Series on Grey System

Yingjie Yang
Sifeng Liu *Editors*

Emerging Studies and Applications of Grey Systems

Springer

Series on Grey System

This series aims to publish books on grey system and various applications in the fields of natural sciences, social sciences and engineering.

This series is devoted to the international advancement of the theory and application of grey system. It seeks to foster professional exchanges between scientists and practitioners who are interested in the models, methods and applications of grey system. Through the pioneering work completed over 40 years, grey data analysis methods have become powerful tools in addressing system with poor information.

Books published with this series will explore the models and applications of grey system, in order to tackle poor information more effectively and efficiently. The series aims to provide state-of-the-art information and case studies on new developments and trends in grey system research and its potential application to solve practical problems.

Coverage includes, but is not limited to:

- Foundations of grey systems theory
- Grey sequence operators
- Grey relational analysis models
- Grey clustering evaluations models
- Techniques for grey system forecasting
- Grey models for decision-making
- Combined grey models
- Grey input-output models
- Techniques for grey control
- Various applications of grey system models in the fields of natural sciences, social sciences and engineering.

Yingjie Yang · Sifeng Liu
Editors

Emerging Studies and Applications of Grey Systems

Springer

Editors
Yingjie Yang
Faculty of Technology
School of Computer Science
De Montfort University
Leicester, Leicestershire, UK

Sifeng Liu
College of Economics and Management
Nanjing University of Aeronautics
and Astronautics
Nanjing, China

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Series Preface

This series will publish the books on grey system theory and various applications in the fields of natural sciences, social sciences and engineering.

It is devoted to the international advancement of the theory and application of grey system theory, and seeks to foster professional exchanges between scientists and practitioners who are interested in the models, methods and applications of grey system theory. Through the pioneering work completed over 40 years, grey system analysis methods have become powerful tools in addressing system with poor information.

Books published with this series will explore the models and applications of grey system theory, in order to tackle poor information more effectively and efficiently. The series aims to provide state-of-the-art information and case studies on new developments and trends in grey system research and its potential application to solve practical problems.

In the era of big data, the grey system theory based on poor information data mining has sprung up. It has become an effective tool for people to extract valuable information from massive data. In the past 40 years, grey system method and model have been widely used in many fields, such as social science, natural science and engineering technology, which has led to innovation and progress in various fields. More and more people interested in grey system theory and a lot of new results have been obtained in recent years. In particular, successful applications in many fields have won the attention of the international world of learning.

Scholars from more than 100 countries and regions in the world have published more than 300,000 documents of grey system research and applications.

On 7 September 2019, Angela Dorothea Merkel, then German Chancellor, praised grey system theory in her speech at Huazhong University of Science and Technology. She said that the work of Prof. Deng Julong, the founder of grey system theory and Prof. Liu Sifeng, the editor of this series, “have made a profound impact on the world.”

The Coverage of this series includes, but is not limited to

- Foundations of grey systems theory
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- Various applications of grey system models in the fields of natural sciences, social sciences and engineering.

If you are interested in the series on grey systems, please contact with Ms. Emily Zhang at emily.zhang@springernature.com or Prof. Sifeng Liu at sfliu@nuaa.edu.cn.

Nanjing, China

Prof. Sifeng Liu, Ph.D.
Editor of the Book Series on Grey System
Director of Institute for Grey Systems Studies
NUAA, President of International Association of Grey System and Uncertain Analysis

Preface

At a time when most researchers and businesses are talking about Big Data, small and incomplete data are never far away from us. In fact, most of our decisions are made under incomplete and small data. There are many areas where small and incomplete data are inevitable, such as health care, traffic management and economic analysis. The theory of grey systems emerged as a novel solution to deal with small and incomplete data, and it is attracting more and more attention from researchers and professionals in different disciplines. It has the ability to establish models from extremely limited data under uncertainty, which is usually difficult for other models.

Although grey systems provide powerful capability in data analysis, their impacts in applications have mainly materialized in China. I have frequently received questions regarding the differences between Grey systems and other related models, and to provide examples of their real-world use. To promote grey systems and their applications, an international research network was formed under the support of the Leverhulme Trust in 2015. The leading experts from UK, China, Canada, Romania and Spain formed the core of the network. A number of activities were conducted to foster the collaboration between members of the network. As a result, the international association of grey systems and uncertainty analysis was established in 2016. Through our discussion in the network and the association, it was deemed necessary to assemble the latest advance in this field to clarify confusions between grey systems and grey sets with other models, and provide a collection of successful application cases as guidance for people interested in applying grey systems to their problems. Therefore, a collection of contributions from our network partners and leading experts in the field is put together to form this book. The contributors include world-leading experts as well as a new generation of research leaders from both Europe and China. The application covers many different disciplines, ranging from social economics to engineering, energy and management.

The book focuses mainly on two themes: 1. The connection between grey systems and other related models, such as fuzzy sets and rough sets (Chaps. 3–5). 2. Successful real-world applications in different disciplines (Chaps. 6–12). To introduce to the readers who have no background in grey systems, a brief introduction is also included (Chaps. 1 and 2).

Chapter 1 is a general overview of the whole field of grey systems and its recent development. It helps the readers to see the development and the state of the art of this field. Chapter 2 then gives a brief introduction to the fundamental concepts in grey systems, especially grey models so as to enable the readers to understand other chapters. These two chapters lay the essential backgrounds for the contents in other chapters. Chapter 3 then moves into the connection between grey systems, especially grey sets with various fuzzy sets and rough sets. It helps to clarify the confusions between grey systems and fuzzy sets and rough sets. Chapter 4 focuses on the hybridization of grey systems with neural-fuzzy systems and illustrates how they can be integrated together. Chapter 5 puts grey systems into a more general level and puts forward the novel concept of grey knowledge. These three chapters demonstrate the distinctive and collaborative features of grey systems in uncertainty modelling. Starting from Chap. 6, the remaining chapters cover various real-world application studies. Chapter 6 reveals how to apply grey systems in economic studies using agent-based systems. Chapter 7 demonstrates how grey systems help with the short-term forecasting of traffic flow. Chapter 8 illustrates how grey models are applied to predict and manage Yellow River ice disasters in China. Chapter 9 showcases the application of grey systems to social network data analysis. Chapter 10 gives some case studies in applying grey systems into the energy-economic system. Chapter 11 explains how to select business strategy using grey stratified decisions model. Finally, Chap. 12 illustrates a cost level analysis for the components of the smartphones using greyness-based quality function deployment. The cases studies in Chaps. 6–12 covers a wider range of disciplines, geographical areas and different culture backgrounds, and it can serve as a useful reference for practitioners challenged by small and incomplete data. Combined with the connections covered in Chaps. 3–5 and the basics in Chaps. 1 and 2, this book will provide a convenient handbook for the practical application of grey systems.

Leicester, UK
March 2022

Yingjie Yang

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Chapter 1
Development of Grey System Research

Sifeng Liu and Yingjie Yang

1.1 The Development of Grey Systems

In human history, there has been a variety of scientific modelling systems which greatly boosted the progress of science and technology. It has enabled us to establish a better understanding of the operation rules of nature and improve our knowledge in the complex interactions among various objects, actions and events. Since the mid-twentieth century, a whole plethora of related theories have evolved, from cybernetics through to hypercycle and dynamic system theory. One of the major challenges for scientific modelling systems is the significant existence of uncertainties. Although the modern technology has been developed in an amazing level of complexity today, our knowledge on the world is still far away from complete. There are still significant incompleteness, inaccuracies and imperfectness associated with our measurement and recoding data, and it has led to the development of a number of methodologies and theories, such as probability theory, fuzzy mathematics and rough set theory.

The theory of grey systems was proposed in 1982 by Julong Deng, following the need to provide more accurate and useful results from small and/or inaccurate data sets. It deals with partially known information by generating and extracting useful information from what is available. Such an operation facilitates the assessment, monitoring and understanding of a system's operational behaviors and their laws of evolution, which is invaluable due to the widespread incidence of poor/insufficient data existing naturally.

S. Liu (✉)
Institute for Grey Systems Studies, Nanjing University of Aeronautics and Astronautics, Nanjing, China
e-mail: sfliu@nuaa.edu.cn

Y. Yang
Institute of Artificial Intelligence, De Montfort University, Leicester, UK

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Since the 1980s, the research on Grey Systems has continued to grow in stature and become an increasingly important methodology in the world of systems and data modelling. The first publication "The Journal of Grey Systems" was released in the UK during 1989 by "Research Information Ltd" and is currently indexed by a number of indexing agencies worldwide which including Web of Science (IF = 1.912 belong to Q2). Since that journal was initially released, many other journals and other such publications have been released across the world and there are currently in excess of a thousand different professional journals worldwide that have published papers in grey systems theory, including releases by big name, world renowned publicists such as "Emerald" which launched a new journal entitled "Grey Systems: Theory and Application" (IF = 3.321 belong to Q1). Many of the publications have become top journals in a variety of fields.

Inevitably, as Grey Systems theory has continued to gain support and recognition, so many universities across the world have set up grey system theory curriculums. At the Nanjing University of Aeronautics and Astronautics (NUAA) for example, the curriculums of the grey system theory are found not only in PhD and Master's programmes, but also in undergraduate programs of many disciplines across the university, as an elective module, and in 2008 was selected as one of China's national level model courses. In 2013, the same course was selected as the national excellent resource sharing course, becoming a free open learning resource for all grey system enthusiasts. In 2017, the same course was selected as one of the online open courses

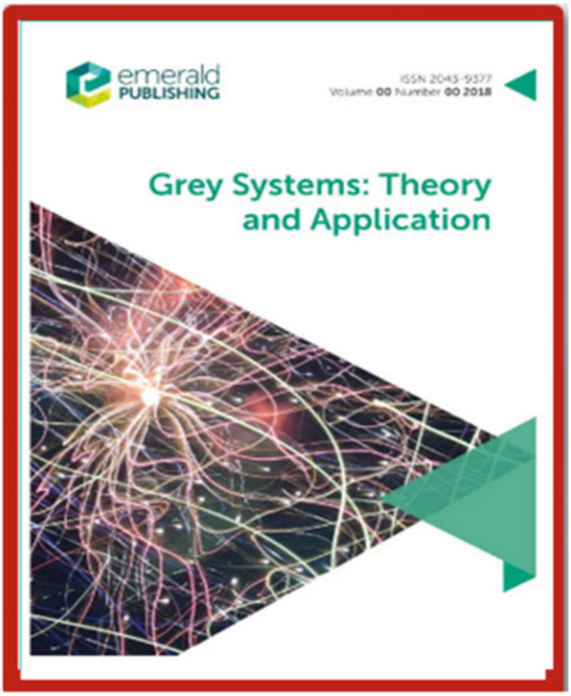

Fig. 1.1 The Journal published by Emerald

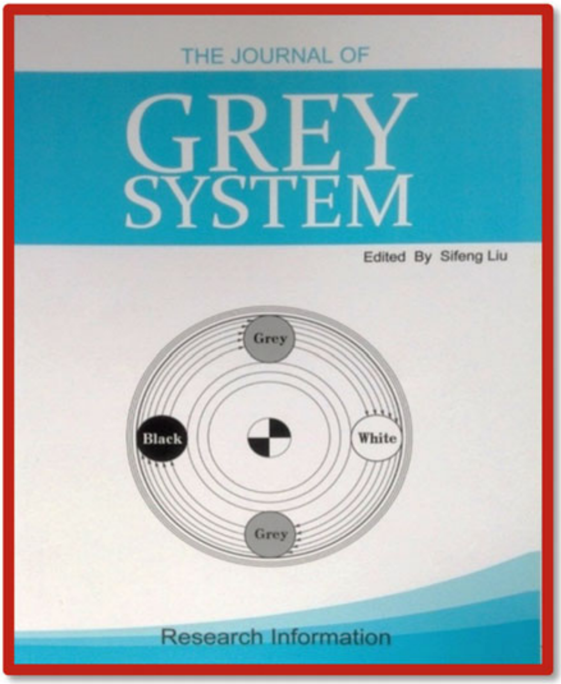

Fig. 1.2 The Journal published by Research Information Ltd

(MOOC of China), where students across China were able to receive credits for a course on grey system theory. In 2020, the same course was selected as online and offline national first-class courses of China. At the same time, Professor Liu Sifeng's team worked with a number of professors from universities in Europe, the United States and Canada, including Keith William Hipel, Former President of the Royal Canadian Academy of Sciences, Professor Yingjie Yang, the Executive President of GSUA and Professor Jeffrey Forrest, the President of IIGSS, to complete the recording of English teaching videos. In January, 2021, Grey Data Analysis—the English video course of grey system theory version has been put in to service on iCourse International officially. It is a new learning channel for global Grey System learners and researchers. You are welcome to join the course through the link as follows: https://www.icourse163.org/en/mooc/course/NUAA1-1461325168.

To begin with, it was mainly in China where universities began recruiting and funding doctoral and postdoctoral researchers in grey system theory and its applications. This process soon spread to other Asian countries or regions, such as Japan, Iran, Turkey, and Taiwan. Today we find universities recruiting and funding Grey Systems research throughout the world from Canada and the USA in North America, across to UK and Europe and down into South Africa. In addition, there are hundreds of thousands more graduate students studying for master and/or Ph.D.'s currently engaged in scientific research, that are applying grey system thinking and methods across the world. As knowledge of Grey Systems Theory has spread across the globe, so have numerous publications of academic works on grey systems been released worldwide by renowned international scientific publishers such as Spring-Verlag, Taylor & Francis Group, Science Press and the IIGSS Academic Publisher to name but a few. They have been released in a number of different languages including Chinese, Japanese, English, Korean, Romanian and Persian, etc. (Fig. 1.3).

In 2014, a series on grey system in Chinese edited by Prof. Sifeng Liu launched by Science Press. At present, 32 copies of this series have been published. In 2021, a series on grey system in English edited by Prof. Sifeng Liu, Prof. Yingjie Yang and Prof. Jeffrey Forrest and launched by Springer-Nature Group. The three books that passed the review in the first phase have completed the signing process of publishing contracts now.

Fig. 1.3 The books published by Springer-Verlag

1.2 The Father of Grey System Theory

The birth of grey system theory is an outcome of Professor Julong Deng who has been working with perseverance for decades.

Julong Deng was born in Lianyuan county, Hunan province, China in 1933. He graduated in electrical machinery from the Huazhong Institute of Technology and became a teaching assistant at the Department of Automatic Control Engineering at the same institute in 1955. Through his academic life, he attached great importance to the learning of math lessons, and kept track of new ideas and new discoveries of mathematics and other relevant science fields, which definitely constituted a solid basis for his later engagement in multi-variable system control problems. In the 1960s, Deng proposed a new method of "Control by remove redundant" based on experiments that were operated by controlling the feed system of domestic T61K heavy machine tools. His paper, "Multivariable linear system shunt calibration device of a comprehensive approach", was published by Acta Automatica Sinica, Vol. 3, No. 1, in 1965. The former Soviet Union Academy of Sciences introduced a summary of his theory at that time. By the early 1970s, "Control by remove redundant" was recognized as a representative methodology at the international conference on control theory in the U.S.

In 1965, L. A. Zadeh, professor of the University of California at Berkeley proposed the Fuzzy Sets Theory (Zadeh, 1965). Prof. Julong Deng began to pay close attention to the work of Prof. Zadeh, and had served as a member of the editorial board member of several journals on fuzzy mathematics. In the late 1970s, the tide of reform and opening up in China was reaching a peak. In serving reform and development, Professor Deng devoted himself to put more effort in the study of "Economic System prediction and control problems". In the face of a class of uncertain systems where "Partial information is known, and partial information is unknown", the main question was how to find an effective method to describe its operation behavior and evolution mechanism? Despite the difficulties, Professor Deng and his colleagues have made very fruitful explorations.

Since the birth of Grey System Theory, it has received positive attention from academic circles at home and abroad and with general practitioners immediately. Many famous scholars and experts have given full affirmation and support. Many young and middle-aged scholars have also joined the Grey System Theory research with great enthusiasm to carry out the theoretical exploration and application research in different fields. Successful application of the Grey System Theory in many fields of science, especially a large number of applications within the process of the country's economic regionalization and regional development strategy planning research, quickly established its academic position of a novel scientific study in a very short period of time. The vigorous vitality and broad prospects for the development of Grey System Theory is also increasingly known by scientists from all kind of disciplines. Successful applications of Grey System Theory have been found in more than 100 countries and regions.

According to retrieval, Prof. Julong Deng's works were cited more than 50 thousand times by scholars both at home and abroad. In 2007, Prof. Julong Deng won the award of founder of Grey System Theory at the first IEEE international conference on grey system and intelligent service (see Figs. 1.1 and 1.2). In 2011, Prof. Julong Deng was elected as the honor fellow of the World Organisation of Systems and Cybernetics at the 15th WOSC International Congress on Cybernetics and Systems (Figs. 1.4 and 1.5).

On 7th September, 2019, during the visit to China, Angela Dorothea Merkel-then, German Chancellor praised Chinese original grey system theory. She said that the work of professor Deng Julong and four other Alumni of HUST "profoundly affecting the world."

Fig. 1.4 Prof. Deng delivering a keynote at GSIS'07

Fig. 1.5 Prof. J. Tien Awarding the Certificate to Prof. Deng

1.3 Institute for Grey System Studies, NUAA

The Institute for Grey System Studies (IGSS) at Nanjing University of Aeronautics and Astronautics (NUAA) was founded in 2000 when Professor Sifeng Liu was affiliated at the NUAA as a distinguished professor.

Professor Sifeng Liu has been devoted to the research of grey system theory for more than 40 years. In 1980s, he put forward a series of new models and new concepts of grey system such as the sequence operator (Liu, 1986), absolute degree of grey incidence (Liu, 1988), and the positioned coefficient of grey number (Liu, 1989). Then in the 1990s, he proposed the buffer operator and its axiom system (Liu, 1991), generalized degree of grey incidence (Liu, 1992), grey clustering evaluation model with fixed weight (Liu, 1993), the measurement of information content of grey number (Liu & Zhu, 1994), LGPG drifting and positioning solution (Liu & Dang, 1997), grey-econometrics model (Liu & Zhu, 1996), etc. All of these achievements earned him great attention and recognition from domestic and foreign counterparts; some results were even specially introduced in many experts' monographs.

In 2000, the doctorial authorization discipline of "Management Science and Engineering" of which professor Sifeng Liu was appointed the first academic leader, was approved by The Academic Degree Commission of the State Council of China. As the first doctorial authorization discipline of the College of Economics and Management at NUAA, this is a huge bonus for the college. The Institute for Grey System Studies opened a new Ph.D. program on Grey System Theory at the same time. Many young scholars from all over the country are joining The Institute for Grey System Studies as Ph.D. students or post-doc. research fellows. Most of them have thrown themselves into the field of grey system research (Fig. 1.6).

As a new theory, grey system theory has received a lot of attention and support from academia. At the same time, like any new thing, the grey system theory is inevitably

Fig. 1.6 Prof. Deng and Prof. Liu with Liu's students

questioned by some people. Some scholars have intermitted their work even though they engaged in grey system research at the beginning. However, Professor Sifeng Liu and many of his colleagues are fighting the argument for Grey Systems Theory through work hard tireless research and continue to make new progress. They've published more than a thousand academic papers, among which, more than 300 are included in ISI database and 100+ of which are published in Q1 journals. More than 60 compositions have been published worldwide by notable publishers, such as Springer-Verlag, Taylor & Francis Group, and Science Press. Both the number of publications on grey systems and the number of citations of IGSS were No. 1 in the database of web of science for more than ten years (from 2004 to present). According to retrieval results by the database of China National Knowledge Infrastructure (CNKI), the literatures on grey systems theory of IGSS have been cited more than 60 thousand times by other scholars, and applied in every walk of life. Their work *Grey Systems and Its Applications* have become a classic in this field. It has been published the Ninth Edition and reprinted for more than 30 times (Liu & Guo, 1991; Liu, 2021).

According to the Annual Report of China Top Cited Books (Edition 2016), the book *Grey System Theory and Its Applications*, which was authored by Prof. Sifeng Liu and published by Science Press, is identified as the No.1 top sited books in the pandect of natural science of China.

The said book has been ranked as one of the top 10 best-selling books of the Electronics and Information field, top 10 most cited books in Economics, and the 8th edition is selected to the Masterpieces Series by Science Press in 2017.

After entering the new millennium, the IGSS scholars proposed the grey algebraic system based on kernel and degree of greyness, the concept of general grey number (Liu et al., 2012), the range suitable for model GM(1, 1) (Liu & Deng, 2000), grey Cobb-Douglass model (Liu et al., 2004), grey game theory (Fang et al., 2010), grey input–output models (Li et al., 2012), and series of discrete grey models (DGM) (Xie & Liu, 2005, 2009). All these models contributed to expanding the GM model systems. Then they structured the new models of grey incidence analysis based on visual angles of similarity and nearness (Liu et al., 2011), and the three dimensional model of grey incidence analysis based on the absolute degree of grey incidence and distance in three dimensional space (Zhang & Liu, 2009). This provided effective methods and tools to analyze and measure the relationship in different sequences based on different angels and relationship in panel data.

In recent years, they proposed the original difference model GM(1, 1) (ODGM) (Liu et al., 2015), the even difference model GM(1, 1) (EDGM) (Liu et al., 2015), fractional order grey models (Wu et al., 2015), self-memory grey model (Guo et al., 2015), spectrum analysis of sequence operators (Lin et al., 2019, 2020; Liu et al., 2020), the negative grey similarity relational analysis model, the negative grey absolute relational analysis model, the negative grey relative relational analysis model, the negative grey comprehensive relational analysis model, and the negative Deng's grey relational analysis model (Liu, 2022), and the new grey clustering evaluation model based on mixed possibility functions (Liu et al., 2015), which included both end-point mixed possibility functions and centre-point mixed possibility functions. It's easy to obtain the possibility functions and solve the evaluation problem of uncertain systems with poor information. They also studied the problems of multi-attribute intelligent grey target decision making, then, constructed four kinds of uniform effect measurement functions in view of different decision making objectives of benefit type, cost type, and moderate type. Accordingly, the decision making objectives with different meaning, different dimension, and/or different nature can be transferred to uniform effect measurements. The critical value of the grey target is designed as the dividing point of positive and negative, which is the zero point. And the two cases of hit the bull's eye or not of the objective effect value are fully considered (Liu et al., 2014). As a result, a new multi-attribute intelligent grey target decision model was proposed. And, to deal with the decision making dilemma of a comparison between the maximum components of two decision coefficient vectors being different from a comparison between the two integrated decision coefficient vectors themselves, they first defined both the weight vector group of kernel clustering and weighted coefficient vector of kernel clustering for decision-making. Then a novel two-stage decision making model with the weight vector group of kernel clustering and weighted coefficient vector of kernel clustering for decision-making is put forward. This method can effectively solve the decision making dilemma and produce consistent results (Liu et al., 2014, 2016, 2017, 2022; Liu & Yang, 2016).

Many scholars from USA, UK, Germany, South Africa, Italy, Korea, Canada, Romania, Poland, Turkey, Iran and Pakistan, etc. have joined IGSS as visiting professors, research fellows or for joint project research. In recent years, some young scholars from different countries have joined the Institute for Grey System Studies as Ph.D. or Master Students, supported by the Chinese government scholarship. This is helpful in the promotion of the popularization and international communication of grey system theory.

Since 2000, more than 200 Ph.D. students, about 400 graduate students for master degree graduated from IGSS, in addition to the inclusion of around eighty post-doc. research fellows. Many of them become academic leaders, named scholars or distinguished professors in different universities or research institutions. Each graduated alumnus become a seed which take root in the new institution, then enlarge and gradually spread one's influence in this field.

The Institute for Grey System Studies has won support from many important academic organizations like China Association for Science and Technology, the Institute of Electrical and Electronics Engineers (IEEE), World Organization of Systems and Cybernetics (WOSC), China Center of Advanced Science and Technology, and Chinese Society for Optimization, Overall Planning and Economic Mathematics, etc. Supported by these organizations, they established the Grey Systems Society of China and Technical Committee on Grey Systems of IEEE SMC. In 2015, initiated by scholars from China, UK, USA, Spain, Romania and Canada, etc., the International Association of Grey System and Uncertainty Analysis (GSUA) was established.

The widespread recognition and application of grey system theory is reflected in its growing acceptance. A number of universities from around the world have adopted Professor Sifeng Liu's monographs as their text books. In 2002, he won the World Organization of Systems and Cybernetics (WOSC) Prize. In 2008, as a preeminent Chinese scholar, he was elected an Honorary WOSC Fellow. In 2013, after a strict review by the European Commission, he was selected to be a Marie Curie International Incoming Fellow (Senior), thus honouring him as the first such Fellow with grey systems expertise. In 2017, Professor Sifeng Liu was selected to be one of the top 10 shortlisted promising scientists in the MSCA 2017 Prizes, the first Chinese scholar who prized in the MSCA since the implementation of the Marie Curie International Incoming Action Program of European Union (Figs. 1.7 and 1.8).

Fig. 1.7 Certificate of award of Marie Curie Fellowship

Fig. 1.8 The prize awards poster of MSCA 2017

1.4 A Summary on the MSCA Project GS-A-DM-DS (629051)

The EU-funded Marie (Skłodowska) Curie Actions (MSCA) project: Grey Systems and Its Application to Data Mining and Decision Support (GS-A-DM-DS, 629051) was completed at the end of 2016. The research fellow, Professor Sifeng Liu has been selected to be one of the 10 shortlisted promising scientists in the MSCA 2017 Prizes.

Combining the expertise of Chinese and European researchers, the GS-A-DM-DS project has led to significant advances in the reliability and applicability of data-mining algorithms based on grey systems theory, which enables valuable insights and predictions to be gained from incomplete data.

Grey systems models developed by the GS-A-DM-DS researchers were successfully used to help select a research and development team to work on China's first domestic-built commercial passenger jet, the C919 that had its maiden flight in May. The grey system approach was chosen because China had no prior experience in building such an aircraft and therefore limited reference data.

A Chinese renewable energy company has also employed models advanced in the project to be able to predict when a gearbox on a wind turbine will fail, enabling it to prolong the life of its turbines by 36%, resulting in fewer shutdowns and saving tens of millions of euros.

"The C919 aircraft and the wind turbines are just two examples of how our models have been used in the real world to great effect," says project manager Professor Yingjie Yang. "As our work focuses on the theory rather than the practical applications, we hope to show that grey systems models are a viable option for companies, governments and policymakers in areas with limited or poor quality data, such as socio-economic analysis, healthcare, climate change and complex R&D projects."

The GS-A-DM-DS team proposed several new prediction and decision-making models to provide more reliable results in complex situations.

They formulated a set of criteria for grey model selection and calibration following systematic research. This will assist in making grey prediction and forecasting easily accessible to new users who have no prior knowledge in grey systems. The criteria will also help to promote the application of grey systems to data mining in Europe.

Project partners developed several grey models in order to achieve more accurate and reliable prediction and forecasting with "small data" and poor information. These will contribute to data mining operations that require high speed and reliability while reducing data requirements. They proposed the even difference model GM(1, 1) (EDGM), the original difference model GM(1, 1) (ODGM), self-memory grey model, and fractional order grey models.

Researchers also developed decision-making models that were validated by simulation and real application case studies. Performance was superior to existing alternatives. These models will enable more realistic and reliable decision-making, and help ensure the uncertainty representation is more accessible for ordinary users (Jahan & Zavadskas, 2019; Kose & Tasci, 2019).

One such example is the new grey clustering evaluation model, based on mixed possibility functions, which includes both end-point mixed possibility functions and centre-point mixed possibility functions. It's easy to obtain the possibility functions and solve the evaluation problems of uncertain systems with poor information. They also studied the problems of multi-attribute intelligent grey target decision-making, and then constructed four kinds of uniform effect measure functions in view of the different decision-making objectives based on benefit type, cost type, and moderate type.

Accordingly, the various decision-making objectives which possess different meanings, dimensions, and/or nature from each other can now be transferred and measured to uniform effect. The critical value of a grey target is designed as the dividing point between positive and negative, which is defined as zero. The objective effect values were fully considered, and as a result, a new multi-attribute intelligent grey target decision-making model was proposed. Dealing with the decision-making dilemma of a comparison between the maximum components of two decision coefficient vectors is different from comparisons between the two integrated decision coefficient vectors themselves. Therefore, both the weight vector group of kernel clustering and weighted coefficient vectors of kernel clustering for decision-making were firstly defined. A novel two-stage decision-making model with the weight vector group of kernel clustering and weighted coefficient vector of kernel clustering for decision-making was then put forward. This method can effectively solve the decision-making dilemma and produce consistent results.

Over 30 research papers were presented in leading international academic journals and conferences, plus 3 books published by Springer, Science Press, and John Wiley & Sons, respectively. In addition, more than 20 visits, seminars and training courses were carried out in China and Europe. One of their research books on grey systems is identified as the No.1 top sited books in the pandect of natural science by China National Knowledge Infrastructure (CNKI) in 2017. The literatures of Prof. Sifeng Liu have been sited 33,059 times in google scholar. His H-Index is 72, i10-index 465.

Today, the models are being used by oil companies to successfully predict where to drill new wells, saving considerable sums of money on speculative excavations; transport authorities are using the methods to estimate traffic on highways before they are built; and multinational brands are exploring the techniques to gain insights into how customers feel about their products.

As a result of the EU-funded Marie (Skłodowska) Curie Actions (MSCA) project, an international association on grey systems and uncertainty analysis was established comprising members from China, Europe and North America.

The GS-A-DM-DS project has demonstrated the feasibility of grey systems in data mining and its great potential for use with limited and poor data. It will have a significant impact on the development of grey systems and data mining in China and Europe.

1.5 Grey Systems—Where We Are Now

Research into Grey Systems Theory has certainly come a long way since its beginnings in the 1980s. We are currently deploying a number of cutting-edge disciplines such as grey hydrology, grey geological geology, grey breeding, and grey medical science, etc. We are witnessing much support from both local and national science funding agencies in furthering active research in grey system theory. There are hundreds of research projects on grey systems and their applications currently receiving support from the National Natural Science Foundation of China, The European Union, The Royal Society of UK, and the Leverhulme Trust, as well as funding streams across other countries Such as Spain, Romania and Canada.

We have now held 52 regional, domestic and international conferences on grey system theory and its applications, since the first conference back in 1984, and these have been supported by many international institutes and universities, including the Shanxi Academy of Agricultural Sciences, Huazhong University of Science and Technology, the Nanjing University of Aeronautics and Astronautics, Wuhan University of Technology, The Leverhulme Trust, Institute for Grey System Studies, De Montfort University, University of Macau, Stockholm University, Educational Society of Pudong, Shanghai, Huawei Technologies Co., Ltd., Thailand and many more. There is now support from "China Center of Advanced Science and Technology" which Mr. Tsung-Dao Lee, a Nobel Prize winner, serves as director and two former presidents of the Chinese Academy of Sciences, Mr. Zhou Guangzhao and Mr. Lu Yongxiang serve as deputy-director. This has helped attract a large number of young scholars to such events.

We see now at the latest conferences that research on Grey Systems theory and its applications are being included in an increasing number of related scientific topics and problem resolution scenarios where various system and data modelling exercises are utilized, such as cybernetics and uncertain system modelling, thus furthering understanding of and promoting, Grey Systems among peers in the world of systems science.

The conferences on grey system theory and applications received a significant number of submissions from many countries and regions including China, the USA, Canada, the UK, Germany, France, Spain, Switzerland, Hungary, Poland, Japan, South Africa, Russia, Turkey, Romania, Holland, Malaysia, Iran, Ukraine, Kazakhstan, Pakistan, Iran, Taiwan, Macao, and Hong-Kong. Many of the articles featured were indexed by the EI database, and were released by various publishers including, Kybernetes, Grey Systems: Theory and Application, The Journal of Grey System, Transaction of Nanjing University of Aeronautics and Astronautics (English version) and Springer-Verlag.

Grey Systems research has attracted much praise and commendation recently by such scholars as Alex Andrew (UK), former Secretary General of the World Organization of Systems and Cybernetics, Professor Keith William Hipel (CA), former President of the Canadian Royal Academy of Sciences, Professor Herman Haken (Germany), the founder of synergetics, Professor Lotfi A. Zadeh (US), the founder of fuzzy mathematics and member of the National Academy of Engineering, United States, Professor James M. Tien (US), former vice-president of IEEE and member of the National Academy of Engineering, United States, Professor Qian Xuesen, winner of China's highest prize award of science and technology, Professor Edmundas Kazimieras Zavadskas, Member of the Lithuanian Academy of Science and Professor Robert Vallee (France), former president of World Organization of Systems and Cybernetics and member of the French-speaking Academy of Engineers.

Grey Systems can now boast a number of committees and associations that have been set up in its name. The most recent of these associations was in 2015 when the International Association of Grey System and Uncertain Analysis (GSUA) was established. However there have been a number over the past few years as the following list shows:

1. 1982, the first paper of grey system theory by Prof. Julong Deng has been published by System and Control Letters.
2. 1989, the first Journal of "The Journal of Grey System" was released in the UK during 1989 by "Research Information Ltd".
3. 2000, Institute for Grey Systems Studies at NUAA was established.
4. 2005, the Grey System Society of China, CSOOPEM, was approved by the Chinese Association for Science and Technology and Ministry of Civil Affairs, China.
5. 2008, the Technical Committee of IEEE SMC on Grey Systems was established.
6. 2011, the Journal of Grey Systems-Theory and Application was launched by Emerald Group.
7. 2012, the first Workshop of European grey system research collaboration network was held by De Montfort University.
8. 2013, Professor Sifeng Liu was selected for a Marie Curie International Incoming Fellowship (FP7-PEOPLE-IIF-GA-2013-629051) of the 7th Research Framework Program of the European Union.
9. 2014, an international network project entitled "Grey Systems and Its Applications" (IN-2014-020) was funded by The Leverhulme Trust.
10. 2014, A book Series of Grey Systems in Chinese were launched by Science Press.
11. 2015, the International Association of Grey System and Uncertain Analysis (GSUA) was established.
12. 2016, Polish Scientific Association of Grey Systems was established.
13. 2018, Grey Systems Society of Pakistan was established.
14. 2019, Turkish Association of Grey Systems Theory was established.
15. 2021, A book Series of Grey Systems were launched by Springer-Nature Group.

Indeed it seems clear that as an emerging discipline, grey systems theory is currently standing firm and is highly regarded in the scientific community.

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Chapter 2
Basic Models in Grey System Theory

Naiming Xie and Baolei Wei

2.1 Introduction

In 40 years since Deng's seminal work on grey system theory, researches have consistently shown that grey system theory has developed as a scientific discipline which is consist of systems analysis, prediction, decision making, control, and optimization (Deng, 1982, 1983, 1984). In this section, framework and mechanism of grey system models will be further summarized. And detailed steps of grey forecasting models, grey relational analysis models, grey cluster models and grey target models will be further summarized.

2.2 General Framework of Grey System Models

2.2.1 Framework of Grey System Models

Grey system theory is proposed for that the sample information people could obtain is always incomplete and limited. Grey system theory mainly focuses on problems

N. Xie (✉) · B. Wei
College of Economics and Management, Nanjing University of Aeronautics and Astronautics, Nanjing, People's Republic of China
e-mail: xienaiming@nuaa.edu.cn

B. Wei
e-mail: weiblei@yeah.net

B. Wei
School of Economics and Management, Nanjing University of Science and Technology, Nanjing, People's Republic of China

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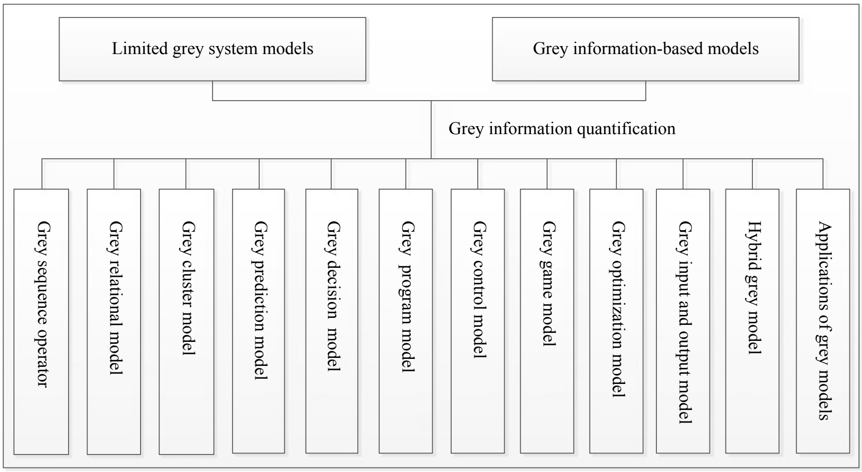

Fig. 2.1 Structure of grey system theory

with limited data and poor information, in which principles of system control are employed to analyze system evolution process and to mine limited information.

Till now, grey system theory has derived many branches, including grey number operation, sequence operator, relational analysis, cluster, forecasting, decision-making, input–output analysis, programming, control, etc., shown in Fig. 2.1. In summary, this theory is mainly about the concepts of system analysis, system forecasting, decision-making and system control under limited data and grey information. In addition, grey system theory has been widely applied in economic, energy, environment, transportation, water resource management, industrial process control, development of complex equipment, post-evaluation of major projects, etc.

2.2.2 Mechanism of Limited Data Grey Modeling

In grey system theory, limited data modeling mainly focus on grey forecasting, and grey sequence operator is for generating optimized modeling sequence while grey relational models are viewed as the testing method of grey forecasting models. As show in Fig. 2.2, there are 8 components in modeling process of limited data grey forecasting models, i.e. modeling variables selection, modeling sequences generation, model form choice, background value generation, parameter calculation, error analysis, properties analysis and forecasting. Modeling variables selection mainly focused on which variable and how many variables should be selected so as to construct single-variable or multi-variable grey forecasting model. Sequence generation often operated with different kinds of sequence operators. Grey relational models

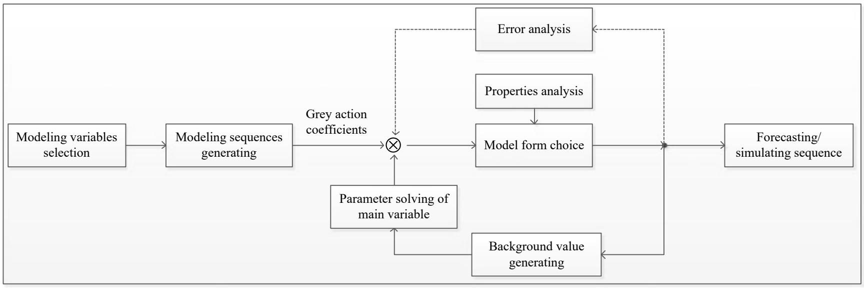

Fig. 2.2 Flow chart of grey forecasting model with limited data

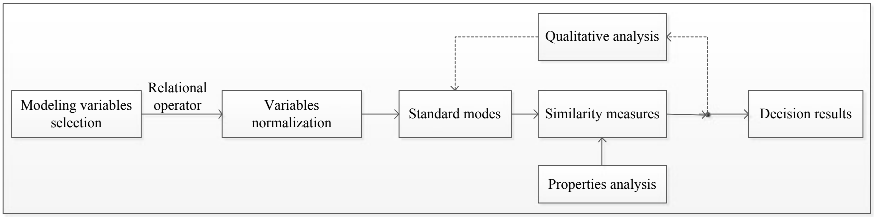

Fig. 2.3 Flow chart of grey target models with limited data

are mainly included in error analysis of different kinds of grey forecasting models (Kumar & Jain, 2010; Xie & Liu, 2009; Xie & Wang, 2017).

Similarly, there are grey relational models, grey cluster models and grey decision making models with limited data. As show in Fig. 2.3, there are 7 steps in modeling process, i.e. modeling variable selection, variable normalization, standard mode construction, similarity measurement, qualitative analysis, properties analysis and decision making (Deng, 1986).

2.2.3 *Mechanism of Grey Number Modeling*

A grey number is a figure that represents a range of values rather than an exact value when the exact value for the said figure is not known. The range of a grey number can be an interval or a general number set. Grey numbers are usually expressed as the symbol "⊗", which is called grey. A grey number represents the degree of information uncertainty in a given system. As the basis of grey systems theory, literatures focused on grey numbers and grey measures has attracted increased attention over the past years (Xie, 2013).

Definition General grey number is defined as

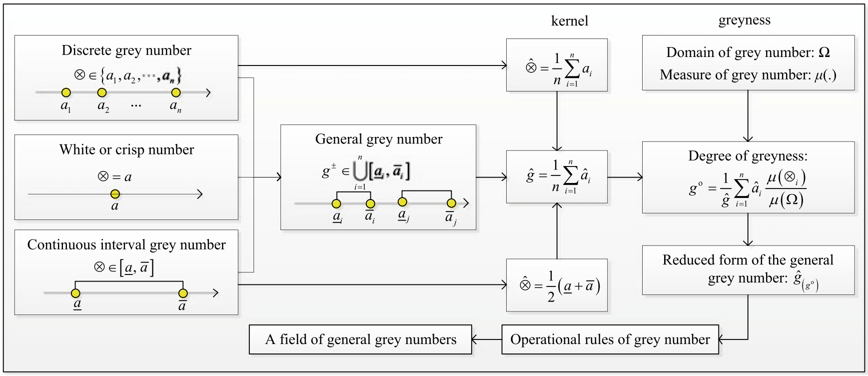

Fig. 2.4 Algebra system based on kernel and degree of greyness of grey numbers

$$g^{\pm} \in \bigcup_{i=1}^{n} [\underline{a}_i,\ \overline{a}_i]$$

where

i. for any interval grey number $\otimes_i \in [\underline{a}_i,\ \overline{a}_i] \subset \bigcup_{i=1}^{n} [\underline{a}_i,\ \overline{a}_i]$;
ii. $\overline{a}_{i-1} \leq \underline{a}_i \leq \overline{a}_i \leq \underline{a}_{i+1}$, and $\overline{a}_{i-1}, \underline{a}_i, \overline{a}_i, \underline{a}_{i+1} \in R$;

then $g^- = \inf_{\underline{a}_i \in g^{\pm}} \{\underline{a}_i\}$ and $g^+ = \inf_{\overline{a}_i \in g^{\pm}} \{\overline{a}_i\}$ are respectively referred to as the lower and upper limits.

Since the degree of greyness of the resulting grey numbers in summation and subtraction operations is not satisfied with the un-reduction axiom, researchers proposed the grey "kernel" and greyness based algebra system of grey numbers is shown in Fig. 2.4.

2.3 Introduction to Grey Forecasting Model

2.3.1 General Evolution of Grey Forecasting Models

The original idea to construct a grey forecasting model is to improve the simulating and predicative ability, especially for the original sequence data which is limited and traditional statistical methods could not be constructed effectively.

In 1983, Prof. Deng firstly mentioned the concept of grey forecasting and in 1984 he proposed the real grey forecasting model which is used to predict the long-term output of grain in China (Deng, 1984). He proposed two models in the paper, one is first order univariate grey dynamic model, and another is first order multivariate grey dynamic model. In 1986, Prof. Deng wrote the first book on grey forecasting models. These early literatures illustrated that the main idea of the grey forecasting model is to dig useful information by accumulating generating of original sequence.

By accumulating generating, the trend of accumulated sequence will embody clearly while the trend of original sequence is vague and could not be caught easily. Random disturbance of the sequence is weakened in the accumulating process. In fact, the sequence is the original information for constructing a grey forecasting model while the sequence is aligned with the time.

Generally speaking, grey forecasting model is a kind of time-varying forecasting model. But the model is different with time series model and linear regression model. Both time series model and linear regression model are constructed with original data while grey forecasting model is constructed with accumulating generating data, therefore, the grey forecasting can catch the result effectively though the limited data while the other two kinds of models need more data in modeling process. The independent variable of linear regression model is selected according the relation between independent variable and dependent variable. The variable of grey forecasting model must organize according to the time, i.e. the data should be collected with the time information, usually with the same time interval. The parameter solving of all these three models is least square method. As a novel forecasting method, grey forecasting model has been used in widely fields, including economic development forecasting analysis, energy demand forecasting, transportation flow forecasting, etc.

