# Robust Fault-Tolerant Formation Control for Multiple UAVs Under Bounded Actuator Faults

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**Abstract.** To improve the robustness and fault-tolerant ability of the multiple unmanned aerial vehicles (UAVs) system under external disturbances and typical faults, the study of this paper considers the robust fault-tolerant control (FTC) problem of the formation cooperation, and proposes the robust fault-tolerant formation control method under bounded dynamic disturbances and actuator faults with the rotorcraft UAV (RUAV) as the controlled plant. For the formation control, with the dynamic decoupling of the RUAV, an outer guidance loop is designed based on the consensus method and leader-follower mode to achieve the formation reference tracking. For the robust FTC, an inner control loop is constructed and synthesized for the RUAV stability, and some linear matrix inequality (LMI) conditions are proposed for the robust fault-tolerance against bounded disturbances and faults of the controlled plant. Eventually, some simulation results are displayed to illustrate the effectiveness of the proposed method, and compare it with the common formation control.

**Keywords:** Rotorcraft unmanned aerial vehicle · Consensus method · Formation control · Robust control · Fault-tolerant control

## 1 Introduction

With the development of the electromechanical technology and control theory, unmanned systems, especially the unmanned aerial vehicles (UAVs), have been widely used in agriculture, transportation, military, and other fields [1]. However, due to the limited load level and flight range of small UAVs, a single UAV is not competent facing complex and wide-range tasks. Consequently, the orderly cooperation of multiple UAVs becomes a popular task mode [2], which aims at enlarging the capability range of the single unmanned system by the scale effect, and improving the completion effectiveness for cooperative tasks.

Similar to the technical orientation of the control law for an unmanned system, the formation control is also the underlying support and key technology for the swarm cooperation, and attracts lots of attention of researchers. According to the formation mode, leader-follower mode [3] and virtual leader mode [4] are


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effective and common for the formation control design. These two modes all use states from followers and actual/virtual leaders, and achieve the absolute reference and relative formation based on the information complementarity. Furthermore, the consensus method provides the formula forms and theoretical analysis for the above formation control by designing distributed consensus protocols of all group members [5], and is applied for multiple UAVs with different structures [6,7].

In addition, with the actuators and sensors of UAVs becoming complicated and diversified, the robustness and fault-tolerant ability against external disturbances and typical faults affect the formation control performance of multiple UAVs obviously. To ensure the stability and acceptable performance of the single UAV under disturbances and faults, robust control [8] and fault-tolerant control (FTC) [9] are always important research directions. Even though, the robust FTC strategies for rotorcraft UAVs (RUAVs) are usually hard to design due to their own unstable dynamics [10], and the RUAV is more sensitive to uncertain factors than the fixed-wing UAV (FWUAV). In the field of robust fault-tolerant formation control, some research results are also proposed for different UAV platforms. For example, [11] and [12] consider actuator faults of multiple FWUAVs, and achieve the disturbance/fault estimation and controller reconstruction by the sliding mode observer, backstepping FTC, and artificial neural network; [13] and [14] consider actuator faults of multiple RUAVs, and reject the virtual disturbances from actual disturbances and faults by the adaptive control augmentation, which improves the formation control performance of all group members.

The study of this paper focuses on the robust FTC problem of the formation cooperation, and proposes the robust fault-tolerant formation control method under bounded dynamic disturbances and actuator faults for multiple RUAVs system. Firstly, the dynamic model of the RUAV is formulated and analyzed with necessary decoupling and simplification, and an outer guidance loop is designed based on the consensus method and leader-follower mode to achieve the position reference and relative formation of multiple RUAVs. Secondly, with the simplified linear RUAV model, an inner control loop is constructed, and some linear matrix inequality (LMI) conditions are proposed to synthesize the robust fault-tolerant controller gain for the closed-loop control performance. Eventually, some simulation results with multiple RUAVs are displayed to illustrate the control effectiveness, and compare the proposed method with the common formation control.

The main contributions of this paper are to combine the common formation control with robustness and fault-tolerance, and construct the robust FTC method for multiple RUAV members with unstable dynamics. Compared with the existing references [11–14], the consensus method is the important theoretical basis for the formation control, and the robust FTC is independent of accurate fault information.

The remaining parts of this paper are organized as follows: Sect. 2 formulates the dynamic model of the RUAV, and analyzes the formation control problem with the model simplification and decoupling; Sect. 3 introduces the robust fault-tolerant formation control method, including the outer-loop formation guidance

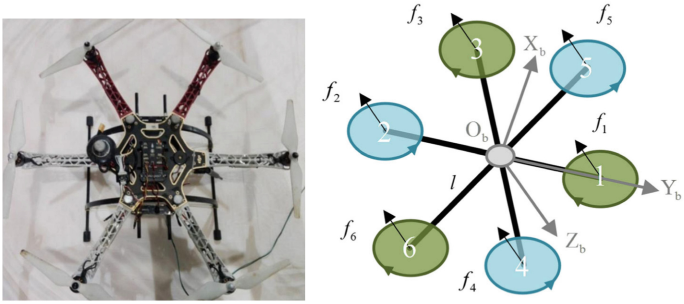


**Fig. 1.** Hexarotor UAV platform and its configuration.

and inner-loop robust FTC; Sect. 4 displays some simulation and comparison results; Sect. 5 ends the whole paper with conclusions.

