Resilient Time-Varying Formation Control of Second-Order Discrete-Time Multi-agent Systems with Actuator Faults and Attacks on Communication Link

Yue Wang[1,2], Liang Han[1], Xiaoduo Li[3(✉)], Xiwang Dong[4], Qingdong Li[4], and Zhang Ren[4]

[1] Sino-French Engineer School, Beihang University, Beijing 100191, People's Republic of China
[2] Ecole CentraleSupélec, 91190 Gif-sur-Yvette, France
[3] Department of Automation, Shanghai Jiao Tong University, Shanghai 200240, People's Republic of China
lixiaoduo@sjtu.edu.cn
[4] School of Automation Science and Electrical Engineering, Beihang University, Beijing 100191, People's Republic of China

Abstract. In this paper, the resilient time-varying formation control for second-order discrete-time multi-agent systems (DMASs) under communication link attacks and actuator faults is investigated. The number or proportion of edge attacks around each agent, including deception attacks and denial-of-service (DOS) attacks, is bounded. A discrete-time distributed formation protocol based on trim of extreme values and fault estimation and compensation is given. Using robust graph properties and discrete stability theory, sufficient conditions for the given DMASs to realize the desired formation with bounded error are derived. Numerical simulation examples are applied to illustrate the effectiveness of the resilient formation strategy.

Keywords: Resilient control · Time-varying formation · Discrete-time multi-agent systems · Communication link attack · Actuator faults

1 Introduction

In the past decades, the research on multi-agent systems (MASs) has attracted much attention due to its extensive applications, such as information fusion [1], source seeking [2], and formation control of unmanned aerial vehicles [3].

Since MASs consist of interconnected agents, they are especially vulnerable to adversarial attacks. The influence of attacks on agents or communication links among agents can spread through the network and degrade the performance of the whole system. Hence, resilient control problems have received increasing attention from the control system community [4].

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In the resilient control literature, a general model to describe adversarial threat is to suppose that the number of compromised agents is bounded for the whole system or around each agent. In the landmark work [5], a novel definition of network robustness, named r-robustness, is introduced, and an improved Mean-Subsequence-Reduced (MSR) algorithm [6] was applied to guarantee leaderless consensus of uncompromised agents under bounded adversarial attacks. The results have been later extended to the case of leader-follower consensus [7], trusted nodes [8], and two-hop information-based approaches [9]. In [10], the authors consider locally bounded edge deception attacks instead of attacks on agents. As for formation control, a generalization of consensus control, the research is limited to first-order time-invariant case [11]. However, the resilient time-varying formation of second-order MAS, which has a more important application value, has not been widely investigated. What's more, first, in the aforementioned works, DOS attacks are not considered, which can reduce network connectivity and robustness, and make it difficult to apply MSR algorithms. Second, most of the existing works only deal with the case where the number of attacked nodes or edges is bounded, while for large-scale systems and for scalability issue, it is also of interest to study the case where their proportion is bounded.

Physical agents are also susceptible to component faults, which is the focus of fault-tolerant control/fault-tolerant cooperative control (FTC/FTCC) [12]. Several actuator faults such as loss of effectiveness, float, lock-in-place, and hard-over-failure [13] have been investigated. In [14], a framework of fault detection, fault isolation, fault estimation (FE), and fault compensation (FC) for continuous-time Takagi-Sugeno (T-S) fuzzy systems is investigated. For discrete-time systems, Gao [15] develops a simultaneous state and fault estimator as well as actuator/sensor compensation scheme to meet prescribed disturbance attenuation performance for linear systems. In [16], a distributed reduced-order fault estimation observer is derived for both continuous-time and discrete-time multi-agent systems. However, reliable information exchange between agents is needed, and the method is not applicable when the network communication links are under adversarial attacks.

In this paper, resilient distributed time-varying formation problem for second-order DMASs under communication link attacks and actuator faults is addressed. A discrete-time time-varying formation protocol based on MSR and FE/FC is developed. Utilizing graph theory, descriptor observer technique, and discrete stability theory, sufficient conditions for DMASs to realize time-varying formation with bounded error are obtained. Finally, simulations are conducted to illustrate the effectiveness of theoretical results.

