Prescribed-Time Multi-target Tracking Control for Second-Order Multi-agent Systems

Kun-Ting Xu[1], Ming-Feng Ge[1,2](✉), Teng-Fei Ding[1], Zhi-Wei Liu[3], Xiang Lu[4], and Jie Liu[5]

[1] School of Mechanical Engineering and Electronic Information, China University of Geosciences, Wuhan 430074, China
{1138469431,gemf,dingtf}@cug.edu.cn

[2] School of Automation, Central South University, Changsha 410083, China

[3] School of Artificial Intelligence and Automation, Huazhong University of Science and Technology, Wuhan 430074, China
zwliu@hust.edu.cn

[4] Key Laboratory of Material Chemistry for Energy Conversion and Storage, Huazhong University of Science and Technology, Ministry of Education, Wuhan 430074, China
luxiang@hust.edu.cn

[5] School of Civil and Hydraulic Engineering, Huazhong University of Science and Technology, Wuhan 430074, China
jieliu@hust.edu.cn

Abstract. This paper investigates the prescribed-time multi-target tracking problem for second-order multi-agent systems (MASs). By employing a time-relevant function as the control gain, a novel control algorithm is proposed to achieve multi-target tracking, where the convergence time is regardless of the changing of the initial condition. Especially, the multi-target tracking control can be achieved based on the assumption that the interaction directed graph has a spanning tree with acyclic partition. The sufficient conditions are established according to Lyapunov stability theory and mathematical induction. Finally, some simulation experiments are proposed to substantiate the presented algorithm.

Keywords: Multi-target tracking · Prescribed-time control · Second-order multi-agent systems (MASs)

1 Introduction

Collaboration control of multi-agent systems is one of the hottest topics in the control field due to the broadly satisfactory applications in formation control [1,2], containment control [3,4], and target tracking [5]. Target-tracking is an

Z. Ren et al. (Eds.): *Proceedings of 2021 5th Chinese Conference on Swarm Intelligence and Cooperative Control*, LNEE 934, pp. 86–94, 2023.
https://doi.org/10.1007/978-981-19-3998-3_9

active research area, which aims to activate multiple follower agents to track the target's trajectory and accomplish the assigned tasks simultaneously.

Specifically, according to the quantity of the tracking target, target-tracking can be divided into single-target tracking and multi-target tracking. Under a leader-follower framework, several single-target tracking problems of MASs have been addressed successfully [2,6]. The results focused on the single-target tracking can not handle with the multi-target tracking problem directly, which motivates researchers to achieve the control objective of multi-target tracking [7]. Different from the single-target tracking, multi-target tracking means that all the followers are divided into several groups and track the trajectories of the multiple targets respectively in complex operation environments. Besides, the difficulties of multi-target tracking lie in the complexity of the construct of the Lyapunov function. Therefore, a significative consideration of coordinated control for MASs lies in multi-target tracking.

It is worth mentioning that the existing research on multi-target tracking control can only be achieved in the asymptotic or finite-time manner. However, the convergence rate is also highly considered when evaluates the designed algorithms [7,8]. Early years, the control methods mainly contain the asymptotic control, finite-time control and fixed-time control [9,10]. None of them can obtain a precise settling time. Then, to make the settling time certainty, the prescribed-time control methods have been provided [11–13]. The prescribed-time stability has been highly considered on account of the performance of the user-defined settling time and the independent of initial conditions. Therefore, the combination of the prescribed-time stability and multi-target tracking becomes a significant but challenging problem.

Inspired by the aforementioned discussions, we propose a prescribed-time multi-target tracking algorithm for second-order MASs under a directed graph. The main contributions can be listed as follows.

1. Unlike the prescribed-time control of single-target tracking [14], the proposed algorithm is designed to achieve multi-target tracking under a leader-follower framework. The tracking targets are time-varying and unknown to the followers.
2. Unlike the existing references on achieving multi-target tracking in the asymptotic or finite-time manner, this is the first work on solving the prescribed-time multi-target tracking problem. The settling time can be prescribed and is unrelated to the initial condition.

