Corroded Stirrups Effects on the Shear Behavior of Reinforced Concrete Slender Beams

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Abstract. Stirrups corrosion causes damage effects on the shear response of reinforced concrete elements. Such damage mainly includes cracking and spalling of concrete in the cover, and reduction of the cross-sectional area of the reinforcing steel. The need for tools able to predict the reduction of the load-bearing capacity of reinforced concrete element has led to several formulations based on the numerical regression of available experimental results. This paper proposes a mechanical model for the prediction of the shear response of reinforced concrete slender beams with corroded reinforcement. The proposed procedure, based on the Modified Compression Field Theory, can reproduce the entire load-displacement curve. The corrosion effects on the effective beam width and rebar cross-section are taken into account. The accuracy of the proposed model is assessed using a database of sixty-two specimens. The proposed model achieves accurate results.

Keywords: Shear · Corrosion · Stirrups · Beams

1 Introduction

The lack of maintenance of Reinforced Concrete (RC) beams have led to deterioration of the mechanical properties of these structural elements, especially in existing bridges [1–3] and buildings [4]. Corrosion causes cracking and spalling of the concrete cover, and reduction of the cross-sectional area of the reinforcing steel [5].

These effects also depend on the type of corrosion (uniform or concentrated) of the reinforcement. Therefore, significant reduction of the ultimate strength and maximum deflection may occur. Moreover, concentrated corrosion (i.e. "pitting") may also change the failure mode of the beam, from ductile for bending to brittle for shear [6–8]. Fernandez et al. [5] observed that pitting is significant for structures where the moment redistributions is achievable.

Corrosion of the longitudinal reinforcing steel has largely been studied, and a large literature is available. By contrast, a few research studies are focused on corrosion of stirrups, and consequently on the reduction of the shear capacity of RC beams. Stirrups confine concrete, and then they reduce the critical shear crack width. Therefore, corrosion of stirrups significantly reduces the plasticity branch of the steel rebars and the shear strength capacity.

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It follows that stirrups corrosion is a critical issue. It affects not only the shear strength but also the ductility of the beam. Therefore, the deformation capacity of RC beams needs to be assessed.

However, most of the models proposed in the literature are able to predict only the load-bearing capacity at the Ultimate Limit State (ULS) of corroded RC beams, and, in many cases, the basic equations are empirical [9].

Recently, Cladera et al. [10] modified the Compression Chord Capacity Model (CCCM) to predict the shear strength of corrosion-damaged RC beams. Predictions were compared to the experimental results of 146 slender and deep beams failing in shear, where stirrups and/or longitudinal reinforcement were subjected to corrosion, and achieved satisfactory results.

This study aims to propose a theoretical model to predict the entire load-deformation curve of corroded RC beams by extending a previously established procedure based on the Modified Compression Field Theory (MCFT) [11]. A crack element containing longitudinal and transverse smeared steel rebars is considered. Moreover, both reduced rebar cross-section of and concrete cover because of corrosion are taken into account. The proposed model allows the calculation of the strain and stress fields by respecting all the equilibrium and compatibility equations. A validation of the proposed model, against an experimental test database reported in the literature, is presented.

2 Stirrups Corrosion Structural Effects

Steel rebars with uniform and/or concentrated corrosion show a reduction in the cross-sectional area (especially due to pitting) and deterioration of the geometric and mechanical properties of the concrete cover due to cracking and spalling.

2.1 Cross-Section Reduction of Steel Rebar

The cross-sectional loss ratio, η_a, can be evaluated as:

$$\eta_a = \frac{A_{s0} - A_s}{A_{s0}} \times 100 = \frac{\phi_0^2 - \phi^2}{\phi_0^2} \times 100 \tag{1}$$

where A_{s0} and ϕ_0 are the cross-sectional area and diameter, respectively, of the steel rebar before corrosion; A_s and ϕ are the cross-sectional area and diameter after corrosion. The maximum value (η_{am}) along a length of the steel rebar is an estimate of pitting. It is safe to use η_{am} for predicting the shear strength of RC members.

