High-order Sliding Mode Formation Control of Multiple Aerial Robotic Vehicle Systems with Time-Varying Disturbances

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Abstract. The robust formation control problems for a group of aerial robotic vehicles (ARVs) with external time-varying disturbances are investigated. Unlike previous analysis methods, the formation control of multiple ARVs in this paper is transformed into the synchronous control problem. Firstly, the synchronous control objective of multiple ARVs is constructed using the cross-coupling synchronization control (CCSC) strategy and formation constraint. Then, to better deal with external time-varying disturbances, minimize the chatter from the controller and implement the synchronization control goal of multi-ARV systems, a robust synchronization controller for multi-ARV systems is developed, which combines high-order sliding mode control (HOSMC) and CCSC. Moreover, the stability of the developed HOSMC-based CCSC scheme is proved by Lyapunov stability theory. Finally, an example is given to verify the validity of the developed HOSMC-based CCSC scheme.

Keywords: Aerial robotic vehicles · Synchronous control · High-order sliding mode control

1 Introduction

Since the outbreak of COVID-19, the application of aerial robotic vehicles (ARVs) in logistics distribution has attracted attention in the context of the soaring pressure of supplies transportation and the outbreak of demand for contactless delivery. However, single ARV has many constraints, such as limited payload capabilities and lower delivery efficiency in complex environments [1,2]. The formation transportation of multiple ARVs is a very meaningful study that can be a solution to these problems.

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In the past decades, some formation methods which involve leader-follower, virtual structure, and behavior-based ones [3–5], have been applied in the field of robotics. Nonetheless, Ref. [6] points out that these methods have obvious shortcomings. Recently, consensus-based approach has been widely used in multi-robot systems and many results have been obtained [6–9]. However, the formation control problem of multi-robot systems with complex disturbances was not studied. To our knowledge, it is hard to investigate the formation control of multi-ARV systems with complex disturbances only using consensus-based approach.

The synchronization control (SC) strategy [10–12], such as cross-coupling SC (CCSC), can be used to research the formation control problem of multiple ARVs with complex disturbances. Using this strategy, each robot interacts with adjacent robots. When one or more robots change due to complex disturbances, other robots can quickly respond to such changes. Hence, this strategy has certain robustness. Moreover, to obtain high-precision formation control of multi-robots, robust synchronization controllers need to be developed. Sliding mode control (SMC) has strong robustness in dealing with complex disturbances, but traditional SMC often produces chattering phenomenon. High-order SMC (HOSMC) [13,14] can be introduced to alleviate the chattering and ensure the formation control accuracy.

Inspired by these discussions, the robust formation control problem for multi-ARV systems with time-varying disturbances by using HOSMC-based CCSC scheme is investigated in the paper. This paper has the following three contributions:

1) The formation control problem is converted to the SC problem of multi-ARV systems. When the formation system suffers from time-varying disturbances, in order to improve the synchronization behavior of multiple ARVs, the CCSC is designed.
2) To better cope with external time-varying disturbances, SMC-based CCSC scheme for multi-ARV systems is presented.
3) A novel HOSMC-based CCSC scheme for multi-ARV systems is presented to alleviate the chattering effect and improve the formation control accuracy.

2 Problem Formulation

2.1 Dynamics of ARV

There is a multi-ARV system with N ARVs. The dynamics of ARV i is given as follows

$$\dot{p}_i(t) = v_i(t) \tag{1}$$

$$\dot{v}_i(t) = ge_3 + T_i(t) + f_{d,i}(p_i(t), v_i(t), t) \tag{2}$$

where $p_i(t), v_i(t), T_i(t)$ are the position, velocity and control input of ARV i, respectively. g is the acceleration of gravity. $f_{d,i}(p_i(t), v_i(t), t)$ denotes the external time-varying disturbances and $e_3 = [0\ 0\ 1]^T$. Here, $\|f_{d,i}(p_i(t), v_i(t), t)\| < f_{d,*}$ is assumed and $f_{d,*}$ is a given constant.