2 Problem Formulation and Preliminaries

2.1 Graph Theory

A directed graph $G = \{V, E, A\}$ is introduced to depict the interaction of MASs, where $V = \{1, 2, \ldots, N\}$, $E \in V \times V$. The weighted adjacent matrix is defined as $A = [a_{ij}] \in R^{N \times N}$, and $a_{ij} > 0$ if $(i, j) \in E$, $a_{ij} = 0$ otherwise. The neighbor set of the ith agent is $N_i = \{j \in V \,|(i, j) \in E\}$. The Laplacian matrix is

$L = [l_{ij}] \in R^{N\times N}$, in which $l_{ii} = \sum_{j=1}^{N} a_{ij}, i = j$ and $l_{ij} = -a_{ij}$, $i \neq j$. Moreover, $B = \text{diag}(b_1, \ldots, b_N)$ is the pining matrix, and $b_i > 0$ if there is a directed path between the ith agent and its leader, $b_i = 0$ otherwise.

Consider the graph can be divided into k subgraphs $\{G_1, G_2, \ldots, G_k\}$, in which $\{V_1, V_2, \ldots, V_k\}$ are the corresponding sets of nodes. $V_1 = \{1, 2, \ldots, o_1\}$, $V_2 = \{o_1 + 1, o_1 + 2, \ldots, o_2\}, \ldots$, $V_k = \{o_{k-1} + 1, o_{k-1} + 2, \ldots, o_k\}$, $o_k = N$ and the number of the agents in each subgroup is defined as n_l, $\forall l \in \{1, 2, \ldots, k\}$.

Assumption 1. *For each subgraph, there is a spanning tree rooted in a leader node.*

Assumption 2. *The sets $\{V_1, V_2, \ldots, V_k\}$ are acyclic partition in directed graph G.*

Under Assumption 2, the Laplacian matrix can be redefined into the following form [16]

$$L = \begin{bmatrix} L_{11} & 0 & \ldots & 0 \\ L_{21} & L_{22} & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ L_{k1} & L_{k2} & \ldots & L_{kk} \end{bmatrix},$$

where L_l is the Laplacian matrix of G_l, and L_{ml} denotes the interaction between G_m and G_l, $\forall l, m \in \{1, 2, \ldots, k\}$.

2.2 Problem Formulation

The considered system is molded as second-order integrator,

$$\begin{cases} \dot{x}_i = v_i, \\ \dot{v}_i = u_i, \end{cases} \tag{1}$$

where $i = 1, 2, \ldots, N$, $x_i \in R^r$ is the position, $v_i \in R^r$ is the velocity, and $u_i \in R^r$ is the control input to be designed.

The sub-leaders can be described as $\dot{x}_{l,0} = v_{l,0}$ and $\dot{v}_{l,0} = a_{l,0}$, where $x_{l,0}, v_{l,0}, a_{l,0} \in R^r$ is position, velocity and acceleration, $\forall l \in \{1, 2, \ldots, k\}$.

The prescribed-time multi-target tracking for (1) will be achieved if there exists a u_i for followers such that the agents can track the sub-leaders' trajectories in a prescribed time respectively.

2.3 Preliminaries

Lemma 1. *[15] Under Assumption 1, there exists a positive-definite matrix $P_l = \text{diag}(\xi_i) = \text{diag}(y_i/x_i)$ such that $Q_l = P_l H_{ll} + H_{ll}^T P_l$, in which $H_{ll} = L_l + B_l$, $x = [x_1, x_2, \ldots x_N]^T = H_{ll}^{-1} 1_N$, $y = [y_1, y_2, \ldots y_N]^T = H_{ll}^{-T} 1_N$, $\forall l \in \{1, 2, \ldots, k\}$.*

Before moving on, a time-relevant function $\eta(t)$ is proposed as

$$\eta(t) = \left(\frac{T_u}{t_0 + T_u - t}\right)^{\rho},$$

where $\rho > 1$ is a positive constant, t_0 and T_u are initial time and the prescribed time.

Lemma 2. *[11] For system (1), if there exists a Lyapunov function $V(y)$ such that $\dot{V}(y) \leq -bV(y) - c\varphi(t)V(y)$, where $b \geq 0, c > 0$, $\varphi(t)$ is given as*

$$\varphi(t) = \begin{cases} \frac{\dot{\eta}(t)}{\eta(t)}, t_0 \leq t < t_0 + T_u, \\ \frac{\rho}{T_u}, t \geq t_0 + T_u. \end{cases} \tag{2}$$

Then, (1) is said to be prescribed-time stability in the prescribed time T_u. Further, it has $V(y) \leq \eta^{-c}(t)\exp^{-b(t-t_0)}V(t_0)$ on $t \in [t_0, t_0 + T_u)$, and $V(y) = 0$ on $t \in [t_0 + T_u, \infty)$.