When only the weight loss ratio is reported, $\eta_w = 100 \times (m_0 - m)/m_0$ (m_0 = mass of a length of the steel rebar before corrosion; and m = mass after corrosion), it should been converted into cross-section loss ratios, η_{am}. With this aim, Cladera et al. [10] recently simplified a discontinued equation between η_{am} and η_w, originally introduced by Lu et al. [9], in the following form:

$$\eta_{am} = 1.36\,\eta_w \tag{2}$$

2.2 Effective Beam Width

Higgins et al. [12] observed that the volumetric expansion of corrosion products of steel stirrups causes spalling of the concrete cover. Thus, the external part of the beam is cracked and not able to carry stress. This corrosion-structural effect depends on stirrup spacing (s_v) and concrete cover (c). Higgins and co-authors proposed two equations, based on empirical data and theoretical computations, to estimate the effective beam width ($b_{w,eff}$). In the following, these equations are reported in the form proposed by Cladera et al. [10], presenting continuity for the case $s_v = 5.5(c + \phi_v)$, where ϕ_v is the stirrup diameter:

$$b_{w,eff} = b_w - 2(c + \phi_v) + \frac{s_v}{5.5} \tag{3}$$

$$b_{w,eff} = b_w - \frac{5.5}{s_v}(c + \phi_v)^2 \tag{4}$$

Equations (3) and (4) are for $s_v \leq 5.5(c + \phi_v)$ and $s_v > 5.5(c + \phi_v)$, respectively. Moreover, the same authors suggested to reduce the beam width (b_w) to the effective value ($b_{w,eff}$) only when stirrups exhibit at least 10% of cross-section loss.

3 Proposed Model for the Shear Response of Corroded RC Beams

The proposed model predicts the shear-displacement response of slender RC beams with corroded rebars. It is based on the MCFT equations, and, as assumed by other researchers in their formulations [13, 14], the flexural model is solved separately by the shear one.

3.1 Flexural Model

A top axial strain (ε_{ct}) value is assumed, then the flexural cross-section strain is estimated by adjusting the longitudinal strain of bottom reinforcement (ε_{sb}) until the internal axial load equals the external one [15]. Flexural analysis provides the bending moment (M), the curvature (χ), and the average axial strain from flexure (ε_{xf}) at the centroid level of the cross-section (see Fig. 1).

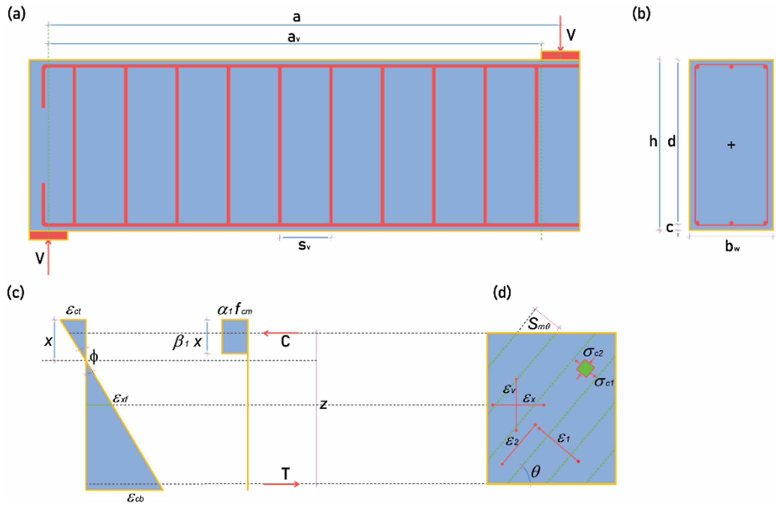

Fig. 1. a) Lateral view and b) cross-section of the RC beam. c) Flexural and d) Shear model.

3.2 Shear Model

The equilibrium conditions of MCFT require that:

$$\sigma_{cv} = \sigma_{c1} - \tau/cot\theta \tag{5}$$

$$\sigma_{c2} = \sigma_{c1} - \tau(1/cot\theta + cot\theta) \tag{6}$$

with σ_{cv} = concrete stress along the vertical direction; σ_{c1} and σ_{c2} = concrete principal tensile and compressive stresses, respectively; τ = shear stress; and θ = strut angle (see Fig. 1). Clamping stress (σ_v) can be obtained as $\sigma_v = \sigma_{cv} + \rho_{sv}\ \sigma_{sv}$; where $\rho_{sv} = A_{sv}/(b_w\ s_v)$ = geometric ratio of the transverse reinforcement; A_{sv} is the total area and s_v spacing of stirrups, respectively; and σ_{sv} is the stirrup stress.

