Similar Formation Problem with Biased Random Measurement Errors

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Abstract. The similar formation problem affected by biased random measurement errors is studied in this paper. In practical application, the measurement data of agents is disturbed by biased random errors, which invalidates the existing similar formation algorithm. Hence, an improved similar formation algorithm is designed in this paper to eliminate the influence of biased errors. Based on the 2-rooted measurement topology, communication edges and measurement edges are appropriately added. Then, using the geometric relationship between agents, agents convert biased random errors into unbiased random errors online. Moreover, the decreasing gain of the algorithm ensures that agents can asymptotically converge to a similar formation of the generic target formation in the sense of mean square. The numerical simulation verifies the conclusion of this paper.

Keywords: Similar formation algorithm · Biased random errors · Mean square convergence

1 Introduction

There is a large body of work on formation control in the multi-agent systems literature, such as commercial performances, area detection, and personnel search and rescue. Depending on whether local coordinate systems of agents are aligned, the formation control algorithm can be divided into the formation control algorithm depending on a common coordinate system and the formation control algorithm not depending on any common coordinate system. With the formation control algorithm depending on a common coordinate system, local coordinate systems of agents are aligned with a common coordinate system or the global coordinate system [1–3]. However, the requirement for the common coordinate system makes these algorithms unable to run in systems that are indoors, underground, and other environments without GPS signals. With formation control algorithms not depending on any common coordinate system, agents do not share a common north. Such formation control algorithms These algorithms reduce the requirements for global information, which allows them to be widely used even in GPS-denied areas. Therefore, formation control algorithms not depending on any common coordinate system attract much attention.

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According to the type of information used, the formation control algorithm not depending on any common coordinate system is divided into: the bearing-based formation algorithm, the distance-based formation algorithm, and the Laplacian-based formation algorithm. In the bearing-based formation algorithm, the agent only uses the relative bearing information with its neighbor agent [4,5]. In [4,5], if bearing constraints of the target formation satisfy certain assumptions of the bearing rigidity, the system locally converges to the target formation. In the distance-based formation algorithm, the agent only uses the relative distance information with its neighbor agent [6,7]. In [6,7], if the target formation satisfies certain assumptions of the rigidity, the system locally converges to the target formation. In the Laplacian-based formation algorithm, the agent uses both the relative bearing information and the relative distance information, i.e. the relative position information, with its neighbor agent, and the Laplacian is designed based on the target formation with respect to a global reference system [8,9]. The article [8] shows that in the complex plane, if the measurement topology is 2-rooted, using the similar formation algorithm, the system globally converges to a similar formation of the target formation. Since only the similar formation algorithm is globally convergent, this paper conducts further research on the algorithm.

The above researches assume that the measurement information is accurate. In fact, the measurement information often contains errors. If the measurement error is an unbiased random error, the measurement error can be described as an unbiased random disturbance in the system model. Regardless of whether the disturbance is additive or multiplicative, as long as the measurement topology and system parameters satisfy certain conditions, the system is convergence in the mean square sense and the almost sure sense [10–12]. The measurement error may also be the biased random error. For example, when the agent measures the relative position with its neighbor, the measurement information of relative distance contains the random error, and the mean value of the error is an unknown non-zero constant. With the biased random error, the article [13] points out that the similar formation algorithm cannot converge to any similar formation of the target formation in expectation sense. As far as the author knows, for the system with biased random measurement errors, there is no further research result on how to ensure the convergence of the formation algorithm.

Based on the similar formation algorithm shown in [8], this paper mainly studies how to design the similar formation algorithm such that the formation system converges to a similar formation of the target formation in the sense of mean square when the relative position measurement is affected by biased random measurement errors. By adding measurement edges to the 2-rooted measurement topology and adding communication edges between agents, the geometric relationship between agents is skillfully used such that the agent can use the information measured by its neighbors to convert biased random errors into unbiased random errors online. Then, by designing decreasing system parameters, this paper proves that the improved discrete-time similar formation algorithm ensures that the formation system asymptotically converges to a similar formation of the target formation in the sense of mean square.

The organization of this article is as follows. The system model and the error model are introduced in Sect. 2. In Sect. 3, the improved discrete-time similar formation algorithm is shown. The construction method of communication topology and measurement topology and the design method of algorithm parameters are introduced in this section. In Sect. 4, the globally mean-square convergence of the improved discrete-time similar formation algorithm is proved. Section 5 shows results of the numerical simulation, which verify the conclusion of this article.

