Multi-target Passive Localization Algorithm Based on UAV Clusters

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Abstract. In order to study the problem of multi-target passive localization of UAV clusters, this paper first uses the traditional pseudo-linear estimation (PLE) method for target coarse positioning. Then for multi-target passive localization scenarios, proposes a resource allocation method of UAV clusters based on the greedy algorithm. Finally uses the improved weighted pseudo-linear estimation (WPLE) method or improved maximum likelihood estimation (MLE) method for target fine positioning. Besides, the influence of different passive localization algorithms and related parameters on the positioning performance of the system is analyzed through simulation. The experimental results show that the positioning algorithm proposed in this paper achieves accurate positioning of multiple targets, and has a higher positioning accuracy than the traditional PLE method, which verifies the effectiveness and feasibility of the algorithm.

Keywords: UAV clusters · Multi-target · Passive positioning

1 Introduction

UAVs have been widely used in the field of military reconnaissance, and have shown a development trend of miniaturization, clustering and intelligence. The UAV clusters can not only make up for the shortcomings of a single UAV, but also broaden the mission fields through the information fusion and resource complementation between multiple drones.

In the field of single-target passive localization, the commonly used method is pseudo-linear estimation (PLE) [1], which lumps the nonlinearity into the noise term, and then obtains the target position estimate by least squares. After considering the statistical characteristics of the noise term in the PLE equation, the weighted pseudo-linear estimation (WPLE) [2, 3] is proposed. But neither PLE nor WPLE can reduce the estimation bias very well. A commonly used method to reduce bias is maximum likelihood estimation (MLE) [4].

However, networked radar has been widely used due to its wider spatial coverage, higher target detection probability and stronger anti-jamming ability. Scenarios with multiple targets like networked radar that needed to be located are more common. Therefore, single-target passive localization technology is difficult to meet actual needs, and it is particularly important to research the problem of multi-target passive localization.

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Under the above background, this paper proposes an algorithm for passive localization of multiple targets by UAV clusters. Firstly, the PLE algorithm is used to obtain the coarse localization information of the targets, and then the UAV resources are allocated to multiple targets based on greedy algorithm. Finally, the improved WPLE method or MLE method is employed in the acquisition of accurate target information.

The main symbols used in the paper are listed as follows (Table 1):

Table 1. The definition of symbols

Symbols	Definition
K	The number of UAVs
N	The number of targets
$\boldsymbol{p} = [\mathbf{x}, \mathbf{y}]^{\mathbf{T}}$	The position of the target
$\boldsymbol{s_k} = [\mathbf{x_k}, \mathbf{y_k}]^{\mathbf{T}}$	The position of the kth UAV
$\boldsymbol{\theta}_{\mathbf{k}}$	The true bearing for the kth UAV
$\tilde{\boldsymbol{\theta}}_{\mathbf{k}}$	The bearing measured by the kth UAV
$\boldsymbol{\sigma}_{\mathbf{nk}}$	The standard deviation of bearing noise for the kth UAV
$\mathbf{r_k}$	The real distance between the kth UAV and the target
$\boldsymbol{A}$	The independent variable matrix of the observation equation
$\boldsymbol{b}$	The dependent variable matrix of the observation equation
$\boldsymbol{\eta}$	The noise term of the observation equation
$\boldsymbol{C}$	The covariance matrix of the noise term
$\boldsymbol{W}$	The weighting matrix
$\boldsymbol{Q}$	The covariance matrix of the bearing noises
$\boldsymbol{f}\left(\tilde{\boldsymbol{\theta}}\vert\boldsymbol{p}\right)$	The likelihood function of the target bearings
$\boldsymbol{f}(\boldsymbol{p})$	The cost function of MLE
$\boldsymbol{F}$	The fisher matrix
$\hat{\boldsymbol{p}} = [\hat{\mathbf{x}}, \hat{\mathbf{y}}]^{\mathbf{T}}$	The estimation of target position
$\mathbf{selected_k}$	The target number assigned to the kth UAV