## 2 RUAV Dynamic Model and Analysis

To design the robust fault-tolerant formation control method for multiple RUAVs, the dynamic model of the controlled plant will be established and simplified in this section, and the control problem will be analyzed further.

With the hexarotor UAV as the controlled plant of the following research [15], as shown in Fig. 1, its fault-free dynamics could be represented as the following equations:

$$\begin{aligned} \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} &= \begin{bmatrix} \cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi \\ \cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi \\ \cos\phi\cos\theta \end{bmatrix} \frac{1}{m}\sum_{i=1}^{6} f_i + \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix}, \\ \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} &= \begin{bmatrix} 1 & \sin\phi\tan\theta & \cos\phi\tan\theta \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi/\cos\theta & \cos\phi/\cos\theta \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix}, \\ \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} &= \boldsymbol{J}^{-1}\left\{ \boldsymbol{B}_f \cdot \left[ f_1\ f_2\ f_3\ f_4\ f_5\ f_6 \right]^{\mathrm{T}} - \boldsymbol{\omega} \times (\boldsymbol{J} \cdot \boldsymbol{\omega}) \right\} \end{aligned} \tag{1}$$

with $\boldsymbol{J} = \begin{bmatrix} I_{xx} & 0 & -I_{xz} \\ 0 & I_{yy} & 0 \\ -I_{xz} & 0 & I_{zz} \end{bmatrix}$ and $\boldsymbol{B}_f = \begin{bmatrix} -l & l & \frac{1}{2}l & -\frac{1}{2}l & -\frac{1}{2}l & \frac{1}{2}l \\ 0 & 0 & \frac{\sqrt{3}}{2}l & -\frac{\sqrt{3}}{2}l & \frac{\sqrt{3}}{2}l & -\frac{\sqrt{3}}{2}l \\ -c & c & -c & c & c & -c \end{bmatrix}$, where $[x\ y\ z]^{\mathrm{T}}$ is the position vector in the north-east-down (NED) coordinate system, $[\phi\ \theta\ \psi]^{\mathrm{T}}$ is the Euler angle vector, $\boldsymbol{\omega} = [p\ q\ r]^{\mathrm{T}}$ is the attitude rate vector, $m$ is the mass, $g$ is the acceleration of gravity, $I_{xx}$, $I_{yy}$, $I_{zz}$, and $I_{xz}$ are the rotational inertias and product of inertial in x-z plane, $l$ is the distance from a rotor to center of gravity, $c$ is a constant parameter for reaction torque, and $f_i$ $(i = 1, ..., 6)$ is the rotor thrust.

As for the flight states regulated by rotor thrusts directly, the vertical dynamics and rotational dynamics in (1) could be linearized and simplified as follows:

$$\dot{v}_z = 0 \cdot v_z - \frac{6}{m} \cdot f_0,$$
$$\left[\begin{array}{c} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \\ \hline \dot{p} \\ \dot{q} \\ \dot{r} \end{array}\right] = \begin{bmatrix} \mathbf{0}_{3\times3} & \boldsymbol{I}_3 \\ \mathbf{0}_{3\times3} & \mathbf{0}_{3\times3} \end{bmatrix} \left[\begin{array}{c} \phi \\ \theta \\ \psi \\ \hline p \\ q \\ r \end{array}\right] + \left[\begin{array}{cccccc} \multicolumn{6}{c}{\mathbf{0}_{3\times6}} \\ \hline -\frac{1}{I_{xx}}l & \frac{1}{I_{xx}}l & \frac{1}{2I_{xx}}l & -\frac{1}{2I_{xx}}l & -\frac{1}{2I_{xx}}l & \frac{1}{2I_{xx}}l \\ 0 & 0 & \frac{\sqrt{3}}{2I_{yy}}l & -\frac{\sqrt{3}}{2I_{yy}}l & \frac{\sqrt{3}}{2I_{yy}}l & -\frac{\sqrt{3}}{2I_{yy}}l \\ -\frac{1}{I_{zz}}c & \frac{1}{I_{zz}}c & -\frac{1}{I_{zz}}c & \frac{1}{I_{zz}}c & \frac{1}{I_{zz}}c & -\frac{1}{I_{zz}}c \end{array}\right] \begin{bmatrix} f_{01} \\ f_{02} \\ f_{03} \\ f_{04} \\ f_{05} \\ f_{06} \end{bmatrix}, \tag{2}$$

where $v_z = \dot{z}$, $f_0 + f_{0i} = f_i$ $(i = 1, ..., 6)$, $\mathbf{0}$ is the zero matrix with the suitable dimension, and $\boldsymbol{I}$ is the identity matrix. As for the position states affected by rotor thrusts indirectly, the pitching angle $\theta$, rolling angle $\phi$, and vertical velocity $v_z$ could be regarded as their virtual control inputs. The reference values of these virtual control inputs are usually designed together with the inner-loop control to construct a hierarchical flight control structure [16].

