



# Trajectory Planning Technology of UAV Swarm Cooperative Stealth Based on Formation Control

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**Abstract.** Aiming at the problem of unmanned aerial vehicle (UAV) swarm stealth and penetration in complex battlefield environment in the future, a trajectory planning method based on cooperative stealth of UAV swarm formation is proposed from the perspective of swarm intelligence cooperation. In the process of cooperative stealth of UAV swarm formation control, the UAV swarm is treated as a three-dimensional sparse array, and the radar cross section (RCS) of the UAV swarm can be controlled by adjusting the space coordinates of each UAV. The functional relationship between the detection probability of radar and the RCS of the UAV swarm is also studied. The detection probability and range of radar can be reduced by aligning the detection radar to the lower RCS direction, which can be used to improve the trajectory planning of the UAV swarm. In this paper, the trajectory planning problem is described as a multi-objective optimization problem, and an ant colony algorithm is used to optimize and simulate the trajectory path of UAV swarm to achieve the shortest path and lowest detection probability of UAV swarm. The simulation results show that the cooperative stealth based on formation control can effectively shorten the trajectory length of UAV swarm, and greatly reduce the detection probability of radar in the process of penetration.

**Keywords:** UAV swarm · Radar cross section (RCS) · Sparse array · Trajectory planning

## 1 Introduction

The UAV (unmanned aerial vehicle) swarm is one of the major weapon systems in the future, which can go deep into the battlefield for reconnaissance and attack. But with the rapid development of radar detection technology, how to improve the ability of UAV swarm stealth penetration, and evade the detection of the radar network, is more and more important. So the trajectory planning with stealth considered needs to be studied.

Trajectory planning is a planning problem to obtain optimal performance under the conditions of initial state, mission objective, threat area and some

known environmental information. It is one of the key technologies of UAV swarm stealth penetration. Currently, the single-machine flight path planning technology has been quite mature. Unlike single UAV, the space and time coordination among UAVs need to be considered in the trajectory planning of UAV swarm, which mainly includes trajectory planning, formation reconstruction and formation maintenance. UAV swarm cooperative trajectory planning is one of the key technologies to improve the effectiveness of UAV swarm performance. Furthermore, increasing interest in UAV swarm has accelerated the integration of trajectory planning.

A constrained unmanned aerial vehicles trajectory optimization problem is formulated in the literature [1]. A hp-adaptive pseudospectral method is introduced to transform the trajectory planning into nonlinear programming. The trajectory planning problem is formulated as a convex programming problem. The results indicate the algorithm can get better performance compared with other typical optimal control solvers.

An approach to trajectory planning of UAV using Invasive Weed Optimization is presented in the literature [2]. The trajectory planning problem is characterized using a simple and elegant model. Invasive Weed Optimization is used as the optimization technique for finding safe and short paths for UAVs for going from source to destination, and B-spline curve is formed to smoothen the resultant trajectory.

In recent years, how to make use of UAV swarm formation to cooperative communication and detection also got more and more attention. In the literature [3,4], the UAV swarm is respectively treated as 2D and 3D MIMO array. By changing the spatial location of each UAV in the UAV swarm coordinates, the main lobe gain and channel capacity of UAV array are improved. Experimental results show that it is an effective method to improve communication performance by UAV swarm formation cooperation. In the literature [5], aiming at the problem of signal occlusion in communication, UAV swarm is designed as a reflection array. By optimizing the position of each UAV in swarm, communication signals are reflected and transmitted to the specified direction, the problem of communication signal occlusion can be solved by UAV swarm. UAV swarm is regarded as antenna array in the literature [6], the application of UAV swarm array in radar imaging is explored. By combining phase compensation with optimization of sparse UAV formation, the resolution of radar imaging is improved with UAV swarm array.

Currently, the research of UAV swarm trajectory planning and formation control was mainly focused on the optimization method and collaborative communication. How to use UAV swarm formation to improve stealth penetration ability is less studied, especially in the trajectory planning process. In most of the current research focused on radar detection threats, the RCS of the whole UAV swarm on radar detection is not considered, they take the RCS of each drone as a basic constant for their calculations. In this article, the RCS control of the UAV swarm and the trajectory planning is combined to reduce the radar detection probability. An algorithm is proposed to comprehensive the whole RCS of UAV swarm and plan the performance by adjusting the formation. The simulation

results shows that this algorithm can greatly improve the performance in UAV swarm trajectory planning.