2.3.2 Steps of GM(1, 1) Model

Definition Assume that $X^{(0)} = \left\{x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n)\right\}$ is an original data sequence or the system main variable data sequence, the accumulated generation sequence of $X^{(0)}$ (AGO) is defined as

$$X^{(1)} = \left\{x^{(1)}(1), x^{(1)}(2), \ldots, x^{(1)}(n)\right\},$$

where

$$x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i), k = 1, 2, \ldots, n.$$

The sequence

$$Z^{(1)} = \left\{-, z^{(1)}(2), z^{(1)}(3), \ldots, z^{(1)}(n)\right\}$$

is called the mean generation sequence of $X^{(1)}$ (MEAN) or the background value of the system, where

$$z^{(1)}(k) = \frac{1}{2}\left(x^{(1)}(k) + x^{(1)}(k-1)\right), k = 2, 3, \ldots, n.$$

Definition The equation

$$x^{(0)}(k) + az^{(1)}(k) = b$$

is called first order and single variable grey forecasting model, abbreviated as GM(1, 1). The equation

$$\frac{d}{dt}x^{(1)}(t) + ax^{(1)}(t) = b$$

is called the continuous form of GM(1, 1) model.

Theorem *By least square method, the parameter matrix* $\left[a\ b\right]^{\mathrm{T}}$ *can be calculated as*

$$\left[a\ b\right]^{\mathrm{T}} = \left(B^{\mathrm{T}}B\right)^{-1}BY,$$

where

$$B = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \vdots & \vdots \\ -z^{(1)}(n) & 1 \end{bmatrix}, Y = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix}.$$

In GM(1, 1) model, a is the parameter of main variable or the developing coefficient of the system, b is the grey action coefficient parameter of GM(1, 1) model.

By setting the boundary condition as $\hat{x}^{(1)}(1) = x^{(1)}(1)$, and substituting parameter a and b into the continuous form, we can get the time response equation of GM(1, 1) model expressed as

$$\hat{x}^{(1)}(k+1) = \left(x^{(0)}(1) - \frac{b}{a}\right)e^{-ak} + \frac{b}{a}, k = 1, 2, \ldots, n-1.$$

Then, the simulative value of the accumulated generation sequence $X^{(1)}$ can be obtained. Consider the inverse accumulated generation, the simulative value of the original sequence $X^{(0)}$ can be computed as

$$\hat{X}^{(0)} = \left\{\hat{x}^{(0)}(1), \hat{x}^{(0)}(2), \ldots, \hat{x}^{(0)}(n)\right\},$$

where

$$\hat{x}^{(0)}(k+1) = \hat{x}^{(1)}(k+1) - \hat{x}^{(1)}(k), k = 1, 2, \ldots, n-1.$$

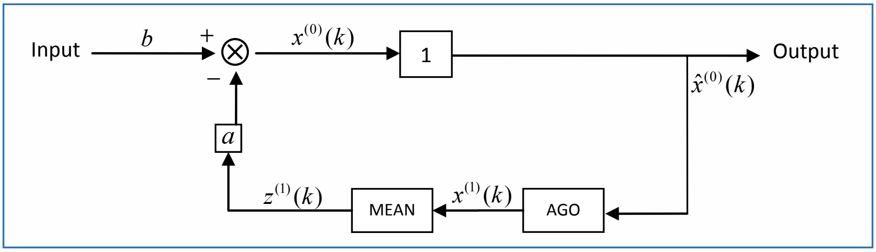

Fig. 2.5 Theoretical transfer diagram of GM(1, 1) model

In order to evaluate the predicting performance of GM(1, 1) model, the mean absolute percentage error (MAPE) is employed. MAPE is defined as follows:

$$MAPE = \frac{1}{n}\sum_{k=1}^{n}\left|\frac{x^{(0)}(k) - \hat{x}^{(0)}(k)}{x^{(0)}(k)}\right| \times 100\%.$$

The theoretical transfer process of GM(1, 1) model is shown in Fig. 2.5. In fact, GM(1, 1) model is a special grey forecasting model because only the main variable sequence is considered in the modeling process. Therefore, the GM(1, 1) model is to analyze the system main variable autoregressive evolution in accordance with the time while the influence factors are not considered in the model.

From the input information and output result, the process of GM(1, 1) model is similar with the trend extrapolation forecasting method and time variable linear regression model. The data of simulative sequence is roll over and over with the time. The difference is that the GM(1, 1) model is constructed on the accumulating generating sequence while the trend extrapolation forecasting method and time variable linear regression model are constructed on the original sequence. Because the random vibration is weakened in the modeling process, the GM(1, 1) model can still catch the trend when the data is less.

Example Let the original sequence be

$$X^{(0)} = (936.5, 938.8, 1033.5, 1239.9, 1327.1, 1425.5, 1593.5, 1861.6),$$

then the computational steps of GM(1, 1) model are described as follows.

Solution:

Step 1 Calculate the accumulated generation sequence:

$$X^{(1)} = (936.5,\ 1875.3,\ 2908.8,\ 4148.7,\ 5475.8,\ 6901.3,\ 8494.8,\ 10356.0).$$

Step 2 Estimate the model parameter

$$\begin{bmatrix} a \\ b \end{bmatrix} = \left(B^{\mathrm{T}}B\right)^{-1}BY = \begin{bmatrix} -0.10928 \\ 792.25 \end{bmatrix},$$

where

$$B = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ -z^{(1)}(4) & 1 \\ -z^{(1)}(5) & 1 \\ -z^{(1)}(6) & 1 \\ -z^{(1)}(7) & 1 \\ -z^{(1)}(8) & 1 \end{bmatrix} = \begin{bmatrix} 2811.8 & 1 \\ 4784.1 & 1 \\ 7057.5 & 1 \\ 9624.5 & 1 \\ 12377 & 1 \\ 15396 & 1 \\ 18851 & 1 \end{bmatrix}, Y = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ x^{(0)}(4) \\ x^{(0)}(5) \\ x^{(0)}(6) \\ x^{(0)}(7) \\ x^{(0)}(8) \end{bmatrix} = \begin{bmatrix} 938.8 \\ 1033.5 \\ 1239.9 \\ 1327.1 \\ 1425.5 \\ 1593.5 \\ 1861.6 \end{bmatrix}.$$

Thus, the model parameters are

$$a = -0.10928, b = 792.25.$$

Step 3 Setting the boundary condition $\hat{x}^{(1)}(1) = x^{(1)}(1) = 936.5$, the time response equation is

$$\hat{x}^{(1)}(k+1) = 818.62e^{0.10928k} - 724.97, k = 2, 3, \ldots, 8.$$

Step 4 Compute the simulative values of $X^{(1)}$ and $X^{(0)}$:

$$\hat{X}^{(1)} = (936.5,\ 1881.8,\ 2936.3,\ 4112.5,\ 5424.6,\ 6888.2,\ 8520.8,\ 10342)$$

and

$$\hat{X}^{(0)} = (936.5,\ 945.3,\ 1054.5,\ 1176.2,\ 1312.1,\ 1463.6,\ 1632.6,\ 1821.1).$$

Step 5 Evaluate the modeling performance based on MAPE

$$MAPE = \frac{1}{8}\sum_{k=1}^{8}\left|\frac{x^{(0)}(k) - \hat{x}^{(0)}(k)}{x^{(0)}(k)}\right| \times 100\% \approx 3.33\%.$$

2.3.3 *Steps of DGM(1, N) Model*

Definition (Xie & Liu, 2009) Assume the system main variable data sequence is

$$X_1^{(0)} = \left(x_1^{(0)}(1), x_1^{(0)}(2), \ldots, x_1^{(0)}(m)\right),$$

the relevant influence factors sequences are

$$X_i^{(0)} = \left(x_i^{(0)}(1), x_i^{(0)}(2), \ldots, x_i^{(0)}(m)\right), i = 2, 3, \ldots, N,$$

the accumulated generation sequences of $X_i^{(0)}$ are

$$X_j^{(1)} = \left(x_j^{(1)}(1), x_j^{(1)}(2), \ldots, x_j^{(1)}(m)\right), i = 1, 2, \ldots, N$$

then

$$x_1^{(1)}(k) = b_0 + b_1 x_1^{(1)}(k-1) + \sum_{i=2}^{N} b_i x_i^{(1)}(k)$$

is called DGM(1, N) model.

Similar with the GM(1, 1) model, b_0 is the grey action coefficient parameter, b_1 is the parameter of main variable or the developing coefficient of the system, b_2, b_3, …, b_N are the grey action coefficients parameters of different independent variables in DGM(1, N) model.

Theorem *By least square method, the parameter matrix can be calculated as*

$$\left[\, b_0\ b_1\ b_2\ b_3\ \cdots\ b_N \,\right]^{\mathrm{T}} = \left(B^{\mathrm{T}}B\right)^{-1}B^{\mathrm{T}}Y,$$

where

$$B = \begin{bmatrix} 1 & x_1^{(1)}(1) & x_2^{(1)}(2) & x_3^{(1)}(2) & \cdots & x_N^{(1)}(2) \\ 1 & x_1^{(1)}(2) & x_2^{(1)}(3) & x_3^{(1)}(3) & \cdots & x_N^{(1)}(3) \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_1^{(1)}(m-1) & x_2^{(1)}(m) & x_3^{(1)}(m) & \cdots & x_N^{(1)}(m) \end{bmatrix}, Y = \begin{bmatrix} x_1^{(1)}(2) \\ x_1^{(1)}(3) \\ \vdots \\ x_1^{(1)}(m) \end{bmatrix}.$$

Then, substituting the model parameter into the model equation, the recursive function of DGM(1, N) model can be obtained as

$$\hat{x}_1^{(1)}(k) = b_0 + b_1 x_1^{(1)}(k-1) + \sum_{i=2}^{N} b_i x_i^{(1)}(k).$$

By setting the boundary condition as $\hat{x}^{(1)}(1) = x^{(1)}(1)$*, and substituting parameter* b_0, b_1, …,b_N *into the DGM(1, N) model, we can get the simulative value of sequence* $X^{(1)}$*. Then, using the inverse accumulated generation, we can obtain the simulative sequence* $\hat{X}_1^{(0)} = \left\{\hat{x}_1^{(0)}(1), \hat{x}_1^{(0)}(2), \ldots, \hat{x}_1^{(0)}(m)\right\}$ *and the mean absolute percentage error (MAPE) which is defined as*

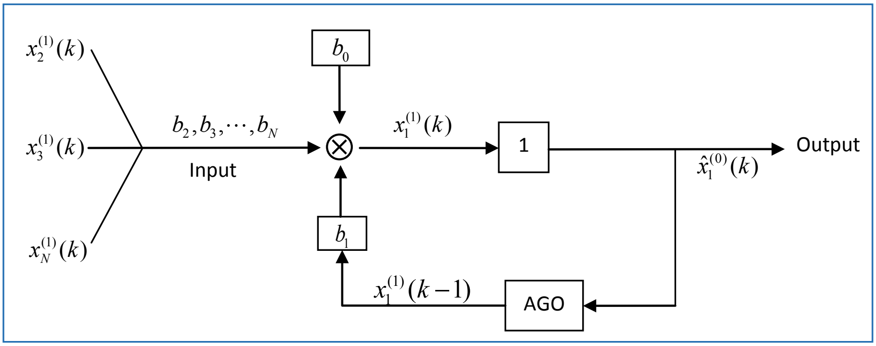

Fig. 2.6 The theoretical transfer diagram of DGM(1, *N*) model

$$MAPE = \frac{1}{m}\sum_{k=1}^{m}\left|\frac{x_1^{(0)}(k) - \hat{x}_1^{(0)}(k)}{x_1^{(0)}(k)}\right| \times 100\%.$$

The theoretical transfer process of DGM (1, N) model is shown in Fig. 2.6. Different with the GM(1, 1) model, more variables are considered in the DGM(1, N) model.

From the input information and output results, the DGM(1, N) model is similar with the multi-variable linear regression model. The main differences between these two models are the time factor and the data sequence generating. The original sequence of DGM(1, N) model should be aligned with the same time interval and the sequences should be accumulating generated while the original data is used in multi-variable linear regression models. Additionally, only the dependent relations between dependent factor and independent factors are considered while the data is not need to be aligned according to the time change.

Example Let the data sequence of a system's characteristic variable be

$$X_1^{(0)} = (2.874, 3.278, 3.307, 3.39, 3.679),$$

and the data sequences of relevant variable be

$$X_2^{(0)} = (7.040, 7.645, 8.075, 8.530, 8.774),$$

then the computational steps of DGM(1, N) model are described as follows.

Solution:

Step 1 Calculate the accumulated generation sequence:

$$X_1^{(1)} = (2.874, 6.152, 9.459, 12.849, 16.528),$$

and

$$X_2^{(1)} = (7.040, 14.685, 22.760, 31.290, 40.064).$$

Step 2 Estimate the model parameter

$$\begin{bmatrix} b_0 \\ b_1 \\ b_2 \end{bmatrix} = \left(B^{\mathrm{T}}B\right)^{-1}BY = \begin{bmatrix} -0.4220 \\ -0.2038 \\ 0.4877 \end{bmatrix},$$

where

$$B = \begin{bmatrix} 1 & x_1^{(1)}(1) & x_2^{(1)}(2) \\ 1 & x_1^{(1)}(2) & x_2^{(1)}(3) \\ 1 & x_1^{(1)}(3) & x_2^{(1)}(4) \\ 1 & x_1^{(1)}(4) & x_2^{(1)}(5) \end{bmatrix} = \begin{bmatrix} 1 & 2.874 & 14.685 \\ 1 & 6.152 & 22.760 \\ 1 & 9.459 & 31.290 \\ 1 & 12.849 & 40.064 \end{bmatrix},$$

$$Y = \begin{bmatrix} x_1^{(1)}(2) \\ x_1^{(1)}(3) \\ x_1^{(1)}(4) \\ x_1^{(1)}(5) \end{bmatrix} = \begin{bmatrix} 6.152 \\ 9.459 \\ 12.849 \\ 16.528 \end{bmatrix}.$$

Thus, the model parameters are

$$b_0 = -0.4220, b_1 = -0.2038, b_2 = 0.4877.$$

Step 3 Setting the boundary condition $\hat{x}^{(1)}(1) = x^{(1)}(1) = 2.874$, the recursive equation is

$$\hat{x}_1^{(1)}(k) = -0.4220 - 0.2038x_1^{(1)}(k-1) + 0.4877x_2^{(1)}(k), k = 2, 3, \ldots, 5.$$

Step 4 Compute the simulative values of $X^{(1)}$ and $X^{(0)}$:

$$\hat{X}^{(1)} = (2.874, 6.154, 9.424, 12.910, 16.499)$$

and

$$\hat{X}^{(0)} = (2.874, 3.280, 3.270, 3.486, 3.588).$$

Step 5 Evaluate the modeling performance based on MAPE:

$$MAPE = \frac{1}{5}\sum_{k=1}^{5}\left|\frac{x^{(0)}(k) - \hat{x}^{(0)}(k)}{x^{(0)}(k)}\right| \times 100\% \approx 1.297\%.$$

2.3.4 *Future Thinking About Grey Forecasting Models*

- **The modeling sequence condition analysis**

Which kind of the sequence could be utilized in constructing grey forecasting models is still concerned by all of scholars in grey forecasting area.

Deng firstly discussed the law of sequence in constructing grey forecasting models. He took the view that grey model sequences should be according to the grey exponential law, especially for the non-negative smooth discrete function. He developed several theorems for homogeneous and non-homogeneous exponential laws and proved that the distribution of the original sequences satisfied to positive and negative distribution are possible to construct grey forecasting model. Almost all of the scholars continue to the hypothesis that modeling sequences approximate to the grey exponential law and some of the research papers focused on analyzing the simulative values adapted to pure exponential trend. This hypothesis is similar with the linear regression models, in which independent variables are assumed have the linear relationship with the dependent variable.

We take the view that the grey modeling sequence has its special law and trend which is different with the exponential law or linear law of linear regression models. In our viewpoint, some of the problems about the modeling sequence condition should be progressed. Such as what are the real sequences developing law for different grey forecasting models? Whether the errors between the real value and the simulative value of modeling sequences have a special distribution? How many of the data should be included in modeling process? And how to choose the suitable sequence interval to construct the grey forecasting models?

- **Construction of novel grey forecasting models**

Most of published papers in grey forecasting claimed novel grey forecasting models were constructed, but most of them were given using the example analysis. In addition, the papers using empirical analysis are not sufficient for the analysis of the empirical research, either. In fact, a large proportion of such papers only improved the parameters solving methods, optimized the background value, optimized the initial value and combined other methods with grey forecasting models while did not construct novel model really. The optimization model proposed in these papers is only for the special case with high simulation accuracy, and there is no good reason to strictly prove that the proposed model is superior to other models.

However, some researchers did the real works, such as grey Verhulst model, non-equip grey model, discrete grey model, non-homogeneous discrete grey model, multi-variable discrete grey model, grey model with time polynomial term, etc.

With time rolling forward, many new system forecasting problems emerged and most of such problems could not provide enough information for constructing traditional forecasting models. To create novel forecasting models become a priority work. We can depict several such new problems like system forecasting with the limited upper value, shock wave system forecasting, system structure forecasting, etc.

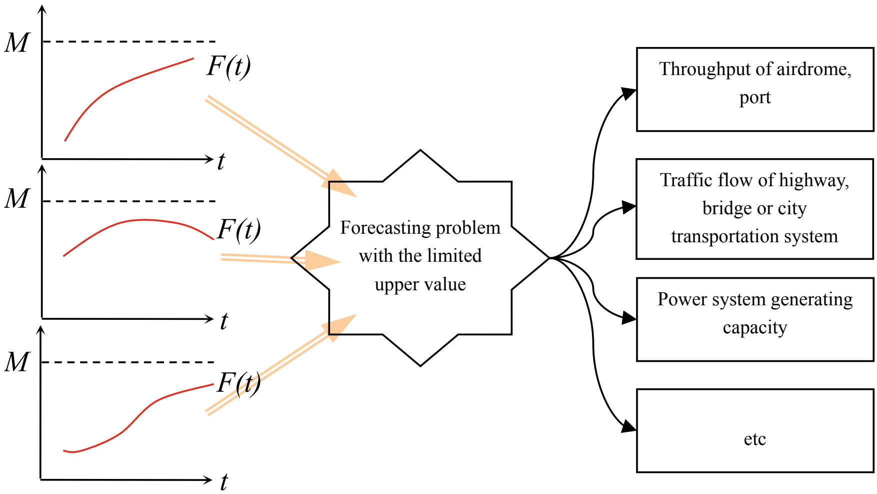

Fig. 2.7 The diagram of system forecasting problem with the limited upper value

System forecasting problem with the limited upper value often emerges in economic system, transportation system, energy system, ecosystem, etc. some of the problems similar with S-curve, exponential curve, converse U-curve, etc. The proposing models can be applied in forecasting throughput of airdrome or port, the traffic flow of highway, power system generating capacity, etc. The diagram of such problems is shown in Fig. 2.7.

Shock wave system forecasting problems are often appeared in the areas which are influenced by the policy or nature calamity. In traditional forecasting models, all of the models admit the hypothesis that the trend of system's main variable will keep on going in the same way in the future. However, this hypothesis will not come into existence by the shock impact on the system main variable.

As is shown in Fig. 2.8, sequence X is the system's main variable and the developing trend change in the neighbor stages. Assume that the turning point is Z, if we construct forecasting model with the sequence X, then we will get the forecasting information C according to the traditional hypothesis of forecasting models. While the fact is that the system will go with the curve B by some shock impact on the turning point. Therefore, if we adopt the traditional method, we will make mistakes.

Assume that we can transfer the sequence X to sequence XD by some kinds of operator D, then construct the forecasting model with generated sequence XD, obviously we will forecasting the trend going with the curve B and could come to the correct conclusions.

System structure forecasting problems are always analyzed by Markov chain model. Markov chain model is usually assumed that the system structure units are stable, and there is no unit inside and outside.

As is show in Fig. 2.9, we maybe analyze the structure with some unit coming into the system, like market share change in which a new company participates in

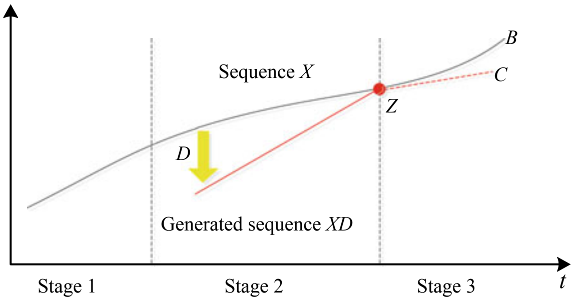

Fig. 2.8 The diagram of shock wave system forecasting problems

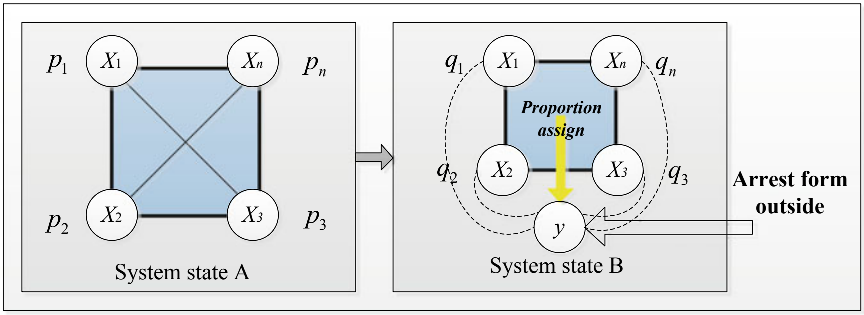

Fig. 2.9 The diagram of system structure forecasting problems

producing same products, like a new airdrome, port or highway is constructed to alter the choice of transportation, etc.

- **The properties analysis of grey forecasting models**

Since a large amount of grey forecasting models and improved grey forecasting models are proposed, the properties of the proposed models should be discussed.

What is the influence of multiple and parallel transformation to other grey forecasting models? What are the influences of parameters of system main variable and influence variables? What is the effect for catching the forecasting trend with the different solution of coefficient of background value? Furthermore, whether new test methods of grey forecasting models should be developed in the forthcoming research?

In the early stage of grey forecasting models researching, scholars utilized the grey relational model to measure the error between simulative value and real value of modeling sequence. In recent years, most of literature indicated that mean absolute percentage error between simulative value and real value are popular applied. The aim to study the models properties is to grasp the strongpoint and drawback of each grey forecasting model and adopt suitable model in special case better. Therefore, the multiple or parallel transformation of sequences, the parameters, the background value, the test methods should be included in the properties analysis.

- **The comparison research of different grey forecasting models**

Since the pool of grey forecasting is so large and plenty of grey forecasting models or improved grey forecasting models can be provide to do forecast work. Frankly in the midst of so many models, we don't know whether that can prove their proposed model is better than the classical model or other improved model under certain conditions. This raises a question: Is it necessary to build a complex grey forecasting model? Whether those complex models make users any better at actually improving the accuracy of prediction remains an open question. Moreover, most of the grey forecasting models are not widely used actually. How to solve these problems is a question.

Authors should express the difference between personal models and the past. Also, they should explain the former models have many constraints on application and cannot meet with the demand of development needs or the novel models they proposed are necessary. Then the users will suddenly see the light and begin to feel clearly. The users will confuse on how to choose the proper model and the researchers should make clear about what are the differences among different grey forecasting models. Although there are literatures focused on the accuracy and applicable conditions of grey models, these results mainly concerned about model performance for specific data, such as exponential data, monotonic data, and so on. General conclusions according to each grey forecasting model are impossible to be got in these papers. Meanwhile, the existent results only compared modeling accuracy of some different grey system models through theoretical study, while there was not only lack of modeling accuracy test method for grey forecasting models, but also few modeling accuracy comparison method between grey models and other prediction methods, such as regression analysis, support vector machine, and neural network, etc.

In our opinion, Monte Carlo method can be utilized to compare the different grey forecasting models and other system forecasting models. The sequences of a prediction problem can be regarded as a population and a large sample from population though random sampling could be generated. Then, these sample sequences could be used in constructing grey forecasting model or other system forecasting models whose modeling accuracy were expected to test. With modeling error calculation, the average error of the model for sample sequences can be obtained. Finally, the modeling accuracy of the model for this kind of problem was acquired by statistical inference testing method. Through the statistical significant test method, the accuracy of any two grey models can be compared, and it can be also compared between grey prediction model and other forecasting methods like regression analysis, support vector machine neural network, etc.

- **Grey forecasting models based on grey information**

Grey number, the basic element of a grey system, is known as a set of interval for lack of information. When information is collected and continuously improved, grey number will transfer to a white number which is a real number.

Although grey number is very important in grey system theory, the factor is not effectively considered in grey forecasting models. The reason is that the grey number operation algorithm is not constructed efficiently. Even so, we consider that the grey number should be combined with grey forecasting models and to construct novel grey forecasting models base on grey number sequences.

As is shown in Fig. 2.10, the grey numbers are describe as intervals while actually a grey number is a real number when enough is got and it has to be expressed as interval for lacking information. So we can get the grey number sequence $(\otimes_1, \otimes_2, \ldots, \otimes_n)$. Traditionally, the grey number sequences with real number, usually mean value of the upper boundary and lower boundary, will be substituted into the existing models respectively. We take the view that the operation of two or more interval grey numbers can be defined with optimized models. With the optimized model, we will actually construct a series of grey forecasting models like the curve 1, 2 …, *m*. Finally we can get the forecasting sequence value with the maximum and minimum in each *t* point.

- **Links with dynamical data analysis**

Dynamical data models take a differential (or difference) equation to describe systems and detect the underlying evolution characteristics from time series observations (Ramsay & Hooker, 2017). In a broad sense, grey system models also belong to this kind of approach. It is worth noting, however, that the grey system model utilizes the cumulative sum (Cusum) operator to visualize and mine the dynamical characteristics hidden in the original time series, which distinguishes this kind of approach from others.

Differential (or difference) dynamic model of time series employs the integral matching method of differential equation parameter estimation as a bridge, connecting the grey prediction model with dynamic data analysis (Wei et al., 2020). By introducing an integral operator, the linear grey prediction model (nonlinear

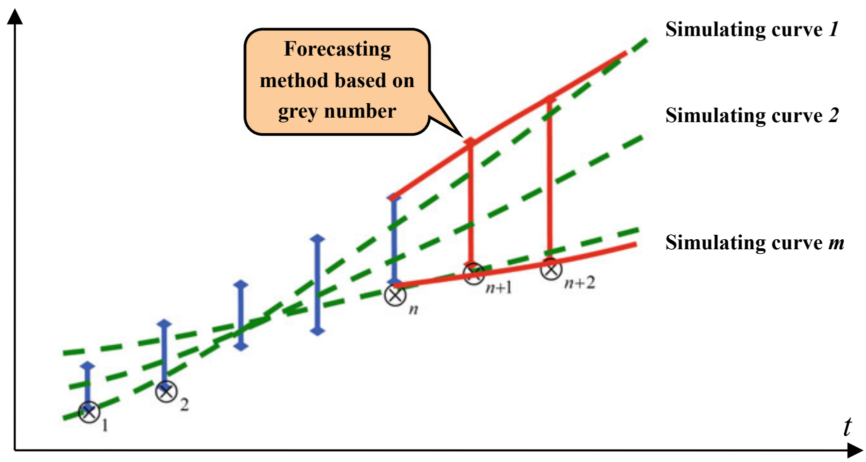

Fig. 2.10 The diagram of grey number sequence forecasting

grey prediction model) is reduced to an equivalent ordinary differential equation (integro-differential equation), which shows that the modelling procedures of integral matching-based models can be viewed as the simplification and reconstruction of grey models (Wei & Xie, 2020, 2022; Yang & Xie, 2021; Yang et al., 2022). Compared with grey models, the integral matching-based models obtain the estimates of structural parameters and initial values simultaneously. As an essential element in grey modelling, the cumulative sum operator proves to be the piecewise left-constant quadrature formula of the integral operator. Besides, the existing variations of Cusum operators, such as the fractional cumulative sum operator and new information priority accumulation, can be regarded as different numerical discretization forms of the integral operator.

2.4 Introduction to Grey Relational Model

2.4.1 General Evolution of Grey Relational Models

Since Deng (1985) proposed the basic principles for grey relational modelling, many discussions about property analysis and exploration about construction of novel models have been conducted, and therefore result in many meaningful results including the simplification and extension of the basic principles. Then generalized grey relation analysis models are subsequently put forward based on the basic idea that the similarity between sequences can be measured by the closeness of the geometrical shape of the interpolating curves corresponding to the original sequences, that is, the sequences are altered to continuous curves by piecewise linear interpolation method, and then the degree of relation is measured by the difference between the interpolating curves.

Different form the early researches where grey relational models are built based on the relation coefficients of each separate point in the whole sample space, the generalized models are constructed form an overall perspective. In addition, the basic elements of research objects are also extended to be a matrix or uncertainty number from the real number.

2.4.2 Steps of Deng's Grey Relational Model

Sequence operator is an efficient pre-processing technique for converting the original sequences to non-dimensional values of roughly equal magnitudes.

Definition Assume that X_i is a system factor and its observation value at the ordinal position k is $x_i(k)$, $k = 1, 2, \ldots, n$; then $X_i = (x_i(1), x_i(2), \ldots, x_i(n))$ is referred to as the behavioral sequence of factor X_i.

Definition Let $X_i = (x_i(1), x_i(2), \ldots, x_i(n))$ be the behavioral sequence of factor X_i, and D a sequence operator such that $X_iD = (x_i(1)d, x_i(2)d, \ldots, x_i(n)d)$, where

(1) if $x_i(k)d = x_i(k)/x_i(1)$, $k = 1, 2, \ldots, n$, then D is referred to as an initialing operator and X_iD is called the initial image of X_i.
(2) if $x_i(k)d = x_i(k)/\overline{X}_i, \overline{X}_i = \frac{1}{n}\sum_{k=1}^{n} x_i(k)$, $k = 1, 2, \ldots, n$, then D is referred to as an averaging operator and X_iD is called the average image of X_i.
(3) if $x_i(k)d = \frac{x_i(k)-\min_k x_i(k)}{\max_k x_i(k)-\min_k x_i(k)}$, $k = 1, 2, \ldots, n$, then D is referred to as an interval operator and X_iD is called the interval image of X_i.
(4) if $x_i(k)d = 1 - x_i(k)$, $k = 1, 2, \ldots, n$, then D is referred to as a reversing operator and X_iD is called the reverse image of X_i.
(5) if $x_i(k)d = 1/x_i(k)$, $x_i(k) \neq 0$, $k = 1, 2, \ldots, n$, then D is referred to as a reciprocating operator and X_iD_1 is called the reciprocal image of X_i.

The relational operators are generally used to normalize or to change the trend of system's behavioral variables.

For example, letting X_0 be an increasing sequence reflecting a system's behavioral characteristics and X_i be the a relevant factor sequence, if X_i is increasing, then X_i and X_0 have a positive incidence; if X_i is decreasing, then X_i and X_0 have a negative incidence. Here, the negative relationship will become positive by using the reversing operator or reciprocating operator, and especially, these two operators should not be mixed or overlapped.

Example Assume that the original sequence is $X = (2.5,\ 2.9,\ 3.2,\ 4.5,\ 5.8,\ 6.3)$, and then the images of sequence X are calculated as follows.

(1) The initial image is

$$XD = (1.00,\ 1.16,\ 1.28,\ 1.80,\ 2.32,\ 2.52).$$

(2) The average image is

$$XD = (0.5952,\ 0.6906,\ 0.7619,\ 1.0714,\ 1.3809,\ 1.5000).$$

(3) The interval image is

$$XD = (0.0000,\ 0.1053,\ 0.1842,\ 0.5263,\ 0.8684,\ 1.0000).$$

(4) The reverse image is

$$XD = (-1.5,\ -1.9,\ -2.2,\ -3.5,\ -4.8,\ -5.3)$$

(5) The reciprocal image is

$$XD = (0.4000, 0.3448, 0.3125, 0.2222, 0.1724, 0.1587).$$

Definition Let $X_0 = (x_0(1), x_0(2), \ldots, x_0(n))$ be the data sequence of a system's behavioral characteristic and the following are relevant factor sequences

$$X_i = (x_i(1), x_i(2), \ldots, x_i(n)), i = 1, 2, \ldots, m.$$

Given real numbers $\gamma(x_0(k), x_i(k))$, $i = 1, 2, \ldots, m$ and $k = 1, 2, \ldots, n$, which denotes the incidence coefficient of X_0 and X_i at point k,

$$\gamma(X_0, X_i) = \frac{1}{n}\sum_{k=1}^{n} \gamma(x_0(k), x_i(k))$$

is referred to as the degree of grey incidence between X_0 and X_i if it satisfies conditions of normality and closeness below:

(1) Normality: $0 < \gamma(X_0, X_i) \leq 1$, $\gamma(X_0, X_i) = 1 \Leftarrow X_0 = X_i$; and
(2) Closeness: the smaller $|x_0(k) - x_i(k)|$; the greater $\gamma(x_0(k), x_i(k))$.

Theorem *Given a system's behavioral sequences* $X_0 = (x_0(1), x_0(2), \ldots, x_0(n))$ *and* $X_i = (x_i(1), x_i(2), \ldots, x_i(n))$, $i = 1, 2, \ldots, m$, *for* $\xi \in (0, 1)$, *it is possible to define*

$$\gamma(x_0(k), x_i(k)) = \frac{\min\limits_i \min\limits_k |x_0(k) - x_i(k)| + \xi \max\limits_i \max\limits_k |x_0(k) - x_i(k)|}{|x_0(k) - x_i(k)| + \xi \max\limits_i \max\limits_k |x_0(k) - x_i(k)|},$$

and

$$\gamma(X_0, X_i) = \frac{1}{n}\sum_{k=1}^{n} \gamma(x_0(k), x_i(k))$$

is referred to as Deng's degree of grey incidence, where ξ *is known as the distinguishing coefficient.*

The computation steps of the Deng's degree of grey incidence can be accomplished as explained below.

Step 1 Calculate the initial image or average image of X_0 and X_i, $i = 1, 2, \ldots, m$; where:

$$X_i' = X_i / x_i(1) = \left(x_i'(1), x_i'(2), \ldots, x_i'(n)\right), i = 0, 1, \ldots, m,$$

or

$$X_i' = X_i / \overline{X}_i = \left(x_i'(1), x_i'(2), \ldots, x_i'(n)\right), i = 0, 1, \ldots, m.$$

Step 2 Compute the difference sequences of X_0 and X_i and write as:

$$\Delta_i(k) = \left|x'_0(k) - x'_i(k)\right|, \Delta_i = (\Delta_i(1), \Delta_i(2), \ldots, \Delta_i(n)),$$
$$i = 1, 2, \ldots, m.$$

Step 3 Find the maximum and minimum differences, and denote as:

$$M = \max_i \max_k \Delta_i(k), m = \min_i \min_k \Delta_i(k).$$

Step 4 Calculate the incidence coefficients:

$$\gamma(x_0(k), x_i(k)) = \frac{m + \xi M}{\Delta_i(k) + \xi M}, \xi \in (0, 1),$$
$$i = 1, 2, \ldots, m, k = 1, 2, \ldots, n.$$

Step 5 Compute the degree of grey incidence:

$$\gamma(X_0, X_i) = \frac{1}{n} \sum_{k=1}^{n} \gamma(x_0(k), x_i(k)), i = 1, 2, \ldots, m.$$

Example Assume that the system's characteristic sequence is

$$X_0 = (x_0(1), x_0(2), x_0(3), x_0(4), x_0(5)) = (1.2,\ 1.6,\ 1.3,\ 2.1,\ 2.0),$$

the relevant sequences are

$$X_1 = (x_1(1), x_1(2), x_1(3), x_1(4), x_1(5)) = (1.5,\ 1.3,\ 1.1,\ 1.8,\ 2.7),$$

$$X_2 = (x_2(1), x_2(2), x_2(3), x_2(4), x_2(5)) = (2.0,\ 1.2,\ 1.8,\ 1.0,\ 2.0),$$

$$X_3 = (x_3(1), x_3(2), x_3(3), x_3(4), x_3(5)) = (1.2,\ 2.0,\ 1.2,\ 2.4,\ 1.0).$$

Solution:

Step 1 The average images of X_i, $i = 0, 1, 2, 3$, are calculated as

$$X'_0 = X_0/\overline{X}_0 = (0.923,\ 0.769,\ 0.615,\ 1.154,\ 1.538),$$

$$X'_1 = X_1/\overline{X}_1 = (0.936,\ 0.813,\ 0.688,\ 1.000,\ 1.563),$$

$$X'_2 = X_2/\overline{X}_2 = (1.250,\ 0.875,\ 1.000,\ 0.625,\ 1.250),$$

$$X_3' = X_3/\overline{X}_3 = (1.000,\ 0.800,\ 1.500,\ 1.200,\ 0.500).$$

Step 2 The difference sequences between X_0' and X_i', $i = 1, 2, 3$, are computed as

$$\Delta_{01} = (0.0144,\ 0.0433,\ 0.0721,\ 0.1538,\ 0.0240),$$

$$\Delta_{02} = (0.3269,\ 0.1058,\ 0.3846,\ 0.5288,\ 0.2885),$$

$$\Delta_{03} = (0.0769,\ 0.0308,\ 0.8846,\ 0.0462,\ 1.0385).$$

Step 3 The extreme values are

$$M = \max_i \max_k \Delta_{0i} = 1.0385, m = \min_i \min_k \Delta_{0i} = 0.0144.$$

Step 4 Let $\xi = 0.5$, and then

$$\gamma(x_0(1), x_1(1)) = 1.000, \gamma(x_0(1), x_2(1)) = 0.631, \gamma(x_0(1), x_3(1)) = 0.895,$$

$$\gamma(x_0(2), x_1(2)) = 0.948, \gamma(x_0(2), x_2(2)) = 0.854, \gamma(x_0(2), x_3(2)) = 0.970,$$

$$\gamma(x_0(3), x_1(3)) = 0.902, \gamma(x_0(3), x_2(3)) = 0.590, \gamma(x_0(3), x_3(3)) = 0.380,$$

$$\gamma(x_0(4), x_1(4)) = 0.793, \gamma(x_0(4), x_2(4)) = 0.509, \gamma(x_0(4), x_3(4)) = 0.944,$$

$$\gamma(x_0(5), x_1(5)) = 0.982, \gamma(x_0(5), x_2(5)) = 0.661, \gamma(x_0(5), x_3(5)) = 0.340.$$

Step 5 Finally,

$$\gamma(X_0, X_1) = \frac{1}{5}(1.000 + 0.948 + 0.902 + 0.793 + 0.982) = 0.925,$$

$$\gamma(X_0, X_2) = \frac{1}{5}(0.631 + 0.854 + 0.590 + 0.509 + 0.661) = 0.649,$$

$$\gamma(X_0, X_3) = \frac{1}{5}(0.895 + 0.970 + 0.380 + 0.944 + 0.340) = 0.706.$$

2.4.3 Steps of Generalized Grey Relational Model

Definition (Liu et al., 2011) Let $X = (x(1), x(2), \ldots, x(n))$ be the behavioral sequence of factor X, and D a sequence operator such that $XD = (x(1)d, x(2)d, \ldots, x(n)d)$, where

(1) if $x(k)d = x(k) - x(1)$, $k = 1, 2, \ldots, n$, then D is referred to as a zero-starting point operator and $X_i D$ is called the zero-starting point image of X_i.
(2) if $x(k)d = \frac{x(k)-x(1)}{x(1)}$, $k = 1, 2, \ldots, n$, then D is referred to as an initialing operator and $X_i D$ is called the initial image of X_i.

The generalized grey relational models are mainly used to analyze the similarity based on the geometric patterns of zigzagged lines corresponding to operator sequences. Assume that X_i and X_j are two behavioral sequences with the same length, same time distances, and equal time intervals, then the generalized grey relational models with different forms are expressed as follows.

Theorem *Let X_i^0 and X_j^0 be the zero-starting point images corresponding to X_i and X_j, then the absolute degree of grey incidence is defined as*

$$\varepsilon_{ij} = \frac{1 + |s_i| + |s_j|}{1 + |s_i| + |s_j| + |s_i - s_j|},$$

where

$$s_i = \int_1^n X_i^0(t)dt = \sum_{k=2}^{n-1} x_i^0(k) + \frac{1}{2}x_i^0(n),$$

$$s_j = \int_1^n X_j^0(t)dt = \sum_{k=2}^{n-1} x_j^0(k) + \frac{1}{2}x_j^0(n), \text{ and}$$

$$s_i - s_j = \int_1^n \left(X_i^0(t) - X_j^0(t)\right)dt = \sum_{k=2}^{n-1} \left(x_i^0(k) - x_j^0(k)\right) + \frac{1}{2}\left(x_i^0(n) - x_j^0(n)\right).$$

This absolute degree of grey incidence quantifies the relationship between the change of sequences, relative to their initial values. The closer the amounts of change are, the greater ε_{ij} is, and vice versa.

Theorem *Let X_i' and X_j' be the initial images corresponding to X_i and X_j, then the relative degree of grey incidence is defined as*

$$\vartheta_{ij} = \frac{1 + |s_i'| + |s_j'|}{1 + |s_i'| + |s_j'| + |s_i' - s_j'|}$$

where

$$s_i' = \int_1^n X_i'(t)dt = \sum_{k=2}^{n-1} x_i'(k) + \frac{1}{2}x_i'(n),$$

$$s_j' = \int_1^n X_j'(t)dt = \sum_{k=2}^{n-1} x_j'(k) + \frac{1}{2}x_j'(n), \text{ and}$$

$$s_i' - s_j' = \int_1^n \left(X_i'(t) - X_j'(t)\right)dt = \sum_{k=2}^{n-1} \left(x_i'(k) - x_j'(k)\right) + \frac{1}{2}\left(x_i'(n) - x_j'(n)\right).$$

This relative degree of grey incidence quantifies the relationship between the rates of change of sequences, relative to their initial values. The closer the rates of change of X_i and X_j are, the greater ϑ_{ij} is, and vice versa.

Theorem *The synthetic degree of grey incidence is defined as*

$$\rho_{ij} = \theta \varepsilon_{ij} + (1 - \theta)\vartheta_{ij}$$

where $\theta \in [0, 1]$ is called synthetic coefficient.

The synthetic degree of grey incidence combines the characteristics of absolute degree and relative degree together, which describes both the similarity and the closeness (Liu et al., 2006). In general, the synthetic coefficient is set as 0.5 when the practitioners have no emphasizes on the absolute quantities or the rates of change. It is obvious that the synthetic coefficient should be set as a real number larger than 0.5 when the focus is on the relationship between absolute quantities, and on the other hand, it should be less than 0.5 when the focus is more on comparison between rates of change.

Theorem *Let X_i^0 and X_j^0 be the zero-starting point images corresponding to X_i and X_j, then the similitude degree of grey incidence is*

$$\xi_{ij} = \frac{1}{1 + \left|s_i - s_j\right|}$$

where

$$s_i - s_j = \int_1^n \left(X_i^0(t) - X_j^0(t)\right)dt = \sum_{k=2}^{n-1} \left(x_i^0(k) - x_j^0(k)\right) + \frac{1}{2}\left(x_i^0(n) - x_j^0(n)\right).$$

The similitude degree of grey incidence measures the geometric similarity of the shapes of sequences X_i and X_j. The more similar the geometric shapes of X_i and X_j, the greater the value of ξ_{ij}, and vice versa.