2.2 Formation Constraint

Definition 1. *The desired formation can be represented by $\ell(\hbar, t)$, where $\hbar$ denotes the time-varying or time-invariant position vector. The desired position $p_i^d(t)$ of ARV i must be located on the boundary curve $\partial\ell(\hbar, t) = 0$ [15].*

The boundary curve of the desired formation can be given as follows [16]:

$$\partial\ell(\hbar, t) = 0 : (p_i(t) = A_i(t)B(t) + C_i(t)) \tag{3}$$

where $A_i(t)$ is the parameter constraint matrix of the ith ARV. $B(t)$ denotes a common vector. $C_i(t)$ is an offset of ARV i.

Assumption 1. *We assume that the designed boundary curve $\partial\ell(\hbar, t) = 0$ is reasonable, which makes sure that the inverse of parameter constraint matrix exists.*

Thus, the formation constraint is described as

$$A_i^{-1}(t)(p_i(t) - C_i(t)) = B(t). \tag{4}$$

According to Definition 1, we have

$$A_i^{-1}(t)(p_i^d(t) - C_i(t)) = B(t). \tag{5}$$

So, one has

$$\alpha_1(t)\nu_1(t) = \alpha_2(t)\nu_2(t) = \cdots = \alpha_N(t)\nu_N(t) \tag{6}$$

where $\nu_i(t) = p_i(t) - p_i^d(t)$ represents the position tracking error vector of ARV i and $\alpha_i(t) = A_i^{-1}(t)$.

2.3 CCSC Strategy

Assumption 2. *Assume that there are only two ARVs adjacent to each one.*

With Assumption 2, the CCSC strategy is adopted to obtain the position synchronization error of ARV i, which is shown as follows

$$\varsigma_{i,i+1}(t) = \alpha_i(t)\nu_i(t) - \alpha_{i+1}(t)\nu_{i+1}(t) \tag{7}$$

$$\varsigma_{i,i-1}(t) = \alpha_i(t)\nu_i(t) - \alpha_{i-1}(t)\nu_{i-1}(t) \tag{8}$$

$$\begin{aligned}\varsigma_i(t) &= \varsigma_{i,i+1}(t) + \varsigma_{i,i-1}(t) \\ &= 2\alpha_i(t)\nu_i(t) - \alpha_{i-1}(t)\nu_{i-1}(t) - \alpha_{i+1}(t)\nu_{i+1}(t).\end{aligned} \tag{9}$$

Then, a coupled position error can be defined as

$$\eta_i(t) = \nu_i(t) + \chi_i\varsigma_i(t) \tag{10}$$

where χ_i =diag$(\chi_{i1}, \chi_{i2}, \chi_{i3})$ and $\chi_{ij}(j = 1, 2, 3)$ is a very small positive constant.

The coupled error is rewritten as

$$\eta(t) = \nu(t) + \chi\varsigma(t) = (I_{3N} + \chi M)\nu(t) \tag{11}$$

where $\eta(t) = [\eta_1(t), \eta_2(t), \cdots, \eta_N(t)]^T$, $\nu(t) = [\nu_1(t), \nu_2(t), \cdots, \nu_N(t)]^T$. I_{3N} denotes the identity matrix with dimension $3N$. χ =diag$(\chi_1, \chi_2, \cdots, \chi_N)$, $\varsigma(t) = M\nu(t)$, $\varsigma(t) = [\varsigma_1(t), \varsigma_2(t), \cdots, \varsigma_N(t)]^T$ and $M = \begin{bmatrix} 2\alpha_1(t) & -\alpha_2(t) & \cdots & -\alpha_N(t) \\ -\alpha_1(t) & 2\alpha_2(t) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -\alpha_1(t) & 0 & \cdots & 2\alpha_N(t) \end{bmatrix}$.

Since χ_{ij} is small enough, and then $I_{3N} + \chi M$ is a positive definite matrix. Hence, when $\lim\limits_{t\to\infty} \|\eta(t)\| = 0$, one has $\lim\limits_{t\to\infty} \|\nu(t)\| = 0$ and $\lim\limits_{t\to\infty} \|\varsigma(t)\| = 0$.