## 2 The Electromagnetic Scattering Model of UAV Swarm

In the cooperative stealth process of UAV swarm, the whole UAV swarm is regarded as a three-dimensional sparse array, the scattering characteristics of UAV swarm can be analyzed and integrated by the array analysis method. The RCS of the UAV swarm can be expressed by:

$$\sigma(\theta, \phi) = \sum_{n=1}^N \sigma_n(\theta, \phi) \cdot P_n \cdot e^{j\{k(\vec{d}_n \cdot \vec{r}_n)\}}. \tag{1}$$

As shown in Fig. 1,  $\theta$  represents the angle between the electromagnetic wave direction and the  $Z$  axis,  $\varphi$  represents the angle between the projection of electromagnetic wave direction on the  $XOY$  plane and the  $X$  axis,  $\sigma(\theta, \varphi)$  is the RCS of the UAV swarm,  $\sigma_n(\theta, \varphi)$  is the RCS of each UAV among the UAV swarm, which is only determined by the shape and material characteristics of the UAV.  $P_n$  represents the normalized scattering current which is excitation by the plane wave shining on each UAV, and can be regarded as the excitation of scattering array.

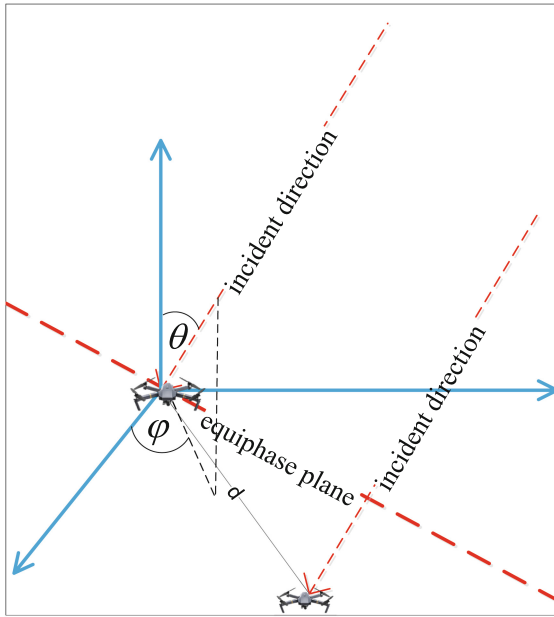


Fig. 1. Schematic diagram of scattering phase analysis.

Unlike the situation in the radiation problem, the phase of the scattering current is determined by the scattering characteristics of each UAV itself and its spatial location. As usual, the distance between the radar and the UAV swarm is far more than the distance between the UAVs, the amplitude and phase of the electromagnetic wave on the surface of the UAVs can be considered equal. Therefore, the amplitude of scattering current generated by plane electromagnetic wave on the surface of UAV can also be considered equal, namely

$$P_n = 1 \cdot e^{j \cdot (\phi_n + \eta_n)}. \quad (2)$$

In (2),  $\phi_n$  represents the phase of the scattering current generated by the relative position of the UAV,  $\eta_n$  represents the phase of the scattering current generated by the scattering characteristics of the UAV itself. As shown in Fig. 1, the equiphase plane of the far-field electromagnetic wave is perpendicular to the incident direction of the electromagnetic wave, and the phase difference of the electromagnetic wave irradiated to the surface of each UAV is mainly determined by the projection of UAV coordinates on the incident direction of the electromagnetic wave.

$$\phi_n = k \cdot \vec{b}_n \cdot \hat{r}_n. \quad (3)$$

Thus, (1) can be written as follows:

$$\sigma(\theta, \varphi) = \sigma_e(\theta, \varphi) \cdot \sum_{n=1}^N e^{j \cdot \eta_n} \cdot e^{j(2k \cdot \vec{b}_n \cdot \hat{r})}. \quad (4)$$

For the UAV swarm composed of the same type of UAV, the values  $\eta_n$  are generally the same. But the scattering phase of each UAV still can be regulated by the active met-amaterials. One feasible way is to use the intelligent skin technology composed of electromagnetic met-amaterials to regulate the phase of echo, making the scattering echo of the UAVs varies between  $0^\circ$  and  $180^\circ$  [7].