Theorem *The close degree of grey incidence is*

$$\zeta_{ij} = \frac{1}{1 + |S_i - S_j|}$$

where

$$S_i - S_j = \int_1^n \big(X_i(t) - X_j(t)\big)dt = \sum_{k=2}^{n-1} \big(x_i(k) - x_j(k)\big) + \frac{1}{2}\big(x_i(n) - x_j(n)\big).$$

The closeness degree of incidence measures the spatial closeness of sequences X_i and X_j. The closer the X_i and X_j sequences, the greater the value of ζ_{ij}, and vice versa.

Example Let sequences X_0 and X_1 be

$$\begin{aligned} X_0 &= (x_0(1), x_0(2), x_0(3), x_0(4), x_0(5), x_0(6), x_0(7)) \\ &= (10,\ 9,\ 15,\ 14,\ 14,\ 15,\ 16), \end{aligned}$$

and

$$\begin{aligned} X_1 &= (x_1(1), x_1(2), x_1(3), x_1(4), x_1(5), x_1(6), x_1(7)) \\ &= (46,\ 58,\ 70,\ 77,\ 84,\ 91,\ 98). \end{aligned}$$

The relative degree of grey incidence ϑ_{01} of sequences X_0 and X_1 are calculated as follows.

Solution:

Step 1 Calculate the initial images of sequences of X_0 and X_1:

$$\begin{aligned} X_0' &= \big(x_0'(1), x_0'(2), x_0'(3), x_0'(4), x_0'(5), x_0'(6), x_0'(7)\big) \\ &= (1.0,\ 0.9,\ 1.5,\ 1.4,\ 1.4,\ 1.5,\ 1.6), \end{aligned}$$

$$\begin{aligned} X_1' &= \big(x_1'(1), x_1'(2), x_1'(3), x_1'(4), x_1'(5), x_1'(6), x_1'(7)\big) \\ &= (1.00,\ 1.26,\ 1.52,\ 1.67,\ 1.83, 1.98, 2.13). \end{aligned}$$

Step 2 Compute the zero-starting point images of sequences X_0' and X_1':

$$\begin{aligned} X_0'^0 &= \big(x_0'^0(1), x_0'^0(2), x_0'^0(3), x_0'^0(4), x_0'^0(5), x_0'^0(6), x_0'^0(7)\big) \\ &= (0.0,\ -0.1,\ 0.5,\ 0.4,\ 0.4,\ 0.5,\ 0.6), \end{aligned}$$

$$X_1'^0 = \big(x_1'^0(1), x_1'^0(2), x_1'^0(3), x_1'^0(4), x_1'^0(5), x_1'^0(6), x_1'^0(7)\big)$$

$$= (0.00,\ 0.26,\ 0.52,\ 0.67,\ 0.83, 0.98, 1.13).$$

Step 3 Calculate $\left|s'\right|$, $\left|s'_j\right|$ and $\left|s'_i - s'_j\right|$:

$$\left|s'_0\right| = \left|\sum_{k=2}^{6} x'^0_0(k) + \frac{1}{2}x'^0_0(7)\right| = 2.00,$$

$$\left|s'_1\right| = \left|\sum_{k=2}^{6} x'^0_1(k) + \frac{1}{2}x'^0_1(7)\right| = 3.828,$$

$$\left|s'_0 - s'_1\right| = \left|\sum_{k=2}^{6} \left(x'_0(k) - x'_1(k)\right) + \frac{1}{2}\left(x'_0(7) - x'_1(7)\right)\right| = 1.925.$$

Step 4 Calculate the relative degree of grey incidence ϑ_{01}:

$$\vartheta_{01} = \frac{1 + \left|s'_0\right| + \left|s'_1\right|}{1 + \left|s'_0\right| + \left|s'_1\right| + \left|s'_0 - s'_1\right|} = \frac{6.825}{8.750} = 0.78.$$

2.4.4 Future Thinking About Grey Relational Models

The further research on grey relational models can be conducted from the following two aspects.

- **Property of grey relational models**

In fact, almost every novel grey relation analysis model is proposed along with the corresponding discussion about properties analysis due to that better knowledge of the properties is helpful for understand the modeling requirements, applicable scope, and even the extension of existing models.

The standardization, initializing treatment, averaging treatment and the value of discriminating coefficient's influence on order of grey relation have been studied, and also the affinity and affine transformation isotonicity of grey relation models.

- **Extension of basic elements**

The basic element of behavior sequence has been extended from real number to grey numbers, interval numbers, matrices and even multi-dimensional matrices, etc., such that the novel grey relational analysis models can be used to analyze the similarity in high dimensional space. Taking the matrix representation as an illustrative example, the behavior matrix is expressed as

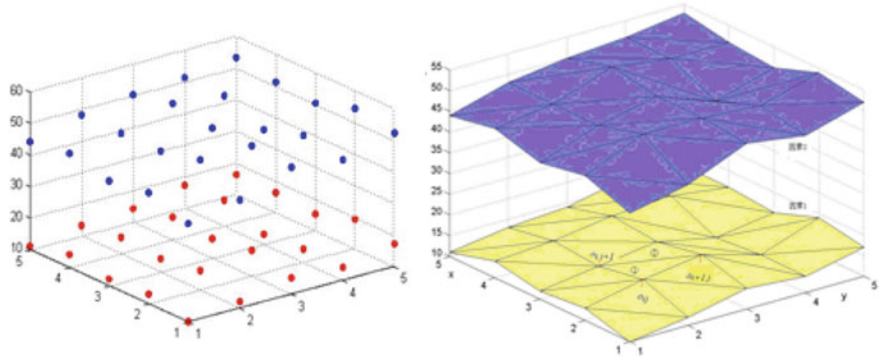

Fig. 2.11 The scatter plot of systematic behavior matrix (left) and its curved surface (right)

$$X = \left(a_{ij}\right)_{m \times n} = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \dots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{bmatrix},$$

and then similar to that in the above case of calculating the absolute degree of grey relation, we can obtain the zero-starting point image and the corresponding curved surface as shown in Fig. 2.11.

The absolute degree of grey relation in three-dimensional space can be constructed as

$$\varepsilon_{pq} = \frac{1 + \left|s_p\right| + \left|s_q\right|}{1 + \left|s_p\right| + \left|s_q\right| + \left|s_p - s_q\right|}$$

where

$$s_p = \iint\limits_D X_p^0(t)dt,\ s_q = \iint\limits_D X_q^0(t)dt,\ s_p - s_q = \iint\limits_D X_p^0(t) - X_q^0(t)dt.$$

$\left|s_p\right|$ and $\left|s_q\right|$ denote the volume between the initial zeroing curved surface and coordinate plane, respectively, $\left|s_p - s_q\right|$ denote the volume between two curved surfaces.

Following the above procedures, similarly, one can construct other grey relational models in three-dimensional space, such as calculating the relative, similitude and close degrees to analyze the relation among panel data.

- **Links with functional data analysis**

Grey relational model describes the relationship between time series and derives time series mining methods such as time series classification and clustering. Different from multivariate statistical analysis, grey relational model is oriented to data with ordinal relationships, which is generally expressed as time series varying with time. The basic idea of grey relational analysis is to use the degree of similarity of the geometric curves of available data sequences to determine whether or not their connections are close. From the perspective of function space, the grey relational model regards each factor in the system factor set as a point in space and regards the observation data of each factor as the coordinates of the point. The model studies the relationship between various elements in a specific space, and then defines the grey relational degree based on the distance in n-dimensional space. Therefore, effectively identifying the dynamic information of time series is the main task of grey relational model.

Similar to the modeling idea of grey relational model, functional data analysis (Wang et al., 2016) regards time series as sampling data of a functional variable and is devoted to studying the principal components and clustering of functional variables. There are four existing functional clustering methods: ① raw-data direct clustering (RDDC). RDDC directly takes discrete data as the analysis object and measures the similarity of time series based on a distance measure. ② two-step tandem clustering (TSTC). TSTC consists of two steps: filtering reduction and traditional cluster analysis. Firstly, the dimension of infinite-dimensional function space is reduced to a finite parameter or coefficient space Then, the fitting coefficients of finite dimensions are classified by the traditional clustering method. ③ distance-based nonparametric clustering (DBNC). This approach constructs the non-parametric clustering of Eigen functions according to the static or dynamic shape characteristics of the curve.

Actually, grey relational model grey relational models are both related and different from functional data analysis. First, their research objects are time series or panel data with time characteristics. Then, their modeling ideas are similar, grey relational model regards time series as geometric shapes in space, which is very similar to functional data analysis that regards time series as sampled data of functional variables. Moreover, their first step is to convert the time series into a continuous function. Here, grey relational model adopts the method of interpolation, while the functional data analysis mostly uses the method of basis function fitting. The difference is that grey relational model and the functional data analysis use different distance measures. Grey relational degree, including Deng's grey relational degree, Grey absolute relational degree, grey relative correlation degree, and grey correlation degree based on proximity and similarity, provide effective methods for the measurement of time series. From a broader research perspective, the main problem to be solved by grey correlation model is the dynamic information mining of time series. However, the application and development of grey relational model are hindered because grey correlation model does not form a relatively perfect system in model hypothesis, distance measurement, and model function. Reconstructing grey relational model with standardized mathematical description is not only of innovative value in the field of grey system but also of great significance in the fields of machine learning, data mining.

2.5 Introduction to Grey Cluster Model

2.5.1 General Evolution of Grey Cluster Models

Grey clustering is to segment a collection of objects or observation indices into clusters by using grey relational matrices or grey possibility functions such that those within each cluster are closely enough related to the others. Different from the unsupervised cluster analysis method, in practical applications there are only a few available characteristic indices for each object such that it is difficult to classify them accurately. The clustering methods can be divided into two categories: grey relational model based cluster and grey possibility function based cluster (Xie et al., 2019).

Grey relational model based cluster can be used to segment both the factors into individual categories and the objects into individual groups. In practice, we often encounter the problems whether the underlying factors belong to the same kind or not, so that we can use less factors or a synthetic factor to represent the system behavior by losing as less information as possible. In other word, this method can be regarded as a concept of variable selection or design in the system analysis process, that is to say that we can reduce the costs spent on the data collection of the unnecessary variables before a large-scale survey is conducted.

Different from the grey relational model based cluster analysis, the grey possibility function based clustering can only be applied to the objects partition, and the possibility functions should be determined in advance. Up to now, this method has been widely used to analyze uncertain systems and developed into two common models: the variable weight one and the fixed weight one, among which the latter is suitable for the case that the attributes have different meanings, dimensions, and observations.

2.5.2 Steps of Variable Weight Grey Cluster Model

On the basis of the observation x_{ij} $(i = 1, 2, \ldots, n, j = 1, 2, \ldots, m)$ where m is the number of indexes, the steps of grey fixed weight clustering method that groups the n objects into s grey classes can be expressed as follows:

Step 1 Determine the s grey classes according to the prior background knowledge.

Step 2 Normalize the attribute data using the appropriate method. If the result is better when the value of index is larger, the values are transformed to be between 0 and 1. For the index j, we have different observation values y_{ij}, the maximum value $y_j^{\max} = \max_{i=1}^{n} y_{ij}$ and minimum value $y_j^{\min} = \min_{i=1}^{n} y_{ij}$, then the index value is normalized as

$$x_{ij} = \frac{y_{ij} - y_j^{\min}}{y_j^{\max} - y_j^{\min}}.$$

If the result is better when the value of index is smaller, then the value of index is normalized as

$$x_{ij} = \frac{y_j^{\max} - y_{ij}}{y_j^{\max} - y_j^{\min}}.$$

Step 3 Set the possibility functions for each index. In fixed weight cluster, the possibility function with respect to the subclass k and index j is set as $f_j^k(\cdot)$, and three of the most employed possibility functions are respectively formulated as follows:

(1) The possibility function of lower measure in Fig. 2.12 is expressed as

$$f_j^k(x) = \begin{cases} 1, & x \in \left[0, x_j^k(3)\right] \\ \frac{x_j^k(4) - x}{x_j^k(4) - x_j^k(3)}, & x \in \left[x_j^k(3), x_j^k(4)\right] \\ 0, & x \notin \left[0, x_j^k(4)\right] \end{cases}.$$

(2) The possibility function of moderate measure in Fig. 2.13 is expressed as

$$f_j^k(x) = \begin{cases} 0 & x \notin \left[x_j^k(1), x_j^k(4)\right] \\ \frac{x - x_j^k(1)}{x_j^k(2) - x_j^k(1)} & x \in \left[x_j^k(1), x_j^k(2)\right] \\ \frac{x_j^k(4) - x}{x_j^k(4) - x_j^k(2)} & x \in \left[x_j^k(2), x_j^k(4)\right] \end{cases}.$$

(3) The possibility function of upper measure in Fig. 2.14 is expressed as

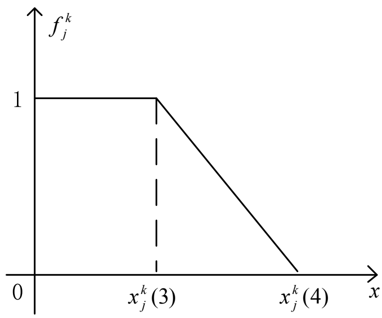

Fig. 2.12 The possibility function of lower measure

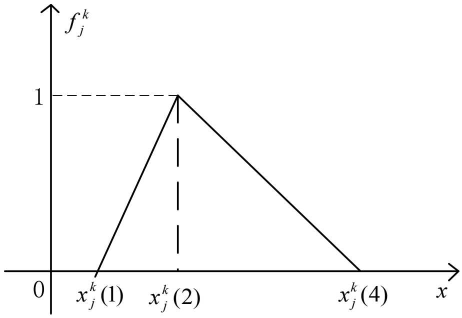

Fig. 2.13 The possibility function of moderate measure

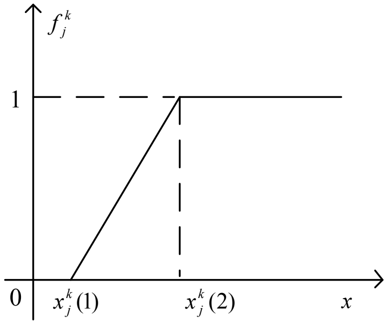

Fig. 2.14 The possibility function of upper measure

$$f_j^k(x) = \begin{cases} 0, & x < x_j^k(1) \\ \frac{x - x_j^k(1)}{x_j^k(2) - x_j^k(1)}, & x \in \left[x_j^k(1), x_j^k(2)\right] \\ 1, & x \geq x_j^k(2) \end{cases}$$

Step 4 Determine the clustering weight η_j $(j = 1, 2, \ldots, m)$ of each index, and the weight is set as $\eta_j = \frac{1}{m}$ when there is no enough information.

Step 5 With the function $f_j^k(\cdot)$ in Step 3, the weight η_j in Step 4 and observation value x_{ij} of object i about index j, calculate the grey fixed weight clustering coefficient

$$\sigma_i^k = \sum_{j=1}^{m} \left(f_j^k(x_{ij}) \times \eta_j\right), i = 1, 2, \ldots, n, j = 1, 2, \ldots, m.$$

Step 6 Calculate the clustering weight vector according to the fixed weight coefficient of each class

$$\begin{aligned} \sigma_i &= \left(\sigma_i^1, \sigma_i^2, \ldots, \sigma_i^s\right) \\ &= \left(\sum_{j=1}^{m} \left(f_j^1(x_{ij}) \times \eta_j^1\right), \ \sum_{j=1}^{m} \left(f_j^2(x_{ij}) \times \eta_j^2\right), \ \cdots, \ \sum_{j=1}^{m} \left(f_j^s(x_{ij}) \times \eta_j^s\right)\right). \end{aligned}$$

Step 7 Calculate the clustering coefficient matrix

$$\sum = \left[\sigma_i^k\right]_{n\times s} = \begin{bmatrix} \sigma_1^1 & \sigma_1^2 & \cdots & \sigma_1^s \\ \sigma_2^1 & \sigma_2^2 & \cdots & \sigma_2^s \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_n^1 & \sigma_n^2 & \cdots & \sigma_n^s \end{bmatrix}.$$

Step 8 Divide all objects into the individual grey class according to the clustering coefficient matrix and the following rule: object i belongs to class k^* if

$$\sigma_i^{k^*} = \max_{1\le k\le s}\left\{\sigma_i^k\right\}.$$

Step 9 Output the priority orders of objects according to the value of clustering coefficient.

Example Table 2.1 shows the statistical recordings of four measures about the 17 main strains of trees planted in China. Geographical measure is the geographical width of the region whose value is represented by the product of longitudes and latitudes; temperature measure describes the adaptability to different temperature conditions and its value is represented by the annual average temperatures of the latitudinal bounds; precipitation measure indicates the adaptability to precipitation conditions and its value is represented by the difference between the maximum and minimum annual average precipitation; arid measure indicates the adaptability to arid conditions and its value is represented by the difference between the maximum and minimum annual average aridities. Show how one can group the following 17 strains of tree into three grey classes: wide adaptability (having extremely strong ability to adapt to natural environments), medium adaptability, and narrow adaptability.

Solution:

Since the physical significance of the four attributes is different and the units corresponding to the observations are various, the fixed weight clustering method is employed.

Step 1 It can be seen that the number of subclasses is $s = 3$ and the possibility functions $f_j^k(\cdot)$ are respectively defined as follows:

$$f_1^1(x) = \begin{cases} 0, & x < 100 \\ \frac{x-100}{200}, & 100 \le x < 300 \\ 1, & x \ge 300 \end{cases}, f_2^1(x) = \begin{cases} 0, & x < 3 \\ \frac{x-3}{7}, & 3 \le x < 10 \\ 1, & x \ge 10 \end{cases},$$

$$f_3^1(x) = \begin{cases} 0, & x < 200 \\ \frac{x-200}{800}, & 200 \le x < 1000 \\ 1, & x \ge 1000 \end{cases}, f_4^1(x) = \begin{cases} 0, & x < 0.25 \\ x-0.25, & 0.25 \le x < 1.25 \\ 1, & x \ge 1.25 \end{cases},$$

Table 2.1 The four measures of the 17 main strains of trees in China

Index	Measure trees	Geo. eco measure	Temp. eco measure	Prec. Eco measure	Arid eco measure
1	Camphor pine	22.50	4.00	0.00	0.00
2	Korean pine	79.37	6.00	600.00	0.75
3	Northeast China ash	144.00	7.00	300.00	0.75
4	Diversiform-leaved poplar	300.00	6.10	189.00	12.00
5	Sacsaoul	456.00	12.00	250.00	12.00
6	Chinese pine	189.00	8.00	700.00	1.50
7	Oriental arborvitae	369.00	8.00	1300.00	2.25
8	White elm	1127.11	16.20	550.00	2.00
9	Dryland willow	260.00	11.00	600.00	1.00
10	Chinese white poplar	200.00	8.00	600.00	1.25
11	Oak	475.00	10.00	1000.00	0.75
12	Huashan pine	314.10	8.00	900.00	0.75
13	Masson pine	282.80	7.40	1300.00	0.50
14	China fir	240.00	8.00	1200.00	0.50
15	Bamboo	160.00	5.00	1000.00	0.25
16	Camphor tree	270.00	8.00	1200.00	0.25
17	Southern Asian pine	9.00	1.00	200.00	0.00

$$f_1^2(x) = \begin{cases} 0, & x < 50 \text{ or } x > 250 \\ \frac{x-50}{100}, & 50 \le x < 150 \\ \frac{250-x}{100}, & 150 \le x \le 250 \end{cases}, f_2^2(x) = \begin{cases} 0, & x < 2 \text{ or } x > 10 \\ \frac{x-2}{4}, & 2 \le x < 6 \\ \frac{10-x}{4}, & 6 \le x \le 10 \end{cases},$$

$$f_3^2(x) = \begin{cases} 0, & x < 100 \text{ or } x > 1100 \\ \frac{x-100}{500}, & 100 \le x < 600 \\ \frac{1100-x}{500}, & 600 \le x \le 1100 \end{cases}, f_4^2(x) = \begin{cases} 0, & x < 0 \text{ or } x > 1 \\ \frac{x-0.5}{0.5}, & 0 \le x < 0.5 \\ \frac{1-x}{0.5}, & 0.5 \le x \le 1 \end{cases},$$

$$f_1^3(x) = \begin{cases} 0, & x < 0 \text{ or } x > 100 \\ 1, & 0 \le x < 50 \\ \frac{100-x}{50}, & 50 \le x \le 100 \end{cases}, f_2^3(x) = \begin{cases} 0, & x < 0 \text{ or } x > 8 \\ 1, & 0 \le x < 4 \\ \frac{8-x}{4}, & 4 \le x \le 8 \end{cases},$$

$$f_3^3(x) = \begin{cases} 0, & x < 0 \text{ or } x > 600 \\ 1, & 0 \le x < 300 \\ \frac{600-x}{300}, & 300 \le x \le 600 \end{cases}, f_4^3(x) = \begin{cases} 0, & x < 0 \text{ or } x > 0.5 \\ 1, & 0 \le x < 0.25 \\ \frac{0.5-x}{0.25}, & 0.25 \le x \le 0.5 \end{cases}.$$

Step 2 Let the weights be

$$\eta_1 = 0.30, \eta_2 = 0.25, \eta_3 = 0.25, \eta_4 = 0.20,$$

where η_1, η_2, η_3 and η_4 are the weights for the geographical, temperature, precipitation, and aridity measures, respectively.

Step 3 Calculate the grey fixed weight clustering coefficient vectors

$$\sigma_1 = \left(\sigma_1^1, \sigma_1^2, \sigma_1^3\right) = (0.0357,\ 0.1250,\ 1.0000),$$

$$\sigma_2 = \left(\sigma_2^1, \sigma_2^2, \sigma_2^3\right) = (0.3321, 0.6881, 0.2488),$$

$$\sigma_3 = \left(\sigma_3^1, \sigma_3^2, \sigma_3^3\right) = (0.3401, 0.6695, 0.3125),$$

$$\sigma_4 = \left(\sigma_4^1, \sigma_4^2, \sigma_4^3\right) = (0.6107, 0.2883, 0.3688),$$

$$\sigma_5 = \left(\sigma_5^1, \sigma_5^2, \sigma_5^3\right) = (0.7656, 0.0750, 0.2500),$$

$$\sigma_6 = \left(\sigma_6^1, \sigma_6^2, \sigma_6^3\right) = (0.6683, 0.5080, 0.0000),$$

$$\sigma_7 = \left(\sigma_7^1, \sigma_7^2, \sigma_7^3\right) = (0.9286, 0.1250, 0.0000),$$

$$\sigma_8 = \left(\sigma_8^1, \sigma_8^2, \sigma_8^3\right) = (0.8594, 0.2250, 0.0417),$$

$$\sigma_9 = \left(\sigma_9^1, \sigma_9^2, \sigma_9^3\right) = (0.7650, 0.2500, 0.0000),$$

$$\sigma_{10} = \left(\sigma_{10}^1, \sigma_{10}^2, \sigma_{10}^3\right) = (0.6536, 0.5250, 0.0000),$$

$$\sigma_{11} = \left(\sigma_{11}^1, \sigma_{11}^2, \sigma_{11}^3\right) = (0.9000, 0.1500, 0.0000),$$

$$\sigma_{12} = \left(\sigma_{12}^1, \sigma_{12}^2, \sigma_{12}^3\right) = (0.7973, 0.3250, 0.0000),$$

$$\sigma_{13} = \left(\sigma_{13}^1, \sigma_{13}^2, \sigma_{13}^3\right) = (0.7313, 0.3625, 0.0375),$$

$$\sigma_{14} = \left(\sigma_{14}^1, \sigma_{14}^2, \sigma_{14}^3\right) = (0.6886, 0.3550, 0.0000),$$

$$\sigma_{15} = \left(\sigma_{15}^1, \sigma_{15}^2, \sigma_{15}^3\right) = (0.4114, 0.6075, 0.3875),$$

$$\sigma_{16} = \left(\sigma_{16}^1, \sigma_{16}^2, \sigma_{16}^3\right) = (0.6836, 0.2250, 0.2000),$$

$$\sigma_{17} = \left(\sigma_{17}^1, \sigma_{17}^2, \sigma_{17}^3\right) = (0.0000, 0.0500, 1.0000).$$

Step 4 From the following results,

$$\max_k\{\sigma_1^k\} = \sigma_1^3 = 1.0000, \max_k\{\sigma_2^k\} = \sigma_2^2 = 0.6881,$$
$$\max_k\{\sigma_3^k\} = \sigma_3^2 = 0.6695,$$

$$\max_k\{\sigma_4^k\} = \sigma_4^1 = 0.6107, \max_k\{\sigma_5^k\} = \sigma_5^1 = 0.7656,$$
$$\max_k\{\sigma_6^k\} = \sigma_6^1 = 0.6683,$$

$$\max_k\{\sigma_7^k\} = \sigma_7^1 = 0.9286, \max_k\{\sigma_8^k\} = \sigma_8^1 = 0.8594,$$
$$\max_k\{\sigma_9^k\} = \sigma_9^1 = 0.7650,$$

$$\max_k\{\sigma_{10}^k\} = \sigma_{10}^3 = 0.6536, \max_k\{\sigma_{11}^k\} = \sigma_{11}^1 = 0.9000,$$
$$\max_k\{\sigma_{12}^k\} = \sigma_{12}^1 = 0.9100,$$

$$\max_k\{\sigma_{13}^k\} = \sigma_{13}^1 = 0.8200, \max_k\{\sigma_{14}^k\} = \sigma_{14}^1 = 0.6886,$$
$$\max_k\{\sigma_{15}^k\} = \sigma_{15}^2 = 0.6075,$$

$$\max_k\{\sigma_{16}^k\} = \sigma_{16}^1 = 0.6836, \max_k\{\sigma_{17}^k\} = \sigma_{17}^3 = 1.0000.$$

It gives the cluster results (I) wide adaptability: diversiform-leaved poplars (no. 4), sacsaouls (no. 5), Chinese pines (no. 6), oriental arborvitaes (no. 7), white elms (no. 8), dryland willows (no. 9), Chinese white poplars (no. 10), oaks (no. 11), Huashan pines (no. 12), masson pines (no. 13), China firs (no. 14), and camphor trees (no. 16); (II) medium adaptability: Korean pine (no. 2), Northeast China Ash (no. 3), and bamboo (no. 15); (III) narrow adaptability: camphor pine (no. 1) and South Asian pine (no. 17).

2.5.3 Future Thinking About Grey Cluster Models

Grey cluster models construction mainly depends on definition of possibility function and weights, and aggregating form for different indices. While possibility function depends on expert's qualitative analysis about different index in real applications. Therefore, how to formulate a scientific possibility function should be further studied on, i.e., whether could decrease dependence of subjective information so as to improve clustering accuracy and efficiency should be further discussed.

2.6 Introduction to Grey Target Model

2.6.1 General Evolution of Grey Target Models

Depending on the actual circumstances and pre-determined goals to decide what actions should be taken is known as decision-making. Thus a basic characteristic of decision-making is to choose action or countermeasure. In general, the procedures of decision-making are consist of the following activities: posting questions, collecting data, establishing goals, designing schemes or actions, choosing schemes, implementing actions, and modifying schemes. But from the narrow point of view, decision-making only includes the steps that choose the specific action satisfying the established goals from the set of schemes.

The decision-making methods which involve grey elements or grey system models are simply called grey decision-making. It is obvious that grey decision-making is also an uncertain decision-making method in which the decision maker's prior knowledge and attitude toward risk have great influence on the decision-making results. In the following context, we are mainly focused on choosing a specific plan from the entire schemes.

2.6.2 Steps of Grey Target Model

Definition Events, countermeasures, objectives, and effects are four key elements of decision-making.

Definition The totality of all events within the range of a research is known as the set of events of the study, denoted as $A = \{a_1, \ a_2, \ \cdots, \ a_n\}$, where $a_i, i = 1, 2, \ldots, n$, denotes the ith event. The totality of all possible countermeasures is known as the set of countermeasures, denoted $B = \{b_1, \ b_2, \ \cdots, \ b_m\}$ with $b_j, j = 1, 2, \ldots, n$, be the jth countermeasure. The Cartesian product $S = A \times B = \{(a_i, b_j), a_i \in A, b_j \in B\}$ of the event set A and the countermeasure set B is known as the set of decision schemes, where each ordered pair $s_{ij} = (a_i, b_j)$ for any $a_i \in A, b_j \in B$, is known as a decision scheme.

Definition Let $S = A \times B = \{(a_i, b_j), a_i \in A, b_j \in B\}$ be a set of decision schemes, $u_{ij}^{(k)}$ the effect value of decision scheme s_{ij} with respect to objective k; and R the set of all real numbers. Then $u_{ij}^{(k)} : S \mapsto R$, defined by $s_{ij} \mapsto u_{ij}^{(k)}$ is known as the effect mapping of S with respect to object k.

If $u_{ij}^{(k)} = u_{ih}^{(k)}$, then we say that the countermeasures b_j and b_h of event a_i are equivalent with respect to objective k, written as $b_j \cong b_h$, and the set $B_i^{(k)} = \{b | b \in B, \ b \cong b_h\}$ is known as the effect equivalence class of countermeasure b_h of event a_i with respect to objective k.

Assume that k is such an objective that the greater the effect value is the better, if $u_{ij}^{(k)} > u_{ih}^{(k)}$, then we say that the countermeasures b_j is superior to b_h in terms of event a_i with respect to objective k, written as $b_j \succ b_h$, and the set $B_i^{(k)} = \{b|b \in B,\ b \succ b_h\}$ is known as the effect superior class of countermeasure b_h of event a_i with respect to objective k.

Similarly, the effect equivalence class of events of the countermeasure b_h with respect to objective k can be expressed as

$$A_{jh}^{(k)} = \{a|a \in A,\ a \cong a_i\},$$

where $a_i \cong a_j$ denotes $u_{ih}^{(k)} = u_{jh}^{(k)}$, and the effect superior class of events of the countermeasure b_h with respect to objective k can be expressed as

$$A_{jh}^{(k)} = \{a|a \in A,\ a \succ a_i\}$$

where $a_i \succ a_j$ denotes $u_{ij}^{(k)} > u_{ih}^{(k)}$.

The effect equivalence class of scheme under objective k is expressed as

$$S^{(k)} = \{s|s \in S,\ s \cong s_{hl}\},$$

where $s_{ij} \cong s_{hl}$ denotes $u_{ij}^{(k)} = u_{hl}^{(k)}$, and the effect superior class of scheme under objective k is expressed as

$$S^{(k)} = \{s|s \in S,\ s \succ s_{hl}\},$$

where $s_{ij} \succ s_{hl}$ denotes $u_{ij}^{(k)} > u_{hl}^{(k)}$.

Definition Assume that $d_L^{(k)}$ and $d_U^{(k)}$ are respectively the lower and upper threshold values of decision effects under objectives k, $k = 1, 2, \ldots, s$, then the following region of the s-dimensional Euclidean space

$$S^s = \left\{ \left(r^{(1)}, r^{(2)}, \ldots, r^{(s)}\right) | d_L^{(k)} \le r^{(k)} \le d_U^{(k)},\ k = 1, 2, \ldots, s \right\}$$

is known as a grey target of an s-dimensional decision-making. If the effect vector of scheme s_{ij} satisfies

$$u_{ij} = \left(u_{ij}^{(1)}, u_{ij}^{(2)}, \ldots, u_{ij}^{(s)}\right) \in S^s$$

where $u_{ij}^{(k)}$ stands for the effect value of the scheme s_{ij} with respect to objective k; then s_{ij} is a desirable scheme with respect to objectives 1, 2, …, s, and b_j is a desirable countermeasure of event a_i with respect to objectives 1, 2, …, s.

We may have an intuition that the grey targets indicate a satisfied solution in terms of relative optimization. In fact, although it is impossible to search the global optimal solution in some practical applications, the decision makers will also be satisfied if a satisfactory outcome is achieved. Certainly, the ultimate optimal effect can be achieved by shrinking the grey targets to a single point and the corresponding scheme is the most desirable one.

The following equation

$$R^s = \left\{ \left(r^{(1)}, r^{(2)}, \ldots, r^{(s)}\right) \Big| \sum_{i=1}^{s} \left(r^{(i)} - r_0^{(i)}\right)^2 \le R^2 \right\}$$

is known as an s-dimensional spherical grey target centered at $r_0 = \left(r_0^{(1)}, r_0^{(2)}, \ldots, r_0^{(s)}\right)$ with radius R, and the vector r_0 is seen as the optimum effect vector.

For $r_1 = \left(r_1^{(1)}, r_1^{(2)}, \ldots, r_1^{(s)}\right) \in R$,

$$|r_1 - r_0| = \sqrt{\sum_{i=1}^{s} \left(r_1^{(i)} - r_0^{(i)}\right)^2}$$

is known as the bull's-eye distance of vector r_1. This distance reflects the superiority of the corresponding decision effect vectors.

Let $u_{ij} = \left(u_{ij}^{(1)}, u_{ij}^{(2)}, \ldots, u_{ij}^{(s)}\right)$ and $u_{hl} = \left(u_{hl}^{(1)}, u_{hl}^{(2)}, \ldots, u_{hl}^{(s)}\right)$ be the effect vectors corresponding to two different schemes s_{ij} and s_{hl}, if

$$\left|u_{ij} - r_0\right| \ge |u_{hl} - r_0|,$$

then scheme s_{hl} is said to be superior to s_{ij}, that is, $s_{ij} \succ s_{hl}$, especially, when the equal sign holds true, $s_{ij} \cong s_{hl}$.

If $u_{ij} \ne r_0$ always holds true for $i = 1, 2, \ldots, n$ and $j = 1, 2, \ldots, m$, then there does not exist an optimum scheme, and the event has no optimum countermeasure. However, if the optimum scheme does not exist, then there are h and l such that $|u_{hl} - r_0| \le \left|u_{ij} - r_0\right|$ holds true for $i = 1, 2, \ldots, n$ and $j = 1, 2, \ldots, m$, i.e., $s_{hl} \succ s_{ij}$ holds for any $s_{ij} \in S$, then s_{hl} is known as a quasi-optimum scheme.

Let $S = A \times B = \left\{\left(a_i, b_j\right), a_i \in A, b_j \in B\right\}$ be a set of decision schemes, and R^s an s-dimensional spherical grey target, and then S becomes an ordered set with "superiority" as its order relation $\succ$.

According to Zorn's Lemma in set theory, it is easy to prove that there exists a quasi-optimum scheme in the set of decision schemes of $(S, \succ)$.

Finally, the computation steps of grey target decision-making model can be summarized as follows, and we demonstrate it by an empirical example.

Example Consider an event a_1 that reconstruct an old building using the following three approaches: b_1 = completely renovate; b_2 = tear down and reconstruct another;

and $b_3 =$ simply fix up minor problems. The three objectives are cost, functionality, and construction speed.

Solution:

Step 1 Denote the cost as objective 1, the functionality as objective 2, and the construction speed as objective 2. Then, we have the following three decision schemes:

$$s_{11} = (a_1, b_1) = (\text{reconstruction, renovation}),$$

$$s_{12} = (a_1, b_2) = (\text{reconstruction, new building}),$$

$$s_{13} = (a_1, b_3) = (\text{reconstruction, maintenance}).$$

Step 2 For functionality and speed, the higher values are the better, but for cost, a lower value is the better. Therefore, the effect vectors corresponding to all the three decision schemes can be defined as follows:

$$u_{11} = \left(u_{11}^{(1)}, u_{11}^{(2)}, u_{11}^{(3)}\right) = (2.00,\ 2.00,\ 2.00),$$

$$u_{12} = \left(u_{12}^{(1)}, u_{12}^{(2)}, u_{12}^{(3)}\right) = (3.00,\ 1.00,\ 3.00),$$

$$u_{13} = \left(u_{13}^{(1)}, u_{13}^{(2)}, u_{13}^{(3)}\right) = (1.00,\ 3.00,\ 1.00).$$

Step 3 Let the bull's eye locate at $r_0 = (1, 1, 1)$ and compute the bull's-eye distances

$$|u_{11} - r_0| = \sqrt{\left(u_{11}^{(1)} - r_0^{(1)}\right)^2 + \left(u_{11}^{(2)} - r_0^{(2)}\right)^2 + \left(u_{11}^{(3)} - r_0^{(3)}\right)^2} = 1.73,$$

$$|u_{12} - r_0| = \sqrt{\left(u_{12}^{(1)} - r_0^{(1)}\right)^2 + \left(u_{12}^{(2)} - r_0^{(2)}\right)^2 + \left(u_{12}^{(3)} - r_0^{(3)}\right)^2} = 2.83,$$

$$|u_{13} - r_0| = \sqrt{\left(u_{13}^{(1)} - r_0^{(1)}\right)^2 + \left(u_{13}^{(2)} - r_0^{(2)}\right)^2 + \left(u_{13}^{(3)} - r_0^{(3)}\right)^2} = 2.00.$$

Step 4 Compare the bull's-eye distances

$$|u_{11} - r_0| < |u_{13} - r_0| < |u_{12} - r_0|.$$

Step 5 Since $|u_{11} - r_0|$ is the smallest, the effect vector $u_{11} = (2, 2, 2)$ of the decision scheme s_{11} enters the grey target. Hence, renovation is a satisfactory decision.

2.7 Conclusions

Grey system models have been widely used in various fields. This chapter proposed the new framework of a series of grey system models so as to unify different kinds of grey models into a common system. Due to the developing of grey system theory framework, all of grey system models were divided into two types, i.e. limited data based grey system models and grey number based grey system models. Then, four types of basic grey system models, i.e. grey forecasting models, grey relational models, grey cluster models and grey target models were further detailed discussed and general steps of these models were given. In addition, future developing directions of these grey system models were further proposed.

As a general introduction of basic grey system models, a great deal of novel grey system models was not included in this chapter. Reader could search and read new models in international journal accordingly. Of course, brand new grey system models could be further proposed.

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Chapter 3
Grey Systems and Uncertainty Modelling

Y. Yang, A. S. Khuman, and S. Liu

3.1 Introduction

Information can, and often is, rather uncertain; with only partial information initially being made available, from which one would be able to *hopefully* provide for a solution. The information itself may contain conflicts that have arisen from the possible different sources used to acquire it. In addition, the information may be viewed and interpreted differently by different cohorts, this in itself can be the cause of extenuating circumstances. These are just some of the issues that one can face with *uncertain* information. These issues can understandably create problems when considering the deployment of applications. Being able to cater for the volatility that is inherently present in uncertainty, becomes an objective with high importance and precedence. This is especially so when considering the domain of complex systems, such as intelligent systems that are designed for the Internet of Things (IOT), and also Complex Adaptive Systems (CAS). With the emergence of *Big Data* and the sheer volume of it, the problem of managing uncertain information is not necessarily improved upon; the addition of information does not naturally improve the quality or reduce the existing uncertainty. This creates ever more complex and vague concepts which in turn involves even more uncertainty, becoming somewhat paradoxical. Because of this, uncertainty modelling is constantly evolving, with its prime purpose being to address uncertain information; employing the use of uncertainty models to obtain understandable conclusions from what limited, incomplete and conflicting information is available.

Y. Yang (✉) · A. S. Khuman
Institute of Artificial Intelligence, De Montfort University, The Gateway, Leicester LE1 9BH, UK
e-mail: yyang@dmu.ac.uk

S. Liu
Institute of Grey Systems Studies, Nanjing University of Aeronautics and Astronautics, Nanjing, China

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The motivation of this chapter comes from the confusion on the differences between grey systems and other models. This chapter serves to quash some of the misunderstanding associated with grey systems. We try to give a concise understanding of what actually sets it apart from other related concepts. In some literature, grey systems are confused with interval-valued fuzzy sets. There is partial overlap between grey systems and fuzzy sets, but not all. A clear review on this topic will help to understand their similarity and differences.

Out of necessity, there have been many proposed models to handle uncertainty, such as; probability theory, fuzzy sets, rough sets and grey sets. As is often the case, the deployment of a specific uncertainty model will generally only consider the area of application for which it was created. There have been efforts made in hybridising combinations of complementary models, such as; fuzzy probability theory, fuzzy-rough, rough-fuzzy, and the like. And as such, these models are typically deployed for specific niches. With regards to grey systems theory, systems can be categorised as either being absolutely known (white), partially known (grey) or absolutely unknown (black), with the main focus being concerned with partially known systems and information. When compared with other approaches, grey systems theory gives more attention to domains concerned with limited data and incomplete information. Therefore, there is a natural harmonic pairing when considering the nature of grey systems theory to that of typical uncertainty based problems, as the information available will often be limited and incomplete. The applicability of grey systems has garnered a lot of popularity as of late, although to the uninformed, this can at times be the cause of confusion, as there are similarities to that of existing approaches. Therefore, there is indeed a need to clarify the relationships that exist between existing models to that of grey systems. In doing so, one will be able to uncover the divergences that set grey systems apart from said models.

In this chapter, reviewing our previous works on grey sets and greyness (Yang & John, 2012), we will describe and discuss the similarities and the differences that exist between the various uncertainty models. This chapter predominately focuses on providing the reader with a detailed understating of the relationships between grey sets and other uncertainty models, such as fuzzy sets and rough sets, as well as the role of grey systems in data analytics.

3.2 Uncertainties and Uncertainty Models

As is the nature of uncertainty, there can be many factors and facets involved which can be associated to the same initial object, at the same time. If one considers each facet as a *variation* of uncertainty, there can then be multiple instances of known uncertainties, these can combine in a combinatorial way to then create emergent uncertainties. Depending on the perspective and angle of consideration one wishes to use; these uncertainties can be categorised. The notion of objective uncertainties should be understood as being caused by objective sources, typically incomplete data sources. Randomness can be seen as an instance of objective uncertainty, as such,

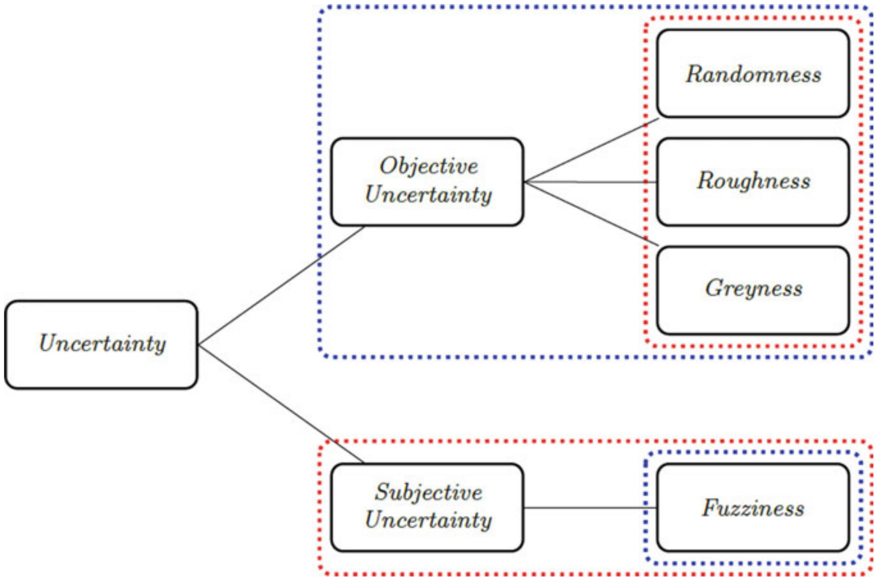


Fig. 3.1 Facets of uncertainty

incompleteness and roughness would, and do belong to the category of objective uncertainties. The complement to this would be that of subjective uncertainties, that derive their uncertainty from human based understanding and reasoning, such as abstract linguistic notions; '*Tall*', '*Warm*', and the like. It can be seen that fuzziness is typically associated to subjective uncertainties, as it is often difficult to universally define meaning and intent when considering human language. To surmise, objective uncertainties are mainly concerned with objects, whereas subjective uncertainties are generally concerned with perception and understanding from humanistic interpretation. Figure 3.1 provides an illustrated classification where it can be seen that greyness can be associated to that of objective uncertainty.