2 The Passive Localization Mechanism of a Single Target by a UAV Cluster

2.1 Scene Model

The single target passive localization model [9] based on UAV clusters is shown in Fig. 1. K UAVs and one target are distributed in a two-dimensional plane space. The position of the kth (k = 1,2…K) UAV $\boldsymbol{s_k} = [\mathrm{x_k}, \mathrm{y_k}]^{\mathrm{T}}$ is known. $\theta_k \in [0, 2\Pi)$ is the real azimuth angle from the target to the kth UAV. The goal of passive localization of a single target is to obtain the unknown real position of the target $\boldsymbol{p} = [\mathrm{x}, \mathrm{y}]^{\mathrm{T}}$. The real azimuth angle, the position of the UAVS, and the position of the target have the following relationships:

$$sin\,\theta_k = \frac{y - y_k}{r_k},\ cos\,\theta_k = \frac{x - x_k}{r_k} \tag{1}$$

Among them, $r_k = ||\boldsymbol{s_k} - \boldsymbol{p}||$ is the real distance between the kth UAV and the target. The model assumes that each UAV is affected by independent zero-mean Gaussian additive noise while observing the target. Then the bearing measured by the kth UAV $\tilde{\theta}_k$ is composed of the true bearing and the bearing noise:

$$\tilde{\theta}_k = \theta_k + n_k,\ n_k \sim N(0, {\sigma_{nk}}^2) \tag{2}$$

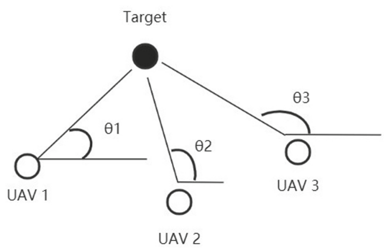

Fig. 1. Scene model

2.2 Principle of PLE Method

For the kth UAV, the following observation equation holds [2]:

$$\boldsymbol{a_k p} = b_k + \eta_k \tag{3}$$

where $\boldsymbol{a_k} = \left[sin\tilde{\theta}_k, -cos\tilde{\theta}_k\right]$, $b_k = \left[sin\tilde{\theta}_k, -cos\tilde{\theta}_k\right]\boldsymbol{s_k}$, $\eta_k = r_k sin\,n_k$ is the error term caused by the bearing noise.

Combine the observation equations of K UAVs together and write them in the form of a matrix:

$$\boldsymbol{Ap = b + \eta} \tag{4}$$

where $\boldsymbol{A} = \left[\boldsymbol{a_1}^{\mathrm{T}}\boldsymbol{a_2}^{\mathrm{T}} \ldots \boldsymbol{a_K}^{\mathrm{T}}\right]^{\mathrm{T}}$, $\boldsymbol{b} = \left[b_1{}^{\mathrm{T}}b_2{}^{\mathrm{T}} \ldots b_K{}^{\mathrm{T}}\right]^{\mathrm{T}}$ and $\boldsymbol{\eta} = \left[\eta_1{}^{\mathrm{T}}\eta_2{}^{\mathrm{T}} \ldots \eta_K{}^{\mathrm{T}}\right]^{\mathrm{T}}$. The PLE of the target position $\hat{\boldsymbol{p}}_{\mathbf{PLE}}$ can be obtained by the least squares method:

$$\hat{\boldsymbol{p}}_{\mathbf{PLE}} = \left(\boldsymbol{A}^{\mathrm{T}}\boldsymbol{A}\right)^{-1}\boldsymbol{A}^{\mathrm{T}}\boldsymbol{b} \tag{5}$$

2.3 Principle of WPLE Method

WPLE is based on the PLE method, using the inverse of the covariance matrix $\boldsymbol{C}$ of the noise term $\boldsymbol{\eta}$ in (4) as the weighting matrix $\boldsymbol{W}$ (that is, trust those observations with low bearing noise). Then weights the bearing information measured by each UAV to obtain the estimate.

The covariance matrix of the noise term is as follows:

$$\boldsymbol{C} = E\left[\boldsymbol{\eta}\boldsymbol{\eta}^{\mathrm{T}}\right] \tag{6}$$

when the bearing noise $\mathrm{n_k}$ is small, η_{k} can be replaced by the following formula:

$$\eta_{\mathrm{k}} = \mathrm{r_k} \sin n_{\mathrm{k}} \approx \mathrm{r_k n_k} \tag{7}$$