In this situation, the scattering current phase control variables  $C_n$  can be defined as follows:

$$\eta_n = C_n \cdot 180^\circ. \quad (5)$$

Since,  $\eta_n = 0^\circ$  or  $180^\circ$ , the scattering current phase control variables  $C_n = 0$  or 1, these variables are mainly used to characterize the change of the scattering phase of each UAV, if  $C_n = 0$ , this means that the scattering phase of UAV is not regulated by the intelligent skin technology, while  $C_n = 1$  means that the scattering and echo phase of UAV changes to  $180^\circ$ .

Therefore, by adjusting the three-dimensional coordinate position of UAV swarm and the scattering phase of each UAV, the scattering characteristics of the whole UAV swarm can be comprehensively designed. The low scattering azimuth can be aligned with the incoming wave direction of the detection radar to avoid detection.

### 3 Probabilistic Model of Radar Detection for UAV Swarm

The factors that affect the trajectory planning of UAV swarm generally include geographical condition, radar detection threat and air defense fire threat, among which radar detection threat is the main threat in the future battlefield environment. How to reduce radar detection is the key problem of the UAV swarm trajectory planning. At present, most of the trajectory planning methods do not consider the RCS of the whole UAV swarm, but only consider the RCS of each UAV individually, which is very inconsistent with the actual situation.

In the process of trajectory planning, the RCS of the whole UAV swarm are need to be considered, It's a function of azimuth and pitch, and these angles are determined by the relative position relation between the UAV swarm and the radar. When the radar is lo-cated in the  $(\theta, \varphi)$  direction of the UAV swarm, according to the radar equation, the signal to noise ratio (SNR) required for radar monitoring can be written as follows:

$$(SNR)_{Pd} = \frac{P_T G^2 \lambda^2 \sigma(\theta, \varphi)}{(4\pi)^3 k T_e B F L R^4}. \quad (6)$$

In (6),  $P_T$  represents the transmitting power,  $G$  represents the gain of the radar antenna,  $\lambda$  represents the wavelength of the electromagnetic wave,  $L$  represents transmission loss of radar signal,  $\sigma(\theta, \varphi)$  represents the RCS in the  $(\theta, \varphi)$  direction of the UAV swarm,  $k$  represents the Boltzmann constant,  $T_e$  represents the Fahrenheit.

The relationship of the signal to noise ratio (SNR) required for radar monitoring ,the detection probability  $P_d$  and false alarm probability  $P_{fa}$  can be simply estimated by Albersheim's (7) [8]:

$$\begin{aligned} A &= \ln \frac{0.62}{P_{fa}} \\ B &= \ln \frac{P_d}{1-P_d} \\ (SNR)_{Pd} &= -5 \log_{10} N + [6.2 + 4.54/\sqrt{N+0.44}] * \\ &\quad \log_{10}(A + 0.12AB + 1.7B). \end{aligned} \quad (7)$$

where,  $N$  is the number of noncoherently integrated sam-ples. According to the (6) and (7), we can get the following equation.

$$\begin{aligned} &-5 \log_{10} N + [6.2 + 4.54/\sqrt{N+0.44}] * \log_{10}(A + 0.12AB + 1.7B) \\ &= \frac{P_T G^2 \lambda^2 \sigma(\theta, \varphi)}{(4\pi)^3 k T_e B F L R^4} \end{aligned} \quad (8)$$

For alerting radar, the false alarm probability is generally set to  $10^{-6}$  or  $10^{-7}$ . Thus, once the radar parameters are determined, the detection probability is determined by the distance and RCS of the UAV swarm.