The control target of this article is to develop a robust synchronization controller for multi-ARV systems in the presence of time-varying complex disturbances such that the coupled position error $\|\eta(t)\|$ to 0, so as to ensure that the position tracking error $\|\nu(t)\|$ and synchronous error $\|\varsigma(t)\|$ converge to 0, when $t \to \infty$.

3 Main Results

The robust formation control of multiple ARVs with time-varying disturbances is studied by combining HOSMC and CCSC scheme, and the stability analysis of the formation system is testified via Lyapunov stability theory in this part.

Using the coupled error (11), the tradition sliding surface is considered as

$$\kappa_i(t) = l_i\eta_i(t) + \dot{\eta}_i(t) \tag{12}$$

where l_i =diag(l_{i1}, l_{i2}, l_{i3}) is a positive definite matrix.

Taking the derivative of (12) and using (10), one can obtain

$$\begin{aligned} \dot{\kappa}_i(t) =&(2l_i\chi_i\dot{\alpha}_i(t) + 2\chi_i\ddot{\alpha}_i(t))\nu_i(t) - (l_i\chi_i\dot{\alpha}_{i-1}(t) + \chi_i\ddot{\alpha}_{i-1}(t))\nu_{i-1}(t) \\ &- (l_i\chi_i\dot{\alpha}_{i+1}(t) + \chi_i\ddot{\alpha}_{i+1}(t))\nu_{i+1}(t) + (l_i + 2l_i\chi_i\alpha_i(t) + 4\chi_i\dot{\alpha}_i(t))\dot{\nu}_i(t) \\ &- (l_i\chi_i\alpha_{i-1}(t) + 2\chi_i\dot{\alpha}_{i-1}(t))\dot{\nu}_{i-1}(t) - (l_i\chi_i\alpha_{i+1}(t) + 2\chi_i\dot{\alpha}_{i+1}(t))\dot{\nu}_{i+1}(t) \\ &+ (2\chi_i\alpha_i(t) + I_3)\ddot{\nu}_i(t) - \chi_i\alpha_{i-1}(t)\ddot{\nu}_{i-1}(t) - \chi_i\alpha_{i+1}(t)\ddot{\nu}_{i+1}(t). \end{aligned} \tag{13}$$

The high-order sliding surface with (12) and (13) can be designed as

$$\kappa_{ho(i)}(t) = \kappa_i(t) + \dot{\kappa}_i(t) = l_i\eta_i(t) + (I_3 + l_i)\dot{\eta}_i(t) + \ddot{\eta}_i(t). \tag{14}$$

The third derivative of coupled error is expressed as

$$\begin{aligned} \dddot{\eta}_i(t) =&2\chi_i\dddot{\alpha}_i(t)\nu_i(t) - \chi_i\dddot{\alpha}_{i-1}(t)\nu_{i-1}(t) - \chi_i\dddot{\alpha}_{i+1}(t)\nu_{i+1}(t) \\ &+ 6\chi_i\ddot{\alpha}_i(t)\dot{\nu}_i(t) - 3\chi_i\ddot{\alpha}_{i-1}(t)\dot{\nu}_{i-1}(t) - 3\chi_i\ddot{\alpha}_{i+1}(t)\dot{\nu}_{i+1}(t) \\ &+ 6\chi_i\dot{\alpha}_i(t)\ddot{\nu}_i(t) - 3\chi_i\dot{\alpha}_{i-1}(t)\ddot{\nu}_{i-1}(t) - 3\chi_i\dot{\alpha}_{i+1}(t)\ddot{\nu}_{i+1}(t) \\ &+ (2\chi_i\alpha_i(t) + I_3)\dddot{\nu}_i(t) - \chi_i\alpha_{i-1}(t)\dddot{\nu}_{i-1}(t) - \chi_i\alpha_{i+1}(t)\dddot{\nu}_{i+1}(t) \end{aligned} \tag{15}$$