In slender beams, the clamping stress along a cross-section enough far from the point load can be neglected [16], and then Eq. (6) allows calculation of the shear stress as follows:

$$\tau = (\sigma_{c1} + \rho_{sv}\sigma_{sv})cot\theta = (\sigma_{c1} - \sigma_{c2})cot\theta/\left(1 + cot^2\theta\right) \tag{7}$$

Hence, the strut angle inclination θ is obtained:

$$cot^2\theta = (-\sigma_{c2} - \rho_{sv}\sigma_{sv})/(\sigma_{c1} + \rho_{sv}\sigma_{sv}) \tag{8}$$

At the same time, the compatibility equations of the MCFT require that:

$$\varepsilon_x + \varepsilon_v = \varepsilon_1 + \varepsilon_2 \tag{9}$$

$$cot^2\theta = (\varepsilon_v - \varepsilon_2)/(\varepsilon_x - \varepsilon_2) \tag{10}$$

with ε_x and ε_v = horizontal and vertical strains, respectively; and ε_1 and ε_2 = principal tensile and compressive strains, respectively.

Equations (8) and (10) return the same value. However, the two equations of the strut angle give the vertical strain (ε_v), which is independent of the crack angle (θ). In any case, elastic or plastic behavior of transverse reinforcement must be distinguished.

If stirrups are elastic, the steel stress is $\sigma_{sv} = E_s\ \varepsilon_v$, and a 2nd-order equation in ε_v is obtained. The close-solution for ε_v is the follow:

$$\varepsilon_v = \left(-B_e + \sqrt{B_e^2 - 4A_eC_e}\right)/(2A_e) \tag{11}$$

where coefficients A_e, B_e, and C_e are:

$$A_e = \rho_{sv}E_s \tag{12}$$

$$B_e = \sigma_{c1} + A_{\varepsilon_v}(\varepsilon_x - 2\varepsilon_2) \tag{13}$$

$$C_e = \sigma_{c2}(\varepsilon_x - \varepsilon_2) - \sigma_{c1}\varepsilon_2 \tag{14}$$

If stirrups yield, ε_v is the solution of a linear equation:

$$\varepsilon_v = -C_y/B_y \tag{15}$$

where coefficients B_y and C_y are:

$$B_y = \sigma_{c1} + \rho_{sv}f_{yv} \tag{16}$$

$$C_y = \sigma_{c2}(\varepsilon_x - \varepsilon_2) - \sigma_{c1}\varepsilon_2 + \rho_{sv}f_{yv}(\varepsilon_x - 2\varepsilon_2) \tag{17}$$

The compression strength of concrete in a diagonally cracked web is reduced as a function of the principal strain. Therefore, the following equations are used [6, 11]:

$$\sigma_{c2} = f_{ce}\left[2\left(\frac{\varepsilon_2}{\varepsilon_{c0}}\right) - \left(\frac{\varepsilon_2}{\varepsilon_{c0}}\right)^2\right] \tag{18}$$

$$\eta = \frac{f_{ce}}{f_c} = \frac{1}{0.8 + 0.34(\varepsilon_1/\varepsilon_{c0})} \leq 1 \tag{19}$$

where f_{ce} = effective compressive strength of concrete; and ε_{c0} = strain at peak stress in concrete.

Cracked concrete is assumed to carry tensile stress by tension stiffening [11]:

$$\sigma_{c1} = f_{ct}/\left(1 + \sqrt{500\,\varepsilon_1}\right) \tag{20}$$

The tensile stress (σ_{c1}) contribution to shear strength [Eq. (7)] has an upper limit ($\sigma_{c1,max}$) due to the capacity of cracked concrete to bridge forces across the crack sides. The maximum admissible value of the tensile stress is the minimum of the two expressions derived from equilibrium of forces across the crack in the longitudinal and transverse directions:

$$\sigma_{c1,max} = min\{\tau_i cot\theta + \rho_{sl}(f_{yl} - \sigma_{sl}); \tau_i/cot\theta + \rho_{sv}(f_{yv} - \sigma_{sv})\} \quad (21)$$

$$\tau_i = \frac{0.18\sqrt{f_c}}{0.31 + \frac{24w}{d_g+16}} \leq |\rho_{sv}(f_{yv} - \sigma_{sv}) - \rho_{sl}(f_{yl} - \sigma_{sl})| sin\theta\, cos\theta \quad (22)$$

where f_{yl} and f_{yv} = yielding strength of the longitudinal and vertical rebars, respectively; τ_i = local shear stress; d_g = maximum coarse aggregate size (in mm); $w = \varepsilon_1 S_{m\theta}$ is the average crack width (in mm); and f_c is in MPa. In Eqs. (21) and (22), the residual reinforcement stresses at the crack are taken as zero when the yield stress is reached.