The innovations of this paper are fourfold. First, we extend the works of [3,10,11] to resilient time-varying formation control of second-order DMASs. Second, DOS attacks on communication links are considered in this paper, whereas this type of attacks is neglected in the works [7–10]. Third, the model in [10] which considers a bounded number of attacks around each agent will be extended to the case with a bounded fraction of attacks to improve the algorithm's scalability capacity. Fourth, the single-agent FE/FC scheme of [15] will be combined with MSR algorithm to be applied for DMASs to attenuate the effects of actuator faults, whereas malfunctions of physical components are not considered in [7–10].

The remainder of this paper is organized as follows. In Sect. 2, preliminaries of graph theory, the attack model and problem description are presented. In Sect. 3, a resilient time-varying formation protocol composed by two parts is given and some theorems that ensure the realization of formation with bounded error is proven. In Sect. 4, a numerical simulation of six agents circular formation is performed. Finally, conclusions are summarized in Sect. 5.

Notations: For a matrix M, $\sigma_M(\cdot)$ and $\sigma_m(\cdot)$ denotes its largest and smallest singular value, M^{T} denotes its transpose. For a matrix, $*$ represents the terms induced by symmetry. Let x be a random variable, $x \sim \mathrm{N}(\mu, \Sigma)$ means that x follows a Gaussian distribution with μ the mean vector and Σ the covariance matrix. $\mathrm{diag}(A, B)$ with square matrices A and B denotes the block diagonal matrix $\begin{bmatrix} A & 0 \\ 0 & B \end{bmatrix}$.

2 Preliminaries and Problem Formulation

2.1 Basic Concepts of Graph Theory

Let $G = (V, E, A)$ denote a directed graph with the node set $V = \{1, \ldots, N\}$, the edge set $E \subseteq V \times V$ and the adjacency matrix $A = [a_{ij}] \in \mathbb{R}^{N \times N}$. $a_{ij} > 0$ if $(j, i) \in E$ and $a_{ij} = 0$, otherwise. If $(j, i) \in E$, j is an in-neighbour of i, i is an out-neighbour of j and (j, i) is called the incoming edge of i from j. We assume $a_{ii} = 0$ for any $i \in V$. Denote $N_i = \{j \in V : (j, i) \in E\}$.

Definition 1 (Set reachability and graph robustness [5])

Given a digraph G and a nonempty subset $\mathcal{S}$ of nodes of G, $\mathcal{S}$ is an r-reachable set (a p-fraction reachable set) if $\exists i \in \mathcal{S}$ such that $|N_i \backslash \mathcal{S}| \geq r$, where $r \in \mathbb{Z}_{\geq 0}$ ($|N_i \backslash \mathcal{S}| \geq p\,|N_i|$, where $0 \leq p \leq 1$).

G is r-robust (p-fraction robust) if for every pair of nonempty, disjoint subsets of V, at least one of the subsets is r-reachable (p-fraction reachable).

2.2 Definition of Time-Varying Formation

Consider the following discrete-time double-integrator dynamics:

$$\bar{x}_i\,[k+1] = A\bar{x}_i\,[k] + Bu_i\,[k]\,, \tag{1}$$

where $\bar{x}_i\,[k] = [x_i[k],\, v_i[k]]^{\mathrm{T}}$ with $x_i[k] \in \mathbb{R}$ and $v_i[k] \in \mathbb{R}$ denoting the position and the velocity of agent i, respectively, $u_i[k] \in \mathbb{R}$ the control input, $A = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}$, and $B = \begin{bmatrix} \frac{T^2}{2} \\ T \end{bmatrix}$ with sampling period T.

Let $h[k] = \left[h_1[k]^{\mathrm{T}}, \cdots, h_N[k]^{\mathrm{T}}\right]^{\mathrm{T}}$ be the desired formation vector for system (1), where $h_i[k] = [h_{xi}[k], h_{vi}[k]]^{\mathrm{T}} \in \mathbb{R}^2$ is the formation vector for agent i.