Acknowledging that these types of uncertainties exist creates challenges with regards to decision-making and planning. As a result, there have been many models proposed to deal with these types of uncertainties, such as; statistical models, fuzzy sets, rough sets and grey systems. Statistical based models like that of probability work well for randomness; rough sets describe the *roughness*, and grey systems convey the *greyness*, or incompleteness. These variations of uncertainties all belong to the objective category of uncertainty, as indicated by Fig. 3.1.

For subjective uncertainties, fuzziness is well suited in that it is able to capture the subjective nature of perception based domains: fuzziness. Understanding that there exists the notion of objective and subjective uncertainties, there have been several proposed integrations between objective and subjective models; e.g. interval fuzzy sets/intuitionistic fuzzy sets (Cornelis et al., 2003), fuzzy rough models (Dubois & Prade, 1990), R-fuzzy sets (Yang & Hinde, 2010), etc. It is noteworthy to mention that

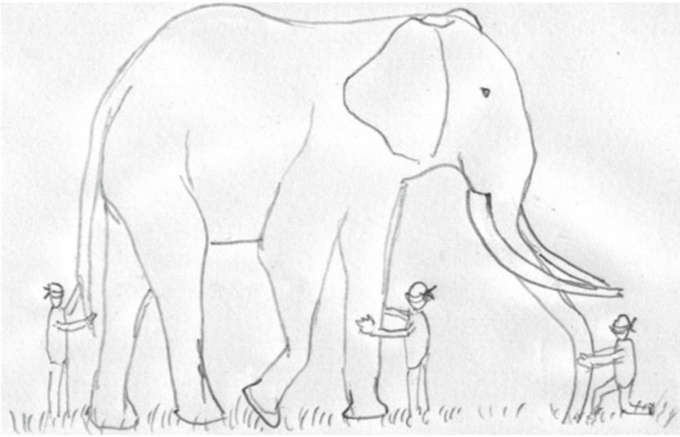


Fig. 3.2 Blind men and the elephant analogy to apply when considering uncertainty models

the aforementioned models are relatively independent from one another, distinctly identifiable, each having its own specialised area of application.

Consider the image depicted in Fig. 3.2, regarding the blind men and the elephant. The parable itself considers a group of blind men, whom have never come across an elephant before. Each blind man feels and touches a single, but different part of the elephant, such as the trunk, tail, tusk and so on. Based on their own relative subjectiveness, they then describe to the others what they are experiencing. As they are describing their subjective understanding of each part they are in contact with, their descriptions of the *elephant* are incomplete and only partial. One of them may be touching a leg and describe it as a tree, another may be touching the tail and describe it as a rope, and so on. The moral of the parable is that humans have a tendency to project their partial experiences as the whole truth, while ignoring the partial experiences of others, implying that one may be partially right and may have partial information. Touching a part without context of the whole can, and often does, provide for a false overall understanding.

The *elephant* is the uncertainty, and each *blind man* is an uncertainty model, describing their understanding from their perspective. Each model will therefore see a variation of the uncertainty being presented, but none of them will be able to see or describe the *whole*. If a new blind man comes along and touches a new part of the elephant, a new model of uncertainty will be created. Only when one connects the various interpretations, does one get the comprehensive whole view of the elephant.

3.3 Fuzzy Sets and Rough Sets

The concept of a fuzzy set was pioneered by Zadeh (1965). In classical set theory, an object can be absolutely described as belonging to the set or not. Applying this crisply, clearly defined boundary to more abstract notions will prove difficult and ineffective. For example, consider the height values that would fall in between *Tall* and *Not Tall*, these values could not simply be categorised as one or the other, in which case one should consider a fuzzy set to describe such instances.

Considering a set A defined in a universe U, if we associate a number $0 \leq \mu_A(x) \leq 1$ with each object x in A to represent its degree of belonging to A, then we have a fuzzy set (Zadeh, 1965).

$$A = \{\langle x, \mu_A(x)\rangle : x \in U\} \tag{3.1}$$

where μ_A: U → [0, 1] is called membership function of A.

The membership function value (Klir & Yuan, 2002) is a real number within [0, 1], which expresses a fuzzy concept or graded set boundary (Dubois & Prade, 1997). A fuzzy set can regress back to a classical set when considering the connotation of the membership value. A returned degree of membership of 1 implies that the object is definitely included in the set, whereas a returned degree of membership of 0 implies that the object has absolutely *no* association to the set. The closer the membership value is to 1, the more *included* the object is to the set. This fuzzy perspective is more indicative of humanistic nature, when compared to the strict and stringent interpretation of a classical set theory approach.

There may indeed be instances where the fuzzy membership value itself is made up of multiple values rather than a single value. In such instances, the fuzzy membership values contained in the membership interval are all valid and legal values to be considered. If we replace the single value membership $\mu_A(x)$ in Eq. 3.1 with an interval $M_A(x) \subseteq D[0, 1]$, then the fuzzy set A becomes an interval-valued fuzzy set (Sambuc, 1975).

$$A = \{\langle x, M_A(x)\rangle : x \in U\} \tag{3.2}$$

with M_A: U → D[0, 1].

Another way to look at fuzzy sets is to define it using two memberships rather than one: the membership $\mu_A(x)$ for object x to belong to the set A and the membership $\nu_A(x)$ for object x to be excluded from the set A. Then the fuzzy set is turned into an intuitionistic fuzzy set (Antanassov, 1999)

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in U\} \tag{3.3}$$

where μ_A: U → [0, 1], ν_A: U → [0, 1] and $0 \leq \mu_A + \nu_A \leq 1 \ \forall x \in U$.

It is noteworthy to mention that an intuitionistic fuzzy set (Antanassov, 1999) is mathematically equivalent to an interval-valued fuzzy set. However, their semantic meanings are different (Beliakova et al., 2011; Burillo & Bustince, 1996; Bustince & Burillo, 1996; Cornelis et al., 2003; Deschrijver & Kerre, 2003; Dubois et al., 2000; Wang & He, 2000). All these various extensions are in fact multi-sets, where an object can have more than one valid membership value. Allowing for additional fuzzy membership values to be contained, makes these models distinctly different from traditional type-1 fuzzy sets defined in Eq. (3.1).

Fuzzy sets describe fuzzy concepts using fuzzy memberships, which introduces an intermediate class between the two extremes, which has association with both sides but does not completely overlap with any one side in particular. A rough set, however, defines an object in its approximation space. Rough sets were first proposed in 1982 by Pawlak (1982).

There are several different interpretations of rough sets (Banerjee & Chakraborty, 2004; Iwinski, 1987; Pawlak, 1991; Wybraniec-Skardowska, 1989; Yao, 1996, 1998b; Yao et al., 1995; Zakowski, 1983). Based on the set-oriented interpretation of rough sets (Iwinski, 1987; Mousavi & Jabedar-Maralani, 2002; Pawlak, 1982, 1991; Rasouli & Davvaz, 2010; Yang & John, 2006a, 2006b; Yao, 1996, 1998a), a rough set can be defined by a pair of crisply definable sets ($\underline{A}$, $\overline{A}$). Here, $\underline{A}$, $\overline{A} \in Def\ (apr)$ and $\underline{A} \subseteq \overline{A}$, $Def\ (apr)$ is the power set of all definable sets in $apr = (U, B)$, and B is an equivalence relation on U.

Here, set A is approximated by a pair ($\underline{A}$, $\overline{A}$), and $\underline{A} \subseteq A \subseteq \overline{A}$. There are two distinct approximations; $\underline{A}$ is referred to as the lower approximation of A, while $\overline{A}$ is referred to as the upper approximation of A. It is clear to see that $\underline{A}$ is a subset of $\overline{A}$.

Both rough and fuzzy sets provide a means to capture two very distinct forms of uncertainty; indiscernibility and vagueness (Dubois & Prade, 1990). In addition to this, we also have the concept of shadowed sets (Pedrycz, 1998) with a close relationship with both fuzzy sets and rough sets. For a fuzzy set $A \in \mathrm{U}$, the membership values which are relatively close to absolute inclusion are elevated to a 1 for its degree of membership, and reduce the degrees of membership for values that are close to 0, to absolute non-inclusion. The shadowed region [0, 1], captures all other values, without applying a specific membership to them, being contained in the shadow region is sufficient enough without having to know to what degree. The *shadowed set* (Pedrycz, 1998) is given as follows:

$$A : U \rightarrow \{0, 1, [0, 1]\} \tag{3.4}$$

Here, each object x in the set is connected with 0, 1 or the interval [0, 1]. The objects whose $A(x)$ is 1 constitutes its core, and the objects whose $A(x)$ is [0, 1] produce a shadow region with uncertainty. Shadowed sets partition the objects of a fuzzy set into three different groups: *Yes* ($\mathrm{A(x)} = 1$), *No* ($\mathrm{A(x)} = 0$) and *Unknown* ($\mathrm{A(x)} = [0, 1]$). Using this understanding one can see the similarities to that to rough set theory, inclusion, non-inclusion and inclusion to the rough approximation.

There are many uncertainties involved in real world applications that make it difficult to determine the crisp values of the membership functions of a typical type-1 fuzzy set (Mendel & John, 2002). Not only do they involve the vagueness, which can be categorised as the absence of sharp definable crisp boundaries, they also involve ambiguity, the incomplete information and inexactness. As a result of these shortcomings, there have been several extensions proposed to overcome this issue; such as Atanassov intuitionistic fuzzy sets (Antanassov, 1999), interval-valued fuzzy sets (Sambuc, 1975), shadowed sets (Pedrycz, 1998). These uncertainty models adopt multiple parameters, interval values and additional fuzzy sets to represent the undetermined membership functions of the involved fuzzy sets. However, the multiple parameters and intervals involved with regards to Atanassov intuitionistic fuzzy sets, interval-valued fuzzy sets and shadowed sets cannot differentiate between values within the same interval or shadowed region. Once a value falls within the interval or shadowed zone, they are indiscernible and their identity lost. The model of rough sets facilitates a different option in expressing the uncertainty of sets, and the hybridisation between them has received extensive research (Bodjanova, 2007; Deng et al., 2007; Dubois & Prade, 1990, 1995; Huynh & Nakamori, 2005; Iwinski, 1987; Jensen & Shen, 2004; Jiang et al., 2005; Komorowski et al., 1999; Nanda & Majumdar, 1992; Obtulowicz, 1987; Pawlak & Skowron, 2007; Pedrycz, 1998; Radzikowska & Kerre, 2002; Salido & Murakami, 2003; Wiweger, 1989; Wu et al., 2003; Yang, 2007a; Yao, 1996, 1997). However, most of these variations focus on the fuzzy equivalence relations and therefore cannot solve the issue of multi-valued approximation. To solve this problem, we proposed the notion of R-Fuzzy sets, created in 2010 (Yang & Hinde, 2010), as a means to allow for uncertain fuzzy membership values to be easily identified when considering upper and lower approximations.

$apr = (J_x, B)$ is an approximation space on a set of values $Jx = \{v_1, v_2, ..., v_n\} \subseteq [0, 1]$ and let J_x/B be the set of all equivalence classes of B. $Def\ (apr)$ is the powerset of all definable sets in apr.$\underline{M}$, $\overline{M} \in Def\ (apr)$ are two given subsets with $\underline{M} \subseteq \overline{M}$. According to our description about rough sets, $(\underline{M}, \overline{M})$ is a rough set. In this way, a set whose exact boundary cannot be precisely identified can be approximated with a rough set using Jx/B, then we have the so called R-Fuzzy sets (Yang & Hinde, 2010):

$$A = \{\langle x, \left(\underline{M_A}(x), \overline{M_A}(x)\right)\rangle | \forall x \in U, \underline{M_A}(x) \subseteq \overline{M_A}(x)\} \subseteq J_x\} \tag{3.5}$$

Similar to fuzzy sets, we can express A as:

$$A = \sum_{x \in U} (\underline{M_A}(x), \overline{M_A}(x))/x \tag{3.6}$$

where $\sum$ indicates union over all objects.

Similar to that of interval-valued fuzzy sets, an R-fuzzy set defines its membership as a set, more specifically a rough set. This rough set contains the values satisfying its descriptor. To this end, there are usually multiple values for the membership values of an R-fuzzy set even if a single value is also possible and completely acceptable. The lower approximation can be seen to contain all values with absolute affinity to the descriptor, whereas the upper approximation will indicate that the specified uncertain fuzzy membership values have some affinity to the descriptor, but definitely not absolution.

It is perfectly plausible to assume that individuals may have differing perceptions from one another when considering the same observation. This difference may be significant for decision-making in some areas, but most models simply ignore it and simplify it into a single group membership value. An R-fuzzy set represents its membership function values using rough sets. It reserves not only the diverse opinion in the same group but also differentiate it into two different groups in the lower and upper approximations. The use of rough approximations provides the functionality to allow for an R-fuzzy approach to model certain situations which would be difficult to undertake using other types of fuzzy sets.

The foundation of the R-fuzzy approach was extended by the works of Khuman, Yang and John by the introduction of a significance measure (Khuman et al., 2015, 2016c). This provided one the means to acquire the conditional probability for every contained uncertain fuzzy membership value for all generated R-fuzzy sets. This allowed for one to inspect the significance of each contained membership value to garner its relative impact on the set, from which the value could be promoted to the lower approximation or demoted to non-inclusion. This was then extended further still by the hybridisation of R-fuzzy and the significance pairing to that of grey analysis (Khuman et al., 2016a, 2016b). This effectively created an equivalence to that of generalised type-2 fuzzy sets, albeit from a R-fuzzy perspective.

3.4 Greyness and Grey Sets

The last chapter has given the definition of a grey number. Now that a grey number has many candidates, its uncertainty will certainly be influenced by its candidate set. Here, we will discuss the measurement of the uncertainty associated with a grey number. The degree of greyness of a grey number is taken to measure the existence of uncertainty in a grey number. There are many available definitions on the degree of greyness for a grey number. Here, we give three available calculations for the degree of greyness of a grey number (Liu et al., 2000, 2012; Yang & John, 2012).

$$g_1^\circ\left(g^\pm\right) = \frac{\mu(g^\pm)}{\mu(\Omega)} \tag{3.7}$$

$$g_2^\circ\left(g^\pm\right) = \frac{|g^+ - g^-|}{|d_{max} - d_{min}|} \tag{3.8}$$

$$g_3^\circ\left(g^\pm\right) = \frac{1}{\hat{g}}\sum_{i=1}^{n} \widehat{a_i}\,\frac{\mu(a_i^\pm)}{\mu(\Omega)} \tag{3.9}$$

$\Omega \subset R$ is the universe and $g^\pm$ is a grey number and $g^\pm \in \bigcup_{i=1}^{n}\left[a_i^-, a_i^+\right] \subseteq \Omega$. $d_{min}, d_{max} \in \Omega$ are the minimum and maximum values of $g^\pm \in \Omega$, and μ is a measurement on Ω. $a_i^\pm$ denotes the component $[a_i^-, a_i^+]$, $\hat{g}$ and $\widehat{a_i}$ represent the mathematical expectation of $g^\pm$ and $a_i^\pm$.

Here, $g_2^\circ\left(g^\pm\right)$ focuses on the gap between the two boundaries whereas $g_3^\circ\left(g^\pm\right)$ stresses more on the components of a grey number. One can see that g_1°, g_2° and g_3° satisfy the following properties

- $g_i^\circ = 0$ if $g_i^+ = g_i^-$ for i = 1, 2, 3

- $g_i^\circ = 1$ if $n = 1$ and $\left[a_i^-, a_i^+\right] = \Omega$ for i = 1, 2, 3
- if $g^\pm$ is a continuous grey number, then $g_1^\circ = g_2^\circ = g_3^\circ$

If a grey number is represented as a continuous interval, there is no clear difference among the three forms. When more than one component is involved, they would be different. g_1° and g_2° could be different but g_2° remains constant if no change with the upper and lower limits. Here, we will use g_2° hereafter.

In a similar way as interval algebra (Yao, 1993), arithmetic operations between grey numbers are conducted through their components. Considering $a^\pm$ and $b^\pm$ as grey numbers, and $*$ represents an arithmetic operation $+, -, \times$ or $\div$. The arithmetic operation $*$ of grey numbers is expressed as:

$$a^\pm * b^\pm = \{x * y | x \in \bigcup_{i=1}^{m}[a_i^-, a_i^+], y \in \bigcup_{j=1}^{n}[b_j^-, b_j^+]\}$$

Obviously, $a^\pm * b^\pm$ is a grey number if $0 \notin \bigcup_{j=1}^{n}[b_j^-, b_j^+]$.

The following two formulas can be easily derived for $a^\pm * b^\pm$ (Yang, 2007b):

$$a^\pm + b^\pm = \bigcup_{i=1}^{m}\bigcup_{j=1}^{n}[a_i^- + b_j^-, a_i^+ + b_j^+] \tag{3.10}$$

$$a^\pm - b^\pm = \bigcup_{i=1}^{m}\bigcup_{j=1}^{n}[a_i^- - b_j^+, a_i^+ - b_j^-] \tag{3.11}$$

$$a^\pm \times b^\pm = \bigcup_{i=1}^{m}\bigcup_{j=1}^{n}\left[\min\left\{a_i^- b_j^-, a_i^- b_j^+, a_i^+ b_j^-, a_i^+ b_j^+\right\}, \max\{a_i^- b_j^-, a_i^- b_j^+, a_i^+ b_j^-, a_i^+ b_j^+\}\right] \tag{3.12}$$

$$a^\pm \div b^\pm = \bigcup_{i=1}^{m}\bigcup_{j=1}^{n}\left[\min\left\{\frac{a_i^-}{b_j^-}, \frac{a_i^+}{b_j^+}, \frac{a_i^-}{b_j^+}, \frac{a_i^+}{b_j^-}\right\}, \max\left\{\frac{a_i^-}{b_j^-}, \frac{a_i^+}{b_j^+}, \frac{a_i^-}{b_j^+}, \frac{a_i^+}{b_j^-}\right\}\right] \tag{3.13}$$

Here, we assume $a_i^- \leq a_i^+$ and $b_j^- \leq b_j^+$. For the division operation, we assume $b_j^- \neq 0$ and $b_j^+ \neq 0$.

Example 3.1 A new engine is to be tested for 5 h in a given date. The test has to be started in the morning either on 9:00 am or 10:00 am. It will run exactly 5 h. The total fuel consumption is evaluated as a value between 50 and 100 L.

Considering the starting time, we get two possible starting time lots: 9:00 am and 10:00 am. The actual starting time must be one of the two candidates, we can express it as a discrete grey number: {9, 10}. The duration is fixed and clear, so we can represent it as a white number 5 (hours). The fuel consumption is unknown and can be any value between 50 and 100 L, so we need a continuous grey number to represent it: [50, 150]. Here, the discrete grey number for starting time and the continuous grey number for fuel consumption represent two single values although they are represented by their candidate set. {9, 10} indicates that the test can only be started in the two optional time, and therefore there is only one starting time. If 9:00 am turned out to be true, then 10:00 am would be false. They cannot be true in the same time. Similar conclusion holds for the fuel consumption [50, 150], however, the possible candidates are much more than 2 in this case: any number between 50 and 150 could turn out to be true although only one of them could be true in the end. If we replaced {9, 10} with [9, 10], then any time between them would be possible, such as 9:30, which disregards the constraints of the example.

Clearly, a grey number has significant potential in representing information incompleteness as a complement to other models. For instance, in database applications, a single white value is required for each attribute of a column. When a value is missing, it is replaced with a null (Codd, 1970), an interval (Lipski, 1981) or a rough set (Sakai, 2002). It is well known that an interval or a rough set can be a multi-set and they do not necessarily mean a single value. A grey number is clearly a better choice under such a situation.

For set A defined on a universe U, a characteristic function serves to identify if an object x is included in A or not. The membership function of a fuzzy set discussed before is a typical characteristic function. As with numbers, sets can be classified into three different groups (Yang & John, 2012) according to the values of their characteristic functions. If the value of the characteristic function is always a single white number, then A is a white set (Yang & John, 2012). As aforementioned, a type-1 fuzzy set is actually a special case of a white set. The reason is that its fuzzy membership value is a single white number. When this value takes its extreme value 0 or 1, then the set turns into a crisp set. On the other hand, if the characteristic function has no known value and it can only be described by a black number, then A is a black set (Yang & John, 2012). In the situation of black sets, there is nothing known about the characteristic function values of the set. It has to be represented with the black number, this is completely opposite to the white sets. Between the white and black sets, a set may have partial information for its characteristic function and its values could be expressed as a grey number, then we have a grey set (Yang & John, 2012). More specifically, a grey set has $\chi_A : U \to D[0, 1]^{\pm}$ ($D[0, 1]^{\pm}$ is the power set of all grey numbers within [0, 1]) as its characteristic function, and it has a grey number as its value: $g_A^{\pm}(x) \in \bigcup_{i=1}^{n} \left[a_i^-, a_i^+\right] \in D[0, 1]^{\pm}$.

A grey set A can be expressed in the form of its relevant objects and their related grey characteristic function values:

$$A = g_A^{\pm}(x_1)/x_1 + g_A^{\pm}(x_2)/x_2 + \cdots + g_A^{\pm}(x_n)/x_n$$

From this definition, it is clear that there is no restriction on the characteristic function. When the characteristic function produces a single white number, we have a white set. In this case, this white set could be a crisp set, but it is not essential. For example, a type-1 fuzzy membership value is also a single white number, and it is still a white set but not a crisp set.

Example 3.2 Based on the previous example concerning an engine test, we now consider 3 engines to be tested: a, b and c. Each engine has 1–5 indicators to be measured during the test. Each engine has a fuel allowance of 150 L for the duration of the test. Therefore, the possible tests are:

- a: 1 or 2 indicators
- b: 2–4 indicators
- c: 5 indicators

Here, the fuel consumption for each indicator could be any value between 20 and 30 L. We have two sets: *A* for the completeness of the test for the 5 indicators, and *B* for the fuel consumption during their test. Their characteristic functions are defined as:

$$f_A = \frac{n}{5} \quad f_B = \frac{l}{150} \tag{3.14}$$

where n is the number of indicators tested for an engine, and l is the quantity of fuel consumed by the engine during the test. *A* and *B* can be expressed as two grey sets. Following the definitions of f_A and f_B, the respective characteristic function value can be derived:

$$\begin{aligned}
f_A(a) &= \left\{\frac{1}{5}, \frac{2}{5}\right\}, f_B(a) = \left[\frac{2}{15}, \frac{2}{5}\right] \\
f_A(b) &= \left\{\frac{2}{5}, \frac{3}{5}, \frac{4}{5}\right\}, f_B(b) = \left[\frac{4}{15}, \frac{4}{5}\right] \\
f_A(c) &= \{1\}, f_B(b) = \left[\frac{2}{3}, 1\right]
\end{aligned}$$

It can be seen that the sets *A* and *B* have two different kind of characteristic function values, where traditional approaches would be rather long winded, the notion of a grey set can easily cater for this requirement.

Example 3.3 Table 3.1 contains the information of five property agents; their names, the level of their service charge, the quality of the service and their profit are listed. The set *A* for evaluating the performance of these property agents is to be derived from these attributes. Assume that a_i is a property agent in the table, and $i = 1, 2, 3, ..., n$, where n is the number of property agents in concern. We can directly define *A* from the *Profit* attribute, and indirectly using *Charge* and *Service Quality*. The *Service Quality* is adopted to setup a grey set.

The *Profit* attribute shows there to be a relationship with *Service Quality*. A characteristic function can be established according to the relationship between *Service Quality* and *Profit*:

Table 3.1 Student information

Name	Charge	Service quality	Profit
Agent1	High	Good	Good
Agent2	Low	Neutral	Good
Agent3	Low	Neutral	Neutral
Agent4	High	Neutral	Poor
Agent5	Low	Poor	Poor

$$f_A^c(a_i) = \begin{cases} 1 & if\ a_i\ Service\ Quality = Good \\ [0, 1] & if\ a_i\ Service\ Quality = Neural \\ 0 & if\ a_i\ Service\ Quality = Poor \end{cases}$$

Under this characteristic function:

$$\begin{aligned} \mathrm{A} &= \frac{[1, 1]}{Agent1} + \frac{[0, 1]}{Agent2} + \frac{[0, 1]}{Agent3} + \frac{[0, 1]}{Agent4} + \frac{[0, 0]}{Agent5} \\ &= \frac{1}{Agent1} + \frac{[0, 1]}{Agent2} + \frac{[0, 1]}{Agent3} + \frac{[0, 1]}{Agent4} + \frac{0}{Agent5} \end{aligned}$$

It can be clearly seen that A is indeed a grey set. Using *Service Quality*, it is not possible to uniquely determine the *Profit* of an agent, hence there are some ill-defined relationship between *Service Quality* and *Profit* which is expressed by a grey number.

The example shows that two different categories of property agents can be identified in the grey set from the values of their characteristic functions: white numbers or grey numbers. Each object in a grey set has therefore an associate degree of greyness which is the degree of greyness of their associated grey number (Yang & John, 2012).

$$g_A^\circ(x) = |g^+ - g^-| \tag{3.15}$$

With the degree of greyness of each object in the set, then a degree of greyness g_A° for the set A can also be derived.

$$g_A^\circ = \frac{\sum_{i=1}^n g_A^\circ(x_i)}{n} \tag{3.16}$$

where i = 1, 2, 3, ..., n and n is the cardinality of U.

In this way, the incompleteness for the evaluation of property agents in the last example can be represented using the degree of greyness.

Example 3.4 Based on the concept of degree of greyness, the uncertainty for the performance evaluation of Agent1, Agent2 and Agent5 in the previous example can be derived:

$g^\circ(\text{Agent1}) = 1 - 1 = 0.$
$g^\circ(\text{Agent2}) = 1 - 0 = 1.$
$g^\circ(\text{Agent5}) = 0 - 0 = 0.$

Table 3.2 Example for degree of greyness

Name	Profit		Service quality		Charge	
	Object	Set	Object	Set	Object	Set
Agent1	0	0	0	0.6	1	1
Agent2	0		1		1	
Agent3	0		1		1 1	
Agent4	0		1		1	
Agent5	0		0		1	

We can also calculate the degree of greyness of A:

$$g^{\circ}_{G} = \frac{0+1+1+1+0}{5} = 0.6$$

The evaluation results from *Profit*, *Service Quality* and *Charge* are shown in Table 3.2. It illustrates that the degree of greyness for a white set is 0, and the degree of greyness for a black set is 1. Different from a white or black set, a grey set may have its degree of greyness located anywhere between the two extremes. A grey set is a single characteristic value set represented with multi-characteristics value sets.

3.5 Relationships Between Different Models

Given that there are many variations of uncertainty models available, there is indeed some overlap between models. It is therefore necessary to investigate their similarities and divergences from grey models so as to apply the right model for the right problem at the right time. Following the description of grey sets, type-1 fuzzy sets, shadowed sets and interval-valued fuzzy sets in previous sections, we can derive the relationship between these sets (Yang & John, 2012).

A grey set can be specified to be a white set, a black set, a crisp set, a type-1 fuzzy set, a shadow set and an interval-valued fuzzy set under special situations. In other words, a grey set can be considered as a generalization of these sets. When the uncertainty of greyness is completely removed, we get a white set from a grey set. It shows that a grey set can be turned into a white set by adding more information to reduce its uncertainty. If no information at all, a grey set becomes a black set. If sufficient information is available and the characteristics function values can only be 0 or 1, a grey set is a crisp set which is certainly a white set as well. When sufficient information available but the characteristic function takes a value between 0 and 1, then it appears as a type-1 fuzzy set. This fact shows that a type-1 fuzzy set is a white set but it still contains fuzziness. It shows that greyness is a different uncertainty from fuzziness, and a white set does not necessarily mean a crisp set. If the available information is not enough to make the grey set white, but its element can be classified into known and unknown, then it is equivalent to a shadowed set. All these situations maintain the meaning of grey characteristic function value: it represents a single value (undetermined) by a set. However, if we changed this interpretation into a multi-valued set, then a grey set could be considered to be an interval-valued fuzzy set. Here, it should be highlighted that a grey set can only be considered to be an interval-valued fuzzy set when its grey value for its characteristic function is interpreted as a set rather than a single value represented by a set. This is a crucial difference. For example, a fuzzy membership [0.6, 0.9] in an interval-valued fuzzy set can represent that all values between 0.6 and 0.9 are valid membership values. In other words, the validity of 0.7 does not exclude the validity of 0.8, they could be the membership values at the same time. However, for a grey set, the validity of 0.7 excludes the validity of 0.8. They cannot be taken at the same time in a grey set.

Similar to type-1 fuzzy sets, a grey set can be equivalent to a rough set under special situations (Yang & John, 2012). The required condition is similar to the condition to convert it into a shadowed set, but there is a difference in that it has to take values in a discrete space {0, 1} rather than the {0, 1, [0, 1]} which include a continuous interval. This result gives also a link between shadowed set and rough set: a rough set is a shadowed set defined in a discrete space of {0, 1}.

A grey set can also be converted into an R-Fuzzy set under special conditions. In an R-fuzzy set A, if the lower approximation $\underline{M}_A(x) = \varnothing$, the upper approximation $\overline{M}_A(x) \neq \varnothing$, and there is only one object in the upper approximation could be the true object belonging to this set, then A is equivalent to a grey set (Yang & Hinde, 2010). The work presented by Khuman (2021), Khuman et al. (2014) comprehensively provides the similarities and divergences that exist between grey and fuzzy theory.

According to these theorems, a grey set can be converted into a crisp set, a type-1 fuzzy set or a rough set under special considerations. An R-fuzzy set and an interval-valued fuzzy set can also be specialised to a grey set under special conditions. Therefore, grey sets can serve as a bridge to connect the different models together. Among these models, grey sets, crisp sets, type-1 fuzzy sets and rough sets are all individually oriented, and they focus mainly on individual objects of each set, and hence their characteristic function values are single values even if grey sets employ a set representation for a single value. We consider these models as individually oriented. However, R-fuzzy sets and interval-valued fuzzy sets highlight characteristic function values of a group of members rather than an individual, and they are group oriented in that their characteristic function values are multi-valued rather than single-valued. To this end, they can represent the diverse values for their group characteristic functions, and they are more group reference oriented. In this way, we can divide these models into two groups; individual reference and group reference. For interval-valued fuzzy sets, there are two possible interpretations of their membership values; a multi-valued set or a single value represented by a set of candidate values. The first interpretation leads to its position in the group reference oriented models, and the second interpretation places it into the individual reference oriented models. The connection and division of these uncertainty models are illustrated in Fig. 3.3.

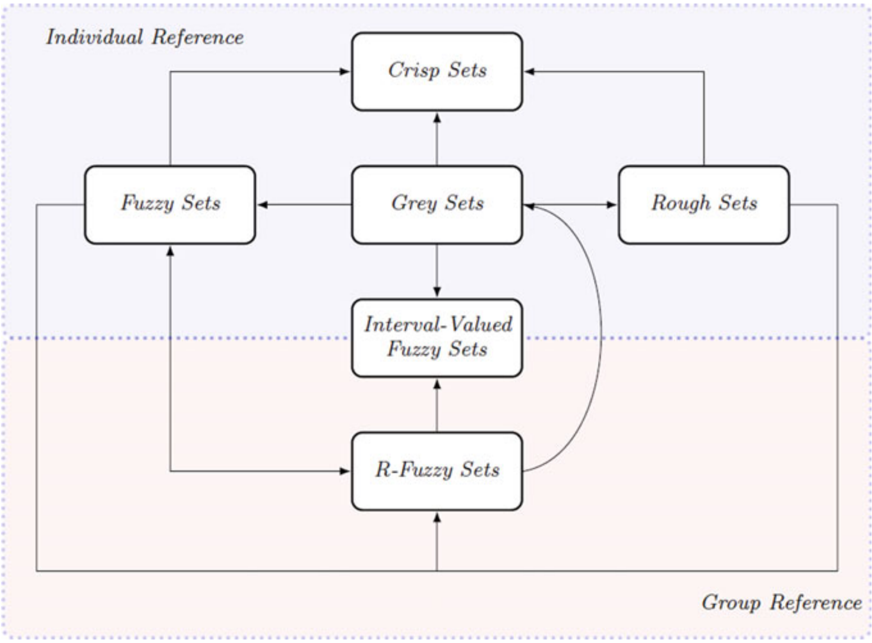


Fig. 3.3 The relationship between different uncertainty models

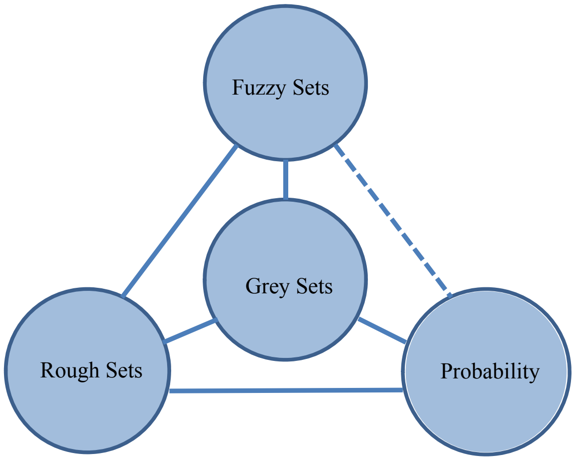
3.6 Roles of Grey Systems in Uncertainty Modelling

3.6.1 A Media for Uncertainty Modelling

Figure 3.3 has demonstrated the connections of grey sets with other uncertainty models. It is clear that grey sets can overlap with type-1 fuzzy sets, rough sets, interval-valued fuzzy sets, R-Fuzzy sets and crisp sets under special situations. Therefore, it provides the feasibility for grey sets to implement these models and provide a synthesized representation of uncertainties to cover a wider range of uncertainties in real world applications. Apart from these models, another important and well known model for uncertainty representation is the probability model for randomness. In fact, a probability distribution is possible to be modelled for the whitenisation function in a grey number. For example, a discrete grey number may have 3 candidates {10, 15, 20}, and we know that the probability for 15 to be the underlined white number is 50%, and the other two have a probability of 25% each. In this way, a grey set can be connected with probability models as well. In our work on the relative uncertainty of grey numbers and grey sets, we have embedded probability models into the uncertainty measurement of grey numbers and grey sets.

Figure 3.4 illustrates the general connections between grey sets and fuzzy sets, rough sets as well as probability models. Grey sets have connections with all the

Fig. 3.4 The connection between grey sets, fuzzy sets and probability models



other three models, and the other models have connections with each other as well. The relationship between grey sets and these models have been explained in Fig. 3.3, here we look at the connections between probability and fuzzy sets and rough sets. In rough sets, the membership function is in fact modeled by probability, and each object in the set is associated with a probability for its relationship with the set. Those with 100% are in the lower approximation, and those with a value greater than 0% are in the upper approximation. For fuzzy sets, probability could be taken as a tool to produce the possibility which in turn brings in the fuzzy membership although its interpretation is not a probability anymore. For this reason, we mark their connection as a dash line rather than a solid line in Fig. 3.4.

From Figs. 3.3 and 3.4, it is possible to construct a synthesized uncertainty model combining greyness, randomness, fuzziness and roughness together using grey sets, as shown in Fig. 3.5.

Figure 3.5 indicates that grey sets can serve as a media to combine greyness, fuzziness, randomness and roughness together in the grey characteristic function values. In this way, different uncertainties are represented in a relative simple model

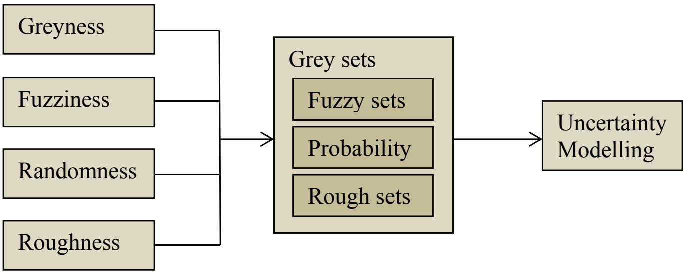


Fig. 3.5 Uncertainty modelling using grey sets to combine different uncertainties

in comparison with those more complicated models. Obviously, this is only a starting point for more advanced representation in the synthesized uncertainty modelling.

3.6.2 Uncertainty Modelling in Data Analytics

As explained in Chap. 2, grey systems provide a powerful tool in dealing with small and poor data. However, the concept of grey sets does not exclude big data in the same time. With the growing application of big data, the challenge in data quality control has increasing significance than ever before. Without an effective uncertainty representation and quantification, the significant existence of uncertainty as a result of the diverse variety of big data would make big data analysis useless in the end. Recently, we proposed the application of grey sets in the uncertainty quantification for data analytics. The idea is to combine the strong facility of grey sets in uncertainty quantification with the distinctive modelling ability of grey modelling for small and incomplete data together to complement the traditional data analytics models such as the statistical models and neural networks models.

No matter small or big data, the data set is composed of individual data samples or rows coming from different data sources. The big volume of big data actually comes from its variety of data sources. It means that the data comes from different locations, time and data collectors under different measurements and interpretations. All these introduce significant problems in data quality: there is a need to measure the existence of uncertainties and their significance. The traditional statistics can help with this, but we need to be able to deal with not only randomness, but also greyness, fuzziness and roughness at the same time. It raises the requirement on a suitable uncertainty representation method. Obviously, grey sets provide a feasible option for this. The data samples or rows in a database can be considered as objects in a universe, and they constitute a grey set where each object is associated with a grey number to represent its uncertainty on reliability. In this way, a degree of greyness (and relative uncertainty) could be defined for each sample or row in the database, and a related degree of greyness (or relative uncertainty) can be derived for each subset and the whole set as shown in Fig. 3.6. In this way, the concept of grey sets can be applied to deal with uncertainty modelling in even big data analysis. Hence it has significant potential to contribute to data analytics not only in small data analysis but also for big data analysis.

The traditional models are good at identifying historical trends from historical data, but they lack of focus on the most recent data which leads to their poor performance in unexpected recent divergence from historical trends. However, due to various unexpected social and economic factors, a divergence from normal developing trends is a common feature in many real world data analysis. We proposed an integration of grey systems with traditional data analysis models as shown in Fig. 3.7.

As shown in Fig. 3.7, each sample data item could be considered as an object in a grey set which cover all data from different sources with different interpretations. As aforementioned, a grey number could be defined with respect to the reliability

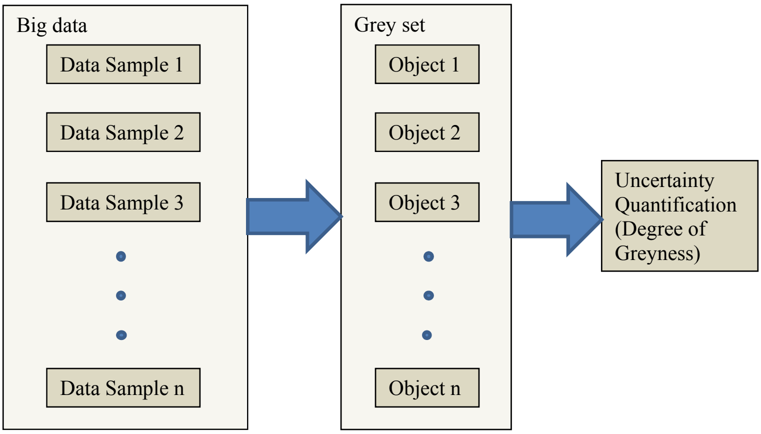
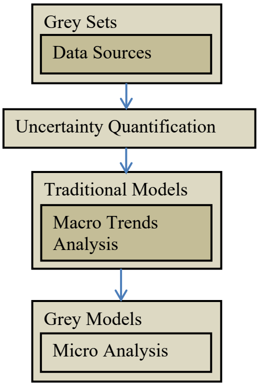


Fig. 3.6 Grey sets and uncertainty in big data

Fig. 3.7 The integration of traditional data analysis with grey data analysis



of each object according to their data source measurement accuracy or reliability. Then a degree of greyness will be derived for each object and then for each subset and the whole set. Depending on the necessity, other measurement, such as the relative uncertainty can also be calculated. Based on the uncertainty quantification, data could be selected for macro trends analysis using traditional models, such as statistical models or neural network models. These models will analyse the data and give a general macro level result, such as the rough period or area. However, the detailed micro level analysis will depend on the grey model analysis. The reason is that while big data is effective for general analysis for macro level developing trends, its consideration of long history and large area often brings it away from accurate local analysis. Guided by the micro analysis, the grey model could focus on the most recent and adjacent data for a local analysis, and it has a better chance to reflect the local trends than the larger area. In this way, we make use of the merits of both models and have potential to improve the data analysis.

3.7 Conclusions

As an emerging model for uncertainties, models in grey systems have strong potential in capturing uncertainties not well captured in other, more conventional models. However, the significance of grey models in uncertainty representation and quantification is still not widely realized as with other related models. There still exists considerable confusion in their similarity and differences from other existing models. It leaves practitioners with a difficult task to know which one to choose for their specific uncertainties. Based on our existing work in this field, we have given a detailed review and discussion on grey numbers, grey sets, their uncertainty representation and quantification, and their relationship with other uncertainty models, describing their connections with fuzzy sets and rough sets. We have demonstrated that grey models can provide the capabilities to represent and quantify uncertainties, not sufficiently covered in other models, such as the relative uncertainty of a grey set. Our discussion has also made it clear that grey sets have close connections with fuzzy sets, rough sets and their extensions. A grey set can be converted into a type-1 fuzzy set, a crisp set or a rough set under specific considerations; an R-fuzzy set and an interval-valued fuzzy set can also be turned into a grey set under special conditions. Furthermore, the discussion reveals the fact that these models can be separated into two different types; individual reference and group reference, and interval-valued fuzzy sets can go either way depending on its interpretation on the semantic meaning of its interval membership values. We continue our research in providing a more detailed means as to why a grey system approach is favourable when compared to other, long established models.

In the end, we discussed the role of grey systems in data analytics and illustrated the significant potential of grey models in uncertainty quantification and data analysis.

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Chapter 4
Extending Neuro-fuzzy Techniques with Grey-Based Hybridisation

Jose L. Salmeron

4.1 Introduction

Real-world problems are commonly composed by interrelated components in many complex ways. They are usually dynamic, that is, they change with time through a series of interactions among related components (Salmeron, 2012). Classical decision-making techniques cannot support these kinds of challenges. For that reason, this paper focused on the extension of neuro-fuzzy techniques as Fuzzy Cognitive Maps with Grey Systems Theory.

Fuzzy Cognitive Maps (FCMs) constitute neuro-fuzzy systems, which are able to include experts' knowledge (Kosko, 1986; Lee et al., 2002; Salmeron, 2009a, 2009b). From an Artificial Intelligence point of view, FCMs are supervised learning neural systems, whereas more and more data is available to model the problem, the system becomes more accurate at adapting itself and reaching a solution (Papageorgiou & Groumpos, 2005; Rodriguez-Repiso et al., 2007). FCM is a modelling technique for complex systems generated from mixing fuzzy logic and artificial neural networks (Salmeron, 2009a, 2009b).

FCMs have some benefits over classical modelling methods; they offer more information in the relationships between variables or concepts. Moreover, the FCM model is dynamic, and customizable and capture nonlinear relationships. The final fuzzy model is worthy for simulation, analysis and check the influence of variables and forecast the behaviour of the whole modelled system.

FCM have been used in many different fields as medicine (Papageorgiou, 2011; Salmeron, 2012), failure modes effects analysis (Pelaez & Bowles, 1995; Salmeron &

J. L. Salmeron (✉)
CUNEF Universidad, Madrid, Spain
e-mail: joseluis.salmeron@cunef.edu

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Gutierrez, 2012) simulation (Fu, 1991), affective computing (Salmeron, 2012), engineering (Salmeron & Papageorgiou, 2014; Stylios & Groumpos, 2000), space exploration (Furfaro et al., 2010), relationship management in airline services (Kang et al., 2004), web-mining inference amplification (Lee et al., 2002) and others. For a deep review of the FCM research see (Papageorgiou & Salmeron, 2013).