Consequently, for the formation control of multiple RUAVs, an outer guidance loop would be designed for the virtual control inputs, and the outputs could be regarded as the reference values of some attitudes and velocities to achieve the formation form. Moreover, for the robust fault-tolerance of multiple RUAVs, the simplified model (2) could be represented as the following linear form:

$$\dot{\boldsymbol{x}}_0 = \boldsymbol{A}_0 \cdot \boldsymbol{x}_0 + \boldsymbol{B}_0 \cdot \boldsymbol{\Gamma} \cdot \boldsymbol{u} + \boldsymbol{B}_{\boldsymbol{w}0} \cdot \boldsymbol{w}_0, \tag{3}$$

where $\boldsymbol{x}_0 \in \mathbb{R}^{n_x}$, $\boldsymbol{u} \in \mathbb{R}^{n_u}$, and $\boldsymbol{w}_0 \in \mathbb{R}^{n_w}$ are the original system state vector, control input vector, and bounded dynamic disturbance vector, $\boldsymbol{A}_0$, $\boldsymbol{B}_0$, and $\boldsymbol{B}_{\boldsymbol{w}0}$ are the system matrix, input matrix, and disturbance input matrix, the diagonal matrix $\boldsymbol{\Gamma} = \mathrm{diag}(\gamma_1, ..., \gamma_i, ..., \gamma_{n_u})$ means the actuator faults, and $0 \leq \gamma_i \leq 1$ represents the partial loss of control effectiveness. The inner-loop robust FTC would be designed for multiple RUAVs with typical form (3).

## 3 Robust Fault-Tolerant Formation Control

Based on the previous dynamic model and control problem analysis, this section will introduce the robust fault-tolerant formation control method for the multiple RUAVs system, as shown in Fig. 2.

### 3.1 Outer-Loop Formation Guidance

The outer loop of the robust fault-tolerant formation control for multiple RUAVs focuses on the virtual control inputs. By designing the references values for the pitching angle, rolling angle, and vertical velocity, the position control could be achieved for the absolute reference value and relative formation form. The consensus method provides the formula forms with the leader-follower mode.

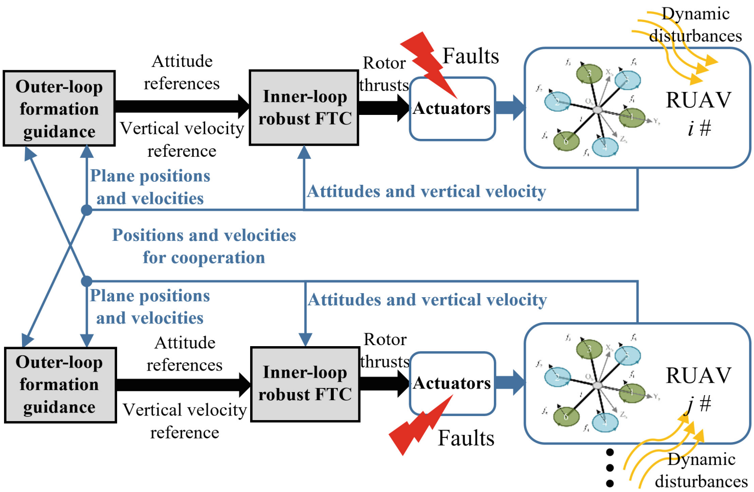


**Fig. 2.** Robust fault-tolerant formation control structure for multiple RUAVs.

As for a vehicle group with $N$ member nodes, its communication topology could be represented as a directed graph $\boldsymbol{G}(\boldsymbol{v}, \boldsymbol{\epsilon}, \boldsymbol{\mathcal{A}})$, where $\boldsymbol{v} = \{v_1, ..., v_N\}$ is a finite nonempty node set, $\boldsymbol{\epsilon} \subseteq \boldsymbol{v} \times \boldsymbol{v}$ is an edge set of ordered pairs of nodes, and $\boldsymbol{\mathcal{A}} = [a_{ij}] \in \mathbb{R}^{N \times N}$ is the adjacency matrix. If $(j, i) \in \boldsymbol{\epsilon}$, the value of $a_{ij}$ is positive that means the information is transmitted from the $j$th node to $i$th node. If $(j, i) \notin \boldsymbol{\epsilon}$, the value of $a_{ij}$ is 0.

With the vertical velocity $v_z$ as the virtual control input of the vertical position $z$, the RUAV could be regarded as the single-integrator system; with the attitudes $\theta$ and $\phi$ as the virtual control inputs of the plane positions $x$ and $y$, the RUAV could be regarded as the double-integrator system. Based on the consensus method and the above directed graph, the formation guidance laws are formulated for the $i$th RUAV member as follows:

$$
\begin{aligned}
v_{z\mathrm{ref}i} =& \dot{z}_{\mathrm{ref}i} + \alpha_{zi} \cdot (z_{\mathrm{ref}i} - z_i) + \sum_{j=1}^{N} a_{ij} \cdot [(z_j - z_{\mathrm{ref}j}) - (z_i - z_{\mathrm{ref}i})] \, , \\
\phi_{\mathrm{ref}i} =& \ddot{\tilde{y}}_{\mathrm{ref}i} + \alpha_{yi} \cdot [\tilde{y}_{\mathrm{ref}i} - \tilde{y}_i + \beta_{yi} \cdot (-\tilde{v}_{yi})] \\
& + \sum_{j=1}^{N} a_{ij} \cdot [(\tilde{y}_j - \tilde{y}_{\mathrm{ref}j}) - (\tilde{y}_i - \tilde{y}_{\mathrm{ref}i}) + \beta_{yi} \cdot (\tilde{v}_{yj} - \tilde{v}_{yi})] \, , \\
\tilde{\theta}_{\mathrm{ref}i} =& \ddot{\tilde{x}}_{\mathrm{ref}i} + \alpha_{xi} \cdot [\tilde{x}_{\mathrm{ref}i} - \tilde{x}_i + \beta_{xi} \cdot (-\tilde{v}_{xi})] \\
& + \sum_{j=1}^{N} a_{ij} \cdot [(\tilde{x}_j - \tilde{x}_{\mathrm{ref}j}) - (\tilde{x}_i - \tilde{x}_{\mathrm{ref}i}) + \beta_{xi} \cdot (\tilde{v}_{xj} - \tilde{v}_{xi})] \, , \\
\theta_{\mathrm{ref}i} =& - \tilde{\theta}_{\mathrm{ref}i} ,
\end{aligned}
\tag{4}
$$