To estimate the average diagonal crack spacing, $S_{m\theta} = 1/(sin\theta/S_{ml} + cos\theta/S_{mv})$, the average crack spacing along the two orthogonal directions S_{ml} and S_{mv} are assumed equal to the stirrups spacing (s_v) and the effective depth (d) of the cross-section, respectively.

3.3 Analytical Procedure

For a generic top-strain value (ε_{ct}), a flexural analysis is performed, and values of bending moment (M), curvature (χ) and flexural axial strain (ε_{xf}) are obtained. For a simply supported beam, the shear force is constant along the shear span (a); thus, the applied shear stress at the critical section - away a_v from the support - is taken as $\tau_{flex} = (M/a_v)/(b_w\, d)$. The shear stress capacity [Eq. (7)] must be $\geq \tau_{flex}$. If this is not the case, shear failure is achieved.

Summarizing, the following steps are involved in the shear model:

1. Set a value of ε_2.
2. Set a value of ε_1, and then calculate η from Eq. (19) and σ_{c2} from Eq. (18).
3. Use the stored values of θ and ε_v to calculate w and σ_{c1}.
4. Calculate ε_v from Eq. (11) or (15).
5. Calculate $\sigma_{sv} = min\{E_s\, \varepsilon_v; f_{yv}\}$.
6. Calculate the new θ by Eq. (10) and $\sigma_{c1,max}$ from Eqs. (21) and (22). If $\sigma_{c1} > \sigma_{c1,max}$, then set $\sigma_{c1} = \sigma_{c1,max}$ and return to Step 3.
7. Check if the value of ε_2 obtained by Eq. (9) is equal to the assumed one. In case not, return to Step 2 and adjust ε_1.
8. Calculate τ from Eq. (7). If $\tau \neq \tau_{flex}$ return to Step 1 and adjust ε_2.

4 Validation of the Proposed Model

A database recently collected by Cladera et al. [10] is used here to validate the proposed model. It includes 62 slender beams with corroded stirrups, which failed in shear.

All the specimens have rectangular cross-sections, and the height of cross-section is less than 350 mm. Moreover, the RC beams are over-reinforced in bending ($\rho_l = 0.99 \div 3.01\%$). The concrete compressive strength (f_{cm}) is ranging between 22.5 and

50 MPa. The geometric percentage of stirrups (ρ_v) is between 0.14 and 0.52%. The corrosion degree of stirrups reaches the maximum value of 97.2%.

The experimental versus numerical results at failure are reported in Fig. 2. They highlight that the proposed model accurately predicts the shear strength of RC beams with corroded stirrups. A mean value equal to 1.01 and a Coefficient of Variation (CoV) equal to 0.29 have been achieved for the ratio of the experimental to numerical shear strength (See Fig. 2).

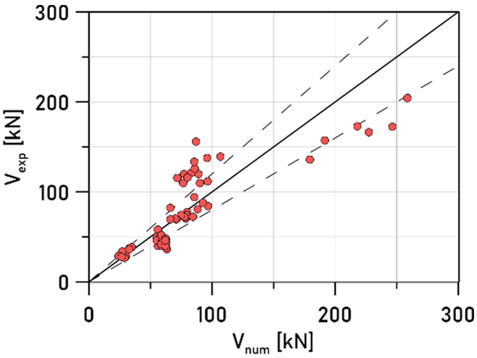

Fig. 2. Comparison between the experimental and numerical shear strength.

5 Conclusions

This research study presents an analytical model to predict the shear behavior of RC beams with corroded stirrups. The proposed procedure take into account the structural effects of corrosion: rebar cross-section loss and effective beam width reduction due to spalling of cover. The numerical results reported in the paper show that the proposed model provides good performance in terms of shear strength prediction (mean = 1.01 and CoV = 0.29). Further investigations, also with the help of sophisticated tools like the Non-Linear Finite Elements Methods, are needed to better figure out the relationship between the inclination angle of strut and the rate of corrosion of stirrups.

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