4.2 Fuzzy Cognitive Maps

4.2.1 Theoretical Background

Fuzzy Cognitive Maps (Kosko, 1996) are a worthy technique for modelling and analysing the systems and people's behaviour. FCMs are a set of nodes linked by edges. The nodes model concepts relevant to a given problem. The causal links between these concepts are modelled by the edges showing the direction of the influence. The other attribute of an edge is its sign, which can be positive (a promoting effect) or negative (an inhibitory effect).

The main aim of a FCM around a problem is to be able to forecast the outcome by letting the critical issues interact with one another. These forecasts can be used for checking whether an initial scenario is consistent with the entire collection of stated causal assertions (Bueno & Salmeron, 2009). Causal relationships between the nodes can have different weights, represented in a range $[-1, +1]$ (or $[0, +1]$). FCM substitutes the signs by a fuzzy value between -1 and $+1$ where the zero value indicates the absence of causality. Moreover, it involves feedback, where the impact of the change in a node may affect other nodes (Lopez & Salmeron, 2013) (Fig. 4.1).

The FCM nodes (c_i) would model such concepts such as heat, radiation, temperature or marketing strategy, among others. An edge linking two nodes models the

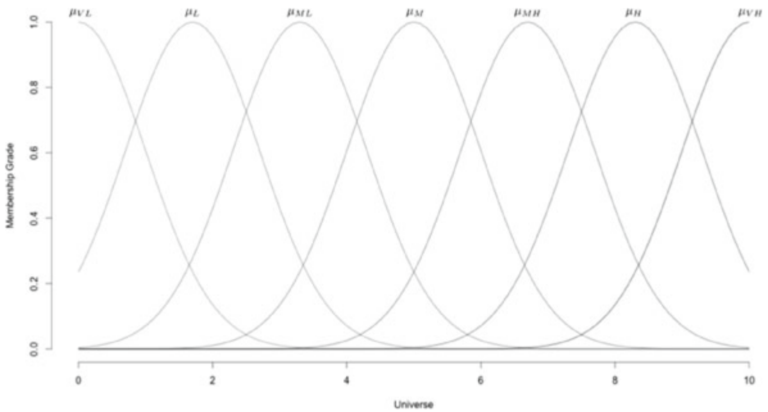

Fig. 4.1 Gaussian membership functions

causal influence of the causal node on the effect node (e.g.: the influence of the temperature to failure).

In a more formal way, an FCM can be represented as a 4-tuple;

$$\Omega = (\mathbf{N}, \mathbf{A}, f, r) \tag{4.1}$$

where N is the set of nodes $N = \{n_i\}_{i=1}^{m}$ with m the number of them, $A = [w_{ij}]_{n \times n}$ is the adjacency matrix modelling the edges between nodes as $\{w_{ij}\}_{i,j=1}^{m}$, $f(\cdot)$ the activation function, and r the nodes' range.

An adjacency matrix A represents the FCM nodes relationships. FCMs measure the strength of the causal relation between two nodes and if no causal relation be it is denoted by 0 in the adjacency matrix.

$$A = \begin{matrix} C_1 \\ \vdots \\ C_n \end{matrix} \overset{\begin{matrix} C_1 & \dots & C_n \end{matrix}}{\begin{pmatrix} w_{11} & \cdots & w_{1n} \\ \vdots & \ddots & \vdots \\ w_{n1} & \cdots & w_{nn} \end{pmatrix}} \tag{4.2}$$

4.2.2 FCM Dynamic Analysis

FCMs are dynamical systems involving feedback, where the effect of change in a node may affect other nodes, which in turn can affect the node initiating the change. The analysis starts with the design of the initial vector state (c(0)), which models the initial value of each variable or concept (node) (Salmeron & Lopez, 2012). The initial vector state c(0) with n nodes is denoted as

$$\mathrm{c}(0) = (\mathrm{c1}(0),\ \mathrm{c2}(0),\ \ldots,\ \mathrm{cn}(0)) \tag{4.3}$$

where $c_1(0)$ is the value of the concept i = 1 at instant t = 0.

The new states of the nodes are computed in an evolutive updating process with an activation function, which is applied to map monotonically the node state into a normalized range [0, 1] or [−1, +1]. The sigmoid function is the most used one (Bueno & Salmeron, 2009) when the node value maps in the range [0, 1]. The vector state c(t + 1) at time t + 1 would be

$$\begin{aligned} c(t+1) &= f(c(t) \cdot A) \\ &= f(c_1(t) \cdot A), \ldots, f(c_n(t) \cdot A)) \\ &= c_1(t+1)), \ldots, (c_n(t+1) \end{aligned} \tag{4.4}$$

The element i of the vector state $c_i(t+1)$ at time $t+1$ can be computed as shown in Eqs. 4.5 and 4.6. Equation 4.5 computes $c_i(t+1)$ just with pre-synaptic computation.

$$\overbrace{C_i(t+1)}^{post-synaptic} = f\left(\overbrace{\sum_{j=1}^{n} W_{ji}.C_j(t)}^{pre-synaptic}\right) \tag{4.5}$$

Equation 4.6 computes the pre-synaptic influence and aggregate the state of the post-synaptic node in the previous iteration. In that situation, the node has memory of its former state.

$$\overbrace{C_i\ (t+1)}^{post-synaptic} = f\left(\overbrace{C_i(t)}^{memory} + \overbrace{\sum_{j=1}^{n} W_{ji}\ .C_j(t)}^{pre-synaptic}\right) \tag{4.6}$$

If $f(\cdot)$ is the unipolar sigmoid function and the updating rule is as Eq. 4.6, then the element i of the vector state c(t) at time t would be

$$C_i(t+1) = \frac{1}{1 + e^{-\lambda.(C_i(t) + \sum_{j+1}^{n} W_{ji}.C_j(t))}} \tag{4.7}$$

where λ is the constant modelling the degree of normalization (function slope). If $f(\cdot)$ is the hyperbolic tangent function and Eq. 4.6 the updating rule, then the element i of the vector state c(t) at time t would be as follows

$$C_i(t+1) = \frac{e^2.\lambda.\left(c_i(t) + \sum_{j=1}^{n} w_{ji}.c_j(t)\right) - 1}{e^2.\lambda.\left(c_i(t) + \sum_{j=1}^{n} w_{ji}.c_j(t)\right) + 1} \tag{4.8}$$

After an inference process, the FCM reaches either one of two states after the iterations. It settles down to a fixed pattern of node values, the so-called hidden pattern or fixed-point attractor (as $c(t) \approx c(t-1)$).

Alternatively, it keeps cycling between several fixed states, known as a limit cycle. Using a continuous activation function, a third possibility known as a chaotic attractor exists. This occurs when, instead of stabilizing, the FCM keeps producing different state vector values for each iteration (Salmeron & Froelich, 2016).

4.2.3 FCM Consensus

It is possible to build FCMs from real-world raw data (e.g.: with evolutionary techniques) or from a panel of experts (Salmeron, 2009a, 2009b). Delphi methodology it is a methodology applied to manage the communication of a panel of experts in order to reach a consensus regarding a problem. One of the main features of Delphi method is when the experts get feedback reports, they could modify their own judgement based on the feedback (Bueno & Salmeron, 2009).

A more analytical way is the Augmented FCM. It doesn't require that experts change their initial judgement for consensus as Delphi method needs. The augmented adjacency matrix is built adding the adjacency matrix of each expert (Salmeron, 2009a, 2009b). Let k FCMs with common nodes. The augmented adjacency matrix (A*) would be computed as follows

$$A* = \begin{pmatrix} w^*_{11=\frac{1}{k}} \cdot \sum_{m=1}^{k} w_{11}.m & \cdots \frac{1}{k} \cdot & \sum_{m=1}^{k} w_{1n}.m \\ \vdots & \ddots & \vdots \\ w^*_{n1=\frac{1}{k}} \cdot \sum_{m=1}^{k} w_{1}.m & \cdots \frac{1}{k} \cdot & \sum_{m=1}^{k} w_{nn}.m \end{pmatrix} \tag{4.9}$$

If the FCMs have not common nodes, then the adjacency matrix would be added including zero rows and columns. Let a couple of FCMs ($\{w'_{ij}\}$ and $\{w''_{ij}\}$) without common nodes, the augmented adjacency matrix would be

$$A* = \begin{pmatrix} w^*_{11=\frac{1}{2}}.w'_{11} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & & w^*_{nn=}\ \frac{1}{2}.w''_{nn} \end{pmatrix} \tag{4.10}$$

Figure 4.2c shows an example of Augmented FCM building process of a couple of FCMs with common and not common nodes. Figure 4.2a, b detail the FCMs from experts A and B and Fig. 4.2c is the Augmented FCM.

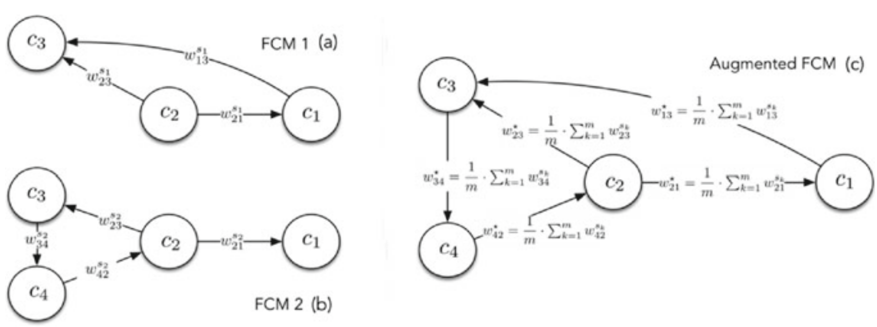

Fig. 4.2 Augmented FCM

4.3 Fuzzy Grey Cognitive Maps

Fuzzy Grey Cognitive Maps (FGCMs) are a FCM generalization (proposed in Salmeron (2010)) for modelling complex systems with high uncertainty, under discrete incomplete and small data sets and it is based on Grey Systems Theory (Deng, 1989).

4.3.1 Theoretical Background

4.3.1.1 Grey Systems Theory

Grey Systems Theory (GST) is a useful approach for solving problems with high uncertainty and discrete small and incomplete datasets (Liu & Lin, 2006). GST has been successfully applied in fields such as military science, agriculture, medicine, meteorology, business, industry, energy, transportation, geology and so on (Yamaguchi et al., 2007).

GST categorizes the information according to the degree of the available knowledge in three-fold. The system is white if the knowledge of that system is fully known (whole understanding), but it is black if the system is completely unknown. Finally, a system with partial knowledge known and partial knowledge unknown is called a grey system (Deng, 1989).

GST includes data fuzziness because it can consider it (Liu & Lin, 2006). Furthermore, the fuzzy theory holds information usually based on experience; while GST is concerned with objective data, they do not need any information different than the datasets to be disposed.

In a more formal way, let Θ be the universal set, then a grey set $\Phi \subset \Theta$ is defined by its both mappings. Note that $\left\{\mu_{\Phi}^{+}(\cdot), \mu_{\Phi}^{-}(\cdot)\right\} \in [0, 1]$, where $\mu_{\Phi}^{-}(\cdot)$ is the lower membership function, $\mu_{\Phi}^{+}(\cdot)$ is the upper one and $\mu_{\Phi}^{-}(\cdot) \leq \mu_{\Phi}^{+}(\cdot)$. Also, GST extends fuzzy logic, since the grey set Φ becomes a fuzzy set when $\mu_{\Phi}^{-}(\cdot) = \mu_{\Phi}^{+}(\cdot)$. The crisp value of a grey number is unknown, but the range in which the value is found is known.

An interval grey number is denoted as $\mathrm{x}^{\pm} \in [\mathrm{x}^{-}, \mathrm{x}^{+}] \mid \mathrm{x}^{-} \leq \mathrm{x}^{+}$ and it has an upper (x^{+}) and a lower (x^{-}) limit (Yang & John, 2012). Both limits are crisp numbers in first order interval grey numbers.

When the grey number $\mathrm{x}^{\pm}$ has just an upper limit is as follows $\mathrm{x}^{\pm} \in (-\infty, \mathrm{x}^{+}]$ and when it has just a lower limit is $\mathrm{x}^{\pm} \in [\mathrm{x}^{-}, +\infty)$. A black number has both unknown limits $\mathrm{x}^{\pm} \in (-\infty, +\infty)$ and it becomes a white number when both limits are the same $\mathrm{x}^{-} = \mathrm{x}^{+}$. The transformation of grey numbers in crisp ones $\hat{x}$ is known as whitenisation (Liu & Lin, 2006) and it is computed as follows

$$\hat{x} = \mathrm{x} + \cdot \xi + (1 - \xi) \cdot \mathrm{x} - | \xi \in [0, 1] \tag{4.11}$$

where ξ is a parameter to control the position of the crisp values according to the limits. If $\xi = 0.5$, then it is called equal mean whitenisation. The length of a grey number is computed as $l(x^{\pm}) = | x^{+} - x^{-}|$. In that sense, if the length of the grey number is zero ($l(x^{\pm}) = 0$), then it is a white number.

Despite the length of a grey number with only one limit known (lower or upper), $x^{\pm} \in [x^{-}, +\infty)$ or $x^{\pm} \in (-\infty, x^{+}]$ being infinite, the grey number is not a black number because it is possible to know one of the two limits.

Equations 4.12–4.15 compute the grey arithmetic operations

$$x\frac{+}{1} + x\frac{+}{2} \in \left[x_1^- + x_1^-, x_1^+ + x_2^+\right] \tag{4.12}$$

$$x\frac{+}{1}\_x\frac{+}{2} \in \left[x_1^- + x_2^-, x_1^+ + x_2\right] \tag{4.13}$$

$$x\frac{+}{1} \cdot x\frac{+}{2} \in [min\left\{x_1^- \cdot x_2^-, x_1^+ \cdot x_2^+, x_1^- \cdot x_2^+, x_1^+ \cdot x_2^-\right\}, max\left\{x_1^- \cdot x_2^-, x_1^+ \cdot x_2^+, x_1^- \cdot x_2^+, x_1^+ \cdot x_2^-\right\}] \tag{4.14}$$

$$\frac{x\frac{+}{1}}{x\frac{+}{2}} \in \left[min\left\{\frac{x_1^-}{x_2^-}, \frac{x_1^+}{x_2^+}, \frac{x_1^-}{x_2^+}, \frac{x_1^+}{x_2^{+-}}\right\}, max\left\{\frac{x_1^-}{x_2^-}, \frac{x_1^+}{x_2^+}, \frac{x_1^-}{x_2^+}, \frac{x_1^+}{x_2^{+-}}\right\}\right] \left| \left\{x_i^-, x_i^+\right\}_{i=1}{}^2 Y \neq 0 \right| \tag{4.15}$$

4.3.1.2 FGCM Fundamentals

The FGCM nodes are concepts or variables and the relationships between them are modelled by grey weighted and directed edges (Nápoles et al., 2021; Salmeron, 2010). Formally, a FGCM is denoted as a 4-tuple

$$\Omega = \left(\zeta,\ W,\ f^{\pm}(\cdot),\ l(\psi)\right) \tag{4.16}$$

where $\zeta = \{\alpha_i^{\pm}\}_{i=1}^{n}$ is the set of nodes with grey numbers as grey states, $W = \{w_{ij}^{\pm}\}$ is the set of edges linking the nodes weighted with grey weights, $f^{\pm}(\cdot)$ the grey activation function and $l(\psi)$ the range of the information space.

An interval grey weight between the nodes $\alpha_i^{\pm}$ and $\alpha_j^{\pm}$ is denoted as $w_{ij}^{\pm} \in [w_{ij}^{-}, w_{ij}^{+}]$ and it has a lower (w_{ij}^{-}) and an upper (w_{ij}^{+}) limit.

FGCM includes greyness as an uncertainty score (Salmeron, 2010; Salmeron & Gutierrez, 2012; Salmeron & Papageorgiou, 2014). If a grey state (or weight) has a high score of greyness, then it means that the grey state have a high uncertainty associated. It is computed as follows

$$\varnothing\left(\alpha\frac{+}{i}\right) = \frac{\left|l(\alpha\frac{+}{i})\right|}{l(\psi)} \tag{4.17}$$

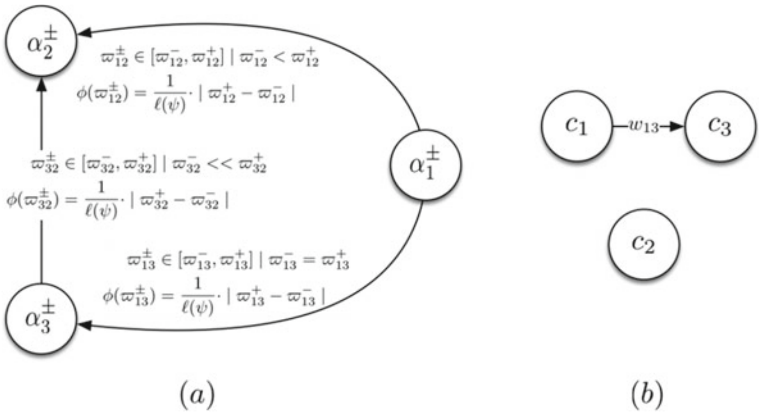

Fig. 4.3 Potential relationships of the nodes in FGCM (**a**) and FCM (**b**)

where $|l(\alpha_i^{\pm})| = |\alpha_i^{+} - \alpha_i^{-}|$ is the length of grey node state $\alpha_i^{\pm}$ in absolute value and $l(\psi)$ is the length of the information space, denoted by ψ. It is computed as follows

$$l(\psi) = \begin{cases} 1if \left\{\alpha_i^{\pm}, \varpi_i^{\pm}\right\} \subseteq [0, 1] \\ 2if \left\{\alpha_i^{\pm}, \varpi_i^{\pm}\right\} \subseteq [-1, +1] \end{cases} \tag{4.18}$$

where $l(\psi) = 1$ for unipolar FGCMs and $l(\psi) = 2$ for bipolar FGCMs.

FGCMs model the human reasoning in a more realistic way than FCM does, because it is able to manage the uncertainty between the concepts and the incomplete information about the modelled system as FCMs cannot do (Salmeron & Palos-Sanchez, 2019).

In Fig. 4.3a, the relationship ($\varpi_{13}^{\pm}$) between $\alpha_1^{\pm}$ and $\alpha_3^{\pm}$ is a white one, between $\alpha_1^{\pm}$ and $\alpha_2^{\pm}$ is a grey one ($w_{12}^{\pm}$), and between $\alpha_3^{\pm}$ and $\alpha_1^{\pm}$ is a black one ($w_{32}^{\pm}$).

FCMs are just able to model white (crisp) relationships (Fig. 4.3b). As a result, a huge amount of information is lost. Another advantage of FGCMs over FCM is related with the uncertainty associated to the elements of the problem (or system) modelled (Salmeron & Papageorgiou, 2012).

4.3.2 FGCM Dynamic Analysis

FGCMs are dynamical systems involving feedback just as FCMs are, where the influence of change in a grey node may affect other grey nodes, which in turn can impact the grey node starting the dynamics. The dynamic analysis begins with an initial grey vector state ($\alpha^{\pm}(0)$), which models the initial grey value of each concept (grey node). The initial grey vector state $\alpha \pm (0)$ with n grey nodes is denoted as

$$\alpha^{\pm}(0) \;=\; \left(\alpha^{\pm}(0),\; \alpha^{\pm}(0), \ldots, \alpha^{\pm}(0)\right) \tag{4.19}$$

where $\alpha^{\pm}(0)$ is the state of the concept i = 1 at time t = 0.

The grey state values of the grey nodes are updated in an evolutive process with a grey activation function, which is applied to map monotonically the grey node state into the information space (Salmeron, 2010). The grey state is updated as follows

$$\begin{aligned} \alpha^{\pm} &= f^{\pm}\left(\alpha^{\pm}(t) \cdot A^{\pm}\right) \\ &= \left(f^{\pm}\left(\alpha_1^{\pm}(t) \cdot A^{\pm}\right), \ldots, f^{\pm}\left(\alpha_n^{\pm}(t) \cdot A^{\pm}\right)\right) \\ &= \left(\alpha_1^{\pm}(t+1), \ldots, \alpha_n^{\pm}(t+1)\right) \end{aligned} \tag{4.20}$$

where $A^{\pm}$ is the grey adjacency matrix and $f^{\pm}(\cdot)$ the grey activation function. Mostly, the grey activation function used to be the grey unipolar sigmoid (Eq. 4.21)

$$\alpha\frac{+}{i}(t+1) = \left[\frac{1}{1+e^{-\lambda.\left(a\frac{\pm}{i}(t)+\sum_{j=1}^{n} w\frac{\pm}{ji}.\alpha\frac{\pm}{j}(t)\right)}}, \frac{1}{1+e^{-\lambda.\left(a\frac{\pm}{i}(t)+\sum_{j=1}^{n} w\frac{\pm}{ji}.\alpha\frac{\pm}{j}(t)\right)}}\right] \tag{4.21}$$

or the grey hyperbolic tangent (Eq. 4.22) as follows

$$\alpha\frac{+}{i}(t+1) = \left[\frac{e^{2.\lambda.\left(a\frac{\pm}{i}(t)+\sum_{j=1}^{n} w_{ji}.\alpha\frac{\pm}{j}(t)\right)}-1}{e^{2.\lambda.\left(a\frac{\pm}{i}(t)+\sum_{j=1}^{n} w_{ji}.\alpha\frac{\pm}{j}(t)\right)}+1}, \frac{e^{2.\lambda.\left(a\frac{\pm}{i}(t)+\sum_{j=1}^{n} w_{ji}.\alpha\frac{\pm}{j}(t)\right)}-1}{e^{2.\lambda.\left(a\frac{\pm}{i}(t)+\sum_{j=1}^{n} w_{ji}.\alpha\frac{\pm}{j}(t)\right)}+1}\right] \tag{4.22}$$

4.3.3 FGCM Consensus

It is possible to build FGCMs from real-world raw data (e.g.: using evolutionary techniques) or from a panel of experts. As far as the author knows, there have been not used Delphi methodology for building FGCMs yet, but augmented FGCMs have been used. The augmented adjacency grey matrix is built adding the adjacency grey matrix of each expert (Salmeron & Palos-Sanchez, 2019).

Let k FGCMs with common grey nodes. The augmented adjacency grey matrix ($A^{\pm *}$) would be computed as follows

$$A^{\pm *} = \begin{pmatrix} \varpi_{11}^{*} = \frac{1}{k}\cdot\sum_{m=1}^{k} \varpi\frac{+}{n1}.m & \cdots & \varpi\frac{*}{1n} = \frac{1}{k}\cdot\sum_{m=1}^{k} \varpi\frac{+}{1n}.m \\ \vdots & \ddots & \vdots \\ \varpi_{n1}^{*} = \frac{1}{k}\cdot\sum_{m=1}^{k} \varpi\frac{+}{n1}.m & \ldots & \varpi\frac{*}{nn} = \frac{1}{k}\cdot\sum_{m=1}^{k} \varpi\frac{+}{nn}.m \end{pmatrix} \tag{4.23}$$

If the FGCMs have not common grey nodes, then the adjacency grey matrix would be added including null rows and columns. Let a couple of FGCMs $\{w_{ij}{}^{\pm(a)}\}$ and $\{w_{ij}{}^{\pm(b)}\}$ without common grey nodes, the augmented adjacency grey matrix would be

$$A^{\pm *} = \begin{pmatrix} \varpi_{11}^{+(a)*} = \frac{1}{2}.\varpi_{11}^{+(a)} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \varpi\,{}^{+(b)*}_{nn} = \frac{1}{2}.\varpi\,{}^{+(b)}_{nn} \end{pmatrix} \tag{4.24}$$

4.4 Conclusions

This work shows the extension of neuro-fuzzy techniques with Grey Systems Theory. FGCMs are a generalization for modelling systems with high uncertainty, under discrete incomplete and small data sets.

FGCMs model the human reasoning in a more realistic way than FCM does, because it is able to process the uncertainty between the nodes and the incomplete information about the modelled problem as FCMs cannot do.

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Chapter 5
The Grey Structure and Evolution of Knowledge Towards a Theory of Grey Knowledge Conception

Emil Scarlat

5.1 What Is the Grey Knowledge?

The fundamental characteristic of grey knowledge is the fact that it is a form of knowledge that fill the continuum which separates the clarity of total explicit knowledge from the opacity of tacit knowledge. This is a form of knowledge that is neither complete tacit, nor totally explicit, because it is more or less **ambiguous, imprecise and/or incomplete**. Yet, grey knowledge is the intermediate form between explicit and tacit knowledge.

Grey knowledge may be closer or more removed from one of these two forms, but never overlap with any of them. Tacit knowledge cannot be explicit, so it is only in the mind of individuals, while explicit knowledge is completely revealed, no longer be ambiguous or uncertain, as expressed through a certain methods of representation of knowledge. For example, the knowledge about dark matter in astrophysics is almost inexistent, but our knowledge in some scientific domains is abundant and new pieces of knowledge appear very rare. It is clear that the knowledge incorporated in different domains differ very much. But in the evolution of these domains there is a continuous accumulation of knowledge.

The real systems like scientific, economic, technological, natural systems etc. revisit continuously their own knowledge. In other words, they are able to transform data in information and information in knowledge.

The **Grey Data** is a fundamental concept for the entire theory of grey systems. "*Data is a certain form of the representation of facts*" (Shi et al., 2015). But there are numerous other definitions of data. For example, in scientific computing, data are the distinct pieces of information which can be processed into a different form giving information; in information theory data is an object that has the self-knowledge

E. Scarlat (✉)
Bucharest University of Economic Studies, Bucharest, Romania
e-mail: emilscarlat@ymail.com

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representation of its state and the state's changing mode over time (Zhong, 2007). In statistics, data are the first result of the process of observation of the reality that are not transformed.

A definition of **grey information** is "*information being incomplete or unknown*" (Liu & Lin, 2011), thus an element from an incomplete knowledge is considered to be a grey element. Furthermore, the relations of incomplete information between systems or elements are taken as being grey. Should there be an unknown sign or incomplete information in the system, it will be considered as grey.

The **grey knowledge** represents "*a dynamic entity that connects tacit knowledge and explicit knowledge within the continuum of knowledge and this knowledge gives a superior meaning in analysis and design of mechanisms through which knowledge is acquired, shared, transmitted, stored and forgotten in systems*" (Scarlat, 2012). The grey knowledge is the most available knowledge in systems. If tacit (black) and explicit (white) knowledge represent extreme limits of this continuum, the grey knowledge is the dynamic form, which turns continuously, so that tacit knowledge becomes explicit knowledge and vice versa.

Generally, knowledge can be viewed as a combination of three types of knowledge:

$$\textbf{Knowledge} = \textbf{BlackKnowledge} + \textbf{GreyKnowledge} + \textbf{WhiteKnowledge}$$

Black Knowledge is the tacit (implicit) knowledge of the system and **White Knowledge** corresponds to the explicit knowledge of the system. The Black Knowledge can be interpreted as a complete lack of information, and the White Knowledge as full of information. Between these two extreme, we have **Grey Knowledge** that is **knowledge when information is incomplete, imprecise or undetermined**.

We define a Grey System as a system that contains **incomplete, imprecise or undetermined information**. But, in real systems, like technological, economical, biological or social systems, information is continuously transformed in knowledge. Principally, we must make a clear distinction between data, information and knowledge. For example, the knowing Knowledge Pyramid says that they are distinctive levels that correspond to the distinct use of them in procedures, decisions or algorithms. These levels are associated with the complexity of systems that is a more complex system has need of a superior level of content. But into the levels of Knowledge Pyramid we have **a finer structure** that include black, white and grey data, black, white and grey information and, black, white and grey knowledge. In the same time, into every category take place dynamic processes that transform a category of knowledge to another using **knowledge feedback mechanisms**. For example, the grey knowledge can be obtained from black knowledge using peer-to-peer conversations, group discussions, contacting experts, chatting etc. In time, the grey knowledge tends to be transformed in white knowledge. The white knowledge is externalized, that means it is collective (attitudes, comportments, rules, laws, norms) or organizational artifacts like handbooks, manuals, designs, records of conversations, reports etc.

All these forms of knowledge are internalized and become black knowledge using experience, reflection, applications of talents, reading, listening. The black knowledge obtained in these internalized processes is located at the personal or organizational level. The black personal knowledge is represented by intentions, heuristics, personal skills, habits, knowhow and black organizational (social) knowledge by routines, culture, history, shared models etc.

It is very common for sets of quantitative and qualitative data to be incomplete, in the sense that not all planned observations are actually made. This is especially true when studies are conducted on systems that contain human subjects.

For the grey systems, the use of grey knowledge represents a fundamental part of their organizational and structural life. Grey knowledge points out the daily reality of the organizational life and its environment, the background upon which main subsystems of the grey systems project the meaning of their actions and decisions. Grey knowledge also reveals new situations, which are still **ambiguous, indistinct and fragmentary**. More specifically, analyzing grey knowledge means noting that the system of beliefs and assumptions that are put to the test in the theories of action. It means, in other words, investigating organizational memory and learning, building explanations in a partially enigmatic way. This is where the creative process can be seen. It is not limited to a reduction of the unknown to the known, but builds new paradigms, metaphors, interpretations, meanings and resolves paradoxes (Table 5.1).

In this perspective, it is very important to have a measure of the degree of greyness that is obtained in evaluating the knowledge flow intensity. We believe that **knowledge intensity** is determined by the ongoing process of knowledge transformation at the level of knowledge feedback mechanisms, and the difference of knowledge intensity between two points (nodes) located on the knowledge flow is determined by

Table 5.1 Influences of the knowledge processes

Knowledge	Tacit (black)	Grey	Explicit (white)
Creation	Action and experience	Routines, objectivation, institutionalisation	Codification, formal representation
Learning	Imitation, learning by doing, internalization	Socialization, speeches, peer-to-peer relations	Traditional learning
Communication	Examples, cases, metaphors, associations of ideas	Routine, traditions, informal communications	Standardized channels of transmission
Localization	Individual	Social group Communities of practice	Tangible support (books, manuals, instructions etc.)
Description	Impossible or to a very limited degree through heuristics	Partial, through speeches, schemata and texts	Formal description
Elaboration	Manual and artistic ability	Argumentation	Symbolic manipulation

the different speed of each feedback loop in the process of knowledge transformation, from implicit into explicit knowledge and reverse.

The speed of knowledge transformation in the process of knowledge discovery is different, so that **the knowledge intensity along a flow will be different**. The knowledge tends to migrate from the points with higher knowledge intensity to the points with lower intensity. In time, the knowledge intensity between two points of a system or between different systems which are connected through knowledge flows tends to equalize (equilibrate), so the knowledge transmitted between them is increasingly becoming smaller or even zero. Higher the difference of intensity is, higher the speed of knowledge flows, defined as the amount of knowledge transferred during the standard unit of time, will be. The difference between the knowledge intensity associated to the different knowledge structures existing in one or more systems determine both the orientation and the dynamics of the knowledge flows that composes a **grey knowledge network**. The measurement of grey knowledge in a system is connected with the knowledge intensity from this network.

5.2 The Representation and Analysis of the Grey Knowledge

The structure of the grey knowledge is represented, generally, by graphs, super graphs and networks that are associated with the pieces of information and the relations between them. A graph has a number of nodes (vertices) and edges that can be denoted with N and, respectively, K. Some graphs have on the edges numbers representing the value of changes between two vertices that are named weights or attribute and are noted with w_k, $k \in K$. Generally, a single attribute cannot precisely describe the node content. This representation is commonly referred to as a *single-attribute graph.* If we used a set of independent attributes to describe the node content, this representation is commonly referred to as an *attributed graph.*

The nodes of a knowledge graphs may include people, computers, databases, data warehouses, specialized software agents etc. The existing knowledge flows between these nodes are oriented in relation to the knowledge intensity associated to each node. Every node is connected with other nodes (or not) using directed flows having different intensities.

The structure of complex graphs induces that the researchers of grey knowledge could not get complete information, i.e., the obtained information is usually inadequate. The missing information make the statistical datum be uncertain when we research the complex graphs.

Then we get some grey numbers and their number-covered sets under correct investigation methods (Zhou & Lu, 2007).

Indeed, in many situations, the properties used to describe the node content may be dependent of knowledge structures. For example, in a citation network each node represents one paper and edges denote citation relationships. It is insufficient to use one or multiple independent attributes to describe detailed information of a paper. Instead, we can represent the content of each paper as a graph with nodes denoting keywords and edges representing contextual correlations between keywords (e.g. co-occurrence of keywords in different sentences or paragraphs). As a result, each paper and all references cited in this paper can form a super-graph with each edge between papers denoting their citation relationships. In this paper, we refer to this type of graph, where the content of the node can be represented as a graph, as a "*super-graph*". Likewise, we refer to the node whose content is represented as a graph, as a "*super-node*".

A complex network is a super graph that is characterised of the following proprieties (Strogatz, 2001):

- **Structural complexity**: the wiring diagram could be an intricate.
- **Network evolution**: the wiring diagram could change over time.
- **Connection diversity**: the links (edges) between nodes (vertices) could have different weights, directions and signs.
- **Node (vertex) diversity**: there could be many different kinds of nodes (vertices).
- **Meta-complication**: the various complications can influence each other.

Generally, there are three types of grey numbers when we define the complex networks:

(1) **The sum of vertices** is a grey number and we denote it as N (⊗). Under the correct investigation methods, we can get its number-covered set [N], i.e., the true number N* of the vertices satisfies $N^* \in [N]$.
(2) **The sum of edges** is a grey number and we denote it as K (⊗). Under the correct investigation methods, we can get its number-covered set [K], i.e., the true number K* of the edges satisfies $K^* \in [K]$.
(3) For the weighted complex networks, **the weight of the edge** l_k is a grey number and we denote it as w_k (⊗). We can get its number-covered set $[w_k]$, i.e., the true weight $w^*{}_k$ of the edge l_k satisfies $w^*{}_{\mathrm{k}} \in [w_k]$.

Then, a **grey network** can be defined as a set $\{N(\otimes), K(\otimes), w_k(\otimes)\}$ of grey nodes, grey edges and grey weights.

A grey network has some numeric characteristics (indexes) that are associated with its proprieties. For example, we can compute:

The grey degree distribution

$\boldsymbol{P_k}(\otimes) = \frac{n_k(\otimes)}{N(\otimes)}$

where n_k (⊗) is the grey sum of vertices whose degree is k. If the number-covered set of n_k (⊗) is $[n_k] = [\mathrm{n^-}_{\mathrm{k}}; \mathrm{n^+}_{\mathrm{k}}]$, then the number-covered set of $P_k(\otimes)$ is $[P_k] = \frac{[n_k]}{N}$.

A knowledge network can have an exceedingly complex structure, as the connections among the nodes can exhibit complicated patterns. The grey degree distribution is a simplified measure that capture some elements of the complex structures in an understandable way.

The grey clustering coefficient of the vertex *v*

$$\boldsymbol{C_v}(\otimes) = \frac{2E_v(\otimes)}{N_v(\otimes)(N_v(\otimes) - 1)}$$

The number-covered set of $\boldsymbol{C_v}(\otimes)$ is $[C_v] = \frac{2[E_v]}{[N_v]([N_v]-1)}$. N_v $(\otimes)$ and E_v $(\otimes)$ are the grey sums of the vertices and, respectively, the existent edges in S_v (S_v is the set of vertices whose element is in the neighbour of a given vertex v).

The average grey clustering degree of the grey network

$$\boldsymbol{C}(\otimes) = \frac{1}{N(\otimes)} \sum_{v \in S} C_v(\otimes)$$

The number-covered set of $\boldsymbol{C}(\otimes)$ is $[\boldsymbol{C}] = \frac{1}{[N]} \sum_{\boldsymbol{v} \in \boldsymbol{S}} [\boldsymbol{C_v}]$, where S is the set of vertices of the grey network. The average clustering coefficient is a measure of the likelihood that two associates of a node are also associates themselves. A higher clustering coefficient indicates a greater associate themselves.

The grey characteristic path length

$$\boldsymbol{L}(\otimes) = \frac{2}{N(\otimes)(\mathrm{N}(\otimes) - 1)} \sum_{i \geq j} d_{ij}(\otimes)$$

The number-covered set of $\boldsymbol{L}(\otimes)$ is

$$[\boldsymbol{L}] = \frac{2}{[\boldsymbol{N}]([\boldsymbol{N}] - 1)} \sum_{\boldsymbol{i \geq j}} [\boldsymbol{d_{ij}}]$$

where d_{ij} $(\otimes)$ is the grey sum of edges of the shortest path which connects the vertices v_i and v_j, which i and j are the order numbers of vertices and $[d_{ij}] = [d^-_{ij}; d^+_{ij}]$ of $[d_{ij}]$.

The grey edge betweenness

$$\boldsymbol{B_i}(\otimes) = \frac{2}{N(\otimes)(\mathrm{N}(\otimes) - 1)} \sum_{v_i, v_j, v_k \in V, j \neq k} \frac{n_{jk}(i)(\otimes)}{n_{jk}(\otimes)}$$

The number-covered set of $\boldsymbol{B_i}(\otimes)$ is

$$[\boldsymbol{B}] = \sum_{v_i, v_j, v_k \in V, j \neq k} \frac{[n_{jk}(i)]}{[n_{jk}]}$$

In these relations, $n_{jk}(\otimes)$ is the grey sum of the shortest paths which connect the vertices v_j and v_k, and $n_{jk}(i)(\otimes)$ is the grey sum of the shortest paths which connect v_j and v_k through the vertex v_i, where i, j and k are the order numbers of vertices.

This measure shows the degree an individual lies between other individuals in the network; it's the number of people who a person is connecting indirectly through their direct links.

The grey closeness is the degree an individual is near all other individuals in a network (directly or indirectly). It is the inverse of the sum of the shortest distances between each individual and every other person in the network.

$$C_l(\otimes) = 1/\boldsymbol{L}(\otimes) = \frac{N(\otimes)(\mathrm{N}(\otimes) - 1)}{2} * \frac{1}{\sum_{i \geq j} d_{ij}(\otimes)}.$$

The number-covered set of $C_l(\otimes)$ is $[C_l] = \frac{1}{[\boldsymbol{L}]}$.

The grey centrality counts of the number of ties to other components in the network. It is a measure of the importance of a node in a network. It assigns relative scores to all nodes in the network based on the principle that connections to nodes having a high score contribute more to the score of the node in question.

$$C_d(\otimes) = \frac{\sum_{i=1}^{K(\otimes)} [C_d(N(\otimes) - C_d(i)]}{[(N(\otimes) - 1)(\mathrm{N}(\otimes) - 2)]}$$

The number-covered set of $C_d(\otimes) is [C_d] = \frac{\sum_{i=1}^{[K]} [C_d(N) - C_d(i)]}{(([\boldsymbol{N}] - 1)([\boldsymbol{N}] - 2))}$.

$C_d(N(\otimes))$ is the maximum value of grey centrality in the network.

These indicators can be used for calculate a general indicator that is the expression of **the grey intensity of the network**. The form of this general indicator is given by the rapport between the current values of these indicators and the maximum values of them.

$$\mathbf{I}(\otimes) = \frac{\left[\sum_{k=1}^{K(\otimes)} deg\, \boldsymbol{P_k}(\otimes) + \boldsymbol{C_v}(\otimes) + \boldsymbol{C}(\otimes) + \boldsymbol{L}(\otimes) + C_l(\otimes) + C_d(\otimes)\right]}{max\left[\sum_{k=1}^{K(\otimes)} deg\, \boldsymbol{P_k}(\otimes) + \boldsymbol{C_v}(\otimes) + \boldsymbol{C}(\otimes) + \boldsymbol{L}(\otimes) + C_l(\otimes) + C_d(\otimes)\right]}$$

(1) A toy example

$\sum_{v=1}^{6} P_{deg}(\mathrm{v}) = 0.21$; $[C_v] = 0.0384$; $[C] = 0.24$; $[L] = 2.5$; $[C_l] = 0.42$; $[C_d] = 1.3$.

The sum of these values is 4.7084.

For the complete network in Fig. 5.1b we have:

Max $\sum_{v=1}^{6} P_{deg}(v) = 0.24$; Max $[C_v] = 0.04371$; Max $[C] = 0.34$; Max $[L] = 3$; $\max[C_l] = 0.48$; $\max[C_d] = 1.54$.

The sum of these values is 5.6437.

Now the value of [I] can be computed: $[I] = 4.7084/5.6437 = 0.8342$.

This index shows that the network represented in Fig. 5.1b has a good intensity, the knowledge network between the members of the community of practice is structured

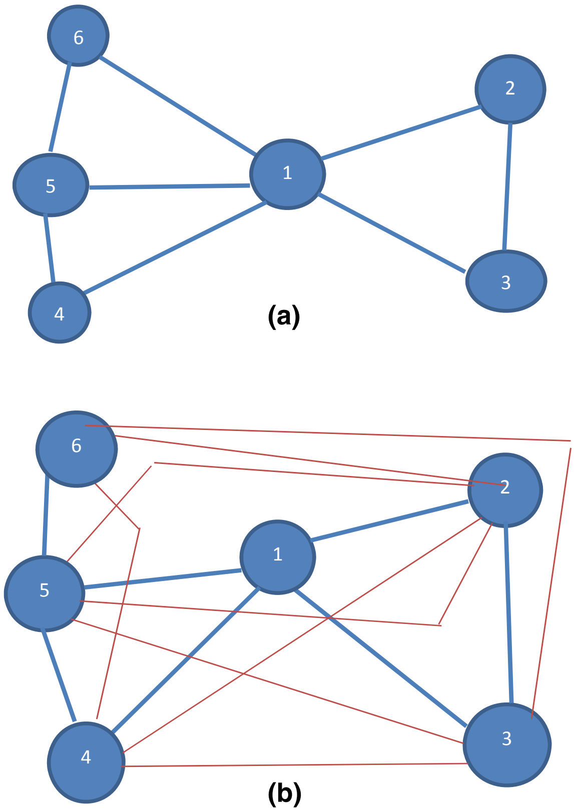

Fig. 5.1 **a** The relationships between the members of a community of practice. **b** The maximal relationships between the members of a community of practice

in such way that the members of the community has access to the knowledge that permit to have good professional performances.

(2) **The evolution of the grey knowledge flows in a health care network**

As an application of grey analysis of a knowledge networks we will intend to give a picture of the knowledge flows that appear in a rehabilitation process of a medical clinic (Institute National of Rehabilitation—INR) from Romania. The aims of this procedure will be:

(a) the identifications of typologies and number of agents involved in the knowledge structure of this process;
(b) the mapping of the relationships among agents with a focus on the exchanges of information and knowledge taking place in the network.

The above goals are realized in the following steps (Cross et al., 2002):

- **Identification of the elements of the network**: a number of four rehabilitation teams (clinics) with their patients were identified at the INR—a unique rehabilitation hospital in Romania. In general, it is recommended to pick up groups that cross physical, functional and even organizational boundaries; from this perspective, the INR meet all these requirements.

- **Design the survey**: in general, different surveys may be designed to identify different kind of networks, such as (i) Information network, (ii) Advice network, (iii) Learning networks, etc. The survey that we have designed particularly for the INR is used to reconstruct the existing interdependent systems between the components of the rehabilitation network.
- **Collect the data**: the survey was distributed through the members of the CoP from the Rehabilitation Clinic no. 3 and 4 within the INR. It is important to secure a very high response rate in the clinics (between 80 and 100%) to not miss key individuals in the mapping.

The following set of attributes is considered in the survey to characterize a node to node link (face-to-face communication):

- *The object of exchange*: which is the nature of exchanged information;
- *The frequency of exchange*: how often interaction happens;
- *The medium of exchange*: how exchange takes place? (face-to-face conversations, telephone, email, formal communication (e.g. standard letter, forms, bulletins etc.)).

The main elements of the network are a number of four rehabilitation teams (clinics) with their patients. They are identified as **A**, **B**, **C** and **D** clinics see the Fig. 5.1). Every clinic has a different number of agents (doctors, medical assistants, physiotherapists; physical-physiotherapist, physical therapist, nurses, statistician, clerk, and patients).