Definition 2. *The system (1) realizes the time-varying formation $h[k]$ if there exists a vector-valued function $r[k] \in \mathbb{R}^2$ called the formation reference function such that $\lim_{k\to+\infty} (\bar{x}_i[k] - h_i[k] - r[k]) = 0, \forall i = 1, 2, \ldots, N$.*

Assumption 1. *The formation vector $h_i[k] = [h_{xi}[k], h_{vi}[k]]^{\mathrm{T}}\, h, i \in V$ satisfies the following conditions,*

1) $h_{xi}[k+1] = h_{xi}[k] + T h_{vi}[k]$.
2) Let $\Delta h_{vi}[k] = h_{vi}[k+1] - h_{vi}[k]$ and $s\,[k] = \max_{i\in V} \left|-\Delta h_{vi}[k] + \Sigma_{d=1}^{k} (2-\beta T)\, (-1)^{d+1}\, \Delta h_{vi}\,[k-d]\right|$, where β will be defined later. $s[k]$ is bounded by $b \in \mathbb{R}, b > 0$.

Remark 1. The first assumption is also used in [17]. And the second one can be easily verified for the simple circular time-varying formation control studied in [17] and [3].

2.3 Attack Model

We suppose that each edge $(i, j) \in E$ is vulnerable to attacks, which can be deception attacks or DOS attacks. The first ones can change the values received by node i from node j, while under DOS attacks, node i cannot receive any information from node j. The set of edges under attacks at time instant k is denoted as $E_A[k]$.

Definition 3 (F-local/f-fraction local edge attacks)
A directed graph $G = \{V, E\}$ is said to be under F-local (f-fraction) edge attacks if for each node $i \in V$, at most F ($\lfloor f|N_i|\rfloor$) of its incoming edges are under edge attacks at each time instant.

Remark 2. F-local edge attacks without DOS attacks is exactly the F-local deception attacks in [18].

2.4 Faults and Disturbances Model

The real dynamics of the agent i with actuator faults and disturbances are as follows:

$$\begin{aligned} \bar{x}_i\,[k+1] &= A\bar{x}_i\,[k] + B\tilde{u}_i\,[k] + d_i[k] \\ y_i\,[k] &= \bar{x}_i\,[k] + w_i\,[k]\,, \end{aligned} \tag{2}$$

where $\tilde{u}_i[k] = u_i[k] + f_i[k] \in \mathbb{R}$ is the real input with $u_i[k]$ denoting the designed input and $f_i[k]$ the fault. $d_i[k]$ and $w_i[k]$ are the disturbances on agent i.

Our aim is to design the control input $u_i[k]$ for each agent $i \in V$ such that MAS (2) realize formation $h[k]$ under F-local or f-fraction local edge attacks with bounded errors related to the actuator faults and disturbances.

3 Main Results

3.1 Overall Structure of the Proposed Approach

Our approach consists of two steps. First, agent i forms a target observer O_i which realizes the desired formation $h_i[k]$ without actuator faults and disturbances and gives a reference model for agent i to track. Second, each agent applies fault estimation and fault compensation strategies to attenuate the influences of faults and disturbances while tracking its reference model.

Define the target observer O_i with the following dynamics

$$\hat{\bar{x}}_i\left[k+1\right] = A\hat{\bar{x}}_i\left[k\right] + Br_i\left[k\right], \tag{3}$$

where $\hat{\bar{x}}_i\left[k\right] = \left[\hat{x}_i\left[k\right], \hat{v}_i\left[k\right]\right]^{\mathrm{T}}$ with $\hat{x}_i[k] \in \mathbb{R}$ and $\hat{v}_i[k] \in \mathbb{R}$ denoting the states of O_i, and $r_i[k] \in \mathbb{R}$ is its input. The initial values are assumed to be $\hat{x}_i[0] = x_i[0]$ and $\hat{v}_i[0] = v_i[0]$.

3.2 Formation Protocol for Target Observers

Define $\delta_{xi}[k] = \hat{x}_i[k] - h_{xi}[k]$, $\delta_{vi}[k] = \hat{v}_i[k] - h_{vi}[k]$, according to dynamics (3) and Assumption 1, we have

$$\begin{aligned} \delta_{xi}\left[k+1\right] &= \delta_{xi}\left[k\right] + T\delta_{vi}\left[k\right] + \frac{T^2}{2}r_i\left[k\right], \\ \delta_{vi}\left[k+1\right] &= \delta_{vi}\left[k\right] + Tr_i\left[k\right] - \Delta h_{vi}\left[k\right]. \end{aligned} \tag{4}$$

Based on RCC algorithm of [10] and inspired by [5], we propose the following edge-attack resilient formation (EARF) algorithm, which consists of three steps for each time step k:

1) Each observer O_i sends its position $\delta_{xi}[k]$ to its out-neighbors.
2) Define the received value set for O_i as $\bar{R}_i[k] = \{\delta^i_{xj}[k], j \in N_i\}$. $d_i[k]$ denotes the number of in-neighbours of O_i from which O_i does not receive information. Define a variable q_i according to the operation mode of the protocol:
   - **F-local mode:** $q_i = F - d_i[k]$
   - **f-fraction local mode:** $q_i = \lfloor f|N_i| \rfloor - d_i[k]$

   If the number of elements in $\bar{R}_i[k]$ whose values are strictly larger than $\delta_{xi}[k]$ is less than q_i, O_i discards all these values. Otherwise, it discards the q_i largest values in $\bar{R}_i[k]$. A similar operation is conducted on the values strictly smaller than $\delta_{xi}[k]$. Denote by $R_i[k]$ the set of rest observers whose values are retained by observer i at time k.
3) The control input for observer i is designed as:

$$\begin{aligned} r_i[k] &= \alpha \sum_{j \in R_i[k]} a_{ij}\left(\delta^i_{xj}[k] - \delta_{xi}[k]\right) - \beta\delta_{vi}[k] + \gamma_i[k], \\ \gamma_i\left[k\right] &= \begin{cases} \frac{2-\beta T}{T}\Delta h_{vi}\left[k-1\right] - \gamma_i\left[k-1\right], & if\ k \geq 1, \\ 0, & if\ k = 0, \end{cases} \end{aligned} \tag{5}$$

where the parameters α and β satisfy

$$\begin{aligned} 0 < \frac{\alpha T^2}{2} \max_{i \in V} \sum_{j \in N_i} a_{ij} < \frac{1}{2}, \\ 1 + \frac{\alpha T^2}{2} \max_{i \in V} \sum_{j \in N_i} a_{ij} < \beta T < 2 - \frac{\alpha T^2}{2} \max_{i \in V} \sum_{j \in N_i} a_{ij}. \end{aligned} \tag{6}$$

Then the observer i update its state value using (5) according to (3).

For $k \in \mathbb{Z}$ and $k \geq 1$, let $M[k] = \max_{i \in V} \{\delta_{xi}[k], \delta_{xi}[k-1]\}$, and $m[k]$ be the corresponding set for the minimal values.

Lemma 1. *Consider MAS (3) under F-local edge attacks (f-fraction local) edge attacks, where each agent applies EARF algorithm using F-local (f-fraction local) mode. Then $M[k]$ is non-increasing and $m[k]$ is non-decreasing.*

Proof. For $k \geq 1$, according to (4) and (5), using the same approach as in *Lemma 1* of [10], $\delta_{xi}[k+1]$ can be expressed as a convex combination of $\delta_{xi}[k]$, $\delta_{xj}^i[k]$ with $j \in R_i[k]$, $\delta_{xi}[k]$, and $\delta_{xj}^i[k-1]$ with $j \in R_i[k-1]$ by using $\gamma_i[k]$ to eliminate the terms related to Δh_{vi}. Therefore, $M[k]$ is non-increasing and $m[k]$ is non-decreasing. □

Theorem 1. *Consider the MAS (3) under F-local (f-fraction local) edge attacks, where each agent applies the EARF algorithm using F-local (f-fraction local) mode. The formation is realized with an error asymptotically bounded by $\frac{b}{\beta T}$ for the velocity state if and only if the communication network is (2F+1)-robust (if the communication network is p-fraction robust with $2f < p \leq 1$).*

Proof. Necessity (for F-local edge attacks case): In the special case where $h_i[k] \equiv 0, i \in V$, the objective becomes realizing the consensus of system (3). In [10], the authors gave a counter-example in which the consensus cannot be reached when the communication network is not (2F+1)-robust.
Sufficiency: Using the same approach as in *Lemma 2* of [10], one can show that there exists $x^* \in \mathbb{R}$ such that $\delta_{xi}[k] \to x^*, \forall i \in V$, then $r_i[k] \to -\beta\delta_{vi}[k] + \gamma_i[k]\Delta h_{vi}[k]$. According to (4), one has $\delta_{vi}[k+1] \to (1-\beta T)\delta_{vi}[k] + s_i[k]$ with $s_i[k] = (T\gamma_i[k] - 1)\Delta h_{vi}[k]$. Using the expression of $\gamma_i[k]$, it is deduced that $s_i[k] = -\Delta h_{vi}[k] + \Sigma_{d=1}^{k} (2-\beta T)(-1)^{d+1} \Delta h_{vi}[k-d]$. According to (6) and Assumption 1, $|\delta_{vi}[k]|$ is bounded by $\frac{b}{\beta T}$ when $k \to \infty$. It is concluded that system (3) reaches the formation $h[k]$ with an error asymptotically bounded by $\frac{b}{\beta T}$ for the velocity state and a formation reference function of $[x^*, 0]^{\mathrm{T}}$. □