Symbols: $\mathbb{C}$ and $\mathbb{R}$ represent the complex number set and the real number set, respectively. $\mathbb{C}^n$ represents the set of the n-dimensional complex column vector. $\mathbb{C}^{n\times n}$ represents set of the $n\times n$-dimensional complex matrix. $\iota = \sqrt{-1}$ represents the imaginary unit. $\mathbf{1}_n$ represents the n-dimensional vector with all elements being 1. I_n represents the $n \times n$-dimensional identity matrix. For $p \in \mathbb{C}$, $||p||$ represents the modulus of p. For a matrix A, its $i-j$th entry is represented by A_{ij}. A^T and A^H represent the transpose and the conjugate transpose of the matrix A, respectively. $||A||$ represents the matrix norm of A induced by Euclidean norm. For the random variable x, $\mathbb{E}(x)$ and $\mathbb{D}(x)$ represent the expectation and variance of x, respectively. For $a \in \mathbb{R}$ and $b \in \mathbb{R}$, $[a \mod b]$ is modulus operation.

2 System Model

Consider a multi-agent system with n agents in a plane. With respect to a global reference frame Σ, the position of each agent $i \in \{1, \cdots, n\}$ is represented by a complex number $z_i \in \mathbb{C}$. And $z = [z_1, \ldots, z_n]^T \in \mathbb{C}^n$ represents the absolute position vector of agents. However, each agent knows neither the global reference frame Σ nor their absolute positions z_i's. Each agent i has a local reference frame Σ_i. With respect to the local reference frame Σ_i, the position of each agent $j \in \{1, \cdots, n\}$ is represented by $z_j^{(i)} \in \mathbb{C}$. θ_i represents the orientation angle of Σ_i, i.e. $z_j^{(i)} - z_i^{(i)} = e^{\iota\theta_i}(z_j - z_i)$. And θ_{ij} represents the orientation difference between Σ_i and Σ_j, i.e. $z_j^{(i)} - z_i^{(i)} = e^{\iota\theta_{ij}}(z_j^{(j)} - z_i^{(j)})$. Especially, $\theta_{ii} = 0$.

$\mathcal{G} = (\mathcal{V}, \mathcal{E})$ represents the measurement topology of the system, where $\mathcal{V} = \{1, \ldots, n\}$ and $\mathcal{E}$ represent the agent set and the measurement edge set, respectively. If $(j, i) \in \mathcal{E}$, agent i measures the relative position of agent j with respect to the local reference frame Σ_i. Let $\mathcal{N}_i = \{j | j \in V \text{ and } (j, i) \in \mathcal{E}\}$. $\mathcal{G}' = (\mathcal{V}, \mathcal{E}')$ represents the communication topology of the system, where $\mathcal{E}'$ represents the communication edge set. If $(i, j) \in \mathcal{E}'$, agent j sends its measurement data to agent i. Let $\mathcal{N}_i' = \{j | j \in V \text{ and } (i, j) \in \mathcal{E}'\}$.

The goal of the similar formation algorithm is to make agents in the system achieve a similar formation of the target formation. Let $\xi = [\xi_1, \ldots, \xi_n]^T \in \mathbb{C}^n$ represent the target formation of the similar formation algorithm with respect to the global reference frame Σ. Suppose the target formation ξ is generic [8]. The set of all similar formations of the target formation ξ is described by

$$S(\xi) = \{c_1\mathbf{1}_n + c_2\xi, c_1, c_2 \in \mathbb{C}\}. \quad (1)$$

Let $y_{ij}^{(i)}(k)$ represents the relative position between agent j and agent i measured by agent i. Then, $y_{ij}^{(i)}(k)$ satisfies that

$$y_{ij}^{(i)}(k) = z_j^{(i)}(k) - z_i^{(i)}(k) + d_{ij}^{(i)}(k), \tag{2}$$

where $d_{ij}^{(i)}(k)$ represents the biased random measurement error.