In the Fig. 5.2, the red circles are agents worked in Clinic 3, the blue squares are agents working in the INR but in other Clinics, and the black up triangles are external experts.

The network that results has around 82 agents, with approximately 132 arcs (links) between them. Specific of this type of networks is that these numbers exchange every day, for example can appear new patients, or external experts that a consulted of the clinics are more or less. The number of medical care agents (physicians and nurses) is relative stable and can vary with one or two agents. They are grey numbers, for example the maximum number of agents that are implied in the rehabilitation process is maximum 200, and the minimum limit of this number is 50 ($N\ (\otimes) = (50; 200]$. The maximum possible number of arcs is give of $N(N - 1)/2 = 3800$. The arcs are oriented but they don't have weights.

Usually, in the complex networks can appear some problems regard:

- Presence of bottlenecks in the flows (e.g. individuals that are too central and suffer from work or information overload);
- Lack of boundary spanners, i.e. individuals providing effective connection among diverse groups;
- Presence of isolated individuals whose more intense involvement in the process could be instead desirable;
- Lower than needed connection density;

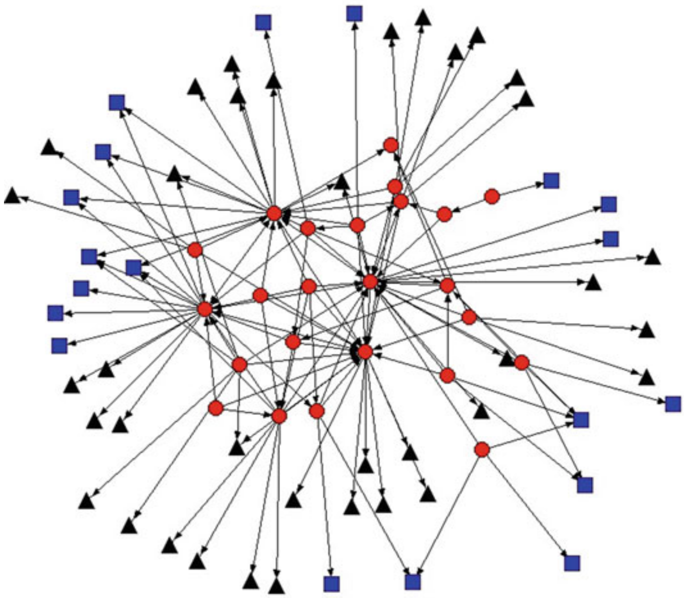

Fig. 5.2 The network of the Clinic 3

– Lack of knowledge hubs;
– Network fragmentation (presence of disconnected clusters).

By analyzing the network of INR, one or more of these problems we expect to face. Given the particularities of this health care organization, other structural deficiencies also could be found. These critical deficiencies must be identified and the network must be redesign, for example by increasing, reorientation, intensifying and introducing of new flows or agents. For instance, we may find out that we need a better communication among specialists in the different phases of the rehabilitation process for the purposes of a better traceability of the care process. Or we may discover that physicians need helps of external agents in some step of the diagnosis. Or that a better patient—physician communication has to be provided to support the rehabilitation in an efficient home care process.

We can extract a sub-graph that display the ways team members of the clinic no 3 seek for general advices in performing their activity (the size of each node is proportional to its degree centrality) (see the Fig. 5.3). Applying the network indicators, we have the following results:

– Also in this case the complex network of the IRC, it is very centralized;
– The agents who are more involved in knowledge sharing process are the following, namely for privacy agents A, actor B, actor C, actor D and actor E (5 principal agents);
– The total number of agents of the network is 15 (including the principal agents);

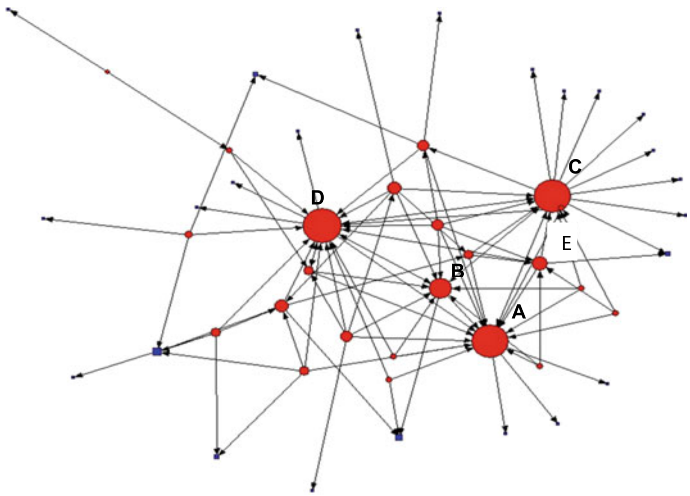

Fig. 5.3 A sub-graph of the clinic network

- There is an intensive use of external experts to get general advice.
- $\sum_{v=1}^{6} P_{deg}(\mathrm{v}) = 0.24$; $[C_v] = 0.0418$; $[C] = 0.28$; $[L] = 2.8$; $[C_l] = 0.47$; $[C_d] = 1.7$.
- The sum of these values is 5.5318.
- For the complete network in Fig. 5.3 we have:
- Max $\sum_{v=1}^{15} P_{deg}(v) = 0.28$; Max $[C_v] = 0.0617$; Max $[C] = 0.34$; Max $[L] = 3.5$; $\max[C_l] = 0.6$; $\max[C_d] = 1.72$. The sum of these values is 6.5017.

Now the value of [I] can be compute:
$[I] = 5.5318/6.5017 = 0.8508$.

This index shows that the last network represented in Fig. 5.3 has a very good intensity, the flows of knowledge between the members of the community of practice are diffused with a good speed and the member of this community send and receive a quantity of knowledge that permit to have good performances in the healthcare of the patients.

5.3 Conclusions and Future Works

The network does not tell us much about the ways agents use the knowledge, the motivations behind knowledge exchange, and the non–human knowledge sources used by subjects to perform their work,

Additional analyses have been carried out on the network humans/knowledge tools and resources, the organizational roles that are more involved in knowledge sharing, and the knowledge tasks that are more or less routinely performed by workers.

Specifically, we will try to answer to the following questions:

- Do the team members use the grey knowledge tools to perform their work?
- Which knowledge tasks they use their knowledge for?
- Which organizational roles are more involved in knowledge exchanges? Concerning the second question, for example, we have found that knowledge resources include a variety of sources: medical journals, books, internet, videos, etc.

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Chapter 6
Agent-Based Modelling in Grey Economic Systems

Camelia Delcea, Yingjie Yang, Sifeng Liu, and Liviu-Adrian Cotfas

6.1 Grey Economic Systems

The economic systems are basically grey systems due to their components and to their interactions which enable the occurrence of uncertainty.

First, the human component plays an important role as a consequence of its usually unpredictable and sometimes irrational behavior, a situation strictly related to the way the humans are thinking and acting. From here, it can easily be demonstrated that when analyzing a system, we are facing grey knowledge. This kind of knowledge exists and it represents that small piece of puzzle needed to successfully fill the gap separating the explicit knowledge form the tacit one, also conducting to uncertainty.

Second, it often happens that the human interactions, which may arise in the economic systems, depend on various, mostly unknown or unpredictable, environmental conditions, including here turbulences or on other conditions that may arise directly from the humans' ability to communicate, act and behave in certain situations.

The classical methods used in cybernetics for modeling these situations can often be considered obsolete as the human behavior is dynamic and certain emergent situations may appear from the human interactions, hard to notice from the very beginning of the economic analysis.

C. Delcea (✉) · L.-A. Cotfas
Bucharest University of Economic Studies, Bucharest, Romania
e-mail: camelia.delcea@csie.ase.ro

Y. Yang
Institute of Artificial Intelligence, De Montfort University, Leicester, UK

S. Liu
College of Economic and Management, Nanjing University of Aeronautics and Astronautics, Nanjing, China

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Therefore, the appearance of the agent-based modeling and its usage in different economic contexts along with the newly developed intelligent-artificial techniques, such as the grey systems theory, can be a powerful tool for many researchers in finding the proper answers in a wide variety of domains.

Thus, in the following we shall discuss a series of elements related to the simple and complex systems, economic systems, grey knowledge and the formation of grey economic systems, grey systems theory and its numbers, types of modeling, agent-based modeling, pros and cons in the use of a specific modeling approach.

In the end, an application made in NetLogo 6.0 using the advantages of the agent-based modeling and grey systems theory is presented for shaping the properties of the agents in online social environments (Gauzente & Roy, 2012), the online consumer behaviour (Horrigan, 2008), and how a firm should proceed in order to choose the appropriate number of sales agents in such an environment.

6.1.1 Simple Versus Complex Systems

Erdi (2007) describes a system as "a delineated part of the universe which is distinguished from the rest by a real or imaginary boundary" which wield a particular influence on the environment and, in the same time, is influenced by the environment (Érdi, 2007)—see Fig. 6.1.

On the other hand, the International Council of Systems Engineering (INCOSE) suggests that a system is a construct or collection of different elements that together produces results not obtainable by the elements alone.

In general, we can have the following types of systems: physical or notional with regard to the existence or inexistence of such a system; open or closed systems when considering the interactions such system can have with the environment—Ludwig von Bertalanffy (Bertalanffy & Sutherland, 1974) believes that the majority of biological and social systems are open systems; simple or complex systems—according to Érdi (2007) the properties of a simple system can be summarized as in Table 6.1. More, the complex systems possess the characteristics in Table 6.2.

Referring to the "butterfly effect" metaphor, one of the best way to describe it is using the Lorenz attractor. Such a simulation can easily be done by any of us through accessing the webpage: http://fractalfoundation.org/OFC/OFC-7-1.html. Figures 6.3, 6.4 and 6.5 present some of the simulations for the Lorenz attractor.

Fig. 6.1 The relation between a system and its environment

Table 6.1 The properties of a simple system

Single cause and single effect	
	When analyzing a system a single cause and a single effect is considered
A small change in the cause implies a small change in the effect	
	This small change is not similar all the time to a linear relationship between the cause and the effect, but it rather suggests that the causality between the cause and the effect won't be surprising Also, small changes in the structure of the system or in the parameters, if we speak about economic systems for example, do not qualitatively alter the system's behavior, so, in more fancy terms we can say that our system is "structurally stable"
Predictability	
	As we are not surprised by the outcome of the small change we have spoken above, we can say that the behavior of the system is predictable

Table 6.2 The properties of a complex system

Circular causality	
	Or, in simple words: the output influence the input For example, we can have A causes B causes C causes D which modifies or causes A again
Feedback loops	
	A feedback loop can be seen as a channel or pathway formed by an output returning to the system as an input and generating either more or less of the same effect In practice, we can have two types of feedback loops: positive and negative The positive feedback loop has a reinforcing effect on the system by driving the change of that system, while the negative feedback loop is balancing, dampening or moderating the system, offering stability Figure 6.2 is presenting the general schema for a feedback loop
Logical paradoxes	
Strange loops	
Small change in the cause implies dramatic effects	
	The expression "butterfly effect" became well-known, even in the popular culture
Emergence	
	Emergence is the process that creates new order together with self-organizing According to Goergen et al. (2010): "Emergence in a human system tends to create irreversible structures or ideas, relationships and organizational forms, which become part of the history of individuals and institutions and in turn affect the evolution of those entities"
Unpredictability	
	As in this case we are surprised by the outcome of the small change which produce dramatic effects in the output, the behavior of the system is unpredictable

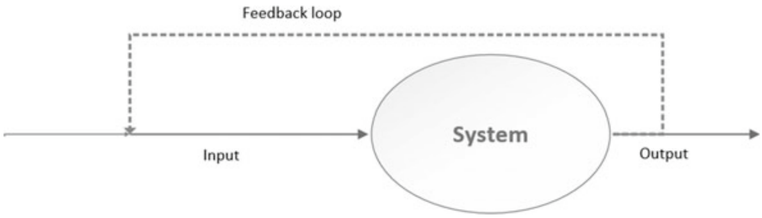

Fig. 6.2 The feedback loop

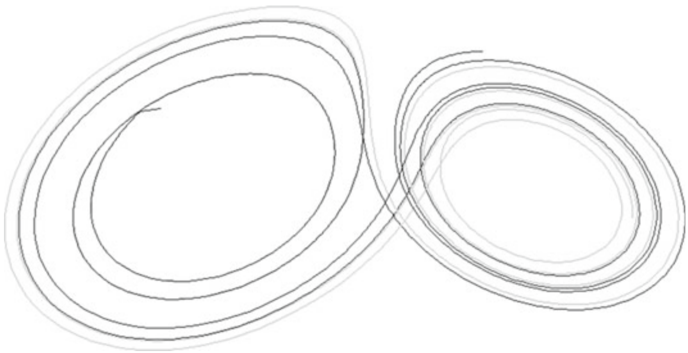

Fig. 6.3 Lorenz attractor (1)

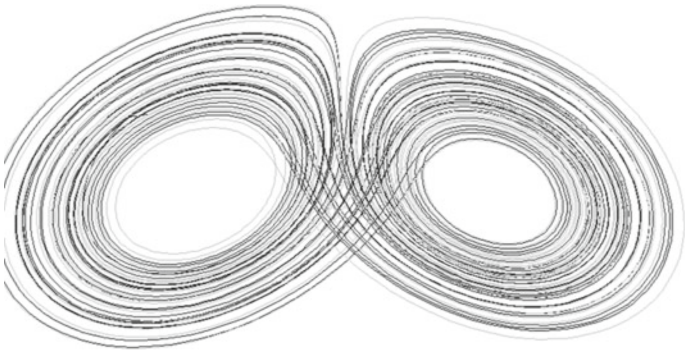

Fig. 6.4 Lorenz attractor (2)

First of all, let's define an attractor. So, the attractor is the solution or the pattern of behaviour that a system can reach. To that extent, we can find either simple attractors (for example a pendulum which will move back and forward when released until it reaches a fix point right in the middle and stops) or strange attractors (such as the Lorenz attractor which derives from the Lorenz's equations for the weather and behaves as in the figures above).

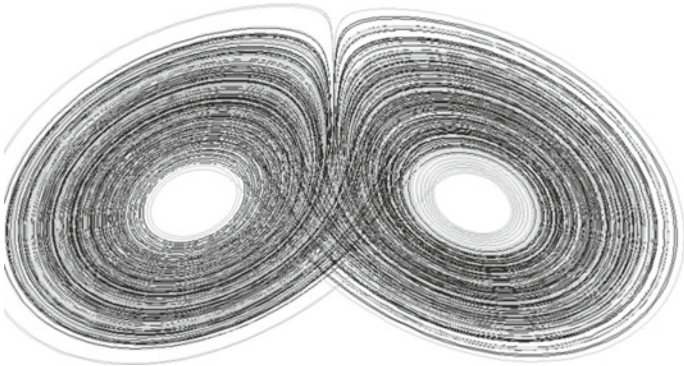

Fig. 6.5 Lorenz attractor (3)

In 1972 during a session on Global Atmospheric Research Program of the American Association for the Advancement of Science meeting, Edward Lorenz presented a lecture on the theme of "Predictability: Does the Flap of a Butterfly's Wings in Brazil set off a Tornado in Texas?" showing that some of the dynamical systems have a "sensitive dependence on initial conditions" and that small errors are subject of dramatic amplification (Érdi, 2007). This is one of the reasons why the effect of a flap in real life can be associated to a round-off error in the Lorenz's weather model.

Even though the "butterfly effect" is more like a hypothesis, it had multiple variations over the time and it has been used in real life to explain the political events of the stock market crashes. Considering the political events, the Palm Beach County Butterfly Ballot Controversy is one of the most well-known butterfly effect (if interested in reading more please go to http://www.authentichistory.com/1993-2000/3-2000election/3-dispute/).

6.1.2 Economic Systems

Taken individually, any part of the economy can be rightfully described as a system as it is formed by individual interconnected economic elements. The very nature of these interconnections that make each economic system emerge.

Properties such as: self-organization, emergence, interdependence, feedback, connectivity are easily to be identified in such complex adaptive system.

Basically, the economic system is formed by institutions, markets, economic unites and individuals strictly related one to another due to the knowledge flow and decisions which melts all the interactions among them—Fig. 6.6. But what can we say more related to the type of the interconnections, to the knowledge flows which are passing through the economic system? Do we know everything about them? Do we, at least, know a part of that knowledge in a specific moment of time? Can we

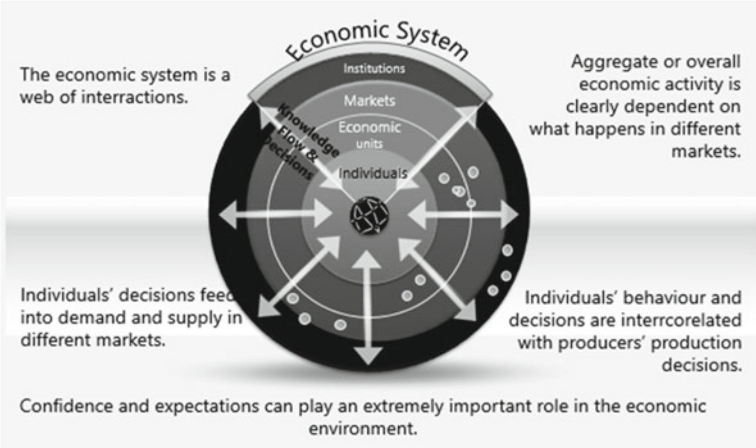

Fig. 6.6 The economic system

know more as the time passes? Are we really facing with tacit or explicit knowledge? Or, is it more there? We shall discuss this in the next section.

6.1.3 Grey Economic Systems

The whole idea of the grey economic systems is related to the knowledge that flows in the economic systems. As in the traditional economy, two main types of knowledge have been extensively used, namely the tacit and the explicit knowledge, with the creation of the grey systems theory an innovative approach may arise and a new type of knowledge: the grey knowledge.

This new type of knowledge is basically associated to the external and internal processes within the economic systems Fig. 6.7. For example, one can encounter this knowledge in externalized processes such as: the face-to-face conversations, sending e-mails, team interaction, online talks, contacting individuals, asking opinions, synchronous discussion, etc. More, the internalized feedback possesses this type of knowledge due to: experience, reflection, listening, observing, reading, emotion, evaluation. As none of these characteristics can be totally associated to implicit nor tacit knowledge, is the duty of the grey knowledge to fill all this gap in knowledge. Therefore, the grey knowledge is characterizing most of the processes in the economic systems, which make, in fact, that all these systems to be rather grey economic systems than just simple economic systems.

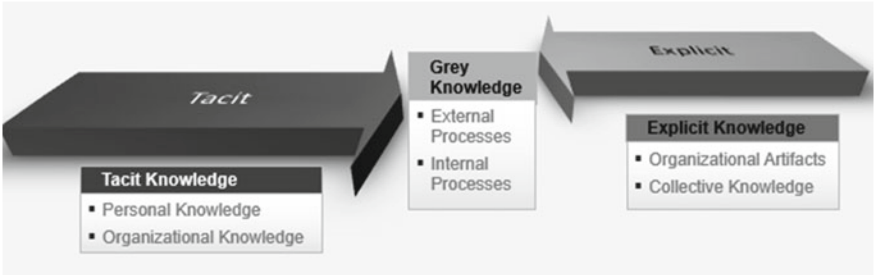

Fig. 6.7 The grey knowledge

6.2 Grey Numbers and Their Operations

Over the years, the grey systems theory has been used by a series of researchers from all over the world, covering different research areas, such as: mathematics, business economics, psychology, computer science, automation control systems, robotics, transportation, geology, engineering, environmental sciences, medicine and physics.

The main advantage provided by this theory is given by the fact that it offers good results on small data sets and in uncertain environment conditions.

As for the economic field, the grey system theory has proven to work well on studies in supply chain management, credit risk, decision-making process, financial performance evaluation, consumer income or investment efficiency, energy consumption, product development (as reported in (webofknowledge.com)).

Nowadays, the grey system theory has been applied along with other intelligent artificial techniques such as fuzzy set theory (Gao et al., 2016), genetic algorithms, neural networks, rough set theory or case-based reasoning in economic systems diagnosis and prediction. By combining the advantages brought by grey systems theory with the benefits offered by the other intelligent-artificial methods, a series of applications have been developed which have succeeded to improve the identified issue from a series of economic situations (Faroqi & Mesgari, 2015; Shqair et al., 2014; Vo et al., 2016).

One of the main features of the grey systems theory is given by the grey numbers. Their specific form and their particular arithmetic are going beyond the traditional modelling approaches, offering to the researcher a way of approaching the modelled system.

First, the grey number has a unique characteristic as while its exact value is not known, it contrives in offering information about a sphere in which it occurs. More specifically, in practical applications, the grey numbers are intervals or sets of numbers, being represented by the notation “$\otimes$”. A grey number is a number whose exact value is not known, but only a range in which that amount can be (Liu et al., 2016a, 2016b).

Second, due to its complex form, the grey number has a specific mathematic associated which, along with the whole grey numbers theory, represents the very base of the grey systems theory (Xie & Liu, 2010).

Considering $\otimes_{\boldsymbol{a}}$ and $\otimes_{\boldsymbol{b}}$, two grey numbers defined as:

$$\otimes_a \in \left[\underline{a}, \overline{a}\right]; \otimes_b \in \left[\underline{b}, \overline{b}\right]$$

The following arithmetic operations may occur (Liu & Lin, 2011):

- Sum: $\otimes_{\boldsymbol{a}} + \otimes_{\boldsymbol{b}} \in \left[\underline{a} + \underline{b}, \overline{a} + \overline{b}\right]$;
- Difference: $\otimes_{\boldsymbol{a}} - \otimes_{\boldsymbol{b}} = \otimes_{\boldsymbol{a}} + (-\otimes_b) \in \left[\underline{a} - \overline{b}, \overline{a} - \underline{b}\right]$;
- Inverse of a number $\otimes_a \in \left[\underline{a}, \overline{a}\right]$, with $\underline{a} < \overline{a}$ and $\underline{a} * \overline{a} > 0$, noted $\otimes_{\boldsymbol{a}}^{-1}$ is defined as: $\otimes_{\boldsymbol{a}}^{-1} \in \left[1/\overline{a}, 1/\underline{a}\right]$;
- Multiplication: $\otimes_{\boldsymbol{a}} * \otimes_{\boldsymbol{b}} \in \left[min\left\{\underline{a}\underline{b}, \underline{a}\overline{b}, \overline{a}\underline{b}, \overline{a}\overline{b}\right\}, max\left\{\underline{a}\underline{b}, \underline{a}\overline{b}, \overline{a}\underline{b}, \overline{a}\overline{b}\right\}\right]$
- Division: $\otimes_{\boldsymbol{a}}/\otimes_{\boldsymbol{b}} = \otimes_{\boldsymbol{a}} * \otimes_{\boldsymbol{b}}^{-1} \in \left[\min\left\{\frac{\underline{a}}{\underline{b}}, \frac{\underline{a}}{\overline{b}}, \frac{\overline{a}}{\underline{b}}, \frac{\overline{a}}{\underline{b}}\right\}, \max\left\{\frac{\underline{a}}{\underline{b}}, \frac{\underline{a}}{\overline{b}}, \frac{\overline{a}}{\underline{b}}, \frac{\overline{a}}{\underline{b}}\right\}\right]$
- Scalar multiplication: for a given positive real number k:

$$k * \otimes_{\boldsymbol{a}} \in \left[k\underline{a}, k\overline{a}\right]$$

Another particular property of the grey number is that adding or multiplying them cannot generally conduct to zero. Basically, the difference between any two grey numbers is not generally zero, unless these are identical. As a result, the division of two grey numbers cannot be equal to 1 only if they are identical (Liu et al., 2016b).

Comparison of Interval Grey Numbers

Comparing grey numbers is also a great feature offered by the grey systems theory and a very useful one as it allows obtaining accurate results for the grey decision-making problems.

To this issue, Xie and Liu have proposed for the first time a method for comparing grey numbers (Xie & Liu, 2010, 2011). In their study, the solution for discrete grey numbers and interval grey numbers is given.

The following premises have been used in their study, conducting to the results listed below:

- $\otimes_a \in \left[\underline{a}, \overline{a}\right]$ and $\otimes_b \in \left[\underline{b}, \overline{b}\right]$ are independent grey numbers;
- f(x) represents the probability density function of $\otimes_a$ and f(y) represents the probability density function of $\otimes_b$. Thus $\int_{\underline{a}}^{\bar{a}} f(x)dx = 1$ and $\int_{\underline{b}}^{\bar{b}} f(x)dx = 1$;
- If $f(x, y)$ represents the joint probability density function of $\otimes_a$ and $\otimes_b$, the probability $p(\otimes_a > \otimes_b)$ that $\otimes_a$ is greater than $\otimes_b$ is given by the formula:

$$p(\otimes_a > \otimes_b) = \begin{cases} 1 & \underline{a} > \overline{b} \\ \frac{(\overline{a}-\underline{b})^2}{2(\overline{a}-\underline{a})(\overline{b}-\underline{b})} & \underline{a} < \underline{b} < \overline{a} < \overline{b} \\ \frac{1-(\overline{b}-\underline{a})^2}{2(\overline{a}-\underline{a})(\overline{b}-\underline{b})} & \underline{b} < \underline{a} < \overline{b} < \overline{a} \\ \frac{(\underline{a}+\overline{a}-2\underline{b})(\overline{a}-\underline{a})}{2(\overline{a}-\underline{a})(\overline{b}-\underline{b})} & \underline{b} < \underline{a} < \overline{a} < \overline{b} \\ \frac{(2\overline{a}-\overline{b}-\underline{b})(\overline{b}-\underline{b})}{2(\overline{a}-\underline{a})(\overline{b}-\underline{b})} & \underline{a} < \underline{b} < \overline{b} < \overline{a} \\ 0 & \underline{b} > \overline{a} \end{cases}$$

For more insight on the grey numbers interval comparison, see Xie and Liu (2011).

6.3 Research on Agent-Based Modelling

Modelling is defined as the process of creating mathematical and conceptual frameworks for describing economic phenomena. In other words, the result of a modelling process is a conceptual or mathematical framework that describes how various concepts in economics actually work, all gathered within a model.

Therefore, a model is a simplified representation of the world (see Fig. 6.8). Or, in other words, we can say that models are approximations of the real world.

The model shouldn't be seen as something fixed and unchangeable, but rather like a continuous change process, where, as a result of repeated observation, simulation

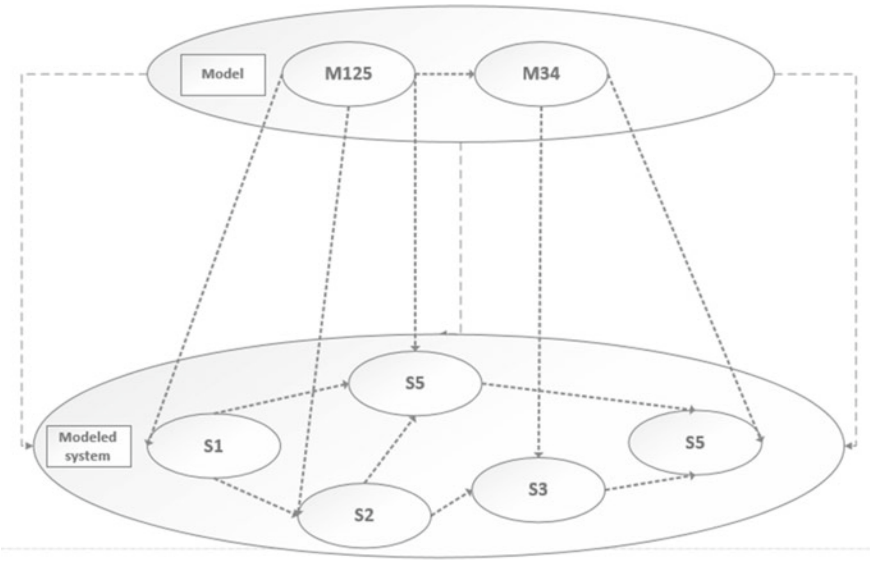

Fig. 6.8 A system and its model

and analysis, one can draw some conclusions and verify and validate his/her research. Also, based on these conclusions one can make various recommendations in order to improve the actual state of the system.

6.3.1 Economic Modelling

Considering the economic behavior of the agents and the fact that it takes time to respond to different changes that may occur in the economic environment, such as: changes in prices, in incomes or in situations, etc., overall, in the economy as a whole, it can easily be seen that it takes time for the effect of those changes to work their way entirely through the system and that the economic behavior is dynamic, no matter that we are referring to the micro or the macro levels (Ferguson & Lim, 2005).

As for the time needed for the economy to reach a new equilibrium after a shock, it can take a long time, and this is due to the fact that, in reality, in economy we do not have only a shock but rather a series of shocks which makes impossible sometimes for the economy to just settle down to a new equilibrium point. The economic reality has also proven over time that things do change and, due to this, even the identification of an equilibrium point can be hard to achieve sometimes, as, in some of the cases, is more probable to face a transitional point rather than an equilibrium one.

On the other side, when referring to the economic analyses, it can easily be seen that the whole modeling process is based on the equilibrium relations and assumptions. Ferguson and Lim believes that this is both a logical and normal thing to happen as "the equilibrium relations are generally determined by solving the optimization problem which drives the economic behavior" (Ferguson & Lim, 2005).

When exposed to shocks, the individual economic agents move towards a new equilibrium through the dynamic behavior they possess. This transition dynamics can be examined by either a discrete or a continuous analysis, which conducts us to the two types of equation-based modeling: with difference equations or with differential equations.

Depending on the values of "i" several situations may appear:

- First-order difference equations (FODE) when i = 1 and $Y_t = f(Y_{t-1})$;
- Second-order difference equations (SODE) when $Y_t = f(Y_{t-1}, Y_{t-2})$;
- Higher-order difference equations when keep adding lags to the Y variable.

More, we can have a case of more than one difference equation when referring to a system, and this is a "system of difference equations" situation.

As for the differential equations, most of the classical economic models are based on the assumption that the transition is dynamically done from one moment of time to another one.

In the recent years, a shift can be encountered from the equation-based modeling, to a new modeling approach based on the agents. The importance of the agent-based models is evident outside academia. Quoting the former president of the European Central Bank, Trichet (2010): "When the crisis came, the serious limitations of existing economic and financial models immediately became apparent. Arbitrage broke down in many market segments, as markets froze and market participants were gripped by panic. Macro models failed to predict the crisis and seemed incapable of explaining what was happening to the economy in a convincing manner. As a policy-maker during the crisis, I found the available models of limited help. In fact, I would go further: in the face of the crisis, we felt abandoned by conventional tools."

6.3.2 Agent-Based Modelling

Agent-based modelling (ABM) represents an alternative method of systems analysis and modeling, which allows solving problems by simulating the complex behavior of agents. ABM can incorporate different characteristics of the agents, that is very difficult to implement using other types of modeling (Rand & Rust, 2011). At the same time, ABM represents a form of computational modeling, in which a phenomenon is modeled in terms of agents and their interactions (Wilensky & Rand, 2015).

ABM has multiple applications being used successfully in various fields. Over the time, studies have been conducted on areas such as: computer science, engineering, environmental sciences, business economics, operations research, management science, mathematics and social science.

Focusing on the economic field, the main areas of interest targeted over the time included, but haven't been limited to: customers flows (Tan et al., 2008), transport systems (Monteiro et al., 2014), supply chain management (Walsh & Wellman, 2000), strategic simulation of the operation of market mechanisms (Wang et al., 2013), operational risk and organizational networks (Schuetze et al., 2006).

Among these economic areas studied using ABM, a particular research area is given by the human systems simulations. This domain allows modeling and simulating the real-world business issues. These models are based on human nonlinear behavior assumption, if–then rules, memory and agents' path-dependence, capacity of learning and adaptation. As a result, the self-organization and emergence behavior is very often encountered in such simulations which make their models to be relatively close to real-economic world we are facing, composed of humans and economic entities with an increased complex behavior. From here, another region of interest appears related to the coupled human and natural systems. The research on this topic is targeting the feedback loops, nonlinearity, thresholds, resilience and time lags (An, 2012).

As for the industry, over the times, the companies have come to understand the important role played by ABM and have used them extensively for improving their performance and revenues (some examples are: Procter and Gamble, Sainsbury, Macy's). Even more, the Government authorities have chosen ABM for traffic management optimization; while in finances the ABM was used in simulations of real stock markets.

6.3.3 Some Examples

According to ISI Web-of-Science (webofknowledge.com) in 2003–2017 period, 368 papers have been written on different topics using NetLogo (Table 6.3). Most of them are in areas such as: computer science, engineering, education and educational research, social sciences, business economics, mathematics, environmental sciences and ecology, operations research and management, etc.

As regarding to the country of the published authors, most of the papers are from USA and China, while 10 of them are from Romania (Table 6.4).

Analyzing their content, we shall shortly present some of the areas in which these papers have been written, what was their main idea, how their authors decided to conduct the research, the employed methods and some of the results, on the purpose of helping the reader to better understand why NetLogo is a popular software when dealing with agent-based modeling and simulations. More, knowing how this program has been used by various researchers, the reader can find his/her own applicability for such model in his/her life.

All the selected papers are extracted from the WoS database (webofknowledge.com) and are presented in the following. We will start first with some papers on the transportation, distance traveled in warehouses and multi-modal adaptive survey systems.

Title: An Alternative Approach for High Speed Railway Carrying Capacity Calculation Based on Multiagent Simulation (Gao et al., 2016).

Main idea: The authors use the multi-agent theory and NetLogo in order to model the presented situation and perform a case study on the Beijing-Shanghai high speed

Table 6.3 NetLogo papers—publication year

Publication year	Record count	% of 368 (%)
2015	60	16.304
2012	51	13.859
2013	50	13.587
2014	43	11.685
2016	39	10.598
2009	36	9.783
2010	23	6.250
2011	21	5.707
2008	14	3.804
2007	12	3.261
2006	5	1.359
2017	5	1.359
2005	4	1.087
2004	3	0.815
2003	2	0.543

Table 6.4 NetLogo papers—authors' country

USA	74	20.108%
Peoples R China	64	17.391%
Germany	21	5.706%
Spain	20	5.435%
Italy	16	4.348%
Australia	14	3.804%
Czech Republic	13	3.533%
England	13	3.533%
France	13	3.533%
India	12	3.261%
Netherlands	11	2.989%
Brazil	10	2.717%
Romania	10	2.717%
Portugal	9	2.446%
Iran	8	2.174%
South Korea	7	1.902%
Greece	6	1.630%
Poland	6	1.630%
Scotland	5	1.359%
Other	36	9.783%

railway in order to validate the proposed model. After conducting the first experiment, it has been concluded that the carrying capacity from Beijing South railway station to Shanghai Hongqiao railway station is of 55 trains. More, average speed seems to reach the maximum value of 320 km/h at the end of first hour, while the average speed falls gradually and reaches the minimum value of 165 km/h in the twelfth hour. As it has been observant that in the first experiment the train departure interval is too small and it leads to train congestion, the authors conduct even a second experiment in which they apply an adjustment strategy: when the average delay is more than 1.5 min, the departure interval is gradually increased. The results obtained show that the carrying capacity is of 77 in this case, while the speed ranges between 320 km/h (its maximum value) and 250 km/h (its minimum value).

Title: Micro-Simulation of Car Drivers' Movements at Parking Lots (Vo et al., 2016).

Main idea: The authors started from the fact that various problems such as: safety, congestion, environmental effects, etc. may be due to the movement the drivers are making across and within the parking facilities. For this, they proposed an agent-based model in NetLogo which allows to the used to easily adjust the parking facilities features. Even though it just considers the interaction between the driver and the parking lot, while excluding other variables such as: pedestrians, drivers' experience and expectations, parking location, the results are encouraging for future developments on this area as they are indicating that the speed limit and maximum parking duration have effects on the parking efficiency.

Title: A statistical study employing agent-based modeling to estimate the effects of different warehouse parameters on the distance traveled in warehouses (Shqair et al., 2014).

Main idea: The paper deals with the cost issues that may be encountered in the manually operated warehouses due to the distance needed to be traveled by the order picker. For this, the authors are proposing an ABM model for which a series of variables have been taken into account: number of aisles, aisle length, number of storage blocks, storage assignment policy, routing policy and order size. More, the study is conducted by considering more than 324 possible designs. As a result, the authors have found that all the considered parameters are affecting the travel distance and suggest that the research can continue by considering a fishbone warehouse configuration or different picking strategies (e.g., zone picking, wave picking).

Title: Agent-based Crowd Simulation Considering Emotion Contagion for Emergency Evacuation Problem (Faroqi & Mesgari, 2015).

Main idea: The authors incorporate emotions into the simulation of an emergency situation and they assume that, in such situation, an emotional contagion may arise. For this, six levels of emotions are considered: calm, alarm, fear, terror, panic and hysteria. Three types of agents are considered: adults (with six levels of emotions), children (with just two levels: calm and hysteria) and security persons (with one level: calm).

The room is represented by a squared space, with three exits. Four situations are considered, having different number of agents from each group. As a conclusion, it has been observed that the needed proportion in order to save as much people as possible in 1/35 (between the number of security members and the number of adults and children).

Title: Agent-based simulation of alternative classroom evacuation scenarios (Liu et al., 2016a).

Main idea: The authors underline the importance of conducting fire drills with students by considering two situations: students evacuating without instructions and students evacuating in good order.

More, the paper is also studying the interrelationships between evacuation efficiency and the classroom layouts (here two types of classrooms have been considered: with one exit or with two exits). One of the results suggests that the classroom with two exits shortens students' evacuation time, while another one suggests that the students who follow preset instructions escape faster and insure better their safety during the evacuation process.

Title: Individual-Oriented Model Crowd Evacuations Distributed Simulation (Gutierrez-Milla et al., 2014).

Main idea: This paper is also about people evacuation and the authors consider that the fire drills are not a realistic situation if one wants to understand people behavior. Therefore, the authors provide a distributed simulator for this problem use a real environment to test it, namely one of the Fira de Barcelona pavilions. A number of

62.820 runs have been used for testing and a number of agents between 3.000 and 15.000. For 15.000 individuals, the authors concluded that the execution time for one simulation was 174 s and that the proposed model works well when one wants to see how a large number of people try to reach the emergency exit of such a building.

A series of differences can be encountered when comparing equation-based modeling (EBM) with agent-based modeling (ABM). First of all, one can see differences regarding the population: in EBM the population is seen as a group of homogenuous individuals, with the same behaviour, which enables the modeling of all these agents as a whole, using a unique set of equations, while in the ABM each individual is characterized by a set of rules of behaviour, which makes the population as a whole to be a set of heterogenous individuals. Second, the connections between variables are different in the two approaches, while in EBM one can encounter continuous interactions between variables, the ABM require uses discrete interactions. Another difference is given by the knowledge of the aggregate phenomena and level of understanding—related to this issue, the EBM requires a good understanding of the aggregate behavior, while ABM demands knowledge of the commonsense behavior and a high level of understanding to stakeholders.

Considering the economic applicability, it can be mentioned that it differs also from one type of modeling to another. Thus, EBM folds mostly on macroeconomic analysis as the population is seen as homogenous while ABM focus on microeconomic analysis as each single agent can be easily characterised and can receive simple rules in order to shape and observe the behaviour of the whole population. As for the approach, the EBM uses a top-down approach, with low-detailed results, starting from the observed aggregate phenomenon to the single individuals behaviour; while the ABM uses bottom-up approach, with detailed results as it considers each individual way of behaving and it observes the aggregate beheviour of a certain group of individuals.

6.4 Influence in Online Social Media Environments

In the following we are presenting an application based on grey systems theory, modelled using ABM, regarding the influence consumers can have in the online social media environments one to another. For this, we have considered the advantages provided by the grey systems theory related to the grey numbers and their arithmetic to better shape the rules of behaviour for the agent in ABM.

6.4.1 Prerequisites

The starting research question for our study is "how many sales agents (in real social environments) or electronic word-of-mouth agents (in the online social environments) are needed in order to influence the buying decisions of the other participants to discussions compared to the total number of persons taking part in a product-related discussion?".

For this, a series of interviews have been conducted with persons working in the sales force management of different Romanian companies on the purpose of better determining the profile of the sales agents. More, questionnaires have been used for shaping the behaviour of the consumers in the online environments. The questionnaires have been used on 367 persons having the age over 18 years-old, from April 2016 to September 2016 and have tried to extract information related to the main triggers that determine a person to buy a specific product, if the buying decision depends on the type of goods bought (regular/luxury goods, soft/durable goods, etc.), to which extent the buying decision is affected by the person's income, or whether an individual can be influenced by another individual in making his/hers own buying decision.

Basic and common-sense observations have been drawn from the results we have received such as: everyone has an opinion related to a product or a good, this opinion can be influenced by other persons depending on their persuasion capacity and even that wealth can contribute to making a buying decision especially if we were talking about luxury/regular goods. On the other side, we have found that each person has an opinion related to a product or good even they have never use it and that each person can, on his/hers turn, influence another one even though the person has only heard about a product and never use it.

These elements have been used in NetLogo 6.0 in order to create the proper rules of behaviour for the agents. Three categories of agents have been created based on the answers received through the questionnaire and through the face-to-face interviews to the sales managers:

Group A—sales agents or electronic word-of-mouth agents—the individuals belonging to this group have the purpose to promote the products in online environments and to initiate discussions in various groups for better promoting the good opinion on a product or to sale it to the other two groups. For this, they are creating positive electronic word-of-mouth in the online environment and they share a good opinion over the products they are selling. More, they are completely not susceptible to be influenced by the other agents—so they cannot be influenced by the agents belonging to the other two groups. They possess a high persuasion capacity which can slightly vary depending on the agent's wellbeing, focus, determination, tiredness and other subjective matters that can affect him at a certain moment of time.

Group B—buying agents with an over-average income—which are both participating to the discussions in the online environments, but they can also initiate new discussions related to a specific good or product. A series of attributes can be associated to the agents from this group such as: opinion (their own opinion over a product which can change due to interaction with other agents), susceptible to be influenced (which express to what extent the influence of another agent on the current agent conducts to its opinion change), persuasion (which catch how influent that agent is and how powerful its persuasion is) and wealth (the value of the wealth belonging to each individual, this value being higher than the value of the same variable in group C—used here just to simply split the agents along the two categories). More, the agents' opinion may change, taking values between a very bad opinion and a very good opinion. Also, the persuasion capacity enables agents to influence other agents based on their level of persuasion. As a result, each agent can be influenced by other agents and can influence on its turn depending on its characteristics.

Group C—buying agents with an average and under-average income—these are also participating and initiating new discussions in online social media environments. Generally, they possess the same attributes as the agents from group B, such as: opinion, susceptible to be influenced, persuasion and wealth.

Considering the questionnaire, it has been observed that making a buying decision is different between group B and C depending on the good or product category we are referring to. For example, in the regular soft or durable goods the agents belonging to the two categories are actin g in the same way: they will buy it based on their need (so we should consider, for each agent, the value recorded in the need variable). On contrary, if we are dealing with a luxury soft good, the B group which have higher values for the wealth, will just buy it based on their need or desire (in this case we shall monitor the values of these two variables), while the C group, with lower wealth values, will buy the product or good only if the desire associated with it exceeds a certain threshold (so, it is just based on the desire level). Last, for the luxury durable goods category, it is reserved just for the group B members which desire to buy these goods based on their desire level, which should exceed a certain threshold. As for group C, the members of this group will decide not to buy it but rather to replace it with a regular durable good as this one has a more affordable place. Thus, even though the group B is not buying, they have an opinion on that product and can influence the opinion of the other agents (being B group members of even C group members), which can lead to different final buying decision for group C members.

All these aspects have been implemented in NetLogo 6.0. as presented in the next section.

6.4.2 NetLogo Implementation

The three groups have been created in NetLogo 6.0. (see Figs. 6.9 and 6.10) and with the attributes from Table 6.5.