Now, taking the derivative of (14) and using (15), one obtains

$$
\begin{aligned}
\dot{\kappa}_{ho(i)}(t) =& (2l_i\chi_i\dot{\alpha}_i(t) + 2\chi_i\ddot{\alpha}_i(t)(I_3 + l_i) + 2\chi_i\dddot{\alpha}_i(t))\nu_i(t) \\
&- (l_i\chi_i\dot{\alpha}_{i-1}(t) + \chi_i\ddot{\alpha}_{i-1}(t)(I_3 + l_i) + \chi_i\dddot{\alpha}_{i-1}(t))\nu_{i-1}(t) \\
&- (l_i\chi_i\dot{\alpha}_{i+1}(t) + \chi_i\ddot{\alpha}_{i+1}(t)(I_3 + l_i) + \chi_i\dddot{\alpha}_{i+1}(t))\nu_{i+1}(t) \\
&+ ((2l_i\chi_i\alpha_i(t) + l_i) + 4\chi_i\dot{\alpha}_i(t)(I_3 + l_i) + 6\chi_i\ddot{\alpha}_i(t))\dot{\nu}_i(t) \\
&- (l_i\chi_i\alpha_{i-1}(t) + 2\chi_i\dot{\alpha}_{i-1}(t)(I_3 + l_i) + 3\chi_i\ddot{\alpha}_{i-1}(t))\dot{\nu}_{i-1}(t) \\
&- (l_i\chi_i\alpha_{i+1}(t) + 2\chi_i\dot{\alpha}_{i+1}(t)(I_3 + l_i) + 3\chi_i\ddot{\alpha}_{i+1}(t))\dot{\nu}_{i+1}(t) \\
&+ (2\chi_i\alpha_i(t) + I_3)(I_3 + l_i)(ge_3 + f_{d,i}(p_i(t), v_i(t), t) - \ddot{p}_i^d(t)) \\
&+ 6\chi_i\dot{\alpha}_i(t)(ge_3 + f_{d,i}(p_i(t), v_i(t), t) - \ddot{p}_i^d(t)) \\
&- \chi_i\alpha_{i-1}(t)(I_3 + l_i)(ge_3 + f_{d,i-1}(p_{i-1}(t), v_{i-1}(t), t) - \ddot{p}_{i-1}^d(t)) \\
&- 3\chi_i\dot{\alpha}_{i-1}(t)(ge_3 + f_{d,i-1}(p_{i-1}(t), v_{i-1}(t), t) - \ddot{p}_{i-1}^d(t)) \\
&- \chi_i\alpha_{i+1}(t)(I_3 + l_i)(ge_3 + f_{d,i+1}(p_{i+1}(t), v_{i+1}(t), t) - \ddot{p}_{i+1}^d(t)) \\
&- 3\chi_i\dot{\alpha}_{i+1}(t)(ge_3 + f_{d,i+1}(p_{i+1}(t), v_{i+1}(t), t) - \ddot{p}_{i+1}^d(t)) \\
&+ (2\chi_i\alpha_i(t) + I_3)\dddot{\nu}_i(t) - \chi_i\alpha_{i-1}(t)\dddot{\nu}_{i-1}(t) \\
&- \chi_i\alpha_{i+1}(t)\dddot{\nu}_{i+1}(t) + T_{hoc(i)}(t).
\end{aligned}
\tag{16}
$$