Therefore, the covariance matrix of the noise term $\boldsymbol{\eta}$ can be calculated:

$$\boldsymbol{C} = diag\left(\mathrm{r_1}^2 \sigma_{\mathrm{n1}}^2, \mathrm{r_2}^2 \sigma_{\mathrm{n2}}^2, \ldots, \mathrm{r_k}^2 \sigma_{\mathrm{nk}}^2\right) \tag{8}$$

where *diag* is a function that converts a row vector into a diagonal matrix. The weighting matrix can be obtained by:

$$\boldsymbol{W} = \boldsymbol{C}^{-1} \tag{9}$$

The WPLE form is:

$$\hat{\boldsymbol{p}}_{\mathbf{WPLE}} = \left(\boldsymbol{A}^{\mathrm{T}} \boldsymbol{W} \boldsymbol{A}\right)^{-1} \boldsymbol{A}^{\mathrm{T}} \boldsymbol{W} \boldsymbol{b} \tag{10}$$

It is worth noting that the distance between the drone and the target needs to be used in (8). However, the target position is unknown so that the distance is also unknown. The traditional WPLE method uses the localization result obtained by PLE to calculate the target distance. The improvement method of this paper is: first use the covariance matrix $\boldsymbol{Q}$ of the bearing noise as the initial value of the covariance matrix $\boldsymbol{C}$:

$$\boldsymbol{C_0} = \boldsymbol{Q} = diag\left(\sigma_{\mathrm{n1}}^2, \sigma_{\mathrm{n2}}^2, \ldots \sigma_{\mathrm{nk}}^2\right) \tag{11}$$

Then, the initial value of the target position is calculated by (10), and this initial value is used to calculate the target distance. Then (8) is used to recalculate the covariance matrix of the noise term η, and (10) is used to calculate the iterative value of the target position. By repeating the above iterative process, the estimated value of WPLE can be obtained.

2.4 Principle of MLE Method

The likelihood function of the target bearings (K observations) can be written as:

$$f\left(\tilde{\boldsymbol{\theta}}|\boldsymbol{p}\right) = \frac{1}{(2\Pi)^{\frac{K}{2}} |\boldsymbol{Q}|^{\frac{1}{2}}} e^{-\frac{1}{2}\left(\tilde{\boldsymbol{\theta}} - \boldsymbol{\theta}(\boldsymbol{p})\right)^{\boldsymbol{T}} \boldsymbol{Q}^{-1}\left(\tilde{\boldsymbol{\theta}} - \boldsymbol{\theta}(\boldsymbol{p})\right)} \tag{12}$$

where $\boldsymbol{\theta}(\boldsymbol{p}) = [\theta_1(\mathrm{p})\theta_2(\mathrm{p})\ldots\theta_K(\mathrm{p})]^T$, $\tilde{\boldsymbol{\theta}} = \left[\tilde{\theta_1}\tilde{\theta_2}\ldots\tilde{\theta}_K\right]^T$. The MLE method is to use the position where the likelihood function is maximized as the estimated value of the target position. Also, the ML problem can be written as:

$$\min_{\boldsymbol{p}} f(\boldsymbol{p}) = \left(\tilde{\boldsymbol{\theta}} - \boldsymbol{\theta}(\boldsymbol{p})\right)^T \boldsymbol{Q}^{-1}\left(\tilde{\boldsymbol{\theta}} - \boldsymbol{\theta}(\boldsymbol{p})\right) \tag{13}$$

Therefore, solving the problem of maximum likelihood function is reduced to solving the problem of minimum $f(\boldsymbol{p})$. We can use this function as a cost function and use the gradient descent method to iterate the estimated value of the target position. Derivation from Eq. (1) can be obtained:

$$\frac{\partial\boldsymbol{\theta}(\boldsymbol{p})}{\partial x} = \left[\frac{-\Delta y_1}{\mathrm{r_1}^2}\frac{-\Delta y_2}{\mathrm{r_2}^2}\ldots\frac{-\Delta y_K}{\mathrm{r_K}^2}\right]^T, \frac{\partial\boldsymbol{\theta}(\boldsymbol{p})}{\partial y} = \left[\frac{\Delta x_1}{\mathrm{r_1}^2}\frac{\Delta x_2}{\mathrm{r_2}^2}\ldots\frac{\Delta x_K}{\mathrm{r_K}^2}\right]^T \tag{14}$$

where $\Delta \mathrm{x_k} = \mathrm{x} - \mathrm{x_k}$, $\Delta \mathrm{y_k} = \mathrm{y} - \mathrm{y_k}$. Combined with (13), the partial derivative of the cost function with respect to the target position can be obtained by:

$$\frac{\partial f(\boldsymbol{p})}{\partial x} = 2\left(\boldsymbol{\theta}(\boldsymbol{p}) - \tilde{\boldsymbol{\theta}}\right)^T \boldsymbol{Q}^{-1}\frac{\partial\boldsymbol{\theta}(\boldsymbol{p})}{\partial x}, \frac{\partial f(\boldsymbol{p})}{\partial \mathrm{y}} = 2\left(\boldsymbol{\theta}(\boldsymbol{p}) - \tilde{\boldsymbol{\theta}}\right)^T \boldsymbol{Q}^{-1}\frac{\partial\boldsymbol{\theta}(\boldsymbol{p})}{\partial \mathrm{y}} \tag{15}$$

Therefore, the iterative formula is:

$$\boldsymbol{p_{new}} = \boldsymbol{p_{old}} - \mathrm{r} * \left[\frac{\partial f(\boldsymbol{p})}{\partial x}\frac{\partial f(\boldsymbol{p})}{\partial y}\right]^T \tag{16}$$

where $\boldsymbol{p_{new}}$ is the position estimate after one iteration and r is the iteration rate. Both the number of iterations and the iteration rate are hyperparameters and should be selected reasonably according to the actual situation.

2.5 Cramer-Rao Lower Bound Calculation

In the unbiased estimation problem, Cramer-Rao lower bound (CRLB) [9] is usually used to measure the validity of an estimate. If the Root Mean Square Error (RMSE) of an estimated value is smaller and closer to the value obtained by CRLB, then the estimate is said to be more effective. In order to calculate CRLB, we introduce the Fisher Matrix (FIM):

$$\boldsymbol{F} = \begin{bmatrix} -\mathrm{E}\left[\frac{\partial^2}{\partial x^2} f\left(\tilde{\boldsymbol{\theta}}|\boldsymbol{p}\right)\right] & -\mathrm{E}\left[\frac{\partial^2}{\partial x \partial y} f\left(\tilde{\boldsymbol{\theta}}|\boldsymbol{p}\right)\right] \\ -\mathrm{E}\left[\frac{\partial^2}{\partial y \partial x} f\left(\tilde{\boldsymbol{\theta}}|\boldsymbol{p}\right)\right] & -\mathrm{E}\left[\frac{\partial^2}{\partial y^2} f\left(\tilde{\boldsymbol{\theta}}|\boldsymbol{p}\right)\right] \end{bmatrix} \tag{17}$$

According to the likelihood function expression of (12), the above formula can be rewritten as:

$$\boldsymbol{F} = \left[\frac{\partial\boldsymbol{\theta}(\boldsymbol{p})}{\partial x}\frac{\partial\boldsymbol{\theta}(\boldsymbol{p})}{\partial y}\right]^T \boldsymbol{Q}^{-1}\left[\frac{\partial\boldsymbol{\theta}(\boldsymbol{p})}{\partial x}\frac{\partial\boldsymbol{\theta}(\boldsymbol{p})}{\partial y}\right] \tag{18}$$

Combining the partial derivative of $\boldsymbol{\theta}(\boldsymbol{p})$ in (14) with respect to the target position, the further expression of Fisher matrix $\boldsymbol{F}$ can be obtained by:

$$\boldsymbol{F} = \sum_{\mathrm{k}=1}^{\mathrm{K}} \frac{1}{\sigma_{\mathrm{nk}}{}^2 \mathrm{r_k}{}^4}\begin{bmatrix} (\Delta y_K)^2 & -\Delta x_K \Delta y_K \\ -\Delta x_K \Delta y_K & (\Delta x_K)^2 \end{bmatrix} \tag{19}$$

The Cramer-Rao lower bound matrix ***CRLB*** can be obtained by the Fisher matrix:

$$CRLB = \boldsymbol{F}^{-1} \tag{20}$$

$\hat{\boldsymbol{p}} = [\hat{\mathrm{x}}, \hat{\mathrm{y}}]^{\mathrm{T}}$ is the estimation of target position. The relationship between RMSE and CRLB is:

$$RMSE = \sqrt{\mathrm{E}\left[(x - \hat{\mathrm{x}})^2 + (y - \hat{\mathrm{y}})^2\right]} \geq \sqrt{\boldsymbol{CRLB}(1, 1) + \boldsymbol{CRLB}(2, 2)} \tag{21}$$

3 Resource Allocation Method Based on Greedy Algorithm

In this paper, the greedy algorithm is used to realize the resource allocation of the UAV clusters to multiple targets. The principle of the greedy algorithm is to split a complex problem into simple problems, and to obtain an optimal solution for each simple problem.