Additionally to the observations in Table 6.5, it should be added that the agents will buy only if their opinion is within the [0.8, 1] interval and for:

- **the regular soft or durable goods**: Need > 0.5 (for each agent in group B or C);
- **the luxury soft goods**: Need > 0.5 or Desire > 0.7 for the agents in group B, while for the agents in group C: Desire > 0.7;
- **the luxury durable goods**: Need > 0.5 or Desire > 0.7 for the group B agents, while group C agents decide not to buy as they are replacing these products with similar durable goods.

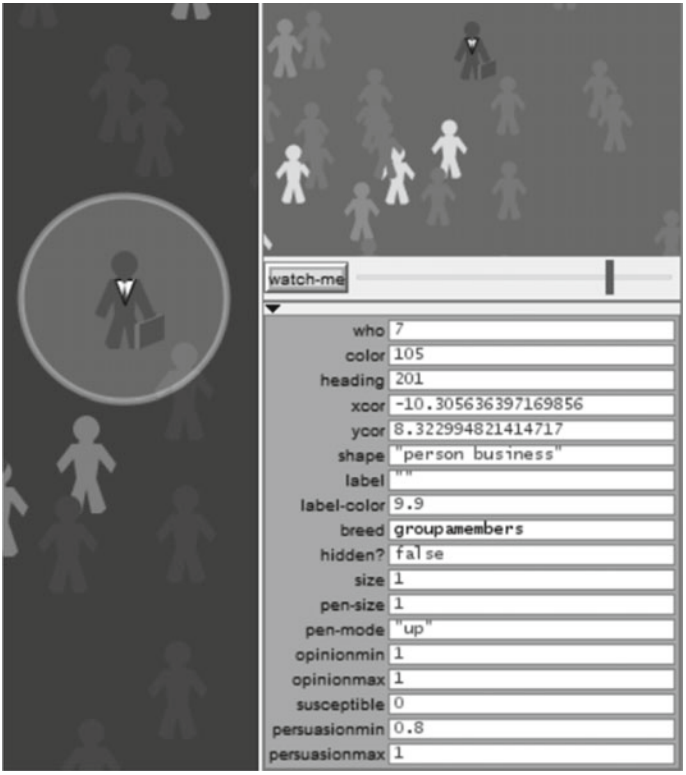

Fig. 6.9 Example of group A member (NetLogo 6.0)

For better shaping the agents, at the beginning of the simulation all the agents from group B and C have a fixed value for opinion. As the time goes by and the agents interact, their opinion can change based on the formula:

$$OpinionMax_X^t = \begin{cases} \min\begin{pmatrix} OpinionMax_X^{t-1} + \\ + OpinionMax_Y^{t-1} * Susceptible_X^{t-1} * PersuasionMax_Y^{t-1}, \\ 1 \end{pmatrix} \\ if\ probability > Interval - Comparison - Probability - Threshold \\ OpinionMax_X^{t-1}, otherwise \end{cases}$$

$$OpinionMin_X^t = \begin{cases} \min\begin{pmatrix} OpinionMin_X^{t-1} + \\ + OpinionMin_Y^{t-1} * Susceptible_X^{t-1} * PersuasionMin_Y^{t-1}, \\ 1 \end{pmatrix} \\ if\ probability > Interval - Comparison - Probability - Threshold \\ OpinionMin_X^{t-1}, otherwise \end{cases}$$

As a result, an interval grey number will be determined for this variable, for each agent in groups B and C.

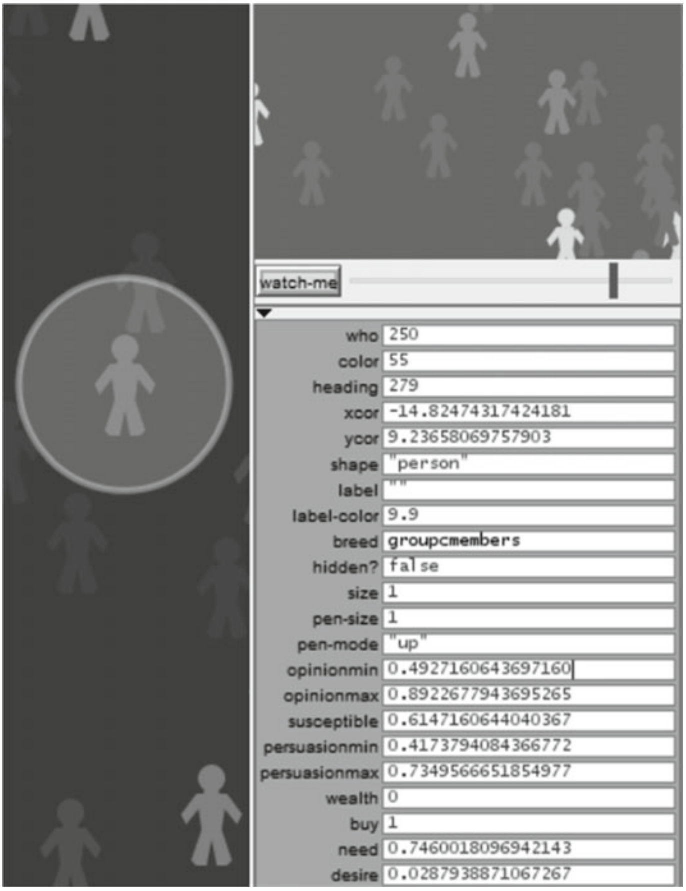

Fig. 6.10 Example of group B member (NetLogo 6.0)

Here $OpinionMax_X^t$ is the maximum value of $\otimes Opinion$ for agent X after discussing with agent Y, while $OpinionMin_X^t$ is the minimum value of $\otimes Opinion$ for agent X after discussing with agent Y at moment t. The probability is determined by comparing the two grey numbers of $\otimes_a \in \left[\underline{a}, \overline{a}\right]$ and $\otimes_b \in \left[\underline{b}, \overline{b}\right]$ (as presented above in the grey numbers section):

$\underline{a} = Susceptible_Y^{t-1} * PersuasionMin_X^{t-1}$

$\overline{a} = Susceptible_Y^{t-1} * PersuasionMax_X^{t-1}$

$\underline{b} = Susceptible_X^{t-1} * PersuasionMin_Y^{t-1}$

$\overline{b} = Susceptible_X^{t-1} * PersuasionMax_Y^{t-1}$

The "Interval-Comparison-Probability-Thresholds" has been modelled through a slider, allowing it to be more suitable for all types of situations (in our case we have set the value of it at 0.7).

Table 6.5 The implied variables

Variable	Type	Observations
Opinion	Grey number $\otimes Opinion = [OpinionMin, OpinionMax]$	Group A: the opinion has been associated to 1 Groups B and C: agents can have different opinions in [−1, 1]
Susceptible	Fixed value/Random value	Group A: cannot be influenced and change their opinion, therefore they will have the value of 1 Groups B and C: a random number in [0, 1)
Persuasion	Grey number $\otimes Persuasion = [PersuasionMin, PersuasionMax]$	Group A: the persuasion is a grey number within [0.8, 1] as it is subjective to their mood Groups B and C: a grey number in [0, 1]
Wealth	Binary value	It is a characteristic of groups B and C and it takes only two values: 0 for group C and 1 for group B
Need	Random value	Specific to groups B and C, having random values in [0, 1), 0 being associated to inexistent need
Desire	Random value	Also for groups B and C, taking random values in [0, 1), 0 being associated to inexistent desire
Buy	Binary value	It is a characteristic of groups B and C and it takes only two values: 0 (the agent is not going to buy the product) and 1 (the agent buys the product)

The Agent-Based Model Simulation

Based on the data provided by the MonitorSocial website (https://monitorsocial.ro/indicator/numarul-contractelor-de-munca-si-salariile-romanilor/), we have extracted the data regarding the Romanian economy and the individuals average wage, which made us consider the following values for simulation: the number of group A members within 0.1 and 0.5%, group B members 12% (held fixed at this value during simulations) and group C members: the rest of them. An extract from the program can be visualized in Figs. 6.11 and 6.12.

More, we have assumed that the total number of population is 1000 persons (also fixed), and the time frame for simulation: 100 time units (at each time unit each agent explores the virtual space and encounters other agents to whom it interacts). The three possible types of goods: regular soft or durable goods, luxury soft goods and luxury durable goods have been considered for simulation—see Fig. 6.13.

The BehaviourSearch software tool (Stonedahl & Wilensky, 2011) was used along with the ABM model for automating the exploration of the model and a faster gathering of the results.

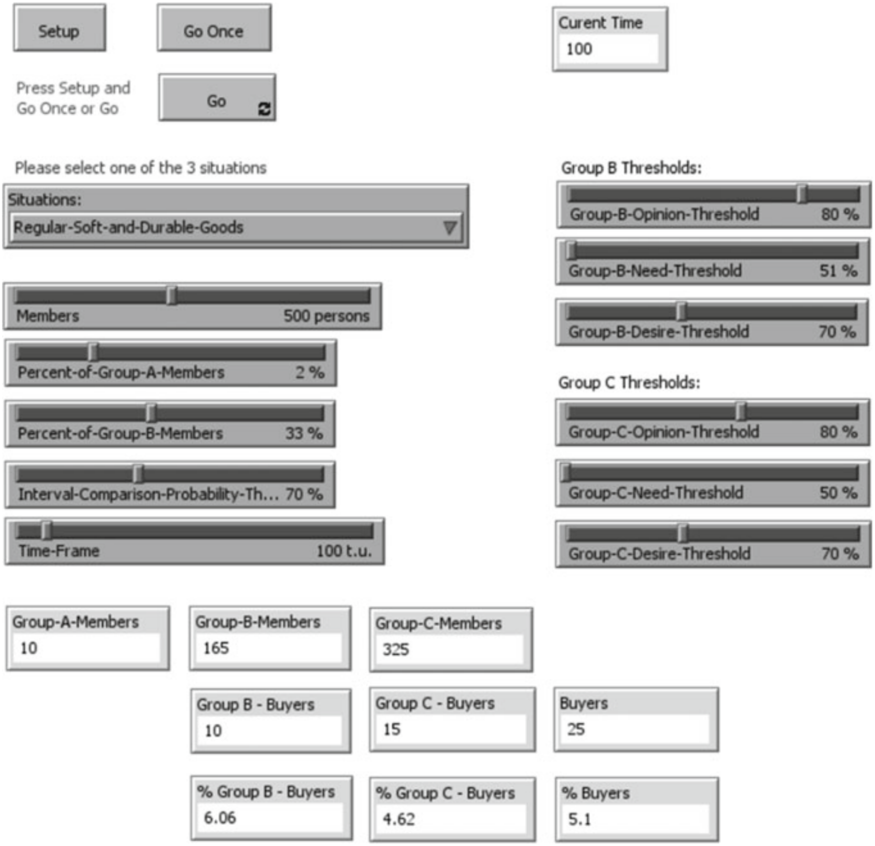

Fig. 6.11 The proposed model (1)

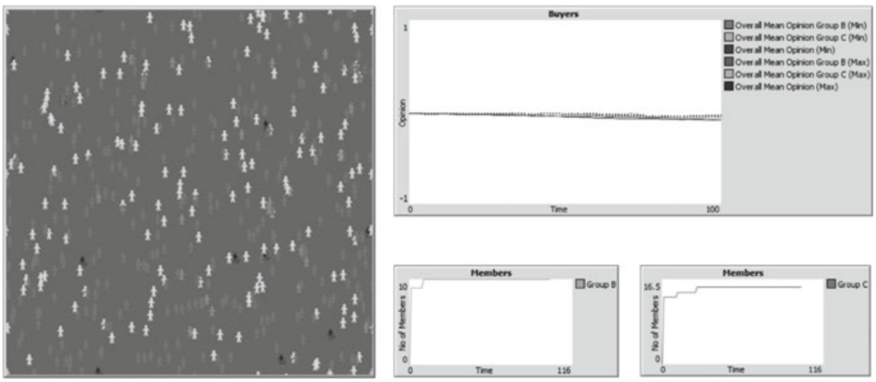

Fig. 6.12 The proposed model (2)

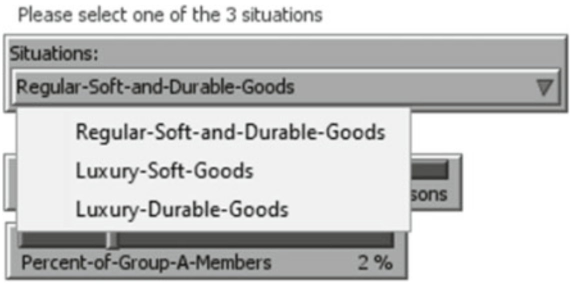

Fig. 6.13 The types of situations considered within the model

Table 6.6 Simulations results

% group A	% group B	% group C	Best	Average
Regular soft and durable goods				
0.2	4.4	95.4	52.91	45.04
0.2	1.6	98.2	51.84	45.61
0.2	10.1	89.7	52.76	45.99
Luxury soft goods				
0.3	10.5	89.2	36.11	30.32
0.4	10.4	89.2	36.26	29.88
0.3	10.7	89.0	36.42	29.51
Luxury durable goods				
0.1	11.7	88.2	9.43	6.95
0.1	12.0	87.9	9.74	7.11
0.1	11.9	88.0	9.34	7.18

It has been proven that this tool offers the proper heuristic techniques and the genetic algorithms for searching the involved parameter space. In our case, this tool gave us the possibility to search within a space of $5 * 120 * 101 = 6060$ possible combinations of parameter settings for each of the three situations. Additionally, we have decided to run each of these simulations in 3 sets, each set containing 9000 times model's run. The average results obtained over the 3 sets are summarized in Table 6.6.

6.5 Concluding Remarks

The present paper explores the modeling techniques that can be employed when facing the grey economic systems. For this, a short introduction to the modeling and its main characteristics is made, along with the explanation of the new idea of "economic grey systems". Elements related to the grey knowledge are exposed and underlined at the beginning of the paper. More, the elements taken from the grey systems theory are representing a valuable base which allows to the researcher a better approach of the economic applications.

When dealing with modeling, two main approaches are possible: the agent-based modeling (ABM) and equation-based modeling (EBM). For better choosing among them, the paper presents some of the pros and cons depending of the expected results, the implied population, the type of the research, the type of approach, etc. More, some of the newest models developed using ABM are presented, all of them created in NetLogo 6.0.

In the end, an application based on grey systems theory and its numbers and on ABM has been created on the purpose of determining the number of sale agents needed for conducting a fruitful marketing campaign in online environments. Looking at the results we can underline the following: contrary on what one could have expected, the data in Table 6.6 prove that, the number of group A agents (namely the sales agents) doesn't need to be at its maximum in order to obtain the highest number of new customers for a good or product.

Also, a few differences among the three considered cases (regular soft or durable goods, luxury soft goods and luxury durable goods) can be underlined: for the regular goods the presence of 0.2% group A agents is enough for achieving the maximum number of customers, while the luxury soft goods need more group A agents: 0.3% as they have to convince even the group C members to buy even though they will buy only based on their desire level. Last, for the durable luxury goods one just need the presence of 0.1% selling agents.

Thus, it can be observed that while for the luxury goods the average number of group B members tends to be at its higher value (10–12% of the population), for the regular goods the percent of group A or group B members doesn't count too much in the final result.

Another interesting result is related to the number of the actual customers in the three situations. First, it can be observed that both the best number and the average number of customers tend to be the same over a certain situation. Looking at the numbers, it can easily be seen that the highest number is registered in the regular goods situation where, on average, in 100 iterations, 45 customers will decide to buy the product, almost 30 in the luxury soft goods and only 7 in the durable soft goods (from 1000 persons considered in the study).

Even though the study has its limitations, the present research can be extended in the future in order to overcome the shortcomings.

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ANNEX—The NetLogo Program for Consumer Opinion

```
breed [ groupAmembers groupAmember ]

breed [ groupBmembers groupBmember ]

breed [ groupCmembers groupCmember ]



turtles-own [

  OpinionMin

  OpinionMax

  Susceptible

  PersuasionMin

  PersuasionMax

]
```

```
groupBmembers-own [
  Wealth
  Buy
  Need
  Desire
]

groupCmembers-own [
  Wealth
  Buy
  Need
  Desire
]

to setup
  clear-all
  ask patches [ set pcolor 4 ]
  set-default-shape groupAmembers "person business"
  set-default-shape groupBmembers "person"
  set-default-shape groupCmembers "person"
  setup-people
```

```
 reset-ticks
end
to setup-people
   create-groupAmembers Percent-of-Group-A-Members * Members
/ 100 [
   setxy random-xcor random-ycor
   set color blue
   set OpinionMin 1
   set OpinionMax 1
   set Susceptible 0
   set PersuasionMin 0.8
  set PersuasionMax 1
  ]
 create-groupBmembers Percent-of-Group-B-Members * Members /
100 [
  setxy random-xcor random-ycor
  set color yellow
  set Wealth 1
  set OpinionMin random-float 2 - 1
  set OpinionMax OpinionMin
  set Susceptible random-float 1
  set PersuasionMax random-float 1
```

```
    set PersuasionMin random-float PersuasionMax

    set Need random-float 1

    set Desire random-float 1

    set Buy 0

  ]

   create-groupCmembers Members - count groupAmembers - count

groupBmembers [

    setxy random-xcor random-ycor

    set color orange

    set Wealth 0

    set OpinionMin random-float 2 - 1

    set OpinionMax OpinionMin
    set Susceptible random-float 1

    set PersuasionMax random-float 1

    set PersuasionMin random-float PersuasionMax

    set Need random-float 1

    set Desire random-float 1

    set Buy 0

  ]

 end


to go

  change-opinion
```

```
  change-buying-decision
  move
  tick
  if ticks = Time-Frame
  [
    stop
  ]
end

to change-buying-decision
  ask groupBmembers
  [
    ifelse Situations: = "Regular-Soft-and-Durable-Goods"
    [
      if OpinionMin >= Group-B-Opinion-Threshold / 100 and Need
>= Group-B-Need-Threshold / 100
      [
       mark-and-buy
      ]
    ]
    [
      ;ifelse Situations: = "Luxury-Soft-Goods" or Situations:
="Luxury-Durable-Goods"
```

```
    if OpinionMin >= Group-B-Opinion-Threshold / 100 and (Need
>= Group-B-Need-Threshold / 100 or Desire >= Group-B-Desire-
Threshold / 100)
     [
       mark-and-buy
     ]
   ]
 ]
 ask groupCmembers
 [
   ifelse Situations: = "Regular-Soft-and-Durable-Goods"
   [
    if OpinionMin >= Group-C-Opinion-Threshold / 100 and Need
>= Group-C-Need-Threshold / 100
    [
     mark-and-buy
    ]
   ]
   [
    if Situations: = "Luxury-Soft-Goods"
    [
      if OpinionMin >= Group-C-Opinion-Threshold / 100 and Desire
```

```
>= Group-C-Desire-Threshold / 100
      [
        mark-and-buy
      ]
    ]
   ]
 ]
end


to mark-and-buy
  set Buy 1
  set color green
end
to change-opinion
  ask  turtles
   [
     ask other turtles-here
     [
       let xOpinionMin [OpinionMin] of myself
       let yOpinionMin OpinionMin
       let xOpinionMax [OpinionMax] of myself
       let yOpinionMax OpinionMax
```

```
let xSusceptible [Susceptible] of myself
let ySusceptible Susceptible
let xPersuasionMin [PersuasionMin] of myself
let yPersuasionMin PersuasionMin
let xPersuasionMax [PersuasionMax] of myself
let yPersuasionMax PersuasionMax
let a1 ySusceptible * xPersuasionMin
let b1 ySusceptible * xPersuasionMax
let a2 xSusceptible * yPersuasionMin
let b2 xSusceptible * yPersuasionMax

let probability 0
ifelse a1 > b2
[ set probability 1
 ]
[ifelse a1 < a2 and a2 < b1 and b1 < b2
 [set probability (b1 - a2) * (b1 - a2)/ (2 * (b2 - a1) * (b2 - a2))
 ]
 [ifelse a2 < a1 and a1 < b2 and b2 < b1
  [set probability (1 - (b2 - a1) * (b2 - a1))/(2 * (b2 - a1) * (b2 -
a2))
  ]
```

```
      [ifelse a2 < a1 and a1 < b1 and b1 < b2
        [set probability (a1 + b1 - 2 * a2) * (b1 - a1) / (2 * (b2 - a1)
* (b2 - a2))
        ]
        [ifelse a1 < a2 and a2 < b2 and b2 < b1
          [set probability (2 * b1 - b2 - a2) * (b2 - a2) / (2 * (b2 - a1)
* (b2 - a2))
          ]
          [set probability 0
          ]
        ]
      ]
    ]
  ]
  if probability > Interval-Comparison-Probability-Threshold
  [set OpinionMax yOpinionMax + xOpinionMax * ySusceptible
* xPersuasionMax
  if OpinionMax > 1
  [set OpinionMax 1
  ]
  set OpinionMin yOpinionMin + xOpinionMin * ySusceptible *
```

```
xPersuasionMin
        if OpinionMin > OpinionMax
        [set OpinionMin OpinionMax
        ]
        if OpinionMin < -1
        [set OpinionMin -1
        ]
        ]
      ]
    ]
end


to move
  ask turtles
  [
    rt random 50
    lt random 50
    fd 1
  ]
end
```

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Chapter 7
Forecasting and Optimization of Short-Term Traffic Flow Dynamics Modeling Based on Grey System Theory

Xinping Xiao, Huan Guo, and Jinwei Yang

7.1 Introduction

With the tremendous development of the economy and the acceleration of urbanization, traffic congestion has become an inevitable problem for all countries in the world. Intelligent transportation system (ITS) is deemed to be one of the effective methods to solve urban traffic problems, while short-term traffic flow analysis of road is the core of ITS. Therefore, it is of great theoretical and practical significance to study the effective modeling theory and method of short-term traffic flow. Using mathematics and mechanics theory, the traffic flow theory is to research the law of road traffic. Short-term traffic flow system is similar to hydrodynamics systems to a large extend, and has the basic characteristics of dynamics system. This system has high uncertainty, and it is difficult for people to grasp the influence range and action mechanism of various factors in the system. It belongs to the information-poor system. If one hour is taken as a cycle and the recording interval is 5 min, only 12 groups of data are collected, which belong to small sample data. Therefore, it is reasonable and feasible to study short-term traffic flow system with dynamics characteristics by using grey system theory which deals with small sample and information-poor systems.

Short-term traffic flow has complex stochastic and non-linear characteristics, which are due to similar trends in data acquisition for several days or weeks at the same location, as well as its strong delays and seasonal characteristics (Mahnke et al., 2005; Vlahogianniet al., 2014). Traffic flow data collected at the same location usually have panel data characteristics. The longitudinal data (daily flow) show a clear intra-day trend. The cross-sectional data show weekly seasonality and limited data characteristics, which consisted of observation data at the same time interval on

X. Xiao (✉) · H. Guo · J. Yang
School of Science, Wuhan University of Technology, Wuhan 430070, China
e-mail: xiaoxp@whut.edu.cn

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different days (Qiu & Yang, 2013; Tan et al., 2009; Tchrakian et al., 2012). Since it can be found that 1 h, 15 and 10 min flow series show outstanding performance for the periodic dynamics, through the quantitative indicators of recursive graph diagonal parallel lines of traffic flow and so on, long-term forecasts can be carried out. When the Lyapunov index is more than zero, 5 min flow sequences exhibit chaotic dynamics and it can be predicted as short-term. When the Lyapunov index is less than zero, 5 min flow sequence exhibit the randomness dynamics and short-term forecasts can be finished (Xiao et al., 2017). Based on the above achievements, with the idea of rapid modeling using limited historical data, based on grey systems theory, the main work and innovations of this paper are to study on the short-term traffic flow dynamics model for forecasting and optimization (Marwan & Romano, 2007).

7.2 The GM(1, 1|τ, r) Model for Forecasting the Urban Road Short-Term Traffic Flow

Considering the delay and nonlinearity of traffic flow in urban road system, we use the grey GM(1, 1|τ, r) model to forecast the short-term traffic flow. Exactly, the delay parameter τ is determined by the speed-flow relationship. The non-linear parameter r is determined by the particle swarm optimization algorithm (PSO). Finally, a prediction case of the traffic flow data of Youyi Avenue and comparative analysis with other methods are studied, which are used to verify the feasibility and rationality of the achievements.

7.2.1 The GM(1, 1|τ, r) Model and Its Properties

Define 7.1 Let $X^{(0)} = (x^{(0)}(k)),\ k = 1, 2, \ldots, n$ be the raw series, $X^{(1)} = (x^{(1)}(k))$ be the accumulated generation sequence of $X^{(0)}$, $Z^{(1)} = (z^{(1)}(k))$ be the background value of $X^{(1)}$, the grey model GM(1, 1|τ, r) be in the following (Deng, 2001, 2002):

$$x^{(0)}(k) + az^{(1)}(k - \tau) = bk^r \tag{7.1}$$

where τ is the delay parameter and r is the nonlinear parameter.

Note

$$B = \begin{bmatrix} -z^{(1)}(2) & (\tau + 2)^r \\ -z^{(1)}(3) & (\tau + 3)^r \\ \vdots & \vdots \\ -z^{(1)}(n - \tau) & n^r \end{bmatrix}, Y = \begin{bmatrix} x^{(0)}(\tau + 2) \\ x^{(0)}(\tau + 3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix}$$

Then parameters a, b are obtained by the least square method as below:

$$[a\ \ b]^T = (B^T B)^{-1} B^T Y \tag{7.2}$$

Further, let

$$C = \sum_{i=2}^{n-\tau} z^{(1)}(i)(i+\tau)^r;\ E = \sum_{i=2}^{n-\tau} [z^{(1)}(i)]^2;\ F = \sum_{i=2}^{n-\tau} (i+\tau)^{2r};$$

$$G = \sum_{i=2}^{n-\tau} z^{(1)}(i)x^{(0)}(i+\tau);\ H = \sum_{i=2}^{n-\tau} x^{(0)}(i+\tau)(i+\tau)^r$$

We have

$$a = \frac{CH - GF}{EF - C^2},\ b = \frac{EH - CG}{EF - C^2} \tag{7.3}$$

As we all known, the numerical value of traffic flow in urban road is very large in spite of five minute time interval. It is likely to cause a singular matrix during the modeling process. So, it's necessary to transform the original series avoiding this situation (Xiao & Mao, 2013). Furthermore, it's necessary for us to determine whether or not multiple transformations will change the prediction precision of the GM(1, 1|τ, r) model.

If $x^{(0)}(k), k = 1, 2, \ldots, n$ is the original series, $x'^{(0)}(k) = \rho x^{(0)}(k), k = 1, 2, \ldots, n$ is the multiple transformation, where ρ is a constant and $\rho \neq 0$. Then, we note the parameters of the model as a', b', and obtain the conclusion as follows.

Theorem 7.1 *Let* $[a\ \ b]^T$, $[a'\ \ b']^T$ *be the parameter vectors of GM(1, 1|*τ*, r) model separately by* $x^{(0)}(k)$ *and* $x'^{(0)}(k)$*, where* $x'^{(0)}(k) = \rho x^{(0)}(k)$*, then* $a' = a, b' = \rho b$.

Proof If $x'^{(0)}(k) = \rho x^{(0)}(k), k = 1, 2, \ldots, n$, then

$$x'^{(1)}(k) = \sum_{i=1}^{k} \rho x^{(0)}(i) = \rho \sum_{i=1}^{k} x^{(0)}(i) = \rho x^{(1)}(k) \tag{7.4}$$

and C, E, F, G, H be changed respectively as following

$$C' = \sum_{i=2}^{n-\tau} z'^{(1)}(i)(i+\tau)^r = \sum_{i=2}^{n-\tau} \rho z^{(1)}(i)(i+\tau)^r = \rho C;$$

$$E' = \sum_{i=2}^{n-\tau} [z'^{(1)}(i)]^2 = \sum_{i=2}^{n-\tau} [\rho z^{(1)}(i)]^2 = \rho^2 E;$$

$$F' = \sum_{i=2}^{n-\tau} (i+\tau)^{2r} = F;\ G' = \sum_{i=2}^{n-\tau} z'^{(1)}(i)x'^{(0)}(i+\tau)$$

$$= \sum_{i=2}^{n-\tau} \rho^2 z^{(1)}(i) x^{(0)}(i+\tau) = \rho^2 G;$$

$$H' = \sum_{i=2}^{n-\tau} x'^{(0)}(i+\tau)(i+\tau)^r = \sum_{i=2}^{n-\tau} \rho x^{(0)}(i+\tau)(i+\tau)^r = \rho H$$

where C', E', F', G', H' are the model parameters by transformation, the results are as follows,

$$a' = \frac{C'H' - G'F'}{E'F' - C'^2} = \frac{\rho^2 CH - \rho^2 GF}{\rho^2 EF - \rho^2 C^2} = a \ b'$$
$$= \frac{E'H' - C'G'}{E'F' - C'^2} = \frac{\rho^3 EH - \rho^3 CG}{\rho^2 EF - \rho^2 C^2} = \rho b$$

The result is to say that the development coefficient is unchanged via transformation. Therefore, the trend of the model prediction value will not be change via multiple transformation.

7.2.2 The Determination for Traffic System Delay Time

When traffic flow is beyond the traffic capacity, according to the classical speed-flow model (see Fig. 7.1), the speed will reduce. The traffic capacity needs to cross with the speed of V_m and $V_m = 0.5V_o$, the rest of the vehicles need to come into the queen and pass when the queue slakes (Dai & Ma, 2006).

The queue length is $q - c$ when q is larger than c in unit time. Assuming that the vehicles are waiting to pass in order of arrival after time t, and there is no congestion caused by unstable traffic flow in the queuing process. Then, the traffic flow intersects with the speed of V_m. So, we have

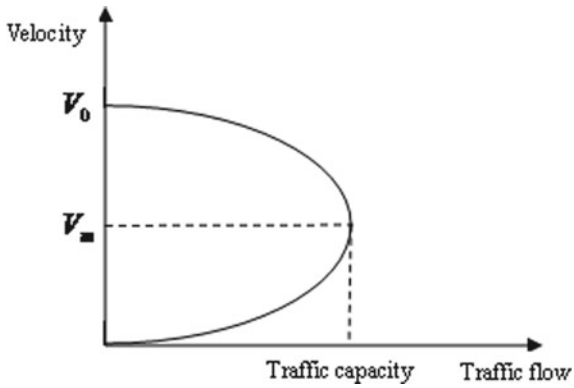

Fig. 7.1 Speed-flow model curve

$$\frac{\tau}{t+\tau} = \frac{q-c}{q} \Rightarrow \frac{\tau}{t} = \frac{q-c}{c} = \frac{q}{c} - 1 \tag{7.5}$$

The velocity by Eq. 7.5 is usually too large, for example if $q/c = 2$, then $\tau = t$ and $V = V_0/2 = V_m$. In the process of the queue dissipation, traffic flow cannot pass at a constant speed. When the traffic flow is greater than the capacity, the traffic flow is in an unstable state. Any impact of traffic conditions can lead to greater delays. Therefore, two parameters α, β are necessary utilized to advance the formula.

$$\frac{\tau}{t} = \alpha\left(\frac{q}{c}\right)^{\beta} - 1 \tag{7.6}$$

During the period t, the arriving traffic flow average delay is $\overline{\tau}$, and

$$\overline{\tau} = \frac{1}{2} \cdot t \cdot \left(\alpha\left(\frac{q}{c}\right)^{\beta} - 1\right) \tag{7.7}$$

Theorem 7.2 *Suppose* $(V_i\,,\ q_i)$ *is the speed-flow sequence measured in the traffic system, we establish the speed-flow model in the following,*

$$V = \frac{V_0}{1+\alpha\left(\frac{q}{c}\right)^{\beta}} \tag{7.8}$$

Parameters α, β *are obtained,*

$$\alpha = \exp\frac{JL - Ik}{nI - J^2}, \beta = \frac{JK - nL}{nI - J^2} \tag{7.9}$$

where

$$I = \sum_i \left(\ln\left|\frac{q_i}{c}\right|\right)^2; J = \sum_i \ln\left|\frac{q_i}{c}\right|; K = \sum_i \ln\left|\frac{V_0}{V_i} - 1\right|;$$
$$L = \sum_i \left(\ln\left|\frac{q_i}{c}\right| \cdot \ln\left|\frac{V_0}{V_i} - 1\right|\right)$$

Proof The driving distance $l = V_m \cdot t$, the average speed in the period $t + \overline{\tau}$ is

$$V = \frac{l}{t+\overline{\tau}} = \frac{V_m \cdot t}{t + \frac{1}{2}\cdot t \cdot \left(\alpha\left(\frac{q}{c}\right)^{\beta} - 1\right)} = \frac{2V_m}{1+\alpha\left(\frac{q}{c}\right)^{\beta}} = \frac{V_0}{1+\alpha\left(\frac{q}{c}\right)^{\beta}} \tag{7.10}$$

Deform the formula (7.8) and take logarithm of both sides,

$$\ln\left|\frac{V_0}{V} - 1\right| = \ln\alpha + \beta\ln\left|\frac{q}{c}\right| \tag{7.11}$$

Put the sequence $(V_i\,,\ q_i)$ into formula (7.11), and the equations are as below,

$$\begin{cases} \ln\left|\frac{V_0}{V_1}-1\right| = \ln\alpha + \beta\cdot\ln\left|\frac{q_1}{c}\right| \\ \ln\left|\frac{V_0}{V_2}-1\right| = \ln\alpha + \beta\ln\left|\frac{q_2}{c}\right| \\ \vdots \\ \ln\left|\frac{V_0}{V_n}-1\right| = \ln\alpha + \beta\ln\left|\frac{q_n}{c}\right| \end{cases} \tag{7.12}$$

If

$$D = \begin{bmatrix} 1 & 1 & \dots & 1 \\ \ln\left|\frac{q_1}{c}\right| & \ln\left|\frac{q_2}{c}\right| & & \ln\left|\frac{q_n}{c}\right| \end{bmatrix}^T, Y = \left[\ln\left|\frac{V_0}{V_1}-1\right|\ \ln\left|\frac{V_0}{V_2}-1\right|\ \dots\ \ln\left|\frac{V_0}{V_n}-1\right|\right]^T$$

The equations are simplified by the least squares method as follows

$$[\ln\alpha\ \ \beta] = (D^T D)^{-1} D^T Y \tag{7.13}$$

Through matrix operation and annotation.

$$I = \sum_i \left(\ln\left|\frac{q_i}{c}\right|\right)^2; J = \sum_i \ln\left|\frac{q_i}{c}\right|; K = \sum_i \ln\left|\frac{V_0}{V_i}-1\right|;$$
$$L = \sum_i \left(\ln\left|\frac{q_i}{c}\right|\cdot\ln\left|\frac{V_0}{V_i}-1\right|\right)$$

The final results comes to be

$$\alpha = \exp\frac{JL - Ik}{nI - J^2}, \beta = \frac{JK - nL}{nI - J^2}.$$

Theorem 7.3 *Suppose $(V_i\,,\ q_i)$ is the speed-flow sequence, τ is the system delay time in one experiment period, then there is*

$$\tau = \frac{L}{V_0}\sum_i \alpha\cdot\left(\frac{q_i}{c}\right)^\beta \tag{7.14}$$

where α, β is are formula (7.9), L is the road length and V_0 is the velocity of free flow.

Proof The required time is T_o $\left(T_o = \frac{L}{V_0}\right)$ to pass a length L road without considering the system delay. However, the traffic problem is an inseparable system, when traffic flow is q_i, the actual passing time is $T(q_i)$. The result is as below form Theorem 7.2,

$$T(q_i) = T_0\left[1 + \alpha\left(\frac{q_i}{c}\right)^{\beta}\right] \tag{7.15}$$

where α, β are in formula (7.9), the delay time is $\tau(q_i)$, and

$$\tau(q_i) = T(q_i) - T_0 = T_0 \cdot \alpha \cdot \left(\frac{q_i}{c}\right)^{\beta} \tag{7.16}$$

So, the delay time of traffic system in one period is τ and

$$\tau = \sum_i \tau(q_i) = \frac{L}{V_0}\sum_i \alpha \cdot \left(\frac{q_i}{c}\right)^{\beta}. \tag{7.17}$$

7.2.3 The Determination for Nonlinear Factor

In order to verify possibility of the model, we use MRE (the mean relative error) and FD (fitting degree between prediction and reality value) as the prediction error index, the formulas are in the following.

$$MRE = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{\hat{x}(i) - x(i)}{x(i)}\right| \tag{7.18}$$

$$FD = 1 - \frac{\sqrt{\sum_{i=1}^{n}\left[\hat{x}(i) - x(i)\right]^2}}{\sqrt{\sum_{i=1}^{n}\hat{x}(i)^2} + \sqrt{\sum_{i=1}^{n}x(i)^2}} \tag{7.19}$$

The effect of prediction will be responded by the values of MRE and FD. The smaller the MRE is, the higher the prediction accuracy is. At the same time, the higher the FD value is, the better the prediction effect is. So, If the delay parameter is known, we establish the optimization model to determine the nonlinear parameter r, and the optimization model is as follows,

$$\min\ \frac{1}{n}\sum_{i=1}^{n}\left|\frac{\hat{x}(i) - x(i)}{x(i)}\right| + \frac{\sqrt{\sum_{i=1}^{n}\left[\hat{x}(i) - x(i)\right]^2}}{\sqrt{\sum_{i=1}^{n}\hat{x}(i)^2} + \sqrt{\sum_{i=1}^{n}x(i)^2}} \tag{7.20}$$

Then we use the PSO algorithm to search best value of the nonlinear parameter r.

7.2.4 Example Analyses

In order to solve the practical problems of transportation system, we do the experiments for a section of Wu Chang District by using vision trace to photograph on May 19, 2010. The length of the experimental road length is $L = 949$ m. The experimental road traffic capacity C = 1600 vehicles/h and the designed speed $V_0 = 60$ km/h, by the city driveway traffic capacity table. We have the capacity $c = 134$ vehicles and the time $T_0 = L/V_0 = 56.94$ s no considering the system delay. The traffic flow and the speed of acquisition point (km/h) are listed in Table 7.1 by SAVA video analysis software using the 5 min time intervals.

Using the data of traffic flow and velocity, we can obtain that $\alpha = 0.161227$, $\beta = 3.092102$ by Theorem 7.2. Then, the delay time can be determined by Eqs. (7.16, 7.17) and $\tau = 705s$. So, we can calculate the delay factor $\tau = 2$ and construct the GM(1, 1|τ, r) model as below.

$$x^{(0)}(k) + a \cdot z^{(1)}(k-2) = bk^r \tag{7.21}$$

Before the modeling, we do multiple transformation for original data to avoid the singular matrix. Then we establish the model by the former 10 data, search $r = 0.9485$ by PSO, and get $a = 0.3893$, $\quad b = 84.5040$. Finally, the GM(1, 1|τ, r) model is in the following.

$$x^{(0)}(k) + 0.3893 \cdot z^{(1)}(k-2) = 84.5040k^{0.9485} \tag{7.22}$$

Then, we have the fitting value of the raw data via Eq. (7.22), and to compare the fitting value with the result via SVM method and GM(1, 1) model (see Table 7.2).

Comparing the fitting results from three methods in Fig. 7.2, the results of GM(1, 1|τ, r) model is substantially better than those results of GM(1, 1) model and SVM.

We forecast the traffic flow by Eq. (7.22) and compare the forecasting results with the rest experimental data in Table 7.3. The forecasting results from four methods are also shown in Fig. 7.3, GM(1, 1|τ, r) model is obviously superior to the other models. It is show that GM(1, 1|τ, r) model can reliably reflect the operation status of the system and provide a satisfactory prediction for ITS.

7.2.5 Conclusions

Aiming at the delay and nonlinearity of road traffic flow, a short-term traffic flow forecasting method is proposed, which is based on the grey system theory. Firstly, the delay parameter τ is determined by the speed-flow relationship. Then, the non-linear parameter r is determined by the particle swarm optimization algorithm. Finally, we verify the model by the traffic flow data of Youyi Avenue and compare the prediction

Table 7.1 Measured traffic flow and speed

Time	15:00–15:05	15:05–15:10	15:10–15:15	15:15–15:20	15:20–15:25	15:25–15:30	15:30–15:35	15:35–15:40
Speed	45.97	49.92	47.75	44.53	42.65	39.90	41.90	42.39
Flow	183	208.5	184.5	180	219	217.5	192	196.5
Time	15:40–15:45	15:45–15:50	15:50–15:55	15:55–16:00	16:00–16:05	16:05–16:10	16:10–16:15	16:15–16:20
Speed	46.70	43.63	46.40	48.51	39.74	46.34	37.92	32.09
Flow	163.5	165	181.5	174	190.5	199.5	198	249

Table 7.2 The comparing results of fitting

Time	Measured flow (X0)	SVM (Yang & Wang, 2006)		GM(1, 1) model (Deng, 2001)		GM(1, 1|τ, r) model	
		Fitting value (X1)	Relative error	Fitting value (X2)	Relative error	Fitting value (X3)	Relative error
15:00–15:05	183	177.4232	3.047432	183.0000	0	183.0000	0
15:05–15:10	208.5	203.5789	2.36024	215.5000	3.357314	208.5000	0
15:10–15:15	184.5	185.0463	0.296098	194.5000	5.420054	184.5000	0
15:15–15:20	180	187.0949	3.941611	199.12461	10.62478	202.9084	12.72689
15:20–15:25	219	193.5760	11.60913	195.3315	10.80753	200.6038	8.400091
15:25–15:30	217.5	210.9623	3.005839	191.6107	11.90313	203.0809	6.629471
15:30–15:35	192	200.4702	4.411563	187.9607	2.103802	198.2126	3.235729
15:35–15:40	196.5	200.9363	2.257659	184.3803	6.167786	185.5101	5.592824
15:40–15:45	163.5	179.2016	9.603425	180.8681	10.62269	177.5979	8.622569
15:45–15:50	165	185.5006	12.42461	177.4338	7.535636	173.3624	5.068121
MRE (%)		5.29576		6.854273		5.02757	
FD		0.967589		0.959862		0.968109	

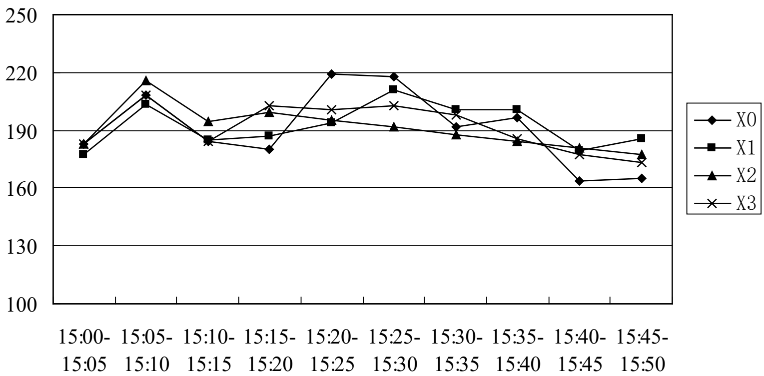

Fig. 7.2 Measured flow and fitting value of model

results of GM(1, 1|τ, r) with other methods, and GM(1, 1|τ, r) model is obviously superior to the other models. The research provide effective information for solving the problems of ITS. When the traffic flow data is uncertain, it is necessary to further study the traffic model.