To illustrate the new contribution in methodological research, the number of noncoherently integrated samples is set to  $N = 1$ , the false alarm probability is set to  $10^{-6}$ , then we can get

$$10 * \log_1 0(13.3 + 3.3B) = \frac{P_T G^2 \lambda^2 \delta(\theta, \varphi)}{(4\pi)^3 k T_e B F L R^4} \quad (9)$$

According to (9),  $B$  can be viewed as a function of  $\delta(\theta, \varphi)$  and the distance  $R$ .

$$B = F_1(\delta(\theta, \varphi), R) = (10^{\frac{P_T G^2 \lambda^2 \sigma(\theta, \varphi)}{10(4\pi)^3 k T_e B F L R^4}} - 13.3)/3.3 \quad (10)$$

where,  $B$  is a monotonically increasing function of  $\sigma(\theta, \varphi)$  and a monotonically decreasing function of the distance  $R$ .

In (7),  $B$  is monotonically increasing function of the detection probability  $P_d$  in its domain  $[0, 1]$ , so the inverse function  $P_d$  is also a monotonically increasing function of  $B$ .

$$P_d = F_2(B) = \frac{e^B}{1 + e^B} \quad (11)$$

The probabilistic model of radar detection for UAV swarm is determined by (10) and (11).

$$P_d = \frac{e^{(10^{\frac{P_T G^2 \lambda^2 \sigma(\theta, \varphi)}{10(4\pi)^3 k T_e B F L R^4}} - 13.3)/3.3}}{1 + e^{(10^{\frac{P_T G^2 \lambda^2 \sigma(\theta, \varphi)}{10(4\pi)^3 k T_e B F L R^4}} - 13.3)/3.3}} \quad (12)$$

In (12), the detection probability  $P_d$  is monotonically increasing with the RCS of UAV swarm and is monotonically decreasing with the distance between UAV swarm and radar. This means that decrease the RCS of the UAV swarm or increase the distance  $R$  can reduce the detection probability and this can be used to the UAV swarm trajectory planning.

## 4 Trajectory Planning Algorithm for UAV Swarm Cooperative Stealth

In the UAV swarm trajectory planning, the UAV trajectory can be divided into a number of segments according to the radar distribution situation. In this paper, the rasterization method is used to discretize the region uniformly. Thus, the trajectory of the UAV swarm is composed of lattice cells, and each lattice cell represents a segment of the trajectory.

In each segment or each lattice cell, the spatial position of each UAV in the formation and the scattering phase state of each UAV in the swarm can be optimized according to the azimuth of the detection radar.

After optimization, the radar wave incident direction is located in the area with small RCS of UAV swarm, so that the detection probability of radar is reduced and can be calculated by (12). The detection probability can be used as a threat price in the trajectory planning algorithm. The whole process can be divided into two main processes.

#### 4.1 UAV Swarm Cooperative Stealth Algorithm for Each Trajectory Segment

When the dispersion is complete, in each segment, the RCS of the UAV swarm can be optimized by using the three-dimensional coordinate and the phase control variables of each UAV.

Assuming that UAV swarm is located in the center of each lattice cell, the threat angle of radar in this segment can be set  $\theta_{ss} < \theta < \theta_{sd}$ ,  $\varphi_{ss} < \varphi < \varphi_{sd}$ , according to the position relationship between radar and UAV swarm. In this angle domain, the RCS of the whole UAV swarm should be as low as possible. Generally, We define the target scattering beampattern  $Sd(\theta, \varphi)$ , and in this angle domain we set it to 0 as shown in (13). The values in other angle domains are generally not specially restricted, indicating that the maximum scattering direction is adjusted to other directions where no radar detects threats.

$$Sd(\theta, \varphi) = 0 \quad \theta_{ss} < \theta < \theta_{sd}, \varphi_{ss} < \varphi < \varphi_{sd}. \quad (13)$$

When the target scattering beampattern  $Sd(\theta, \varphi)$  is determined, the scattering beampattern  $S(\theta, \varphi)$  of the whole swarm is optimized to approximate the target scattering beampattern by adjusting the relative coordinate position  $x_i, y_i, z_i$  and scattering phase control variables of UAV swarm. The optimization problem can be written as follows

$$\begin{aligned} \min f &= |S(\theta, \varphi) - Sd(\theta, \varphi)| \\ \text{st :} & \quad \theta_{ss} < \theta < \theta_{sd} \\ & \quad \varphi_{ss} < \varphi < \varphi_{sd} \end{aligned} \quad (14)$$

Evolutionary algorithms such as genetic algorithm can be used to approximate the problem. When the approximation error is minimized, and the optimal RCS of UAV swarm in this segment can be obtained.