Remark 3. For the problem of simple circular time-varying formation control in [17], when the sampling period T is short and the desired formation varies slowly, one can show that b will be small enough to be ignored, which will be illustrated by the simulation part of this paper.

The above analysis focused on the total number or fraction of the two types of attacks around an agent. We may also deal with the case where the numbers of DOS attacks and deception attacks are bounded separately, given in the following theorem. The proof is similar to that of Theorem 1 and is therefore omitted.

Theorem 2. *If we revise the EARF algorithm to no longer consider DOS attacks, i.e., we always set $D_i[k] = \emptyset$, then if for each node $i \in V$, at most F_{Dos} ($f_{Dos}|N_i|$) of its incoming edges are under DOS attacks and at most F_{Dec} ($f_{Dec}|N_i|$) of its incoming edges are under deception attacks at each time instant, using the revised EARF algorithm in F_{Dec}-local mode (f_{Dec}-fraction local mode), the formation with the same performance as in Theorem 1 of system (3) is achieved if and only if the communication network is $(F_{Dos} + 2F_{Dec} + 1)$-robust (if the communication network is p-fraction robust with $f_{Dos} + 2f_{Dec} < p \leq 1$).*

3.3 Actuator Fault Estimation and Compensation

According to (2) and (3), one can obtain

$$\begin{aligned} \tilde{\bar{x}}_i\left[k+1\right] &= A\tilde{\bar{x}}_i\left[k\right] + B\left(u_i\left[k\right] + f_i[k] - r_i\left[k\right]\right) + d_i\left[k\right], \\ z_i[k] &= \tilde{\bar{x}}_i\left[k\right] + w_i\left[k\right], \end{aligned} \tag{7}$$

where $z_i[k] = y_i[k] - \hat{\bar{x}}_i[k]$.

Since we are now dealing with a specific agent $i \in V$, for simplicity, the index i is omitted in this section.

Define an extended state $x_e[k] = [\tilde{\bar{x}}[k]^{\mathrm{T}} \; f[k] \; w[k]^{\mathrm{T}}]^{\mathrm{T}}$ and a reformulated input $u_r[k] = u[k] - r[k]$. Let $\Delta f[k] = f[k+1] - f[k]$, $M \in \mathbb{R}^{2\times 2}$ be a non-singular matrix, and $L_e = [0 \; 0 \; M^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{R}^{5\times 2}$. Note that $S_e = E_e + L_e C_e$ is non-singular, and define the following matrices

$$\begin{aligned} E_e &= \begin{bmatrix} I_2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, A_e = \begin{bmatrix} A & B & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -I_2 \end{bmatrix}, B_e = \begin{bmatrix} B \\ 0 \\ 0 \end{bmatrix}, d_{de}[k] = \begin{bmatrix} d[k] \\ \Delta f[k] \\ w[k] \end{bmatrix}, \\ C_e &= \begin{bmatrix} I_2 & 0 & I_2 \end{bmatrix}, N_e = \begin{bmatrix} I & 0 & 0 \\ 0 & 1 & 0 \\ -I & 0 & I \end{bmatrix}, w_e\left[k\right] = \begin{bmatrix} d\left[k\right] \\ \Delta f\left[k\right] \\ M^{-1}w\left[k\right] \end{bmatrix}. \end{aligned} \tag{8}$$

Theorem 3. *(Observer and feedback law design for fault estimation and compensation)*