It can be seen from (3) that when the distance between the UAV and the target is large, the error term η_k brought by the bearing noise will be correspondingly larger, which is unfavorable for the estimation. It can be seen from (8) that when the distance between the UAV and the target is far, the corresponding value in the covariance matrix element of the noise term $\boldsymbol{\eta}$ will also be larger, so the bearing measured by the UAV is not so credible. Therefore, the optimal solution for the resource allocation of a UAV to multiple targets depends on the distance between the drone and the target. Then, the mathematical model for the allocation of K UAVs to N target resources is:

$$\mathrm{selected_k} = \min_i \left\| \boldsymbol{s_k} - \boldsymbol{p_i} \right\| \tag{22}$$

where $\mathrm{i} = 1,2\ldots\mathrm{N}$, $\mathrm{selected_k}$ is the target number assigned to the kth UAV, and $\mathrm{p_i}$ is the position coordinate of the i-th target. To prevent drones from being overallocated to the same target, resulting in waste of resources, the constraint below should be satisfied while allocating resources:

$$\mathrm{N_i} \leq \frac{\mathrm{K}}{\mathrm{N}} \tag{23}$$

where $\mathrm{N_i}$ is the number of UAVs assigned to the i-th target.

4 Passive Localization Algorithm Flow of UAV Clusters for Multi-target

In the above, we have discussed the passive localization mechanism of the UAV clusters for a single target. The method of resource allocation is analyzed. Combining the two can realize the passive localization of the UAV clusters for multiple targets.

But pay attention to (22), when using the greedy algorithm in resource allocation, we need to use the target position before passive positioning, which is what needs to be measured and estimated. Therefore, this paper proposes an algorithm and shows it in Table 2. That is, the released UAV cluster first uses the PLE method to coarsely

Table 2. Multi-target passive localization algorithm

Multi-target passive localization algorithm based on UAV clusters
Step1: Target coarse positioning
Uses PLE to get the coarse position of each target.
Step2: UAV resource allocation
Adopts greedy algorithm to allocate UAV resources.
Step3: Target fine positioning
Uses improved WPLE: Takes $\boldsymbol{Q}$ as the initial value of $\boldsymbol{C}$ and iteratively calculates the target position by (10).
Or uses improved MLE: Employs the PLE result as the start value of iteration. Iteratively gets the localization results according to (16). The initial rate is larger (to increase the convergence speed), and the rate will be reduced after a period of iteration (to reduce the risk of not being able to converge to the optimal point).

locate each target. The estimated values of the PLE method are used to replace the real targets position. Then the greedy algorithm is adopted for UAVs resource allocation. The improved WPLE or MLE method is used for fine positioning.

The PLE algorithm has low computational complexity and does not require iteration. But its positioning accuracy is poor. Therefore, other passive positioning methods (such as WPLE and MLE) need to be used to ensure the positioning accuracy. However, these methods generally have high time complexity and space complexity. The reasonable way for multi-target localization is: after resource allocation, the UAVs assigned to the same target use these methods for positioning so that the accurate passive localization of each target can be performed at the same time. The PLE method used in the algorithm for coarse positioning is only to quickly obtain the approximate position coordinates of the targets, and then the greedy algorithm can be used for resource allocation.

5 Simulation Result Analysis

5.1 System Performance Analysis

In the designed simulation experiment, K = 100, N = 4, that is, a cluster of 100 UAVs passively locates 4 targets. The initially released UAVs and targets are uniformly and randomly distributed in a two-dimensional plane (200x200), as shown in Fig. 2. The standard deviation of the bearing noise for each UAV is 8Π/180. According to the algorithm flow, 100 UAVs first adopt PLE method for coarse positioning of 4 targets in turn, and then adopt the greedy algorithm for resource allocation. The result of resource allocation is shown in Fig. 3, and each target is allocated 25 UAVs equally according to the constraint (23). Subsequently, the UAVs assigned to the same target use WPLE or MLE to accurately locate the target.