Lemma 3. *For any vectors x, y, there exists $\sigma > 0$, then*

$$\|x\| \, \|y\| \leq \sigma \|x\|^2 + \frac{1}{4\sigma}\|y\|^2. \tag{3}$$

3 Main Results

In this section, we propose an algorithm to force followers to track their leaders' trajectory in the prescribed time over the directed graph. Further, we demonstrate the prescribed-time stability of system (1) under the control of the designed algorithm.

3.1 Prescribed-Time Multi-target Tracking Control Algorithm

For the prescribed-time multi-target tracking problem, we propose a novel algorithm for (1), namely,

$$\begin{aligned} u_i = a_{l,0} - \alpha_1 \varphi^2(t)(\sum_{j \in N_i} a_{ij}(x_i - x_j) + b_i(x_i - x_{l,0})) \\ -\alpha_2 \varphi(t)(\sum_{j \in N_i} a_{ij}(v_i - v_j) + b_i(v_i - v_{l,0})), \end{aligned} \tag{4}$$

where α_1, α_2 are positive parameters, $\varphi(t)$ is defined in (2).

3.2 Analysis for Prescribed-Time Multi-target Tracking

Theorem 1. *Under Assumptions 1-2, the prescribed-time multi-target tracking of (1) is achieved under the control algorithm (4) with the following limitation.*

$$\begin{aligned} &2+\alpha_1\rho-\alpha_2+bT_u+c\rho-\alpha_2\rho\tfrac{\lambda_{\max}(Q_l)}{\lambda_{\max}(P_l)}\leq 0,\\ &\rho\alpha_1-\alpha_2\leq 0, \end{aligned} \tag{5}$$

and the prescribed time is $T=k(t_0+T_u)$, $\forall l\in\{1,2,\ldots,k\}$.

Proof. The tracking errors are $\bar{x}_i=x_i-x_{l,0}$ and $\bar{v}_i=v_i-v_{l,0}$. The related compact form are $\bar{x}=\mathrm{col}(\bar{x}_1,\bar{x}_2,\ldots,\bar{x}_N)$ and $\bar{v}=\mathrm{col}(\bar{v}_1,\bar{v}_2,\ldots,\bar{v}_N)$.

Define the following auxiliary variables

$$\begin{aligned} \tilde{x}&=(H\otimes I_r)\varphi(t)\bar{x},\\ \tilde{v}&=(H\otimes I_r)\bar{v}, \end{aligned} \tag{6}$$

where

$$H=L+B=\begin{bmatrix} h_{11} & 0 & \ldots & 0\\ h_{21} & h_{22} & \ldots & 0\\ \vdots & \vdots & \ddots & 0\\ h_{k1} & h_{k2} & \ldots & h_{kk} \end{bmatrix},$$

and $h_{ll}=L_l+B_l$. Similarly, the compact forms are $\tilde{x}=\mathrm{col}(\tilde{x}_1,\tilde{x}_2,\ldots,\tilde{x}_N)$ and $\tilde{v}=\mathrm{col}(\tilde{v}_1,\tilde{v}_2,\ldots,\tilde{v}_N)$.

Combining (6) with (4), it obtains the following newly closed-loop system

$$\begin{cases} \dot{\tilde{x}}=\bar{\varphi}(t)\tilde{x}+\varphi(t)\tilde{v},\\ \dot{\tilde{v}}=-\varphi(t)(H\otimes I_r)(\alpha_1\tilde{x}+\alpha_2\tilde{v}), \end{cases}$$

where

$$\bar{\varphi}(t)=\frac{\dot{\varphi}(t)}{\varphi(t)}=\begin{cases} \frac{\varphi(t)}{\rho}, t_0\leq t\leq t_0+T_u,\\ 0, t\geq t_0+T_u. \end{cases}$$

Let $z=\alpha_1\tilde{x}+\alpha_2\tilde{v}$. Differentiating z yields that

$$\dot{z}=-\alpha_2\varphi(t)(H\otimes I_r)z+\alpha_1\bar{\varphi}(t)\tilde{x}+\alpha_1\varphi(t)\tilde{v}.$$