The biased measurement error $d_{ij}^{(i)}(k)$ is described by

$$d_{ij}^{(i)}(k) = \delta_{ij}^{(me)}(k)e^{\iota\vartheta_{ij}(k)}. \tag{3}$$

In Eq. (3), $\delta_{ij}^{(me)}(k)$ represents the distance measurement error. $\vartheta_{ij}(k)$ represents the azimuth angle of the agent j relative to the agent i, i.e. $\vartheta_{ij}(k) = \frac{(z_j^{(i)}(k)-z_i^{(i)}(k))}{||(z_j^{(i)}(k)-z_i^{(i)}(k))||}$. In this paper, we only consider the biased distance measurement error. Hence, the azimuth angle measured by agent is accurate. The biased distance measurement error $\delta_{ij}^{(me)}(k)$ satisfies the following assumption.

Assumption 1. *$\delta_{ij}^{(me)}(k)$ is a random real number, which is unknown to any agent. The expectation and the variance of $\delta_{ij}^{(me)}(k)$ are $\delta_c > 0$ and $\sigma_c^2 > 0$, respectively. $\delta_{ij}^{(me)}(k)$ is independent of each other for different i, j and k.*

To compensate the effect of the biased measurement error, this paper shows an improved discrete-time similar formation algorithm which requires relative position measurement information measured by agents and their neighbors. The model of the improved similar formation algorithm is built as

$$z_i^{(i)}(k+1) - z_i^{(i)}(k) = g_i\big(\cdots, y_{sj}^{(i)}(k), \cdots\big), \tag{4}$$

where the function g_i is a homogeneous linear function corresponding to the measurement digraph $\mathcal{G}$, $i \in \mathcal{V}$, $j \in \mathcal{N}_i$, $s \in \mathcal{N}_i \cup \{i\}$ and $k = 1, 2, \ldots$.

In Eq. (4), when $s = i$, $y_{sj}^{(i)}(k) = y_{ij}^{(i)}(k)$; when $s \in \mathcal{N}_i$, we have

$$y_{sj}^{(i)}(k) = \theta_{is} y_{sj}^{(s)}(k). \tag{5}$$

When the agent s satisfies that $s \in \mathcal{N}_i$ and $s \in \mathcal{N}_i'$, the agent i use the following distributed method [14] to calculate the orientation difference θ_{is}:

$$\theta_{is} = [(\vartheta_{si}(k) - (\vartheta_{is}(k)) \mod 2\pi] - \pi. \tag{6}$$

The definition of the discrete-time similar formation algorithm (4) converging to a similar formation of the target formation in the mean-square sense is given as follows.

Definition 1. *The distributed similar formation algorithm (4) globally mean-square converges to a similar formation of the target formation ξ for an arbitrary initial coordinate z_0, if there exist $c_1, c_2 \in \mathbb{C}$ such that*

$$\mathbb{D}(z(k) - c_1\mathbf{1}_n - c_2\xi) \to 0,\ k \to \infty. \tag{7}$$

3 Improved Discrete-Time Similar Formation Algorithm

In this section, the improved algorithm is described in detail. The agent satisfies the following assumption.

Assumption 2. *Each agent knows its neighbors' target positions, i.e., for any* $j \in \mathcal{N}_i$, ξ_j *is available to agent* i.

Then, the building method of the measurement topology and the communication topology is shown as follows. At first, build the measurement topology $\mathcal{G}$, which satisfies the following assumption.

Assumption 3. *The measurement digraph* $\mathcal{G}$ *is 2-rooted.*

According to Assumption 3, we have a 2-rooted measurement digraph. Then, combining the arbitrary generic formation $\xi \in \mathbb{C}^n$, we can get the complex Laplacian matrix $\widetilde{L} \in \mathbb{C}^{n\times n}$ of digraph $\mathcal{G}$. In the matrix $\widetilde{L}$, its $i-j$th entry satisfies that $\widetilde{L}_{ij} \neq 0$ if $j \in \mathcal{N}_i$; $\widetilde{L}_{ij} = 0$, otherwise. Its $i-i$th diagonal entry satisfies that $\widetilde{L}_{ii} = \sum_{j\in\mathcal{N}_i} -\widetilde{L}_{ij}$. And the matrix $\widetilde{L}$ satisfies that $\widetilde{L}\mathbf{1}_n = 0$ and $\widetilde{L}\xi = 0$. By Theorem 4.1 in [8], there exists a diagonal invertible matrix $\widetilde{D}$ such that eigenvalues of $\widetilde{D}\widetilde{L}$ in the left-half open plane in addition to two fixed eigenvalues at the origin. Hence, we can find a small enough $a \in (0, 1)$ such that except for the two eigenvalues of $I + a\widetilde{D}\widetilde{L}$ which are fixed at 1, the other eigenvalues of $I + a\widetilde{D}\widetilde{L}$ are all in the unit circle. Let $L = a\widetilde{D}\widetilde{L}$.