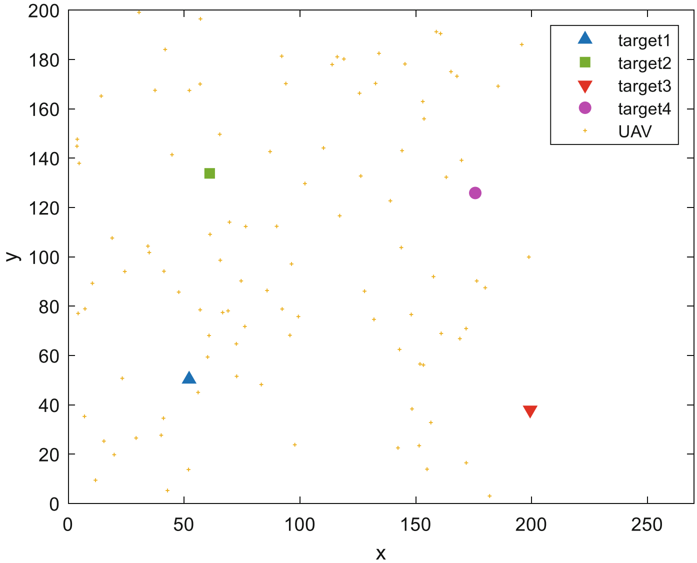

Fig. 2. Initial release of UAV cluster and targets

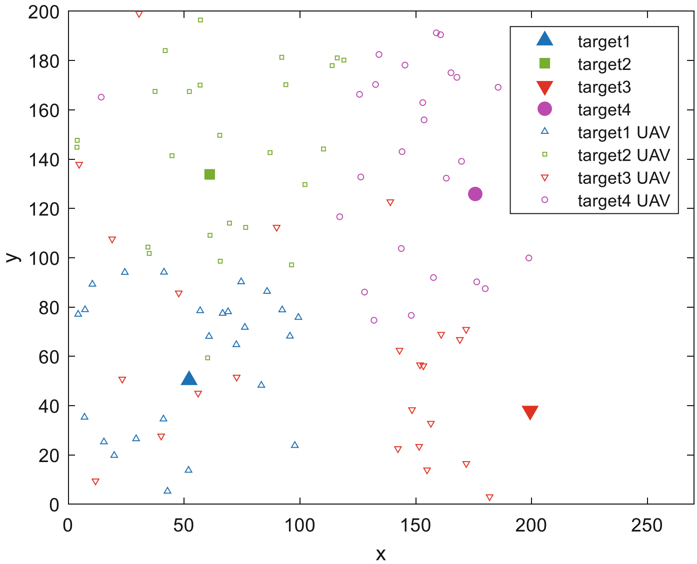

Fig. 3. UAV cluster and targets after resource allocation

The results of coarse positioning and fine positioning are shown in Table 3. The data is the deviation between the estimated value and the true position coordinate. In order to avoid the contingency of the experiment, both coarse positioning and fine positioning were performed 1000 times, and the average value of 1000 estimated values is used as the positioning result.

The UAV cluster passively locates each target again after allocating resources. From the data in Table 3, it can be seen that no matter whether the WPLE method or the MLE method is used, although the observations of a single target is reduced (reduced from 100 to 25), its positioning accuracy is greatly improved compared to the initial positioning accuracy of the PLE method for coarse positioning.

Noted from the calculation process, the time complexity of WPLE and MLE is related to the observation times. Due to the resource allocation, the number of observations is reduced and the fine positioning of each target is carried out at the same time. Therefore, the algorithm greatly reduces the time complexity in multi-target localization scenario.

Table 3. Coarse positioning and fine positioning results

	Target1	Target2	Target3	Target4
Coarse positioning (PLE)	2.7985	4.8116	9.5662	2.1886
Fine positioning (WPLE)	0.1674	0.0385	1.6264	0.3552
Fine positioning (MLE)	0.1302	0.0090	0.3042	0.0297

5.2 Performance Comparison of Different Passive Localization Methods

In this part of the experiment, to compare the performance of different passive localization methods, 25 UAVs locate a single target using PLE, WPLE and MLE. Among them, it costs 0.000602 s using PLE method for a single run, while the WPLE method takes 0.012833 s, and the MLE method takes 0.017344 s.