Specifically, it follows that

$$\begin{aligned} \dot{z}_1&=-\alpha_2\varphi(t)(h_{11}\otimes I_r)z_1+\alpha_1\bar{\varphi}(t)\tilde{x}_1+\alpha_1\varphi(t)\tilde{v}_1,\\ \dot{z}_2&=-\alpha_2\varphi(t)(h_{21}\otimes I_r)z_1-\alpha_2\varphi(t)(h_{22}\otimes I_r)z_2\\ &\quad+\alpha_1\bar{\varphi}(t)\tilde{x}_2+\alpha_1\varphi(t)\tilde{v}_2,\\ &\vdots\\ \dot{z}_k&=-\alpha_2\varphi(t)\sum_{l=1}^{k-1}(h_{kl}\otimes I_r)z_l-\alpha_2\varphi(t)(h_{kk}\otimes I_r)z_k\\ &\quad+\alpha_1\bar{\varphi}(t)\tilde{x}_k+\alpha_1\varphi(t)\tilde{v}_k, \end{aligned} \tag{7}$$

where $z_l, \tilde{x}_l, \tilde{v}_l, \forall l \in \{1, 2, \ldots, k\}$ are associated with the lth subgroup.

The Lyapunov function candidate is given as

$$V_l = \frac{1}{2} z_l^T (P_l \otimes I_{n_l}) z_l, l \in \{1, 2, \ldots, k\}.$$

The remaining proof is based on mathematical induction and the Lyapunov argument.

Step 1: Suppose that $l = 1$, it follows

$$V_1 = \frac{1}{2} z_1^T (P_1 \otimes I_{n_1}) z_1. \tag{8}$$

For $t \in [t_0, t_0 + T_u)$, taking the derivative of V_1 yields that

$$\begin{aligned} \dot{V}_1 = &-\frac{1}{2}\alpha_2 \varphi(t) z_1^T (Q_1 \otimes I_{n_1}) z_1 + \bar{\varphi}(t) z_1^T (P_1 \otimes I_{n_1})(z_1 - \alpha_2 \tilde{v}_1) \\ &+\alpha_1 \varphi(t) z_1^T (P_1 \otimes I_{n_1}) \tilde{v}_1. \end{aligned} \tag{9}$$

According to Lemma 3, we have

$$\begin{aligned} \lambda_{\min}(Q_l)\tilde{x}^T\tilde{x} \le \tilde{x}^T (Q_l \otimes I_{n_1})\tilde{x} &\le \lambda_{\max}(Q_l)\tilde{x}^T\tilde{x}, \\ \tilde{z}_1^T (P_l \otimes I_{n_1})\tilde{v} &= \sum\nolimits_{i=1}^{N} \xi_i \tilde{z}_{1i}^T \tilde{v}_i \\ &\le \lambda_{\max}(P_l)(\sigma \tilde{z}_1^T \tilde{z}_1 + \frac{1}{4\sigma}\tilde{v}^T\tilde{v}). \end{aligned} \tag{10}$$

Let $R(t) = \dot{V}_1 + (b + c\varphi(t))V_1$. Then we can obtain that

$$\begin{aligned} R(t) \le &\frac{1}{2}\varphi(t)\lambda_{\max}(P_1 \otimes I_{n_1}) \left[\frac{2}{\rho} + \alpha_1 - \frac{\alpha_2}{\rho} + \frac{b}{\varphi(t)} + 2c\right] z_1^T z_1 \\ &-\frac{1}{2}\varphi(t)\alpha_2\lambda_{\max}(Q_1 \otimes I_{n_1}) z_1^T z_1 \\ &+\frac{1}{2}\varphi(t)\lambda_{\max}(P_1 \otimes I_{n_1})(\alpha_1 - \frac{\alpha_2}{\rho})\tilde{v}_1^T\tilde{v}_1, \end{aligned}$$

It can be concluded that $R(t) \le 0$ if (5) holds.

Based on (2), it provides that

$$\begin{aligned} \|\tilde{x}\|^2 + \|\tilde{v}\|^2 &= H^2(\varphi^2(t)\|\bar{x}\|^2 + \|\bar{v}\|^2) \\ &\ge \varepsilon_1(\|\bar{x}\|^2 + \|\bar{v}\|^2), \end{aligned} \tag{11}$$

where $\varepsilon_1 = \min\left\{\rho^2/{T_u}^2, 1\right\}$.

In addition, it follows that

$$\|\tilde{x}\|^2 + \|\tilde{v}\|^2 \le \frac{1}{\varepsilon_2}\eta^{-c}(t)\exp^{-b(t-t_0)}V_1(t_0), \tag{12}$$

where $\frac{1}{\varepsilon_2} = \frac{1}{2}\lambda_{\max}(P_1)\min(\alpha_1^2, \alpha_2^2)$.