The limited maneuverability of UAV formation is the positioning accuracy of each UAV in the swarm, to ensure control accuracy of the RCS, the positioning accuracy should be less than a tenth of the wave length. For instance, when the frequency of the radar is 1 GHz, the positioning accuracy should be less than 30mm.

#### 4.2 Trajectory Planning Algorithm for UAV Swarm

When the RCS of UAV swarm is optimized in each cell, the radar detection probability  $P_d$  can be calculated by (12), which can be used as one of the costs of going through this segment.

In the process of UAV swarm trajectory planning, on the one hand, it is necessary to ensure the lowest probability of being detected by radar in the penetration process, on the other hand, it is necessary to minimize the whole penetration distance, which means less stagnation time and fuel consumption.

Therefore, the trajectory planning problem of UAV swarm can be modeled as a multi-objective optimization problem in the following equation:

$$\begin{aligned} \min f &= \alpha \sum_{i=0} P_i + \beta \sum_{i=0} d_i \\ \text{st} : \quad &\alpha + \beta = 1, \alpha, \beta > 0 \end{aligned} \quad (15)$$

In (15),  $d_i$  represents length of the  $i$ th segment in the trajectory,  $P_i$  represents the detection probability of the  $i$ th segment in the trajectory.  $\alpha$ ,  $\beta$  represents the weights of the length and the detection probability.

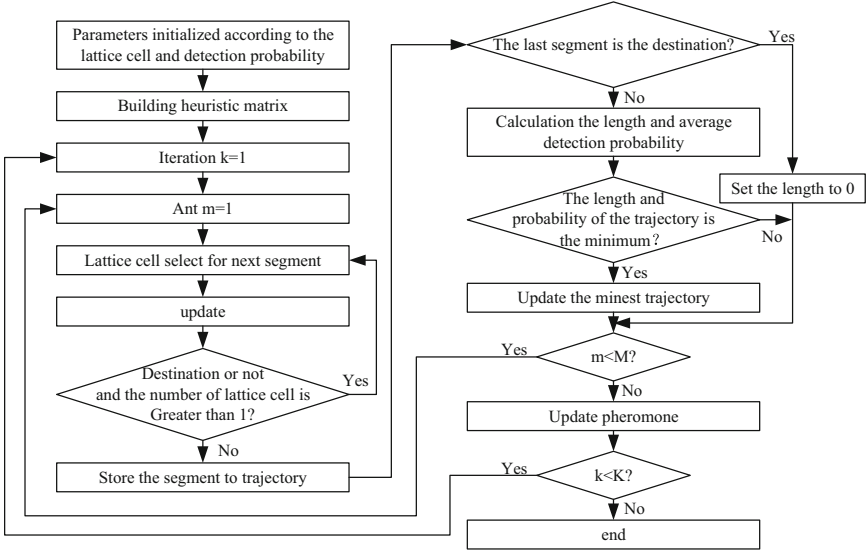


Fig. 2. Ant colony algorithm with the Trajectory Planning.

This multi-objective optimization problem can be solved with ant colony algorithm in the process as shown in Fig. 2.

## 5 Simulation and Analysis

### 5.1 Simulation and Analysis of UAV Swarm Trajectory Planning

In the process of UAV swarm penetration, the distance between UAV and detection radar is relatively far, and the UAV swarm generally flies at a low alti-tude. In this situation, the pitching angle between UAV swarm and the radar on the ground is approximately  $0^\circ$ . Therefore, in order to simplify the calculation, a two-dimensional scenario is adopted for simulation test.

In the simulation, the size of the penetration region is  $200 \text{ km} * 200 \text{ km}$ . The takeoff point is located near the origin of coordinates, and the destination point



is located at (200,200). Six radars are distributed in the region as shown in Fig. 3, and the frequency is set to 1GHz, bandwidth is 10MHz. Number of UAVs is set to 31. The weights of the trajectory length and the detection probability are set equal  $\alpha = \beta = 0.5$ .