where the subscript $i$ or $j$ means the member number, $\alpha_{zi}$, $\alpha_{xi}$, $\alpha_{yi}$, $\beta_{xi}$, and $\beta_{yi}$ are all positive parameters, $x_{\text{ref}i}$, $y_{\text{ref}i}$, and $z_{\text{ref}i}$ are all position references satisfying some formation forms, and

$$\begin{bmatrix} \tilde{x}_{\text{ref}i} \\ \tilde{y}_{\text{ref}i} \end{bmatrix} = \begin{bmatrix} \cos\psi_i & \sin\psi_i \\ -\sin\psi_i & \cos\psi_i \end{bmatrix} \begin{bmatrix} x_{\text{ref}i} \\ y_{\text{ref}i} \end{bmatrix}, \begin{bmatrix} \tilde{x}_i \\ \tilde{y}_i \end{bmatrix} = \begin{bmatrix} \cos\psi_i & \sin\psi_i \\ -\sin\psi_i & \cos\psi_i \end{bmatrix} \begin{bmatrix} x_i \\ y_i \end{bmatrix}, \begin{bmatrix} \tilde{v}_{xi} \\ \tilde{v}_{yi} \end{bmatrix} = \begin{bmatrix} \dot{\tilde{x}}_i \\ \dot{\tilde{y}}_i \end{bmatrix}.$$

Eventually, the above formation guidance laws obtain the references values $v_{z\text{ref}i}$, $\phi_{\text{ref}i}$, and $\theta_{\text{ref}i}$ for the vertical velocity, rolling angle, and pitching angle to achieve absolute reference and relative formation.

According to the form of (4), the outer-loop formation guidance relies on the own states and states from adjacent members. In the special case with only one information transmitter, (4) is with the typical leader-follower mode. However, if a member receives information from many adjacent members, it would own more than one nominal leaders, and their contributions depend on the adjacency matrix of the communication topology.

### 3.2 Inner-Loop Robust FTC

The inner loop of the robust fault-tolerant formation control for multiple RUAVs focuses on the actual control inputs and linear model. A robust fault-tolerant controller and its synthesis conditions would be proposed to track the reference values from the outer-loop formation guidance. Passive fault-tolerant methods provide the controller form and theoretical basic.

To ensure some states $\boldsymbol{y}_0 = \boldsymbol{C} \cdot \boldsymbol{x}_0 \in \mathbb{R}^{n_y}$ in (3) for the reference vector $\boldsymbol{y}_{\text{ref}}$ (as the member of multiple RUAVs, flight states $[v_y \ \phi \ \theta \ \psi]^{\text{T}}$ should track the reference values $[v_{y\text{ref}} \ \phi_{\text{ref}} \ \theta_{\text{ref}} \ \psi_{\text{ref}}]^{\text{T}}$), the integrations of tracking errors are defined as follows:

$$\dot{\boldsymbol{e}}_{\boldsymbol{y}} = \boldsymbol{y}_{\text{ref}} - \boldsymbol{C} \cdot \boldsymbol{x}_0.$$

Together with (3), an augmented linear model is formed as follows:

$$\underbrace{\begin{bmatrix} \dot{\boldsymbol{x}}_0 \\ \dot{\boldsymbol{e}}_{\boldsymbol{y}} \end{bmatrix}}_{\dot{\boldsymbol{x}}} = \underbrace{\begin{bmatrix} \boldsymbol{A}_0 & \boldsymbol{0}_{n_x \times n_y} \\ -\boldsymbol{C} & \boldsymbol{0}_{n_y \times n_y} \end{bmatrix}}_{\boldsymbol{A}} \underbrace{\begin{bmatrix} \boldsymbol{x}_0 \\ \boldsymbol{e}_{\boldsymbol{y}} \end{bmatrix}}_{\boldsymbol{x}} + \underbrace{\begin{bmatrix} \boldsymbol{B}_0 \\ \boldsymbol{0}_{n_y \times n_u} \end{bmatrix}}_{\boldsymbol{B}} \boldsymbol{\Gamma} \cdot \boldsymbol{u} + \underbrace{\begin{bmatrix} \boldsymbol{B}_{w0} & \boldsymbol{0}_{n_w \times n_y} \\ \boldsymbol{0}_{n_y \times n_y} & \boldsymbol{I}_{n_y} \end{bmatrix}}_{\boldsymbol{B}_w} \underbrace{\begin{bmatrix} \boldsymbol{w}_0 \\ \boldsymbol{y}_{\text{ref}} \end{bmatrix}}_{\boldsymbol{w}}.$$