Combining (11) with (12), it follows

$$\|\bar{x}\|^2 + \|\bar{v}\|^2 \leq \frac{1}{\varepsilon_1 \varepsilon_2} \eta^{-c}(t) \exp^{-b(t-t_0)} V_1 \left(t_0\right).$$

Based on Lemma 2, it concludes that $\lim_{t\to t_0+T_u} \eta^{-c}(t) = 0$. Then, it follows that $\lim_{t\to t_0+T_u} \|\bar{x}\| = 0$ and $\lim_{t\to t_0+T_u} \|\bar{v}\| = 0$.

Similarly, for $t \geq t_0 + T_u$, if (5) holds, we can easily obtain that

$$\begin{aligned} \dot{V}_1 &\leq -bV_1 - c\varphi(t)V_1 \\ &= -\left(b + c\rho/T_u\right) V_1 \\ &\leq 0. \end{aligned} \tag{13}$$

Hence, it yields that $\lim_{t\to t_0+T_u} \|\bar{x}\| = 0$ and $\lim_{t\to t_0+T_u} \|\bar{v}\| = 0$ for $t \geq t_0 + T$, $\forall i \in V_1$.

Step 2: Suppose that $l = 2$. When $t \geq t_0 + T_u$, it follows that $z_1 = 0$. Then $\dot{z}_2$ can be rewritten as

$$\dot{z}_2 = -\alpha_2 \varphi(t)(h_{22} \otimes I_r) z_2 + \alpha_1 \bar{\varphi}(t) \tilde{x}_2 + \alpha_1 \varphi(t) \tilde{v}_2.$$

By employing the similar manipulation as presented in (8)-(13), it can be obtained that $\bar{x}_i$ and $\bar{v}_i$ converge to zero as $t \geq 2(t_0 + T_u)$, $\forall i \in V_2$.

Step 3: For $l = k$, when $t \geq (k-1)(t_0 + T_u)$, it can be concluded that

$$\dot{z}_k = -\alpha_2 \varphi(t)(h_{kk} \otimes I_r) z_k + \alpha_1 \bar{\varphi}(t) \tilde{x}_k + \alpha_1 \varphi(t) \tilde{v}_k.$$

Similarly, the prescribed-time convergence of $\bar{x}_i, \bar{v}_i$ will be achieved in $k(t_0+T_u)$, $\forall i \in V$.

Based on the mathematical induction, it obtains that $\bar{x}_i, \bar{v}_i$ approach zero within the prescribed time $k(t_0 + T_u)$. This ends the proof.

4 Simulation Results

In this section, the effectiveness of the proposed algorithm is proved through simulation experiments.

The studied MASs contain thirteen agents, including three sub-leaders and ten followers. The interaction network is shown in Fig ??, in which nodes $L1, L2, etc$ are the sub-leaders, and nodes $1-5, 6-9, 10-13$ are the corresponding followers. Specifically, the pinning matrix is $B = \mathrm{diag}(0,1,1,0,0,0,0,5,0,6,0,4,0)$. The trajectories of sub-leaders are selected as

$$\begin{cases} x_{1,0} = [2 + \cos(0.2t), -1 + \sin(0.2t)]^T, \\ v_{1,0} = [-0.2\sin(0.2t), 0.2\cos(0.2t)]^T, \\ a_{1,0} = [-0.04\cos(0.2t), -0.04\sin(0.2t)]^T, \end{cases}$$

$$\begin{cases} x_{2,0} = [0.3 + \cos(0.3t), 0.5 + \sin(0.3t)]^T, \\ v_{2,0} = [-0.3\sin(0.3t), 0.3\cos(0.3t)]^T, \\ a_{2,0} = [-0.09\cos(0.3t), -0.09\sin(0.3t)]^T, \end{cases}$$

$$\begin{cases} x_{3,0} = [2\sin(t), -2\cos(t)]^T, \\ v_{3,0} = [2\cos(t), 2\sin(t)]^T, \\ a_{3,0} = [-2\sin(t), 2\cos(t)]^T. \end{cases}$$

The control parameters of (4) are set as follows. To satisfy the conditions (5), let $\alpha_1 = \alpha_2 = 5, \rho = 7,\ t_0 = 0.1, T_u = 4$, and then the prescribed time is $T = 12.3s$. The simulation results are shown in Figs. ??. For more details, it can be easily observed from Fig. ?? that all the followers are divided into three subgroups and track the corresponding trajectories of the sub-leaders in the prescribed time T.