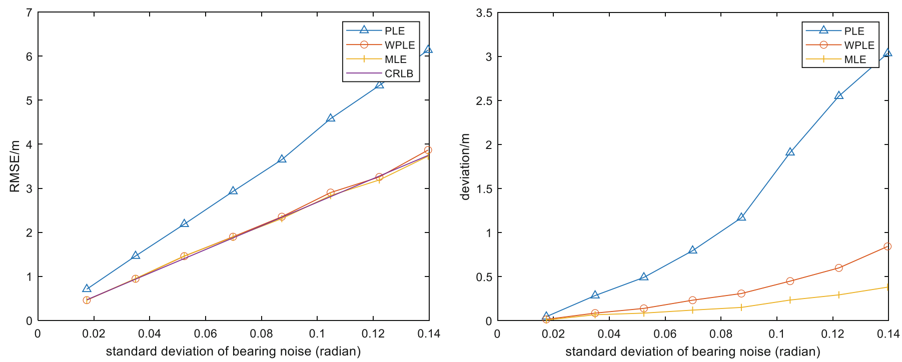

Fig. 4. RMSE and deviation under different bearing noise standard deviations

Figure 4 shows the RMSE and deviations of the three passive localization methods under different bearing noise standard deviations. After fixing the bearing noise standard deviation, three passive localization methods were used to estimate the position of the target 1000 times to obtain the RMSE and deviation of the estimators. We noticed that as the standard deviation of the bearing noise increases, the estimation performance of the three passive localization algorithms is decreasing (the RMSE and the deviation are increasing). At the same time, it can be seen that the estimation performance of the PLE algorithm is the worst, and both the RMSE and the deviation are the largest; the RMSE of WPLE and MLE can approach CRLB, but the deviation of MLE is smaller than WPLE. It can be concluded that the positioning performance of MLE is better than the other two algorithms under the same scene conditions.

Figure 5 shows the effects of the three passive localization methods under different numbers of UAVs. The standard deviation of the bearing noise is 8Π/180, and the three passive localization methods are all performed 1000 times to obtain the RMSE and deviation of the estimators. We can find that RMSE gradually decreases as the number of UAVs increases, but the deviation is almost unchanged. Therefore, the increase in the number of UAVs can improve the estimation performance of the three positioning methods to a certain extent due to the decrease in RMSE.

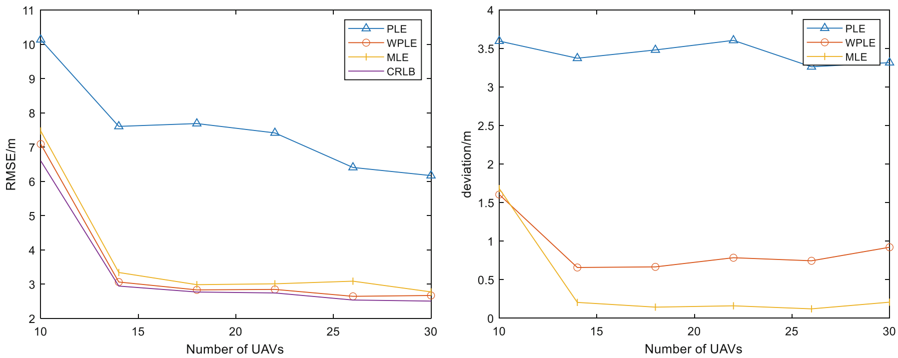

Fig. 5. RMSE and deviation under different numbers of UAVs

6 Conclusion

The main contribution of this paper is to propose a multi-target passive localization algorithm based on UAV clusters and do theoretical and experimental analysis of the algorithm. The proposed algorithm uses PLE for coarse positioning, then uses greedy algorithm for resource allocation, and finally uses improved WPLE or improved MLE for fine positioning. Experiment results shows that the proposed algorithm not only achieves rapid multi-target localization, but also improves the positioning accuracy compared to the traditional PLE method. Therefore, the algorithm is feasible and effective. At the same time, among the three passive positioning methods used in the proposed algorithm, MLE has the excellent estimation performance that both the RMSE and the deviation are the smallest. Otherwise, the estimation performance of the three passive positioning methods improves as the standard deviation of bearing noise decreases and the number of UAVs increases.

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