Using (16), we design the HOSMC-based CCSC law for multi-ARV systems as follows

$$
T_{hoc(i)}(t) = T_{hoc(i1)}(t) + T_{hoc(i2)}(t) \tag{17}
$$

$$
\begin{aligned}
T_{hoc(i1)}(t) =& - (2l_i\chi_i\dot{\alpha}_i(t) + 2\chi_i\ddot{\alpha}_i(t)(I_3 + l_i) + 2\chi_i\dddot{\alpha}_i(t))\nu_i(t) \\
&+ (l_i\chi_i\dot{\alpha}_{i-1}(t) + \chi_i\ddot{\alpha}_{i-1}(t)(I_3 + l_i) + \chi_i\dddot{\alpha}_{i-1}(t))\nu_{i-1}(t) \\
&+ (l_i\chi_i\dot{\alpha}_{i+1}(t) + \chi_i\ddot{\alpha}_{i+1}(t)(I_3 + l_i) + \chi_i\dddot{\alpha}_{i+1}(t))\nu_{i+1}(t) \\
&- ((2l_i\chi_i\alpha_i(t) + l_i) + 4\chi_i\dot{\alpha}_i(t)(I_3 + l_i) + 6\chi_i\ddot{\alpha}_i(t))\dot{\nu}_i(t) \\
&+ (l_i\chi_i\alpha_{i-1}(t) + 2\chi_i\dot{\alpha}_{i-1}(t)(I_3 + l_i) + 3\chi_i\ddot{\alpha}_{i-1}(t))\dot{\nu}_{i-1}(t) \\
&+ (l_i\chi_i\alpha_{i+1}(t) + 2\chi_i\dot{\alpha}_{i+1}(t)(I_3 + l_i) + 3\chi_i\ddot{\alpha}_{i+1}(t))\dot{\nu}_{i+1}(t) \\
&- ((2\chi_i\alpha_i(t) + I_3)(I_3 + l_i) + 6\chi_i\dot{\alpha}_i(t))(ge_3 - \ddot{p}_i^d(t)) \\
&+ (\chi_i\alpha_{i-1}(t)(I_3 + l_i) + 3\chi_i\dot{\alpha}_{i-1}(t))(ge_3 - \ddot{p}_{i-1}^d(t)) \\
&+ (\chi_i\alpha_{i+1}(t)(I_3 + l_i) + 3\chi_i\dot{\alpha}_{i+1}(t))(ge_3 - \ddot{p}_{i+1}^d(t)) \\
&- (2\chi_i\alpha_i(t) + I_3)\dddot{\nu}_i(t) + \chi_i\alpha_{i-1}(t)\dddot{\nu}_{i-1}(t) \\
&+ \chi_i\alpha_{i+1}(t)\dddot{\nu}_{i+1}(t)
\end{aligned}
\tag{18}
$$

$$
T_{hoc(i2)}(t) = -\mu_{hoc(i)}(t)sign(\kappa_{ho(i)}(t)) - k_i\kappa_{ho(i)}(t) \tag{19}
$$

where $\mu_{hoc(i)}$ =diag$(\mu_{hoc(i1)}, \mu_{hoc(i2)}, \mu_{hoc(i3)})$ and $\mu_{hoc(ij)}(j = 1, 2, 3)$ is a positive constant. k_i =diag(k_{i1}, k_{i2}, k_{i3}) is a diagonal positive definite matrix.

Theorem 1. *With the designed controller (17)-(19), the formation control system of multiple ARVs is asymptotically stable, on condition that the designed parameters meet the following conditions.*

$$
\|f_{d,i}(p_i(t), v_i(t), t)\| < f_{d,*} \tag{20}
$$

$$\frac{\left\|\mu_{hoc(i)}\right\|}{f_{d,*}} \geq \|(2\chi_i\alpha_i(t)+I_3)(I_3+l_i)\| + \|6\chi_i\dot{\alpha}_i(t)\| + \|\chi_i\alpha_{i-1}(t)(I_3+l_i)\| + \|3\chi_i\dot{\alpha}_{i-1}(t)\| + \|\chi_i\alpha_{i+1}(t)(I_3+l_i)\| + \|3\chi_i\dot{\alpha}_{i+1}(t)\|. \tag{21}$$

Proof. To testify that the coupled position error is convergent, we define a Lyapunov function

$$V(t) = \frac{1}{2}\sum_{i=1}^{N}\kappa_{ho(i)}^T(t)\kappa_{ho(i)}(t) \tag{22}$$