1) ***Estimation.*** *Define the following extended state estimator*

$$\hat{x}_e\left[k+1\right] = S_e^{-1}\left(\left(A_e - K_e C_e\right)\hat{x}_e\left[k\right] + K_e z\left[k\right] + B_e u_r\left[k\right] + L_e z\left[k+1\right]\right), \tag{9}$$

where $K_e \in \mathbb{R}^{5\times 2}$. Let $\tilde{x}_e = x_e - \hat{x}_e$. then the H_∞ performance $||\tilde{x}_e||_2 \leq \gamma_e||w_e||_2, \gamma_e \in \mathbb{R}, \gamma_e > 0$ is satisfied if the following LMIs are solvable:

solve $\bar{K}_e \in \mathbb{R}^{5\times 2}, P_e \in \mathbb{R}^{5\times 5}$ *subject to* $P_e = P_e^{\mathrm{T}} > 0$ *and*

$$\begin{bmatrix} -P_e + I & * & * \\ 0 & -\gamma_e^2 I & * \\ P_e S_e^{-1} A_e - \bar{K}_e C_e & P_e N_e & -P_e \end{bmatrix} < 0. \tag{10}$$

Then K_e *is designed to be* $S_e P_e^{-1} \bar{K}_e$.

2) ***Compensation.*** *Design the control input* $u_r[k] = -F_e \hat{x}_e[k]$ *with* $F_e = [F \quad 1 \quad 0] \in \mathbb{R}^{1\times 5}$ *with* $F \in \mathbb{R}^{1\times 2}$. *The tracking* H_∞ *performance requirement* $\|\tilde{\bar{x}}\|_2^2 \le \gamma_{c1}^2 \|\tilde{x}_e\|_2^2 + \gamma_{c2}^2 \|d\|_2^2, \gamma_{c1} > 0, \gamma_{c2} > 0$ *is satisfied if:*

   a) *The following LMIs are solvable: solve* $\bar{F} \in \mathbb{R}^{1\times 2}, \bar{P}_c \in \mathbb{R}^{2\times 2}$ *subject to* $\bar{P}_c = \bar{P}_c^{\mathrm{T}} > 0$ *and*

$$\begin{bmatrix} -\bar{P}_c & * & * & * \\ 0 & -\gamma_{c2}^2 I & * & * \\ A\bar{P}_c - B\bar{F} & I & -\bar{P}_c & * \\ I & 0 & 0 & -I \end{bmatrix} < 0. \tag{11}$$

   b) *The following LMIs are satisfied:*

$$\begin{bmatrix} -\bar{P}_c & * & * & * & * \\ 0 & -\gamma_{c1}^2 I & * & * & * \\ 0 & 0 & -\gamma_{c2}^2 I & * & * \\ A\bar{P}_c - B\bar{F} & B\bar{F}_e & I & -\bar{P}_c & * \\ I & 0 & 0 & 0 & -I \end{bmatrix} < 0, \tag{12}$$

   where $\bar{F}_e = [\bar{F}\bar{P}_c^{-1} \ 1 \ 0 \ 0]$.

Then F *is designed to be* $\bar{F}\bar{P}_c^{-1}$.

Proof.

1) *Using the extended state form,* (7) *can be rewritten as*

$$\begin{aligned} E_e x_e[k+1] &= A_e x_e[k] + B_e u_r[k] + d_{de}[k], \\ z[k] &= C_e x_e[k]. \end{aligned} \tag{13}$$

Combining (13) *and* (9), *we have*

$$\tilde{x}_e[k+1] = S_e^{-1}\left((A_e - K_e C_e)\tilde{x}_e[k] + d_{de}[k]\right). \tag{14}$$

If LMIs (10) *is solvable, according to* $K_e = S_e P_e^{-1} \bar{K}_e$ *and Schur complement theorem, we can get*

$$\Omega_e = \begin{bmatrix} \Omega_{e1} & (A_e - K_e C_e)^{\mathrm{T}} S_e^{-T} P_e N_e \\ P_e N_e^{\mathrm{T}} P_e S_e^{-1}(A_e - K_e C_e) & N_e^{\mathrm{T}} P_e N_e - \gamma_e^2 I \end{bmatrix} < 0 \tag{15}$$