5 Conclusion

In this paper, by employing a time-relevant function, the prescribed-time multi-target tracking problem of second-order MASs has been solved successfully under the directed graph. The proposed algorithm has been demonstrated that the error states between the follower agents and the corresponding leaders converge to zero within a prescribed time. Further, combining the Lyapunov stability theory with the mathematical induction, the necessary conditions for the achievement of the designed protocol are obtained. Moreover, the simulation results have been presented to verify the prescribed-time performance and multi-target tracking ability of the proposed method. Future works will be concentrated on solving the prescribed-time multi-target tracking for nonlinear physical models that agree with industrial machining practice.

Acknowledgement. This work was supported by the National Key Technology R&D Program of China (Grant: 2020YFB1709301) and the National Natural Science Foundation of China (Grant 62073301).

References

1. Li, D., Ge, S.S., He, W., Ma, G., Xie, L.: Multilayer formation control of multi-agent systems. Automatica **109**, 108558 (2019)
2. Ding, T.-F., Ge, M.-F., Liu, Z.-W., Wang, Y.-W., Karimi, H.R.: Lag-bipartite formation tracking of networked robotic systems over directed matrix-weighted signed graphs. IEEE Trans. Cybern. (2020). https://doi.org/10.1109/TCYB.2020.3034108

3. Liang, H., Zhang, L., Sun, Y., Huang, T.: Containment control of semi-Markovian multiagent systems with switching topologies. IEEE Trans. Syst. Man Cybern. Syst. **51**(6), 3889–3899 (2021)
4. Qin, H., Chen, H., Sun, Y., Chen, L.: Distributed finite-time fault-tolerant containment control for multiple ocean bottom flying node systems with error constraints. Ocean Eng. **189**, 106341 (2019)
5. Altan, A., Hacolu, R.: Model predictive control of three-axis gimbal system mounted on UAV for real-time target tracking under external disturbances. Mech. Syst. Signal Process. **138**, 106548 (2020)
6. Huang, J., Song, Y.D., Wang, W., Wen, C., Li, G.: Smooth control design for adaptive leader-following consensus control of a class of high-order nonlinear systems with time-varying reference. Automatica **83**, 361–367 (2017)
7. Han, G.S., Guan, Z.H., Li, J., He, D.X., Zheng, D.F.: Multi-tracking of second-order multi-agent systems using impulsive control. Nonlinear Dyn. **84**, 1771–1781 (2016)
8. Liang, C.D., Ge, M.F., Liu, Z.W., Wang, Y.W., Karimi, H.R.: Output multiformation tracking of networked heterogeneous robotic systems via finite-time hierarchical control. IEEE Trans. Cybern. **51**(6), 2893–2904 (2020)
9. Zuo, Z., Han, Q.L., Ning, B., Ge, X., Zhang, X.M.: An overview of recent advances in fixed-time cooperative control of multiagent systems. IEEE Trans. Ind. Inf. **14**(6), 2322–2334 (2018)
10. Wang, L., Zeng, Z., Ge, M.F.: A disturbance rejection framework for finite-time and fixed-time stabilization of delayed memristive neural networks. IEEE Trans. Syst. Man Cybern. Syst. **51**(2), 905–915 (2019)
11. Ren, Y., Zhou, W., Li, Z., Liu, L., Sun, Y.: Prescribed-time cluster lag consensus control for second-order non-linear leader-following multiagent systems. ISA Trans. **109**, 49–60 (2021)
12. Munoz-Vazquez, A.J., Sanchez-Torres, J.D., Jimenez-Rodriguez, E., Loukianov, A.G.: Predefined-time robust stabilization of robotic manipulators. IEEE Trans. Mechatron. **24**(3), 1033–1040 (2019)
13. Liang, C.D., Ge, M.F., Liu, Z.W., Ling, G., Zhao, X.W.: A novel sliding surface design for predefined-time stabilization of Euler-Lagrange systems. Nonlinear Dyn. **106**, 445–458 (2021)
14. Xu, C., Wu, B., Zhang, Y.: Distributed prescribed-time attitude cooperative control for multiple spacecraft. Aerosp. Sci. Technol. **113**, 106699 (2021)
15. Zhang, H., Li, Z., Qu, Z., Lewis, F.L.: On constructing Lyapunov functions for multi-agent systems. Automatica **58**, 39–42 (2015)
16. Qin, J., Yu, C.: Cluster consensus control of generic linear multi-agent systems under directed topology with acyclic partition. Automatica **49**(9), 2898–2905 (2013)