Taking the derivative of (22), using (16) and (17), one obtains that

$$\begin{aligned}
\dot{V}(t) \leq & \sum_{i=1}^{N}\|(2\chi_i\alpha_i(t)+I_3)(I_3+l_i)\|\,\|f_{d,i}(p_i(t),v_i(t),t)\|\,\left\|\kappa_{ho(i)}(t)\right\| \\
&+\sum_{i=1}^{N}\|6\chi_i\dot{\alpha}_i(t)\|\,\|f_{d,i}(p_i(t),v_i(t),t)\|\,\left\|\kappa_{ho(i)}(t)\right\| \\
&+\sum_{i=1}^{N}\|\chi_i\alpha_{i-1}(t)(I_3+l_i)\|\,\|f_{d,i-1}(p_{i-1}(t),v_{i-1}(t),t)\|\,\left\|\kappa_{ho(i)}(t)\right\| \\
&+\sum_{i=1}^{N}\|3\chi_i\dot{\alpha}_{i-1}(t)\|\,\|f_{d,i-1}(p_{i-1}(t),v_{i-1}(t),t)\|\,\left\|\kappa_{ho(i)}(t)\right\| \\
&+\sum_{i=1}^{N}\|\chi_i\alpha_{i+1}(t)(I_3+l_i)\|\,\|f_{d,i+1}(p_{i+1}(t),v_{i+1}(t),t)\|\,\left\|\kappa_{ho(i)}(t)\right\| \\
&+\sum_{i=1}^{N}\|3\chi_i\dot{\alpha}_{i+1}(t)\|\,\|f_{d,i+1}(p_{i+1}(t),v_{i+1}(t),t)\|\,\left\|\kappa_{ho(i)}(t)\right\| \\
&-\sum_{i=1}^{N}\left\|\mu_{hoc(i)}\right\|\,\left\|\kappa_{ho(i)}(t)\right\| - \sum_{i=1}^{N}\kappa_{ho(i)}^T(t)k_i\kappa_{ho(i)}(t)
\end{aligned} \tag{23}$$

According to (20) and (21), we have

$$\begin{aligned}
\dot{V}(t) < & \sum_{i=1}^{N}(\|(2\chi_i\alpha_i(t)+I_3)(I_3+l_i)\| + \|6\chi_i\dot{\alpha}_i(t)\|)\left\|\kappa_{ho(i)}(t)\right\| f_{d,*} \\
&+\sum_{i=1}^{N}(\|\chi_i\alpha_{i-1}(t)(I_3+l_i)\| + \|3\chi_i\dot{\alpha}_{i-1}(t)\|)\left\|\kappa_{ho(i)}(t)\right\| f_{d,*} \\
&+\sum_{i=1}^{N}(\|\chi_i\alpha_{i+1}(t)(I_3+l_i)\| + \|3\chi_i\dot{\alpha}_{i+1}(t)\|)\left\|\kappa_{ho(i)}(t)\right\| \\
&-\sum_{i=1}^{N}\left\|\mu_{hoc(i)}\right\|\,\left\|\kappa_{ho(i)}(t)\right\| - \sum_{i=1}^{N}\kappa_{ho(i)}^T(t)k_i\kappa_{ho(i)}(t) \leq 0
\end{aligned} \tag{24}$$

Hence, it can be obtained that the coupled position error $\|\eta(t)\|$ asymptotically converges to zero as time $t \to \infty$, that is

$$\lim_{t\to\infty} \|\eta(t)\| = 0 \tag{25}$$

Here, the proof is done. □

4 Numerical Simulation

In this part, an example is given to testify the effectiveness of the designed SC scheme, where four ARVs are required to move a rectangle cargo. The weight of each ARV is about 0.26 kg. Time-varying disturbances act on the multi-ARV systems when t =15 s, which are shown in Table 1. The initial states of four ARVs are given as $p10(t) = [6, 4, 0]^T$, $p20(t) = [4, 6, 0]^T$, $p30(t) = [2, 4, 0]^T$ and $p40(t) = [4, 2, 0]^T$, respectively. The expected trajectory of ARV i is denoted by (2) with $A_i(t) = diag(A_{i(11)}(t), A_{i(22)}(t), t)$, $B(t) = [2\ 2\ 0.5]^T$, $C_i(t) = [0.2t\ 0\ 0]^T$, $A_{i(11)}(t) = 2+\cos(0.2t+(i-1)\pi/2)$ and $A_{i(22)}(t) = 2+\sin(0.2t+(i-1)\pi/2)$, $i = 1, 2, 3, 4$.