with $\Omega_{e1} = (A_e - K_eC_e)^{\mathrm{T}} S_e^{-T} P_e S_e^{-1} (A_e - K_eC_e) - P_e + I$*. Define the Lyapunov candidate function* $V_e(\tilde{x}_e[k]) = \tilde{x}_e[k]^T P_e \tilde{x}_e[k]$*, the time increment of* V_e *along* (14) *is* $\Delta V_e[k] = \tilde{x}_e[k+1]^T P_e \tilde{x}_e[k+1] - \tilde{x}_e[k]^T P_e \tilde{x}_e[k] \leq -\|\tilde{x}_e[k]\|_2^2 + \gamma_e^2 \|w_e[k]\|_2^2$*. Then under zero initial condition,* $\sum_{k=0}^{\infty} \|\tilde{x}_e[k]\|_2^2 - \gamma_e^2 \|w_e[k]\|_2^2 \leq \sum_{k=0}^{\infty} \Delta V_e[k] + \|\tilde{x}_e[k]\|_2^2 - \gamma_e^2 \|w_e[k]\|_2^2 \leq 0$*, which verifies the* H_∞ *performance.*

2) Substituting $u_r[k] = -F_e\hat{x}_e[k]$ *into* (7)*, we can get*

$$\tilde{\bar{x}}[k+1] = (A - BF)\tilde{\bar{x}}[k] + BF_e\tilde{x}_e[k] + d[k]. \tag{16}$$

If LMIs (11) *are satisfied, let* $P_c = \bar{P}_c^{-1}$ *and by left-multiplying and right-multiplying* (11) *by* $\mathrm{diag}(P_c, I, I, P_c, I)$ *and using Schur complement theorem, with* $F = \bar{F}\bar{P}_c^{-1}$*, we have*

$$\Omega_c = \begin{bmatrix} Q + I & (A - BF)^{\mathrm{T}} P_c BF_e & (A - BF)^{\mathrm{T}} P_c \\ * & (BF_e)^{\mathrm{T}} P_c BF_e - \gamma_{c1}^2 I & (BF_e)^{\mathrm{T}} P_c \\ * & * & P_c - \gamma_{c2}^2 I \end{bmatrix} < 0, \tag{17}$$

where $Q = (A - BF)^{\mathrm{T}} P_c (A - BF) - P_c$*. Define the Lyapunov candidate function* $V_c(\tilde{\bar{x}}[k]) = \tilde{\bar{x}}[k]^{\mathrm{T}} P_c \tilde{\bar{x}}[k]$*, using the same approach as in the last section, one can show that the demanded* H_∞ *performance is verified.* □

Remark 4. From the definition of w_e, we can show that the effect of $w[k]$ can be attenuated if the matrix M is chosen as a reasonably high-gain non-singular matrix.

We focus only on the case where the dimension of x_i is one, but the proposed approach can be easily extended to multi-dimension cases by treating each dimension independently.

4 Simulation

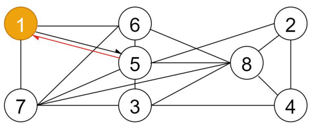


Fig. 1. Network topology

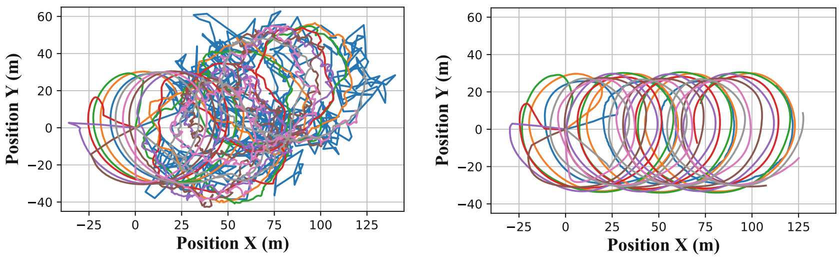
In this section, numerical simulations are designed to study the aforementioned results. Consider a formation task on a two-dimension X-Y plane with $N = 8$ agents. The desired formation for agent i is

$$h_{iX} = \begin{bmatrix} r\cos\left(\frac{T}{5}k + \frac{2\pi(i-1)}{N}\right) + Tk \\ -\frac{rT}{5}\sin\left(\frac{T}{5}k + \frac{2\pi(i-1)}{N}\right) + k \end{bmatrix}, h_{iY} = \begin{bmatrix} r\sin\left(\frac{T}{5}k + \frac{2\pi(i-1)}{N}\right) \\ \frac{rT}{5}\cos\left(\frac{T}{5}k + \frac{2\pi(i-1)}{N}\right) \end{bmatrix} \quad (18)$$

for the dimension of X and Y, respectively, with the circular formation radius $r = 30\,\text{m}$ and the sampling time $T = 0.1\,\text{s}$. The network topology is given in Fig. 1. It can be verified that the digraph is 3-robust and 0.5-fraction robust. The edge weights are set to 1. $\alpha = 16.6, \beta = 15$ satisfies the conditions (6). The simulation starts at $k = 0$ and ends at $k = 1000$. The initial positions and velocities of the agents are set to zero.