After we get the matrix L, we need to add some measurement edges to the 2-rooted measurement topology and add some communication edges between agents to compensate the effect of the biased random measurement error. Assumption 3 shows that for each agent i, there exist two neighbors $j_1, j_2 \in \mathcal{N}_i$. Based on Assumption 3, the method of adding measurement and communication edges satisfies the following assumptions.

Assumption 4. *For each agent* i *and its two neighbors* $j_1, j_2 \in \mathcal{N}_i$, *add two measurement edges* $(j_2, j_1), (i, j_1)$ *to measurement edge set* $\mathcal{E}$.

Assumption 5. *For each agent* i *and its neighbor agent* j_1 *mentioned in Assumption 4, add one communication edge* (i, j_1) *to communication edge set* $\mathcal{E}'$, *i.e., the agent* j_1 *sends measurement messages to the agent* i.

Note: Assumption 5 shows that the communication topology is only used to transmit information between agents, and it does not need to be connected. And according to Assumptions 4 and 5, in the system with n agents, at most $2n$ measurement edges and n communication edges need to be added. Since some of the measurement edges that need to be added according to Assumptions 4 already exist in the 2-rooted measurement digraph, the actual number of added measurement edges is less than $2n$.

After we building topologies, by period $T > 0$, each agent $i \in V$ measures the relative position information $y_{ij}^{(i)}(k)$ of the neighbor agent $j \in \mathcal{N}_i$; and for each agent $i \in V$, its neighbor agent j_1 shown in Assumptions 4 and 5 sends the relative position measurement information $y_{j_1 j_2}^{(j_1)}(k)$ and the azimuth angle measurement information $\vartheta_{si}(k)$ to the agent i. Then, at the kth iteration, the position of the agent $i \in V$ changes according to the following equation:

$$z_i^{(i)}(k+1) - z_i^{(i)}(k) = \sum_{j \in \mathcal{N}_i, s \in \mathcal{N}_i \cup \{i\}} a(k)\omega_{js}^{(i)}(k)\big(y_{sj}^{(i)}(k)\big), \tag{8}$$

where the decreasing gain $a(k)$ satisfies that for any integer $k \geq 0$, $a(k) \in (0,1]$, $\sum_{k=0}^{\infty} a(k) \to \infty$ and $\sum_{k=0}^{\infty} a(k)^2 < \infty$.

In Eq. (8), $\omega_{js}^{(i)}(k)$ satisfies the following equation

$$\omega_{js}^{(i)}(k) = \begin{cases} L_{ij} & j \in \mathcal{N}_i - \{j_1, j_2\}, s = i \\ L_{ij_1} - \omega_i(k) & j = j_1, s = i \\ L_{ij_2} + \omega_i(k) & j = j_2, s = i \\ -\omega_i(k) & j = j_2, s = j_1 \\ 0 & \text{others} \end{cases}, \tag{9}$$

and $\omega_i(k)$ in Eq. (9) satisfies the following equation

$$\omega_i(k) = -\frac{\sum_{j \in \mathcal{N}_i} L_{ij} e^{\iota \vartheta_{ij}(k)}}{e^{\iota \vartheta_{ij_2}(k)} - e^{\iota \vartheta_{ij_1}(k)} - e^{\iota \vartheta_{j_1 j_2}(k)}}. \tag{10}$$

It is worth noting that the parameter $\omega_i(k)$ is calculated only by the information currently obtained by the agent i. Hence, the time-varying parameter $\omega_{js}^{(i)}(k)$ is calculated by agent i online.