Table 1. Time-varying disturbances of multi-ARV systems ($15 \le t \le 30\,s$)

NO	Time-varying disturbances
ARV1	$f_{d,1}(p_1(t), v_1(t), t) = [0.05\sin(2t), 0.05\sin(0.5t), 0.05\sin(4t)]^T$
ARV2	$f_{d,2}(p_2(t), v_2(t), t) = [0.05\sin(0.5t), 0.05\sin(4t), 0.05\sin(2t)]^T$
ARV3	$f_{d,3}(p_3(t), v_3(t), t) = [0.04\sin(4t), 0.05\sin(2t), 0.05\sin(0.5t)]^T$
ARV4	$f_{d,4}(p_4(t), v_4(t), t) = [0.03\sin(6t), 0.05\sin(3t), 0.05\sin(0.5t)]^T$

In addition, for the purpose of comparison, the SMC-based CCSC law is designed, which is given as follows

$$T_{c(i)}(t) = T_{c(i1)}(t) + T_{c(i2)}(t) \tag{26}$$

$$\begin{aligned} T_{c(i1)}(t) = &- (2l_i\chi_i\dot{\alpha}_i(t) + 2\chi_i\ddot{\alpha}_i(t))\nu_i(t) + l_i\chi_i\dot{\alpha}_{i-1}(t)\nu_{i-1}(t) \\ &+ \chi_i\ddot{\alpha}_{i-1}(t)\nu_{i-1}(t) + (l_i\chi_i\dot{\alpha}_{i+1}(t) + \chi_i\ddot{\alpha}_{i+1}(t))\nu_{i+1}(t) \\ &- (l_i + 2l_i\chi_i\alpha_i(t) + 4\chi_i\dot{\alpha}_i(t))\dot{\nu}_i(t) + l_i\chi_i\alpha_{i-1}(t)\dot{\nu}_{i-1}(t) \\ &+ 2\chi_i\dot{\alpha}_{i-1}(t)\dot{\nu}_{i-1}(t) + (l_i\chi_i\alpha_{i+1}(t) + 2\chi_i\dot{\alpha}_{i+1}(t))\dot{\nu}_{i+1}(t) \\ &- (2\chi_i\alpha_i(t) + I_3)(ge_3 - \ddot{p}_i^d(t)) + \chi_i\alpha_{i-1}(t)(ge_3 - \ddot{p}_{i-1}^d(t)) \\ &+ \chi_i\alpha_{i+1}(t)(ge_3 - \ddot{p}_{i+1}^d(t)) \end{aligned} \tag{27}$$

$$T_{c(i2)}(t) = -\mu_{c(i)}sign(\kappa_i(t)) - k_i\kappa_i(t) \tag{28}$$

where $\mu_{c(i)}$ =diag$(\mu_{c(i1)}, \mu_{c(i2)}, \mu_{c(i3)})$ and $\mu_{c(ij)}(j = 1, 2, 3)$ is a positive constant. $\chi_{ij} = 0.01$ and $k_{ij} = 0.1$ are adopted for position synchronous controllers.