For the above linear controlled plant with bounded actuator faults $\boldsymbol{\Gamma}$ and disturbances $\boldsymbol{w}$, a state-feedback controller $\boldsymbol{u} = \boldsymbol{K} \cdot \boldsymbol{x}$ is designed for the following closed-loop system:

$$\dot{\boldsymbol{x}} = (\boldsymbol{A} + \boldsymbol{B} \cdot \boldsymbol{\Gamma} \cdot \boldsymbol{K})\boldsymbol{x} + \boldsymbol{B}_w \cdot \boldsymbol{w}, \tag{5}$$

where $\boldsymbol{K}$ is the unknown controller gain. The following theorem would synthesize the controller gain to ensure the closed-loop robustness and fault-tolerance.

**Theorem 1.** *Under the partial loss of actuator effectiveness $\gamma \cdot \boldsymbol{I}_{n_u} \leq \boldsymbol{\Gamma} \leq \boldsymbol{I}_{n_u}$ and bounded disturbance vector $\boldsymbol{w}$, the closed-loop system (5) would be robustly* *stable, if there exist a positive definite matrix* $\boldsymbol{X}$ *and a suitable matrix* $\boldsymbol{M}$ *satisfying the following LMI condition:*

$$\begin{bmatrix} \boldsymbol{X}\cdot\boldsymbol{A}^{\mathrm{T}}+\boldsymbol{A}\cdot\boldsymbol{X}+\boldsymbol{M}^{\mathrm{T}}\cdot\boldsymbol{B}^{\mathrm{T}}+\boldsymbol{B}\cdot\boldsymbol{M} & (1-\gamma)\boldsymbol{B} & \boldsymbol{M}^{\mathrm{T}} & \boldsymbol{B}_w \\ * & -\boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} \\ * & * & -\boldsymbol{I} & \boldsymbol{0} \\ * & * & * & -\boldsymbol{I} \end{bmatrix} < 0, \tag{6}$$

where $*$ means the transposed element in the symmetric position of a matrix. The controller gain satisfying the above conditions is $\boldsymbol{K}=\boldsymbol{M}\cdot\boldsymbol{X}^{-1}$.

*Proof.* For the closed-loop system (5), a Lyapunov function is designed as follows:

$$V(\boldsymbol{x})=\boldsymbol{x}^{\mathrm{T}}\cdot\boldsymbol{P}\cdot\boldsymbol{x},$$

where $\boldsymbol{P}>0$ is a positive definite matrix. Consider its derivative with respect to time:

$$\begin{aligned}\dot{V}(\boldsymbol{x}) =&\dot{\boldsymbol{x}}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{x}+\boldsymbol{x}^{\mathrm{T}}\boldsymbol{P}\dot{\boldsymbol{x}}\\ &=\left[(\boldsymbol{A}+\boldsymbol{B}\boldsymbol{\Gamma}\boldsymbol{K})\boldsymbol{x}+\boldsymbol{B}_w\boldsymbol{w}\right]^{\mathrm{T}}\boldsymbol{P}\boldsymbol{x}+\boldsymbol{x}^{\mathrm{T}}\boldsymbol{P}\left[(\boldsymbol{A}+\boldsymbol{B}\boldsymbol{\Gamma}\boldsymbol{K})\boldsymbol{x}+\boldsymbol{B}_w\boldsymbol{w}\right]\\ &=\boldsymbol{x}^{\mathrm{T}}\left[(\boldsymbol{A}^{\mathrm{T}}+\boldsymbol{K}^{\mathrm{T}}\boldsymbol{B}^{\boldsymbol{T}})\boldsymbol{P}+\boldsymbol{P}(\boldsymbol{A}+\boldsymbol{B}\boldsymbol{K})+2\boldsymbol{P}\boldsymbol{B}(\boldsymbol{\Gamma}-\boldsymbol{I})\boldsymbol{K}\right]\boldsymbol{x}+2\boldsymbol{x}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{B}_w\cdot\boldsymbol{w}\\ &\le\boldsymbol{x}^{\mathrm{T}}\left[(\boldsymbol{A}^{\mathrm{T}}+\boldsymbol{K}^{\mathrm{T}}\boldsymbol{B}^{\boldsymbol{T}})\boldsymbol{P}+\boldsymbol{P}(\boldsymbol{A}+\boldsymbol{B}\boldsymbol{K})+2\boldsymbol{P}\boldsymbol{B}(\boldsymbol{\Gamma}-\boldsymbol{I})\boldsymbol{K}+\right.\\ &\quad\left.\boldsymbol{P}\boldsymbol{B}_w\boldsymbol{B}_w^{\mathrm{T}}\boldsymbol{P}\right]\boldsymbol{x}+\boldsymbol{w}^{\mathrm{T}}\boldsymbol{w}.\end{aligned}$$