With paraments $\omega_{js}^{(i)}(k)$ shown in Eq. (9), the expectation of Eq. (8) satisfies that

$$\begin{aligned} &\mathbb{E}\big(z_i^{(i)}(k+1) - z_i^{(i)}(k)\big) \\ &\quad = a(k) \sum_{j \in \mathcal{N}_i} L_{ij} \mathbb{E}\big(z_j^{(i)}(k) - z_i^{(i)}(k)\big) + a(k) \sum_{j \in \mathcal{N}_i - \{j_1, j_2\}} L_{ij} \delta e^{\iota \vartheta_{ij}(k)} \\ &\quad + a(k)(L_{ij_1} - \omega_i(k))\delta e^{\iota \vartheta_{ij_1}(k)} + a(k)(L_{ij_2} + \omega_i(k))\delta e^{\iota \vartheta_{ij_2}(k)} \\ &\quad - a(k)\omega_i(k)\delta e^{\iota \vartheta_{j_1 j_2}(k)} \\ &\quad = a(k) \sum_{j \in \mathcal{N}_i} L_{ij} \mathbb{E}\big(z_j^{(i)}(k) - z_i^{(i)}(k)\big). \end{aligned} \tag{11}$$

Equation (11) shows that $\mathbb{E}\big(z_i^{(i)}(k+1) - z_i^{(i)}(k)\big)$ is not affected by the expectation of biased measurement error δ. In other words, by adding the measurement edge and the communication edge, and using the geometric relationship between the agents, this algorithm successfully converts the biased random measurement error into the unbiased random error.

4 Globally Mean-Square Convergence of the Improved Discrete-Time Similar Formation Algorithm

In this section, the globally mean-square convergence of the improved discrete-time similar formation algorithm (8) is proved.

Multiply $e^{\iota\theta_i}$ on both sides of Eq. (8), then Eq. (8) is equivalent to

$$\begin{aligned} z_i(k+1) - z_i(k) \\ &= \sum_{j\in\mathcal{N}_i} a(k)L_{ij}(y_{ij}(k)) \\ &+ e^{-\iota\theta_i}a(k)\omega_i(k)\big(d^{(i)}_{ij_2}(k) - d^{(i)}_{ij_1}(k) - d^{(i)}_{j_1j_2}(k)\big) \\ &= \sum_{j\in\mathcal{N}_i} a(k)L_{ij}(y_{ij}(k)) + a(k)\omega_i(k)\big(d_{ij_2}(k) - d_{ij_1}(k) - d_{j_1j_2}(k)\big), \end{aligned} \tag{12}$$

where for any $j \in \mathcal{N}_i$, $y_{ij}(k) = z_j(k) - z_i(k) + d_{ij}(k)$, $d_{ij}(k) = e^{-\iota\theta_i}d^{(i)}_{ij}(k)$; and $d_{j_1j_2}(k) = e^{-\iota\theta_i}d^{(i)}_{j_1j_2}(k)$.

Then, Eq. (12) can be written as

$$z(k+1) = (I_n + a(k)L)z(k) + a(k)\Big(\sum_{i\in\mathcal{V}} \tilde{A(i)}\big(d_i(k) + b_i(k)\big)\Big), \tag{13}$$

where $\tilde{A(i)} \in \mathbb{C}^{n\times n}$, if $j \neq i$, $\tilde{A(i)}_{ij} = L_{ij}$; the other entries of $\tilde{A(i)}$ are 0. $d_i(k) = [d_{i1}(k), \ldots, d_{in}(k)]^T \in \mathbb{C}^n$, if $j \notin \mathcal{N}_i$, $d_{ij}(k) = 0$. $b_i(k)$ satisfies that

$$b_i(k) = \frac{\omega_i(k)}{\sum_{j\in\mathcal{N}_i} L_{ij}}\big(d_{ij_2}(k) - d_{ij_1}(k) - d_{j_1j_2}(k)\big)\mathbf{1}_n. \tag{14}$$

The main theorem of this paper are as follows.

Theorem 1. *Under Assumptions 1, 2, 3, 4 and 5, for any generic formation $\xi \in \mathbb{C}^n$, there exist paraments $\omega^{(i)}_{js}(k)$, complex constants c_1 and c_2 such that the improved discrete-time similar formation algorithm* (8) *converges to the similar formation $c_1\mathbf{1}_n + c_2\xi$ of the generic formation ξ.*

Proof. For proving the theorem, the properties of the mean and variance of $z(k) - c_1\mathbf{1}_n - c_2\xi$ need to be studied. Based on Eq. (13), we have

$$\begin{aligned} z(k+1) - c_1\mathbf{1}_n - c_2\xi = (I_n + a(k)L)z(k) - c_1\mathbf{1}_n - c_2\xi \\ + a(k)\Big(\sum_{i\in\mathcal{V}} \tilde{A(i)}\big(d_i(k) + b_i(k)\big)\Big), \end{aligned} \tag{15}$$

