division Sardar Patel Bhavan , Sansad Marg New Delhi-110001. (2016)
4. Farnum, N.R., Stanton, L.W.: Some results concerning the estimation of beta distrubution parameters in PERT. J. Oper. Res. Soc. **30**, 287–90 (1987). http://www.jstor.org/stable/3009290
5. Golenko-Ginzburg, D.: On the distribution of activity time in pert. J. Oper. Res. Soc. **39**(8), 767–771 (1988)
6. Golenko-Ginzburg, D.: A new approach to the activity-time distribution in pert. J. Oper. Res. Soc. **40**(4), 389–393 (1989)
7. Badiru, A.B.: STARC 2.0: an improved pert network simulation tool. Comput Ind Eng. **20**(3):389–400 (1991)
8. Badiru, A.B.: A simulation approach to PERT network analysis. SIMULATION **57**(4), 245–255 (1991)
9. Mohan, S., Gopalakrishnan, M., Balasubramanian, H., Chandrashekar, A.: A lognormal approximation of activity duration in PERT using two time estimates. J. Oper. Res. Soc. **58**(6), 827–831 (2007)
10. Hajdu, M., Bokor, O.: The effects of different activity distributions on project duration in PERT Networks. Procedia - Soc. Behav. Sci. **2014**(119), 766–775 (November 2015)
11. Hassan, A., El-Rayes, K., Attalla, M.: Optimizing the scheduling of crew deployments in repetitive construction projects under uncertainty. Eng. Constr. Archit. Manag. 3, (2020)
12. Parks, W.H., Ramsing, K.D.: The use of the compound poisson in Pert. Manag. Sci. **15**(8), B-397-B-402 (1969)
13. Abdelkader, Y.H.: Evaluating project completion times when activity times are Weibull distributed. Eur. J. Oper. Res. **157**(3), 704–715 (2004)
14. Hahn, E.D.: Mixture densities for project management activity times: a robust approach to PERT. Eur. J. Oper. Res. **188**(2), 450–459 (2008)
15. Hahn, E.D., Del Mar López Martín, M.: Robust project management with the tilted beta distribution. Sort. **39**(2), 253–72 (2015)
16. Soroush, H.M.: The most critical path in a PERT network. J. Oper. Res. Soc. **45**, 287–300. (1994). http://www.jstor.org/stable/3009290