Note that, if design $\boldsymbol{P}$ and $\boldsymbol{K}$ to ensure

$$(\boldsymbol{A}^{\mathrm{T}}+\boldsymbol{K}^{\mathrm{T}}\boldsymbol{B}^{\boldsymbol{T}})\boldsymbol{P}+\boldsymbol{P}(\boldsymbol{A}+\boldsymbol{B}\boldsymbol{K})+2\boldsymbol{P}\boldsymbol{B}(\boldsymbol{\Gamma}-\boldsymbol{I})\boldsymbol{K}+\boldsymbol{P}\boldsymbol{B}_w\boldsymbol{B}_w^{\mathrm{T}}\boldsymbol{P}<0, \tag{7}$$

there would be a small positive value $\lambda$ satisfying $\dot{V}(\boldsymbol{x})\le-\lambda\cdot\boldsymbol{x}^{\mathrm{T}}\boldsymbol{x}+\boldsymbol{w}^{\mathrm{T}}\boldsymbol{w}$. In this case, if $\boldsymbol{x}^{\mathrm{T}}\boldsymbol{x}>\boldsymbol{w}^{\mathrm{T}}\boldsymbol{w}/\lambda$, there would be $\dot{V}(\boldsymbol{x})<0$, the value of $\boldsymbol{x}^{\mathrm{T}}\boldsymbol{x}$ would be reduced, and the bounded stability with $\boldsymbol{x}^{\mathrm{T}}\boldsymbol{x}\le\boldsymbol{w}^{\mathrm{T}}\boldsymbol{w}/\lambda$ would be achieved finally. So (7) is the sufficient condition for the closed-loop robustness under actuator faults.

By pre- and post-multiplying (7) by $\boldsymbol{X}=\boldsymbol{P}^{-1}$, the left hand side is same as the following form with $\boldsymbol{M}=\boldsymbol{K}\cdot\boldsymbol{P}^{-1}$:

$$\begin{aligned}&(\boldsymbol{A}^{\mathrm{T}}+\boldsymbol{K}^{\mathrm{T}}\boldsymbol{B}^{\boldsymbol{T}})\boldsymbol{P}+\boldsymbol{P}(\boldsymbol{A}+\boldsymbol{B}\boldsymbol{K})+2\boldsymbol{P}\boldsymbol{B}(\boldsymbol{\Gamma}-\boldsymbol{I})\boldsymbol{K}+\boldsymbol{P}\boldsymbol{B}_w\boldsymbol{B}_w^{\mathrm{T}}\boldsymbol{P}\\ &=(\boldsymbol{X}\boldsymbol{A}^{\mathrm{T}}+\boldsymbol{A}\boldsymbol{X}+\boldsymbol{M}^{\mathrm{T}}\boldsymbol{B}^{\mathrm{T}}+\boldsymbol{B}\boldsymbol{M})+2\boldsymbol{B}(\boldsymbol{\Gamma}-\boldsymbol{I})\boldsymbol{M}+\boldsymbol{B}_w\boldsymbol{B}_w^{\mathrm{T}}\\ &\le(\boldsymbol{X}\boldsymbol{A}^{\mathrm{T}}+\boldsymbol{A}\boldsymbol{X}+\boldsymbol{M}^{\mathrm{T}}\boldsymbol{B}^{\mathrm{T}}+\boldsymbol{B}\boldsymbol{M})+\boldsymbol{B}(\boldsymbol{I}-\boldsymbol{\Gamma})(\boldsymbol{I}-\boldsymbol{\Gamma})\boldsymbol{B}^{\mathrm{T}}+\boldsymbol{M}^{\mathrm{T}}\boldsymbol{M}+\boldsymbol{B}_w\boldsymbol{B}_w^{\mathrm{T}}\\ &\le(\boldsymbol{X}\boldsymbol{A}^{\mathrm{T}}+\boldsymbol{A}\boldsymbol{X}+\boldsymbol{M}^{\mathrm{T}}\boldsymbol{B}^{\mathrm{T}}+\boldsymbol{B}\boldsymbol{M})+(1-\gamma)^2\boldsymbol{B}\boldsymbol{B}^{\mathrm{T}}+\boldsymbol{M}^{\mathrm{T}}\boldsymbol{M}+\boldsymbol{B}_w\boldsymbol{B}_w^{\mathrm{T}}.\end{aligned}$$

So the sufficient condition for the closed-loop robustness and fault-tolerance is $(\boldsymbol{X}\boldsymbol{A}^{\mathrm{T}}+\boldsymbol{A}\boldsymbol{X}+\boldsymbol{M}^{\mathrm{T}}\boldsymbol{B}^{\mathrm{T}}+\boldsymbol{B}\boldsymbol{M})+(1-\gamma)^2\boldsymbol{B}\boldsymbol{B}^{\mathrm{T}}+\boldsymbol{M}^{\mathrm{T}}\boldsymbol{M}+\boldsymbol{B}_w\boldsymbol{B}_w^{\mathrm{T}}<0$, which is equivalent to (6) based on Schur complement. □