Consider a matrix $Q \in \mathbb{C}^{(n-2)\times n}$. The matrix Q satisfies that $Q\mathbf{1}_n = 0$, $Q\xi = 0$ and $QQ^H = \mathbf{1}_{n-2}$. Let $Qz(k) = x(k)$. Left multiply Q on both sides of Eq. (15), we have

$$x(k+1) = (I_{n-2} + a(k)QLQ^H)x(k) + a(k)Q\Big(\sum_{i\in\mathcal{V}} \tilde{A(i)}\big(d_i(k) + b_i(k)\big)\Big). \tag{16}$$

Let $\lambda_{max}(k)$ represents the eigenvalue of $I_{n-2} + a(k)QLQ^H$ with the maximum norm. And we have that for any integer $k \geq 0$, $||\lambda_{max}(k)|| < 1$.

Since

$$\begin{aligned}\mathbb{E}\big(\tilde{A(i)}\big(d_i(k) + b_i(k)\big)\big) &= \mathbb{E}\big(\sum_{j\in\mathcal{N}_i} L_{ij}\delta_{ij}^{(me)}(k)e^{\iota(\vartheta_{ij}(k))}\big) \\ &+ \mathbb{E}\big(\omega_i(k)\big(d_{ij_2}(k) - d_{ij_1}(k) - d_{j_1j_2}(k)\big) \\ &= \big(\sum_{j\in\mathcal{N}_i} L_{ij}e^{\iota(\vartheta_{ij}(k))}\big)\delta_c - \big(\sum_{j\in\mathcal{N}_i} L_{ij}e^{\iota(\vartheta_{ij}(k))}\big)\delta_c \\ &= 0,\end{aligned} \tag{17}$$

when $k \to \infty$, we have

$$\begin{aligned}\mathbb{E}(x(k+1)) &= (I_{n-2} + a(k)QLQ^H)\mathbb{E}(x(k)) \\ &= \prod_{i=0}^{k}(I_{n-2} + a(i)QLQ^H)\mathbb{E}(x(0)) \\ &\leq \prod_{i=0}^{k}\lambda_{max}(i)\mathbb{E}(x(0)) \\ &\to 0.\end{aligned} \tag{18}$$

Now, consider the property of the variance of $x(k)$. Due to Eq. (17), the second-order moment of $x(k)$ satisfies that

$$\begin{aligned}&\mathbb{E}\big(x(k+1)^H x(k+1)\big) \\ &= \mathbb{E}\Big(x(k)^H\big(I_{n-2} + a(k)QLQ^H\big)^H\big(I_{n-2} + a(k)QLQ^H\big)x(k)\Big) \\ &+ (a(k))^2\mathbb{E}\Big\{\Big[Q\Big(\sum_{i\in\mathcal{V}}\tilde{A(i)}\big(d_i(k) + b_i(k)\big)\Big)\Big]^H\Big[Q\Big(\sum_{i\in\mathcal{V}}\tilde{A(i)}\big(d_i(k) + b_i(k)\big)\Big)\Big]\Big\}.\end{aligned} \tag{19}$$

Since the norm of any element of $\tilde{A(i)}\big(d_i(k)+b_i(k)\big)$ is less than δ_c, then Eq. (19) satisfies that

$$\begin{aligned}&\mathbb{E}\big(x(k+1)^H x(k+1)\big) \\ &\leq \lambda_{max}(k)^2\mathbb{E}\big(x(k)^H x(k)\big) + \big(a(k)\big)^2\delta_c^2 \\ &= \prod_{i=0}^{k}\lambda_{max}(i)^2 + \sum_{s=1}^{k-1}\prod_{i=s}^{k}\lambda_{max}(i)^2\big(a(s-1)\big)^2\delta_c^2 + \big(a(k)\big)^2\delta_c^2 \\ &\to 0 \text{ as } k \to \infty.\end{aligned} \tag{20}$$

Due to Eqs. (18) and (20), we get that $\mathbb{D}(z(k) - c_1\mathbf{1}_n - c_2\xi)$ tends to 0 as k tends to infinity. Hence, the improved discrete-time similar formation algorithm (8) satisfies Definition 1.