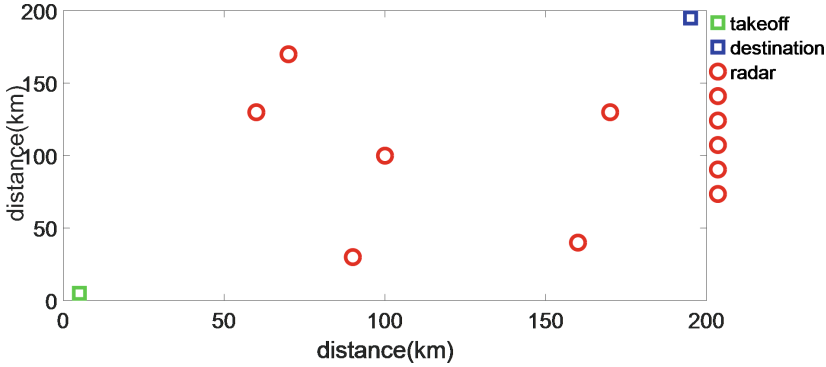


Fig. 3. Detection radar distribution.

In order to better reflect the effect of cooperative stealth, the ant colony algorithm is used to carry out the trajectory planning respectively for the cooperative stealth and non-stealth situations. The results are shown in Table 1.

Table 1. The trajectory planning results

	The length of trajectory (km)	Maximum detection probability	Mean detection probability
Cooperative stealth	262.4	0.012	0.0008
Non-stealth	303.8	0.23	0.0711

Table 1 shows that using formation optimization to control RCS of UAV swarm can not only shorten the trajectory length, but also greatly reduce the detection probability. The reason why the cooperative stealth can shorten the length of the trajectory can be explain by (12). To reduce the probability of radar detection, reducing the RCS of the UAV swarm by cooperative stealth has the same effect with increase the distance between the radar and UVA swarm. The comparison of the two trajectories is shown in Fig. 4.

In Fig. 4, the detection probability and detection distance of radar can be greatly reduced through the cooperative stealth. And the trajectory is basically a straight line which is the main reason to reduce the length of the trajectory. While the trajectory with no stealth optimized has to increase the distance

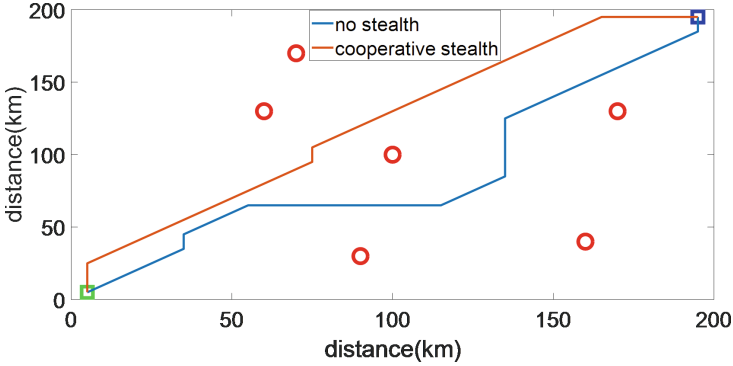


Fig. 4. The comparison of two trajectories.

between radar and UAV swarm to reduce the detection probability, resulting in longer flight trajectory and longer penetration time.

Table 2. The statistical results

	Average length of trajectory (km)	Average maximum detection probability
Cooperative stealth	266.7	0.017
Non-stealth	347.2	0.21

To illustrate the effectiveness of the method better, a more general simulation is carried out with the radar randomly distributed over the area. The takeoff point and the destination point remain the same. The total number of the simulation is 40, and the statistical results is shown in Table 2. In the simulation the trajectory planning with cooperative stealth can always find a low detection trajectory with increasing less distance between the radar and the UAV swarm.