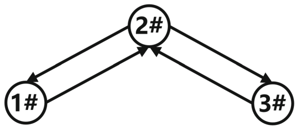


**Fig. 3.** Directed graph of communication topology.

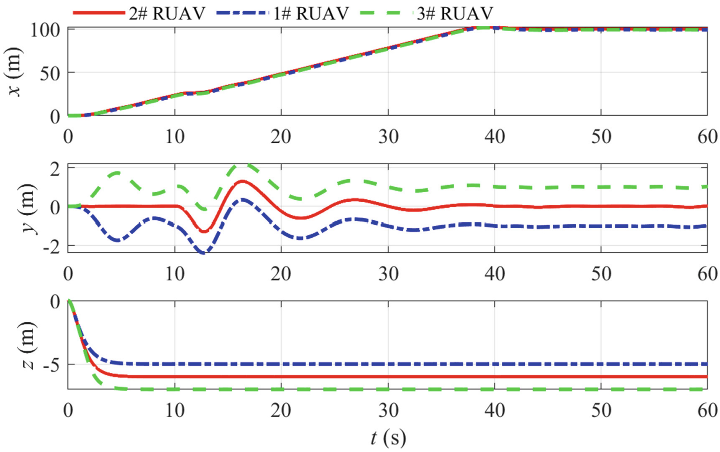


**Fig. 4.** Position curves of multiple RUAVs with robust fault-tolerant formation control.

## 4 Simulation Validation

To validate the effectiveness of the above robust fault-tolerant formation control method, three hexarotor UAVs are applied for simulation results, which are distinguished as 1#–3#, and their communication topology is shown in Fig. 3. In the simulation validation, the following actuator faults are introduced as the partial loss of rotor thrusts:

$$\gamma_2 = \begin{cases} 1 & t < 10\,\text{s} \\ 0.5 & t \geq 10\,\text{s} \end{cases}, \quad \gamma_4 = \begin{cases} 1 & t < 10\,\text{s} \\ 0.5 & t \geq 10\,\text{s} \end{cases}$$

Some random process noises are also introduced as the bounded dynamic disturbances of RUAVs.

For the formation control, 1# RUAV and 3# RUAV apply their own states and states from 2# RUAV to achieve "Λ" formation form. Figures 4 and 5 display the position curves of multiple RUAVs with the robust fault-tolerant formation control. These curves indicate that three members track absolute reference values and keep a certain relative formation form. The formation control performance is ensured effectively.

The above formation form is affected by the attitude control performance of RUAVs towards reference values from the out-loop formation guidance. Figure 6 shows the attitude curves of multiple RUAVs under faults and disturbances.

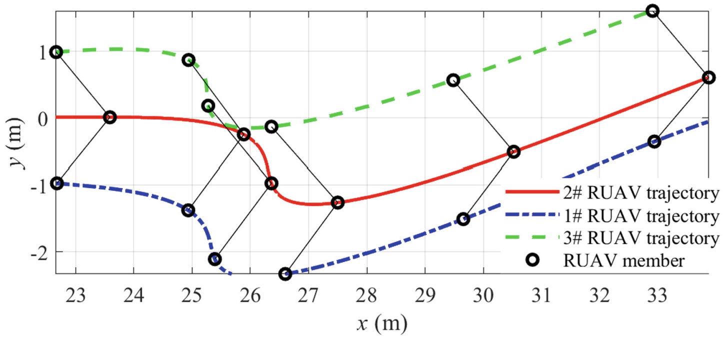


**Fig. 5.** Plane positions of multiple RUAVs with robust fault-tolerant formation control.

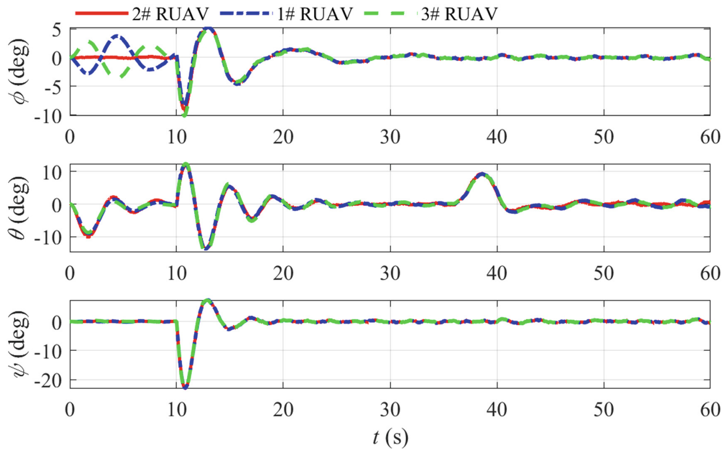


**Fig. 6.** Attitude curves of multiple RUAVs with robust fault-tolerant formation control.

Although RUAV attitudes vibrate in a degree after actuator faults, the inner-loop robust FTC stabilizes controlled plants rapidly due to its own fault-tolerant ability and robust control performance.

In the case with the common formation control without fault-tolerant ability, the attitude controller is usually incompetent to stabilize RUAVs under actuator faults, as shown in Fig. 7. The closely related aftermath is that multiple RUAVs are hard to track reference values or keep a formation form, as shown in Fig. 8.