5 Numerical Simulation

Consider the similar formation system with 9 agents in a plane. The target formation ξ of the 9 agents is shown in Fig. 1, which is generic. Figure 2 shows the measurement topology and the communication topology. In Fig. 2, black curves and black dashed curves represent measurement edges in the measurement topology, red curves represent communication edges in the communication topology. If a black (dashed) curve points from agent i to agent j, then $(j, i) \in \mathcal{E}$; if a red curve points from agent i to agent j, then $(j, i) \in \mathcal{E}'$.

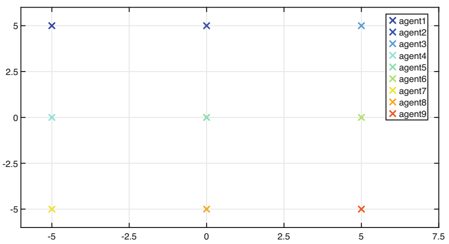

Fig. 1. The target formation ξ.

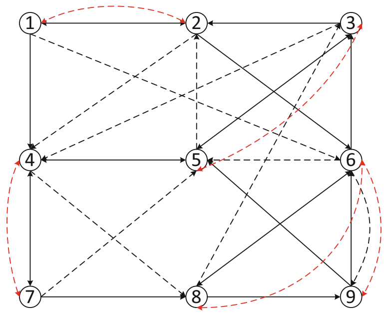

Fig. 2. The topology.

Next, we introduce the composition process of the topology shown by Assumptions 3, 4 and 5 in the paper.

In the first step, according to Assumption 3 to build a 2-rooted measurement digraph, and measurement edges of the 2-rooted digraph are black curves in Fig. 2. Then, based on the 2-rooted digraph, build the matrix L according to the way shown in Sect. 3.

In the second step, according to Assumption 4, add measurement edges to the 2-rooted digraph, which is represented by black dashed curves. And according to Assumption 5, add communication edges between agents, which is represented by red curves.

According to Assumptions 4 and 5, in the system with 9 agents, at most 18 measurement edges and 9 communication edges need to be added. Figure 2 shows that only 9 measurement edges and 9 communication edges are added, actually.

Initial positions $z_i(0), i \in \mathcal{V}$ of 9 agents are randomly selected. The numerical simulation results of the improved discrete-time similar formation algorithm (8) as follows.

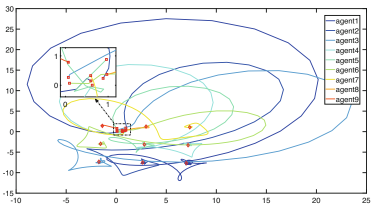

Fig. 3. Movement trajectories.

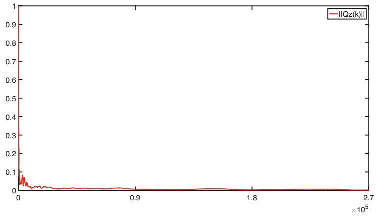

Fig. 4. The curve of $||Qz(k)||$.

In Fig. 3, the starting points of the 9 agents, i.e. the initial formation, are represented as red squares, which is surrounded by a dashed line frame. The end points of the 9 agents, i.e. the final formation, are represented as red diamonds. And the 9 colored curves represent the trajectories of the 9 agents respectively. Figure 3 shows that the 9 agents move from the initial random formation to the final formation, which is a similar formation of the target formation ξ.

In Fig. 4, the red curve represents the curve of $||Qz(k)||$ with the increase of k. Figure 4 shows that $||Qz(k)|| \to 0$ as $k \to \infty$, which means that the formation of the 9 agents asymptotically tends to the similar formation of the target formation ξ.

6 Conclusion

To solve the similar formation problem with relative measurements between agents affected by biased random errors, the improved discrete-time similar formation algorithm is designed. By adding some measurement edges to the 2-rooted measurement topology and adding some communication edges between agents, each agent can use the geometric relationship between agents to convert biased random errors into unbiased random errors online. Then, the decreasing system parameters ensures that the formation system asymptotically converges to a similar formation of the target formation in the sense of mean square. Finally, the numerical simulation result supports the conclusion in this paper. The optimization of the formation algorithm and physical experiments will be the following research direction.

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