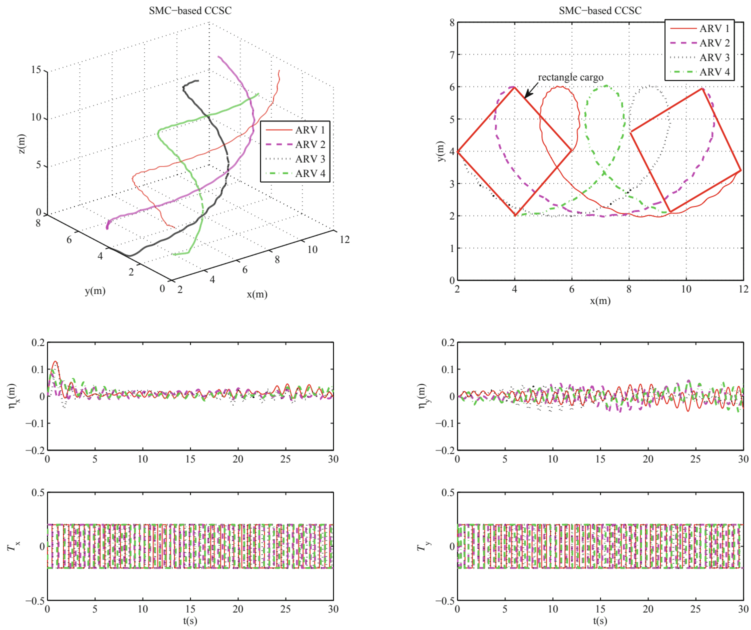

Fig. 1. Simulation results of multi-ARV systems with SMC-based CCSC

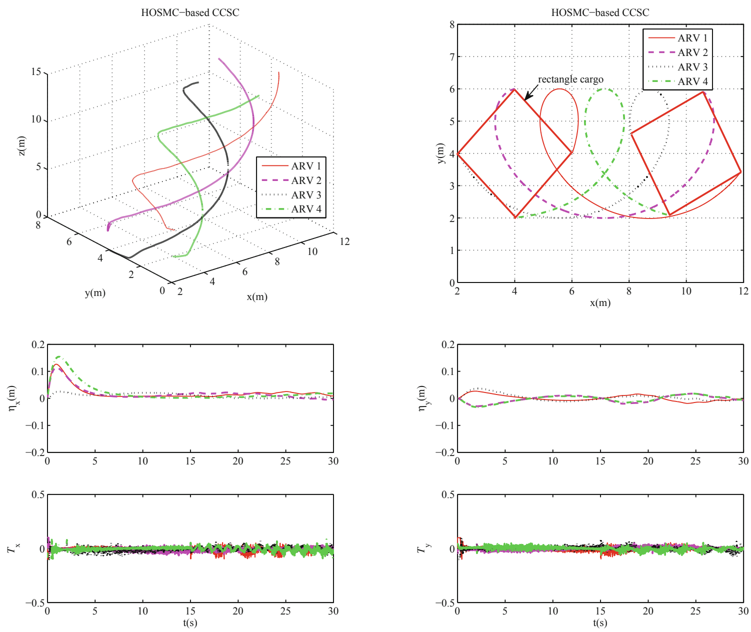

Fig. 2. Simulation results of multi-ARV systems with HOSMC-based CCSC

The parameters of SMC-based CCSC law and HOSMC-based CCSC law are selected as $l_{ij} = 1$, $\mu_{hoc(ij)} = 10$ and $l_{ij} = 1$, $\mu_{c(ij)} = 12$, respectively.

The comparison results, which are shown in Fig. 1 and Fig. 2, include moving trajectory of multiple ARVs, the coupled errors η_x and η_y, and the control efforts T_x and T_y. In order to quantify the comparison results, the RMSEs of the coupled errors of four ARV systems are given, which are shown in Table 2.

Table 2. RMSEs of coupled errors ($\times 10^{-2}$ m)

Parameters	SMC-based CCSC	HOSMC-based CCSC
η_x	1.41	1.12
η_y	2.32	0.95

Figure 1 shows the simulation curves of four ARVs moving a rectangle cargo with SMC-based CCSC strategy. Figure 2 shows the simulation results of multi-ARV systems with HOSMC-based CCSC strategy. It can be observed that both schemes can stably transport the rectangular goods. But, the moving trajectory of four ARVs with HOSMC-based CCSC is smoother than SMC-based CCSC and the coupled errors are smaller. When the time varying disturbances are added to multi-ARV systems at $t = 15$ s, chattering phenomenon appears in the control input based on HOSMC-based CCSC scheme, but it is very small, which indicates that the designed SC scheme has high robustness in anti-disturbance.

Table 2 shows the RMSEs of coupled errors of four ARV systems. One can see that the synchronous control accuracy of multiple ARVs with HOSMC-based CCSC strategy is much higher than SMC-based CCSC one despite external complex disturbances. Thus, it can be concluded that HOSMC-based CCSC scheme has better control effect.

5 Conclusions

The robust formation control problems for multiple ARVs with time-varying disturbances are studied. In this article, the formation control of multiple ARVs is converted to the SC problem. The synchronization control goal of multiple ARVs is constructed based on the CCSC strategy and formation constraint. In order to accomplish the anti-disturbance control of multi-ARV systems, minimize the chatter from the controller and implement the synchronization control goal, a robust synchronization controller for multi-ARV systems is developed, which combines HOSMC and CCSC. And then the stability of the developed HOSMC-based CCSC scheme is proven by using Lyapunov stability theory. The future research direction involves extending the designed SC scheme to investigate robust formation control of nonlinear multiagent systems.

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