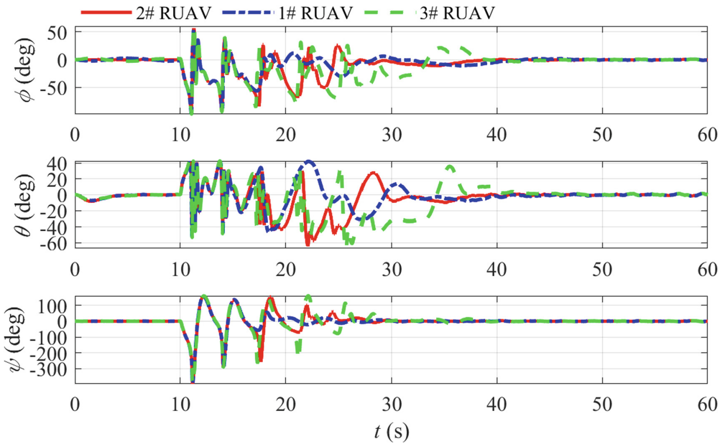


**Fig. 7.** Attitude curves of multiple RUAVs with common formation control.

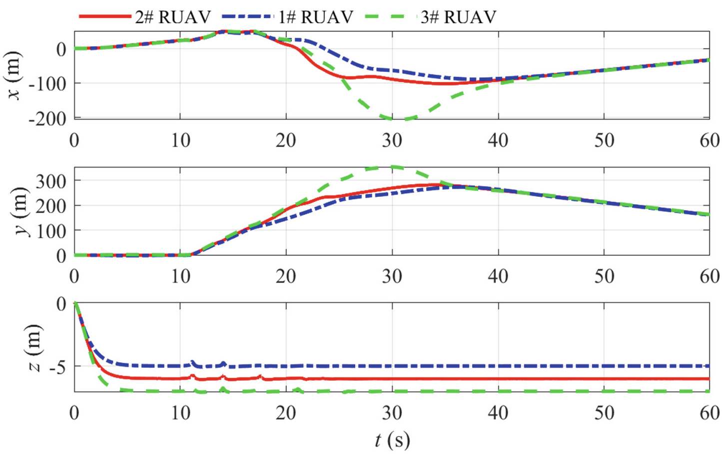


**Fig. 8.** Position curves of multiple RUAVs with common formation control.

## 5 Conclusion

To improve the robustness and fault-tolerant ability of multiple UAVs under external disturbances and typical faults, this paper designs an outer guidance loop based on the consensus method and an inner control loop against bounded dynamic disturbances and actuator faults, and proposes the robust fault-tolerant formation control method for the multiple RUAVs system. Simulation results indicate that the proposed method stabilizes controlled plants under actuator faults effectively, and ensures every member in the group to track the reference position and keep the formation form.

In the future work, the multiple FWUAVs system under wind disturbances and destroyed aerodynamic surfaces will be considered to validate the proposed method further. Moreover, based on the current formation control structure, the general robust FTC method for multiple UAVs will be researched against different faults to improve the formation control performance in real applications.

## References

1. List of Unmanned Aerial Vehicle Applications. http://en.wikipedia.org/wiki/List_of_unmanned_aerial_vehicle_applications. Accessed 3 Oct 2021
2. Office of the Secretary of Defense: Unmanned Aircraft Systems Roadma, pp. 2005–2030. DoD, Washington (2005)
3. Li, G., Jiang, Z., Jia, Q.: Simulation of distributed formation control system for multiple UAVs. Comput. Simul. **27**(2), 101–104 (2010)
4. Wang, D., Zong, Q., Zhang, B., et al.: Fully distributed finite-time formation control for multiple UAVs. Control Decis. **34**(12), 2656–2660 (2019)
5. Ren, W., Beard, R.W.: Distributed Consensus in Multi-Vehicle Cooperative Control: Theory and Applications. Springer, Cham (2008). https://doi.org/10.1007/978-1-84800-015-5
6. Cheng, H., Qi, X., Yang, S., et al.: A method for obstacle avoidance of consistency based quadrotor UAV formation. Flight Dyn. **37**(2), 51–55 (2019)
7. Zhu, X., Zhang, X., Yan, M., et al.: UAV formation control strategy based on consensus. Comput. Simul. **33**(8), 30–34 (2016)
8. Mei, S., Shen, T., Liu, K.: Modern Robust Control Theory and Application, 2nd edn. Tsinghua University Press, Beijing (2008)
9. Zhang, Y., Jiang, J.: Bibliographical review on reconfigurable fault-tolerant control systems. Annu. Rev. Control. **32**(2), 229–252 (2008)
10. Qi, X., Qi, J., Theilliol, D., et al.: A review on fault diagnosis and fault tolerant control methods for single rotor aerial vehicles. J. Intell. Robot. Syst. **73**(1–4), 535–555 (2014)
11. Li, B., Dong, W., Ma, X.: Back-stepping-fault-tolerant control for keeping the formation of unmanned aerial vehicles. Acta Armamentarii **11**, 2172–2184 (2018)
12. Yu, Z., Zhang, Y., Jiang, B., et al.: Distributed adaptive fault-tolerant close formation flight control of multiple trailing fixed-wing UAVs. ISA Trans. **106**, 181–199 (2020)
13. Zhao, X., Zong, Q., Tian, B., et al.: Finite-time fault-tolerant formation control for multiquadrotor systems with actuator fault. Int. J. Robust Nonlinear Control **28**, 5386–5405 (2018)
14. Liu, D., Liu, H., Xi, J.: Fully distributed adaptive fault-tolerant formation control for octorotors subject to multiple actuator faults. Aerosp. Sci. Technol. **109**, 1–13 (2021)
15. Liu, Z., Theilliol, D., Yang, L.: Interconnection and damping assignment passivity-based control design under loss of actuator effectiveness. J. Intell. Robot. Syst. **100**, 29–45 (2020)
16. Dai, B., He, Y., Gu, F., et al.: Acceleration feedback enhanced controller for wind disturbance rejection of rotor unmanned aerial vehicle. Robot **42**(1), 79–88 (2020)