## 5.2 Simulation and Analysis of Cooperative Stealth of UAV Swarm Formation

Taking the first lattice cell as an example, the nearest radars are mainly in the direction  $\varphi = 18^\circ$ ,  $\varphi = 65^\circ$  of the aircraft coordinate system. In order to avoid radar detection, it is necessary to minimize the RCS of the UAV swarm in these directions. As shown in Fig. 5, normalized scattering pattern  $Sd(\theta, \varphi)$  is set as:

$$Sd(0^\circ, \varphi) = 0 \quad 0^\circ < \varphi < 20^\circ, 63^\circ < \varphi < 67^\circ. \quad (16)$$

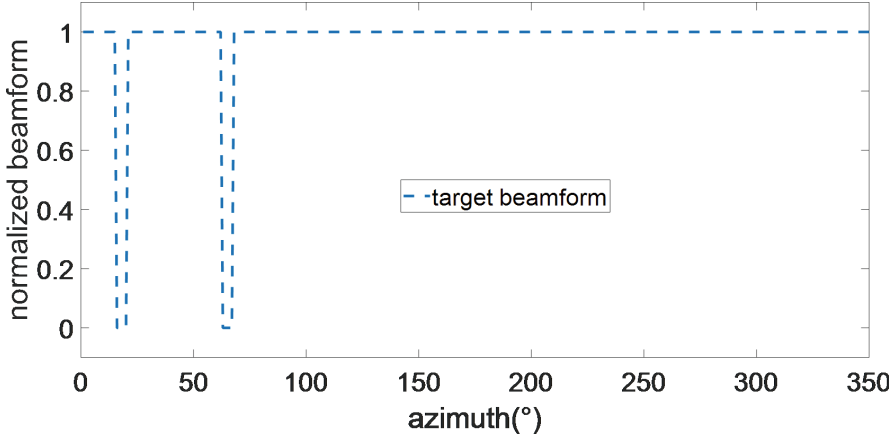


Fig. 5. Normalized scattering pattern.

In this segment of trajectory, normalized scattering pattern  $Sd(\theta, \varphi)$  in Fig. 5 is taken as the optimization target, and the genetic algorithm is used to synthesize the swarm array. In the optimization process, only the spatial coordinates of UAVs are used as the optimization variables, and the iteration is set to 300. The normalized scattering pattern  $S(\theta, \varphi)$  obtained is shown in Fig. 6 below.

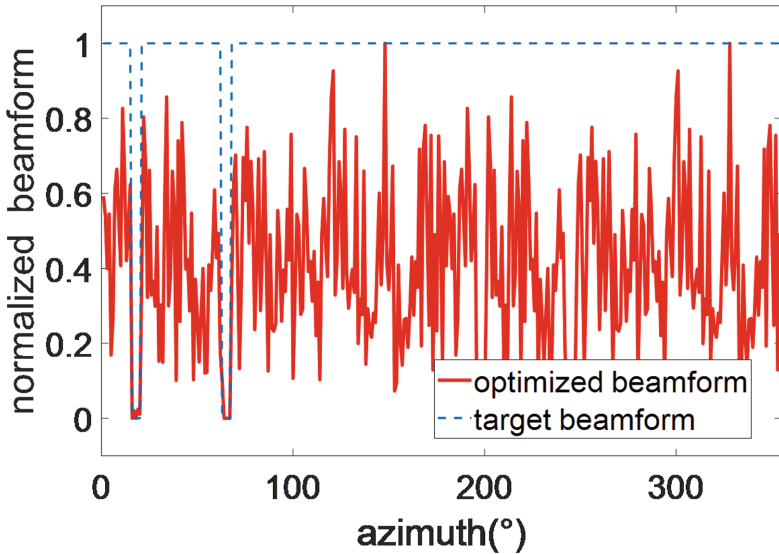


Fig. 6. The optimized scattering pattern.

After array optimization, the scattering pattern  $S(\theta, \varphi)$  is effectively reduced in the direction of detection radar, with an average value less than 0.02 in this range. Compared with other directions, the mean value of RCS is reduced by more than 15 dB, which can effectively reduce the detection probability of this segment trajectory.

## 6 Conclusion

In this paper, a trajectory planning method is proposed from the perspective of UAV swarm cooperative stealth. The UAV swarm is regarded as a three-dimensional sparse array, and the RCS of the whole swarm is optimized by array synthesis method, the relationship between radar detection probability and RCS of the swarm is also considered. Through simulation analysis, this method can significantly reduce the trajectory length and radar detection probability of UAV swarm.

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