In the first scenario, we suppose that there are no disturbances and actuator faults, and agent 1 is under false data injection on the edge from agent 5. The attack starts at time instant $k = 300$ and the attack value is generated by a uniform distribution on $[-100, 100]$ for each one of the X and Y dimensions. Using EARF without step two, the result is given in Fig. 2(a). One can show that during the first 300 time instants, the formation is formed but then totally destroyed by the edge attack.

Then using the EARF algorithm with 1-local mode, the result is given in Fig. 2(b), where the attack effect is largely attenuated.



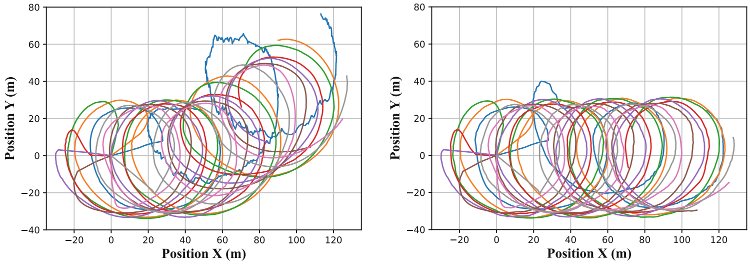
(a) (b)

Fig. 2. MAS under edge attack without (a)/with (b) EARF algorithm

In the second scenario, in addition to edge attacks, each agent is subjected to disturbances with $d_i \sim \text{N}(0, \text{diag}(0.05^2, 0.05^2))$ and $w_i \sim \text{N}(0, \text{diag}(0.05^2, 0.05^2))$. What's more, agent 1 is under actuator fault in the Y dimension from the time instant $k_f = 400$ with $f_{1Y}[k] = 500(1 - e^{\frac{k-k_f}{100}})\,\text{m/s}^2$ if $k > k_f$, and 0, otherwise. The result is shown in Fig. 3(a). Agent 1 is dragged away from other agents and the formation is disturbed.

To mitigate the fault effect, the fault estimation and compensation scheme established in this paper is used. $M = \text{diag}(3, 3)$. The feasibility problems of LMIs (10) and (11) can be transformed to optimization problems as illustrated in [15] to find optimal disturbance attenuation performance. Using Matlab, the design matrix are therefore solved as follows:

$$K_e = \begin{bmatrix} 1.1332 & 1.1979 \\ 0.6680 & 4.6035 \\ 1.1866 & 7.9114 \\ 0.0724 & 0.7771 \\ 0.5494 & 2.8124 \end{bmatrix}, F = \begin{bmatrix} 3.4840 & 4.6744 \end{bmatrix}. \tag{19}$$



(a) (b)

Fig. 3. MAS under actuator faults and without (a)/with (b) fault compensation

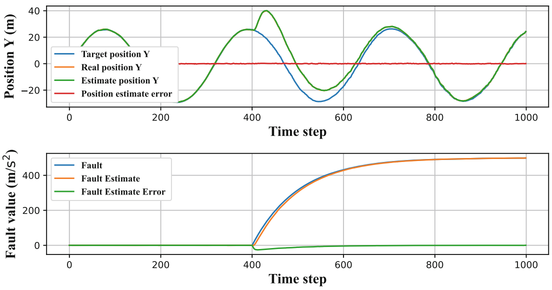


Fig. 4. State/fault estimate of agent 1

Figure 4 shows the state/fault estimate of agent 1, which are then used for feedback control. Figure 3(b) shows that agent 1 is firstly dragged away from MAS by the fault but then it mitigates the fault effects and gets back to the team. Therefore, the desired formation is realized under actuator faults and attacks on communication link with the developed resilient control strategy.

5 Conclusion

Resilient time-varying formation problem of second-order DMASs under communication link attacks and actuator faults was addressed in this paper. A discrete-time formation control protocol based on MSR and FE/FC was developed. By applying properties of robust graphs and discrete stability theory, sufficient conditions for DMASs to realized the desired formation were presented. Simulation results have verified the feasibility of the formation strategy.

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