Table 7.3 The comparing results of forecasting

Time	Measured flow (X0)	SVM (Yang & Wang, 2006)		GM(1, 1) model (Deng, 2001)		GM(1, 1|τ, r) model	
		Forecasting value (X1)	Relative error	Forecasting value (X2)	Relative error	Forecasting value (X3)	Relative error
15:50–15:55	181.5	179.0765	1.335262	174.0431	4.108485	174.3067	3.963251
15:55–16:00	174	190.7663	9.635805	170.7278	1.880575	181.0498	4.051609
16:00–16:05	190.5	217.6704	14.26268	167.4756	12.0863	183.9864	3.419213
16:05–16:10	199.5	178.9312	10.31018	164.2854	17.65143	184.8178	7.359499
16:10–16:15	198	225.1602	13.71727	161.1559	18.60813	183.7086	7.217879
16:15–16:20	249	214.2145	13.97008	158.0861	36.51161	217.3962	12.69229
MRE (%)		10.53855		15.14109		6.450623	
FD		0.940827		0.880682		0.958437	

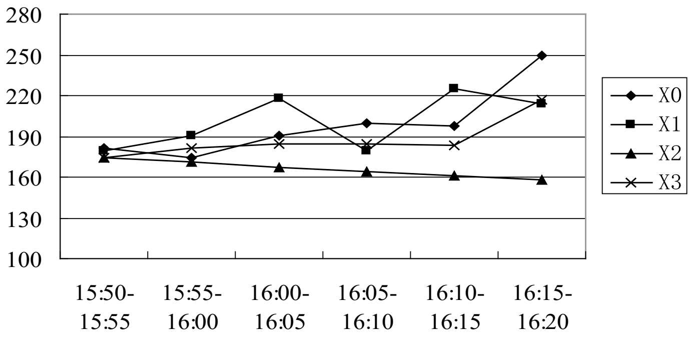

Fig. 7.3 Measured flow and forecasting value of model

7.3 Grey Coupled Prediction Model of Traffic Flow with Panel Data Characteristics

Considering the traffic data with panel data characteristics, we study the grey coupled prediction model. Traffic flow data is the panel data which we collected continuously at the same site. The longitudinal data (daily flow) show a clear intra-day trend. The cross-sectional data show weekly seasonality and limited data characteristics, which consisted of observation data at the same time interval on different days. We can use autoregressive integrated moving average (ARIMA) model to predict the longitudinal data, use the rolling seasonal grey model (RSDGM(1, 1)) to predict the cross-sectional data, and use matrix perturbation analysis to determine the length of the rolling sequence. On the basis of ARIMA model and RSDGM(1, 1) model, we construct the coupled model. From numerical experiments, we can see that RSDGM(1, 1) model can forecast the traffic flow with good adaptability and stability.

7.3.1 DGM(1, 1) Model

Let $x^{(0)} = (x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n))^T$ be a non-negative and original sequence. The first-order accumulated generating operation (1-AGO) sequence is $x^{(1)} = (x^{(1)}(1), x^{(1)}(2), \ldots, x^{(1)}(n))^T$.

$$x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i), \quad k = 1, 2, \ldots n. \tag{7.23}$$

Then, DGM(1, 1) model (Xie & Liu, 2009) is defined as follows.

$$x^{(1)}(k+1) = \beta_1 x^{(1)}(k) + \beta_2. \tag{7.24}$$

and $\hat{\beta} = [\beta_1, \beta_2]^T = (B^T B)^{-1} B^T Y$ is the parameter estimation, where

$$Y = \begin{bmatrix} x^{(1)}(2) \\ x^{(1)}(3) \\ \vdots \\ x^{(1)}(n) \end{bmatrix}, B = \begin{bmatrix} x^{(1)}(1) & 1 \\ x^{(1)}(2) & 1 \\ \vdots & \vdots \\ x^{(1)}(n-1) & 1 \end{bmatrix}.$$

We can have the solution of DGM(1, 1) model in the following.

$$\hat{x}^{(1)}(k+1) = \left(x^{(0)}(1) - \frac{\beta_2}{1-\beta_1}\right)\beta_1^k + \frac{\beta_2}{1-\beta_1}. \tag{7.25}$$

Also, we can get the restored values of $x^{(0)}(k)$ by

$$\hat{x}^{(0)}(k+1) = (\beta_1 - 1)\left(x^{(0)}(1) - \frac{\beta_2}{1-\beta_1}\right)\beta_1^{k-1}. \tag{7.26}$$

7.3.2 *Research on the CTAGO Operator*

When the raw data appear the seasonal changes, the data oscillation makes the raw sequence unable to meet the smoothness ratio for grey modeling condition effectively, which results in a large deviation of the prediction results. So, we use CTAGO to obtain a better smoothness ratio for grey modeling.

Suppose the periodic value of the seasonal raw data $x^{(0)}$ is q, define the CTAGO as follows (Xia & Wong, 2014):

$$y^{(0)}(k) = \text{CTAGO}(x^{(0)}(k)) = \sum_{j=1}^{q} x^{(0)}(k+j-1)\,,\ \forall k = 1, 2, \ldots n-q+1. \tag{7.27}$$

We denote that $y^{(0)} = (y^{(0)}(1), y^{(0)}(2), \ldots, y^{(0)}(r))^T$ is the CTAGO sequence, when $r = n - q + 1$.

For getting the relationship between CTAGO data and original data, we have

$$\begin{aligned} y^{(0)}(k+1) - y^{(0)}(k) &= \sum_{j=1}^{q} x^{(0)}(k+j) - \sum_{j=1}^{q} x^{(0)}(k+j-1) \\ &= x^{(0)}(k+q) - x^{(0)}(k). \end{aligned}$$

So

$$y^{(0)}(k+1) - y^{(0)}(k) = x^{(0)}(k+q) - x^{(0)}(k). \tag{7.28}$$

From the Eq. (7.28), we can see that the difference information of the corresponding data in original seasonal sequence is equal to the difference information of the $y^{(0)}$.

For $\forall k = 1, 2, \ldots n-q+1$, when $x^{(0)}(k+q) > x^{(0)}(k)$, then $y^{(0)}(k+1) > y^{(0)}(k)$; When $x^{(0)}(k+q) < x^{(0)}(k)$, then $y^{(0)}(k+1) < y^{(0)}(k)$.

In conclusion, the CTAGO operator has periodic oscillation as well as the original sequence. However, the CTAGO operator can fade the oscillation and make the CTAGO sequence relatively flat. Thus, the smooth ratio of CTAGO sequence is satisfied for grey modeling in a given condition. We can get some properties of the sequence as follows:

Property 7.1 *Suppose* $\forall k = 1, 2, \ldots, n-q$*,* $q \geq 3$*,*$\overline{M} = \frac{1}{q+1}(x^{(0)}(k) + x^{(0)}(k+1) + \cdots + x^{(0)}(k+q))$*,* $M = \max\{x^{(0)}(k), \ldots, x^{(0)}(k+q)\}$*, and* $m = \min\{x^{(0)}(k), \ldots, x^{(0)}(k+q)\}$.

When $M < \frac{3}{2}\overline{M}$*,* $m > \frac{1}{2}\overline{M}$*, and* $\rho_y(k) = \frac{y^{(0)}(k)}{\sum_{i=1}^{k-1} y^{(0)}(i)} \in (\frac{1}{q-0.5}, \frac{1}{2})$*, then we have:*

$$\frac{\rho_y(k+1)}{\rho_y(k)} < 1, k = 3, 4, \ldots n-q-1,$$

The CTAGO sequence, which satisfies the conditions for establishing the DGM(1, 1) grey model, is a quasi-smooth sequence.

Proof $\frac{\rho_y(k+1)}{\rho_y(k)} = \frac{y^{(0)}(k+1)}{\sum_{i=1}^{k} y^{(0)}(i)} / \frac{y^{(0)}(k)}{\sum_{i=1}^{k-1} y^{(0)}(i)} = \frac{y^{(0)}(k+1)}{y^{(0)}(k)} \cdot \frac{\sum_{i=1}^{k-1} y^{(0)}(i)}{\sum_{i=1}^{k-1} y^{(0)}(i)+y^{(0)}(k)}$

To prove $\frac{\rho_y(k+1)}{\rho_y(k)} < 1$, that is, $y^{(0)}(k+1) \cdot \sum_{i=1}^{k-1} y^{(0)}(i) < y^{(0)}(k)(\sum_{i=1}^{k-1} y^{(0)}(i) + y^{(0)}(k))$, it is necessary to prove

$$\frac{y^{(0)}(k+1) - y^{(0)}(k)}{y^{(0)}(k)} < \frac{y^{(0)}(k)}{\sum_{i=1}^{k-1} y^{(0)}(i)} = \rho_y(k).$$

By the formula (7.28), we have

$$\frac{y^{(0)}(k+1) - y^{(0)}(k)}{y^{(0)}(k)} = \frac{x^{(0)}(k+q) - x^{(0)}(k)}{x^{(0)}(k) + \cdots + x^{(0)}(k+q-1)}.$$

When $x^{(0)}(k+q) - x^{(0)}(k) < 0$, the result is clearly right;
When $x^{(0)}(k+q) - x^{(0)}(k) > 0$, and $M < \frac{3}{2}\overline{M}$, $m > \frac{1}{2}\overline{M}$, then

$$\frac{y^{(0)}(k+1) - y^{(0)}(k)}{y^{(0)}(k)} = \frac{x^{(0)}(k+q) - x^{(0)}(k)}{(q+1)\overline{M} - x^{(0)}(k+q)} < \frac{M - m}{(q+1)\overline{M} - M}$$

$$< \frac{1.5\overline{M} - 0.5\overline{M}}{(q+1)\overline{M} - 1.5\overline{M}} = \frac{1}{q-0.5}.$$

As $\rho_y(k) \in (\frac{1}{q-0.5}, \frac{1}{2})$, we can get $\frac{y^{(0)}(k+1)-y^{(0)}(k)}{y^{(0)}(k)} < \frac{y^{(0)}(k)}{\sum_{i=1}^{k-1} y^{(0)}(i)} = \rho_y(k)$.

Under the Property 7.1's conditions, the cross-sectional data cycle of traffic flow is $q = 7$, and the fluctuation range is $m > \frac{1}{2}\overline{M}, M < \frac{3}{2}\overline{M}$. When $\rho_y(k) \in (\frac{1}{6.5}, \frac{1}{2}) = (0.1538, 0.5)$, the CTAGO sequence which satisfies for establishing the DGM(1, 1) grey model, is a quasi-smooth sequence.

7.3.3 DGM(1, 1) Model of the CTAGO Operation

Let $y^{(0)} = (y^{(0)}(1), y^{(0)}(2), \ldots, y^{(0)}(r))^T$ be the CTAGO sequence, then $y^{(1)} = (y^{(1)}(1), y^{(1)}(2), \ldots, y^{(1)}(r))^T$ is the 1-AGO sequence, where

$$y^{(1)}(k) = \sum_{i=1}^{k} y^{(0)}(i)\ , \quad k = 1\ ,\ 2\ ,\ \ldots\ ,\ r. \tag{7.29}$$

From the Eqs. (7.27) and (7.29), we have

$$y^{(1)}(k) = \sum_{i=1}^{k} y^{(0)}(i) = \sum_{i=1}^{k}\sum_{j=1}^{q} x^{(0)}(i+j-1), k = 1\ ,\ 2\ ,\ \ldots\ ,\ r. \tag{7.30}$$

Suppose $A = \begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \\ 1 & 1 & 0 & \cdots & 0 \\ 1 & 1 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & 1 & 1 & \cdots & 1 \end{pmatrix}_r$ and $G = \begin{pmatrix} 1 & 1 & \cdots & 1 & 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & 1 & 1 & \cdots & 1 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \cdots & 1 & 1 & \cdots & 1 \end{pmatrix}_{r \times n}$, the corresponding matrix expressions are $y^{(0)} = Gx^{(0)}$ and $y^{(1)} = Ay^{(0)} = AGx^{(0)}$, the DGM(1, 1) model for the CTAGO sequence $y^{(0)}$ is established as follows

$$y^{(1)}(k+1) = \beta_1 y^{(1)}(k) + \beta_2. \tag{7.31}$$

Also, we can get the following theorems for parameter identification:

Theorem 7.1 *Let* $P = (\beta_1, \beta_2)^T, Y = \begin{pmatrix} y^{(1)}(2) \\ y^{(1)}(3) \\ \vdots \\ y^{(1)}(r) \end{pmatrix}_{r-1}, B = \begin{pmatrix} y^{(1)}(1) & 1 \\ y^{(1)}(2) & 1 \\ \vdots & \vdots \\ y^{(1)}(r-1) & 1 \end{pmatrix}_{(r-1)\times 2}$, *the parameter identification for DGM(1, 1) model of the sequence* $y^{(0)}$ *is*

$$P = (B^T B)^{-1} B^T Y = ((A_2 G_2 M)^T A_2 G_2 M)^{-1} (A_2 G_2 M)^T A_1 G X,$$

where

$$B = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 1 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \cdots & 1 \end{pmatrix}_{r-1} \begin{pmatrix} 1 & 1 & \cdots & 1 & 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 1 & 1 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & & \vdots & \vdots & \ddots & \vdots & \vdots & & \vdots \\ 0 & 0 & & 0 & 0 & \cdots & 1 & 1 & \cdots & 1 \end{pmatrix}_{(r-1)\times(n-1)} \begin{pmatrix} x^{(0)}(1) & 1 \\ x^{(0)}(2) & 0 \\ \vdots & \vdots \\ x^{(0)}(n-1) & 0 \end{pmatrix}$$
$$= A_2 \cdot G_2 \cdot M,$$

$$Y = \begin{pmatrix} 1 & 1 & 0 & \cdots & 0 \\ 1 & 1 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & 1 & 1 & \cdots & 1 \end{pmatrix}_{(r-1)\times r} \cdot \begin{pmatrix} 1 & 1 & \cdots & 1 & 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & 1 & 1 & \cdots & 1 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & & 0 & 0 & \cdots & 1 & 1 & \cdots & 1 \end{pmatrix}_{r\times n} \begin{pmatrix} x^{(0)}(1) \\ x^{(0)}(2) \\ \vdots \\ x^{(0)}(n) \end{pmatrix}$$
$$= A_1 \cdot G \cdot X.$$

Proof Because $y^{(1)}(k+1) = \beta_1 y^{(1)}(k) + \beta_2, k = 1, 2, \ldots, r-1,$

We can obtain $Y = BP$, that is to say,

$$\begin{bmatrix} y^{(1)}(2) \\ y^{(1)}(3) \\ \vdots \\ y^{(1)}(r) \end{bmatrix}_{r-1} = \begin{bmatrix} y^{(1)}(1) & 1 \\ y^{(1)}(2) & 1 \\ \vdots & \vdots \\ y^{(1)}(r-1) & 1 \end{bmatrix}_{(r-1)\times 2} \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix},$$

As

$$Y = \begin{pmatrix} y^{(1)}(2) \\ y^{(1)}(3) \\ \vdots \\ y^{(1)}(r) \end{pmatrix}_{r-1} = \begin{pmatrix} 1 & 1 & 0 & \cdots & 0 \\ 1 & 1 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & 1 & 1 & \cdots & 1 \end{pmatrix}_{(r-1)\times r} \begin{pmatrix} y^{(0)}(1) \\ y^{(0)}(2) \\ \vdots \\ y^{(0)}(r) \end{pmatrix}_r$$

$$= \begin{pmatrix} 1 & 1 & 0 & \cdots & 0 \\ 1 & 1 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & 1 & 1 & \cdots & 1 \end{pmatrix}_{(r-1)\times r} \begin{pmatrix} 1 & 1 & \cdots & 1 & 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & 1 & 1 & \cdots & 1 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \cdots & 1 & 1 & \cdots & 1 \end{pmatrix}_{r\times n} \begin{pmatrix} x^{(0)}(1) \\ x^{(0)}(2) \\ \vdots \\ x^{(0)}(n) \end{pmatrix}$$
$$= A_1 \cdot G \cdot X,$$

and

$$B = \begin{pmatrix} y^{(1)}(1) & 1 \\ y^{(1)}(2) & 1 \\ \vdots & \vdots \\ y^{(1)}(r-1) & 1 \end{pmatrix}_{(r-1)\times 2} = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 1 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \cdots & 1 \end{pmatrix}_{r-1} \begin{pmatrix} y^{(0)}(1) & 1 \\ y^{(0)}(2) & 0 \\ \vdots & \vdots \\ y^{(0)}(r-1) & 0 \end{pmatrix}_{(r-1)\times 2}$$
$$= \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 1 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \cdots & 1 \end{pmatrix}_{r-1} \begin{pmatrix} 1 & 1 & \cdots & 1 & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & \cdots & 1 & \cdots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \cdots & 1 & 1 & \cdots & 1 \end{pmatrix}_{(r-1)\times(n-1)} \begin{pmatrix} x^{(0)}(1) & 1 \\ x^{(0)}(2) & 0 \\ \vdots & \vdots \\ x^{(0)}(n-1) & 0 \end{pmatrix}$$
$$= A_2 \cdot G_2 \cdot M$$

Then $Y = BP$ can be noted as below.

$$A_1GX = A_2G_2M \cdot P.$$

And the parameter solution can be transformed as

$$\min\|A_1GX - A_2G_2M \cdot P\|^2 = \min(A_1GX - A_2G_2M \cdot P)^T(A_1GX - A_2G_2M \cdot P).$$

By the derivation formula, we get the parameter identification

$$P = \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix} = ((A_2G_2M)^TA_2G_2M)^{-1}(A_2G_2M)^TA_1GX.$$

Theorem 7.2 *Let* $B = A_2G_2M$, $Y = A_1 \cdot G \cdot X$, *and* $(\beta_1, \beta_2)^T = ((A_2G_2M)^TA_2G_2M)^{-1}(A_2G_2M)^TA_1GX$, *we have*

(1) *The solution of DGM(1, 1) model is*

$$\hat{y}^{(1)}(k+1) = \left(y^{(0)}(1) - \frac{\beta_2}{1-\beta_1}\right)\beta_1^k + \frac{\beta_2}{1-\beta_1}. \tag{7.32}$$

(2) *The time response function of* $y^{(0)}$ *is*

$$\hat{y}^{(0)}(k+1) = (\beta_1 - 1)(y^{(0)}(1) - \frac{\beta_2}{1-\beta_1})\beta_1^{k-1}. \tag{7.33}$$

(3) *The solution of the original sequence* $x^{(0)}$ *is*

$$\hat{x}^{(0)}(k+1) = (\beta_1 - 1)(y^{(0)}(1) - \frac{\beta_2}{1-\beta_1})\beta_1^{k-q} - y^{(0)}(k-q+1) + x^{(0)}(k-q+1), k = q, q+1, \ldots, n. \tag{7.34}$$

Proof

(1) By the Eq. (7.25), the result is obvious.
(2) By the Eq. (7.32), we have

$$\begin{aligned}\hat{y}^{(0)}(k+1) &= \hat{y}^{(1)}(k+1) - \hat{y}^{(1)}(k) \\ &= (\beta_1 - 1)(y^{(0)}(1) - \frac{\beta_2}{1-\beta_1})\beta_1^{k-1}, k = 1, 2, \ldots, n-1.\end{aligned}$$

(3) According to the formula (7.33) and Property 7.1, we can obtain

$$\begin{aligned}\hat{x}^{(0)}(k+1) &= \hat{y}^{(0)}(k-q+2) - y^{(0)}(k-q+1) + x^{(0)}(k-q+1) \\ &= (\beta_1 - 1)\left(y^{(0)}(1) - \frac{\beta_2}{1-\beta_1}\right)\beta_1^{k-q} \\ &\quad - y^{(0)}(k-q+1) + x^{(0)}(k-q+1), k = q, q+1, \ldots, n.\end{aligned}$$

In general, we call the formula (7.34) the solution of SDGM(1, 1)) model.

7.3.4 Rolling Discrete Grey Model(RDGM(1, 1))

When we research on the traffic flow for traffic system, the latest information should be updated timely. It's obviously that the latest data can reflect the recent trends and characteristics of the system, and the rolling model can improve the prediction accuracy in most cases (Wu et al., 2015). So, the rolling DGM(1, 1) model is the extension of DGM(1, 1) model (Akay & Atak, 2007), which keep the length of the sequence unchanged by introducing new data and deleting old data. The rolling DGM(1, 1) model can be established in the following.

Step 1: We establish the DGM(1, 1) model for the sequence of $\{x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(p)\}$, and forecast $\hat{x}^{(0)}(p+1)$;

Step 2: We accept new observation $x^{(0)}(p+1)$ and remove the old information $x^{(0)}(1)$ in real time, and the new sequence is $\{x^{(0)}(2), \ldots, x^{(0)}(p), x^{(0)}(p+1)\}$, which is for establishing the new DGM(1, 1) model, and forecasting $\hat{x}^{(0)}(p+2)$;

Step 3: We repeat the Step 2 until all needed forecasting data points are obtained.

In the above process, when the length of the rolling sequence is too small, it may result in prediction distortion. On the opposite, the overlong sequence may lead to data redundancy. So, we will research on the rolling sequence length in RSDGM(1, 1), which is an important factor.

7.3.5 Rolling Seasonal Discrete Grey Model(RSDGM(1, 1))

The rolling forecasting model has been successfully applied in the areas of financial, electricity, and energy (Akay & Atak, 2007; Zhao & Guo, 2016). We also establish the corresponding equal dimension rolling metabolism grey model (RSDGM(1, 1)) for the seasonal traffic flow sequence of more elements, based on the thought of rolling metabolic forecasting. In the improved forecasting model, how to determine the rolling sequence length is the key problem, so that it not only meets the small data sample for modeling requirement but also contains the seasonal information of the data sequence.

When the length of $x^{(0)}$ is n with a period of q, its CTAGO sequence length is $r = n - q + 1$. We need $r = n - q + 1 \geq q$ by considering periodicity. Because r needs to be identified as the minimum value q to meet the new information priority principle, thus $n = 2q - 1$. For explaining the small data size of grey model, matrix perturbation theory was used to research the problem (Wu et al., 2013, 2015). For explain the weight preference of the previous period in SDGM(1, 1) model, we obtain the Lemma 1 as follows.

Lemma 7.1 (Wu et al., 2013) *Assuming that x and $x + h$ satisfy*

$$\|Bx - Y\|_2 = \min, \ \|(B + \Delta B)x - (Y + \Delta Y)\|_2 = \min,$$

where $B, \Delta B \in C^{m\times n}$ with $m \geq n$, and $Y \neq 0, \Delta Y \in C^{m\times n}$.

Suppose $k_+ = \left\|B^\dagger\right\|_2 \|B\|, \gamma_+ = 1 - \left\|B^\dagger\right\|_2 \|\Delta B\|_2$, and $r_x = Y - Bx$, in which $B^\dagger$ is the pseudo-inverse of matrix B. If $rank(B) = rank(B + \Delta B) = n$ and $\left\|B^\dagger\right\|_2 \|\Delta B\|_2 < 1$, then

$$\|h\| \leq \frac{k_+}{\gamma_+}\left(\frac{\|\Delta B\|_2}{\|B\|}\|x\| + \frac{\|\Delta Y\|}{\|B\|} + \frac{k_+}{\gamma_+}\frac{\|\Delta B\|_2}{\|B\|}\frac{\|r_x\|}{\|B\|}\right).$$

Theorem 7.3 *Let the length of $x^{(0)}$ be $n = 2q - 1$ and the length of its CTAGO sequence $y^{(0)}$ be $r = q$. When ε is a disturbance of $x^{(0)}(t)$,$L(x^{(0)}(t))$ is the perturbation bound. If $\varepsilon \neq 0$ and $\left\|B^\dagger\right\|_2 \|\Delta B\|_2 < 1$, we have that*

$$\begin{aligned} &L(x^{(0)}(1) < L(x^{(0)}(2) < \cdots < L(x^{(0)}(q)), \\ &L(x^{(0)}(q)) > L(x^{(0)}(q+1) > \cdots L(x^{(0)}(2q-1); \end{aligned}$$

that is to say, $L(x^{(0)}(q)) = \max\{L(x^{(0)}(1)), L(x^{(0)}(2)), \ldots, L(x^{(0)}(2q-2)), L(x^{(0)}(2q-1))\}$.

Proof

(i) If we give a disturbance ε to $x^{(0)}(1)$, by Theorem 7.1,

$$Y + \Delta Y_1 = A_1 \cdot G \cdot \begin{pmatrix} x^{(0)}(1)+\varepsilon \\ x^{(0)}(2) \\ \vdots \\ x^{(0)}(n) \end{pmatrix} = Y + A_1 \cdot G \cdot \begin{pmatrix} \varepsilon \\ 0 \\ \vdots \\ 0 \end{pmatrix},$$

$$\Delta Y_1 = A_1 \cdot G \cdot \begin{pmatrix} \varepsilon \\ 0 \\ \vdots \\ 0 \end{pmatrix} = \begin{pmatrix} \varepsilon \\ \varepsilon \\ \vdots \\ \varepsilon \end{pmatrix}_{r-1},$$

$$B + \Delta B_1 = A_2 \cdot G_2 \cdot \begin{bmatrix} x^{(0)}(1)+\varepsilon & 1 \\ x^{(0)}(2) & 0 \\ \vdots & \vdots \\ x^{(0)}(n) & 0 \end{bmatrix} = B + A_2 \cdot G_2 \cdot \begin{bmatrix} \varepsilon & 0 \\ 0 & 0 \\ \vdots & \vdots \\ 0 & 0 \end{bmatrix},$$

$$\Delta B_1 = A_2 \cdot G_2 \cdot \begin{bmatrix} \varepsilon & 0 \\ 0 & 0 \\ \vdots & \vdots \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} \varepsilon & 0 \\ \varepsilon & 0 \\ \vdots & \vdots \\ \varepsilon & 0 \end{bmatrix}_{(r-1)\times 2}, \ \Delta B_1^T \cdot \Delta B_1 = \begin{pmatrix} (r-1)\varepsilon^2 & 0 \\ 0 & 0 \end{pmatrix}.$$

So, $\|\Delta B_1\|_2 = \sqrt{\lambda_{\max}(\Delta B_1^T \Delta B_1)} = \sqrt{r-1}|\varepsilon| = \sqrt{q-1}|\varepsilon|$, $\|\Delta Y_1\|_2 = \sqrt{r-1}|\varepsilon| = \sqrt{q-1}|\varepsilon|$.

Therefore,

$$\begin{aligned} L(x^{(0)}(1)) &= \frac{k_+}{\gamma_+}\left(\frac{\|\Delta B\|_2}{\|B\|}\|x\| + \frac{\|\Delta Y\|}{\|B\|} + \frac{k_+}{\gamma_+}\frac{\|\Delta B\|_2}{\|B\|}\frac{\|r_x\|}{\|B\|}\right) \\ &= \frac{k_+}{\gamma_+}|\varepsilon|\left(\frac{\sqrt{q-1}}{\|B\|}\|x\| + \frac{\sqrt{q-1}}{\|B\|} + \frac{k_+}{\gamma_+}\frac{\sqrt{q-1}}{\|B\|}\frac{\|r_x\|}{\|B\|}\right). \end{aligned} \tag{7.35}$$

(ii) If we give a disturbance ε to $x^{(0)}(2)$,

$$\Delta Y_2 = \begin{pmatrix} 2\varepsilon \\ 2\varepsilon \\ \vdots \\ 2\varepsilon \end{pmatrix}_{r-1}, \ \Delta B_2 = \begin{bmatrix} \varepsilon & 0 \\ 2\varepsilon & 0 \\ \vdots & \vdots \\ 2\varepsilon & 0 \end{bmatrix}_{(r-1)\times 2},$$

$$\|\Delta B_2\|_2 = \sqrt{4q-7}|\varepsilon|, \|\Delta Y_2\|_2 = 2\sqrt{q-1}|\varepsilon|.$$

In the same way,

$$L(x^{(0)}(2)) = \frac{k_+}{\gamma_+}|\varepsilon|\left(\frac{\sqrt{4q-7}}{\|B\|}\|x\| + \frac{2\sqrt{q-1}}{\|B\|} + \frac{k_+}{\gamma_+}\frac{\sqrt{4q-7}}{\|B\|}\frac{\|r_x\|}{\|B\|}\right). \tag{7.36}$$

(iii) If we give a disturbance ε to $x^{(0)}(t)(t = 3, \ldots, q-1, q)$, ΔY will be changed as well as ΔB, and

$$\|\Delta B_t\|_2 = \sqrt{\sum_{j=1}^{t} j^2 + t^2(q-t-1)}|\varepsilon|, \|\Delta Y_t\|_2 = \sqrt{\sum_{j=2}^{t} j^2 + t^2(q-t)}|\varepsilon|.$$

When $t = 3, \ldots, q-1, q$,

$$L(x^{(0)}(t)) = \frac{k_+}{\gamma_+}|\varepsilon|\left(\frac{\sqrt{\sum_{j=1}^{t} j^2 + t^2(q-t-1)}}{\|B\|}\|x\| + \frac{\sqrt{\sum_{j=2}^{t} j^2 + t^2(q-t)}}{\|B\|} + \frac{k_+}{\gamma_+}\frac{\sqrt{\sum_{j=1}^{t} j^2 + t^2(q-t-1)}}{\|B\|}\frac{\|r_x\|}{\|B\|}\right). \tag{7.37}$$

Let $f(t) = \sum_{j=2}^{t} j^2 + t^2(q-t)$ and $g(t) = \sum_{j=1}^{t} j^2 + t^2(q-t-1)$.
When $2 \le t \le q$ and $f(t) - f(t-1) = (2t-1)(q-t+1) > 0$, we have $g(t) - g(t-1) = (2t-1)(q-t) \ge 0$.
From the formula (7.35)–(7.38), we can get $L(x^{(0)}(1) < L(x^{(0)}(2) < \cdots < L(x^{(0)}(q))$.

(iv) If we give a disturbance ε to $x^{(0)}(t)(t = q+1, q+2, \ldots, 2q-1,)$, ΔY will be changed as well as ΔB, and

$$\|\Delta B_t\|_2 = \sqrt{\sum_{j=1}^{n-t} j^2}|\varepsilon|(t = q+1, q+2, \ldots, 2q-2),$$

$$\|\Delta B_{2q-1}\|_2 = 0, \|\Delta Y_t\|_2 = \sqrt{\sum_{j=1}^{n-t+1} j^2}|\varepsilon|;$$

Then

$$L(x^{(0)}(t)) = \frac{k_+}{\gamma_+}|\varepsilon|\left(\frac{\sqrt{\sum_{j=1}^{n-t} j^2}}{\|B\|}\|x\| + \frac{\sqrt{\sum_{j=1}^{n-t+1} j^2}}{\|B\|} + \frac{k_+}{\gamma_+}\frac{\sqrt{\sum_{j=1}^{n-t} j^2}}{\|B\|}\frac{\|r_x\|}{\|B\|}\right),$$

$$L(x^{(0)}(2q-1)) = \frac{k_+}{\gamma_+}\frac{|\varepsilon|}{\|B\|}. \quad (7.38)$$

We can get the flowing results by contrasting the formulas (7.37) and (7.38)

$$L(x^{(0)}(q)) > L(x^{(0)}(q+1) > \cdots L(x^{(0)}(2q-1).$$

In conclusion, $L(x^{(0)}(q)) = \max\{L(x^{(0)}(1)), L(x^{(0)}(2)), \ldots, L(x^{(0)}(2q-2)), L(x^{(0)}(2q-1))\}$. □

As $\hat{x}^{(0)}(2q) = \hat{y}^{(0)}(q+1) - y^{(0)}(q) + x^{(0)}(q)$, the parameter perturbation bound of $x^{(0)}(q)$ is the largest at the same perturbation condition by Theorem 7.3. Thus, $x^{(0)}(q)$ has the maximum weight, which has the greatest impact for estimating the parameter. It need more attention to the priority of new information and the periodic law of original data for establishing the equal dimension RSDGM(1, 1) rolling model.

The procedure of RSDGM(1, 1) is displaying in Fig. 7.4 when the length of the sequence is determined.

7.3.6 RSDGM(1, 1)-ARIMA Model

As the traffic flow sequence is made up of time-series data and cross-sectional data, ARIMA model is usually used to forecast the time-series data with the intra-day trend of traffic flow, RSDGM(1, 1) model is applied to forecast the cross-sectional data with its typical characteristics of limited data and seasonal fluctuations. So, we establish the RSDGM(1, 1)-ARIMA model which coupled on the intersection point of the time-series and cross-sectional data by the grey relational analysis.

7.3.6.1 ARIMA Model

We usually use the ARIMA model to determine the regression relationship between the historical data and the future data of traffic flow, and apply the differencing technique for the non-stationary data. For non-stationary sequence x_t, the ARIMA model by differencing is usually denoted as ARIMA (p, d, q) (Hamilton, 1994):

$$\varphi(B)(1-B)^d x_t = C + \theta(B)\varepsilon_t \quad (7.39)$$

where B is a backshift operator; $\varphi(B)$ is the polynomial of autoregressive coefficient and $\varphi(B) = 1 - \varphi_1 B - \cdots - \varphi_P B^P$; $\theta(B)$ is the polynomial of moving smooth coefficient and $\theta(B) = 1 - \theta_1 B - \cdots - \theta_q B^q$; p is the lag order of AR; q is the lag order of MA; d is the frequency difference; $\{\varepsilon_t\}$ is white noise sequence of zero mean and C is a constant.

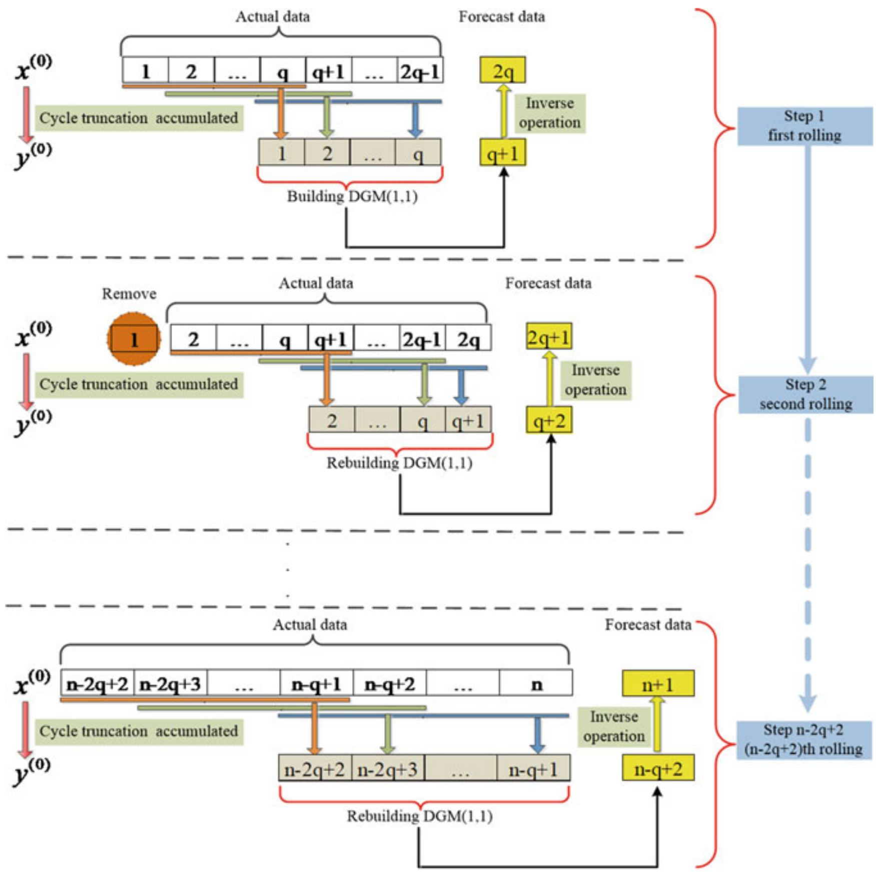

Fig. 7.4 Forecasting procedure of RSDGM(1, 1) models

7.3.6.2 RSDGM(1, 1)-ARIMA Model

For traffic flow panel data, we forecast the cross-sectional data by RSDGM(1, 1) model, and forecast the time-series data by ARIMA model, then couple the forecast results of the two models at the intersection. At time $t+1$, the cross-sectional forecasting result is Q_{t+1}^{s} and its weight is w_t^s, the time-series forecasting result is Q_{t+1}^{a} and its weight is w_t^a. Then, we have the coupled furcating model is in the following:

$$Q_{t+1} = w_t^s Q_{t+1}^s + w_t^a Q_{t+1}^a \tag{7.40}$$

For coupling the model, we identify the weight by the nearness grey relational degree, which is defined as follows:

Definition 7.1 (Liu et al., 2016) Let $X_i = (x_i(1), x_i(2), \ldots, x_i(n))$ and $X_j = (x_j(1), x_j(2), \ldots, x_j(n))$.

Suppose $S_i - S_j = \int_1^n (X_i - X_j)dt$; then,

$$\rho_{ij} = \frac{1}{1 + |S_i - S_j|} \tag{7.41}$$

ρ_{ij} is the nearness grey relational degree between X_i and X_j.

The bigger the grey relation degree between the fitting data and the actual data of the single model is, the bigger the weight of the coupled model is; otherwise, its weight is smaller.

The literature (Wang et al., 2014) takes the predictive nearness grey relational degree of the q phase before $t + 1$ as the weight index for time-series and achieved the good results with $q = 3, 5$ or 7. So, We use the nearness grey relational degree of the q phase before the $t + 1$ moment as the weight, and take $q = 7$ phase before $t + 1$.

The procedure of coupled prediction model is in the following:

(1) We respectively extract the fitting and real value sequence of the 7 time intervals before $t + 1$ from RSDGM and ARIMA model:

$$Q^s = (Q^s(t-q), Q^s(t-q+1), \ldots, Q^s(t)), \hat{Q}^s = (\hat{Q}^s(t-q), \hat{Q}^s(t-q+1), \ldots, \hat{Q}^s(t)).$$

$$Q^a = (Q^a(t-q), Q^a(t-q+1), \ldots, Q^a(t)), \hat{Q}^a = (\hat{Q}^a(t-q), \hat{Q}^a(t-q+1), \ldots, \hat{Q}^a(t)).$$

(2) According to formula (7.41), we get ρ_t^s and ρ_t^a:

$$\rho_t^s = \frac{1}{1 + \left|Q^s - \hat{Q}^s\right|}, \rho_t^a = \frac{1}{1 + \left|Q^a - \hat{Q}^a\right|};$$

(3) The weight coefficients in RSDGM(1, 1)-ARIMA model are determined by ρ_t^s and ρ_t^a:

$$w_t^s = \rho_t^s/(\rho_t^s + \rho_t^a), w_t^a = \rho_t^a/(\rho_t^s + \rho_t^a);$$

(4) We use the formula (7.40) to forecast the time-series and cross-sectional sequence:

$$Q_{t+1} = w_t^s Q_{t+1}^s + w_t^a Q_{t+1}^a.$$

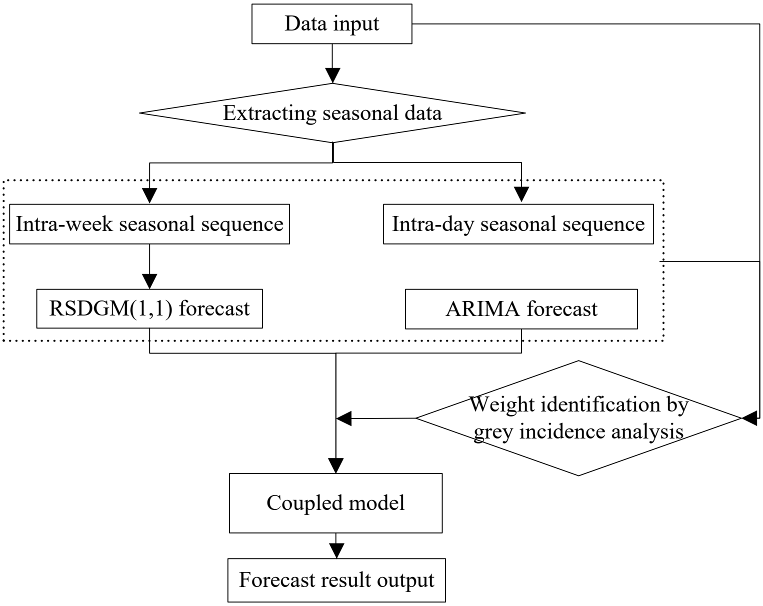

Fig. 7.5 Flowchart of the coupled forecasting model

The forecasting process diagram of the coupled model is in the following (Fig. 7.5).

7.3.7 Numerical Examples

7.3.7.1 The Description for Data

We use four loop detectors to measure the required data on Shaoshan Road with 4 lanes in each direction (only considering from south to north), which is a busy arterial road in Changsha of China. Via the SCATS Traffic Reporter system (Li, 2016), each detector collected 288 traffic data with 5-min acquisition cycle. Flow data were collected from 21 days (October 14 to November 3, 2013) and used for model development. For convenience, we converted the raw traffic flow data into hourly data points. Figure 7.6 shows the 3D display of the panel data. Figure 7.7 shows that the cross-sectional data have a significant weekly seasonality with a period of 7, and the time-series data have significant intra-day trends with a period of 24.

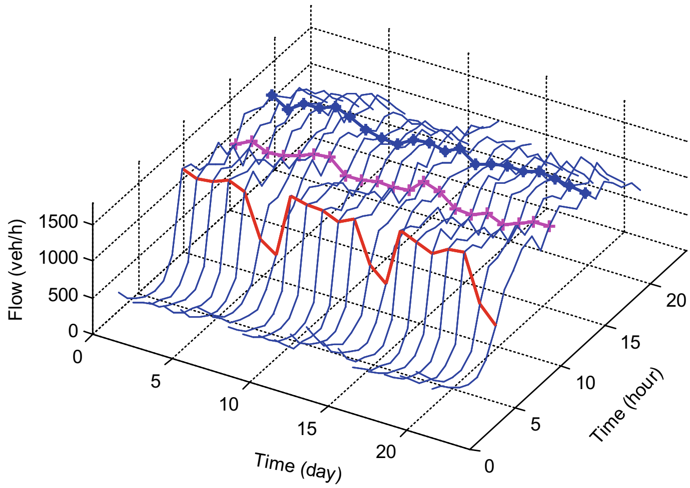

Fig. 7.6 3D display of time-series and cross-sectional traffic data

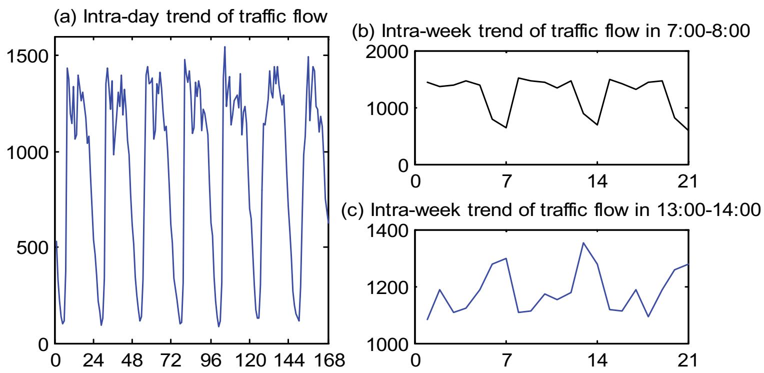

Fig. 7.7 Intra-day seasonal traffic trends and intra-week traffic seasonality

We use absolute percentage error (APE) and mean absolute percentage error (MAPE) to describe the model predictive performance. At the same time, the equal coefficient (EC) is used to describe the fitting degree, $x(k)$ is the measured data, $\hat{x}(k)$ is the predicted data, and N is the data points number. Thus,

$$APE = \frac{\left|x(k) - \hat{x}(k)\right|}{x(k)} \times 100\% \tag{7.42}$$

$$MAPE = \frac{1}{N} \sum_{k=1}^{N} \frac{\left|\hat{x}(k) - x(k)\right|}{x(k)} \times 100\% \tag{7.43}$$

$$EC = 1 - \frac{\sqrt{\sum (\hat{x}(k) - x(k))^2}}{\sqrt{\sum (x(k))^2} + \sqrt{\sum (\hat{x}(k))^2}} \tag{7.44}$$

7.3.7.2 Forecasting Results Analysis of RSDGM(1, 1) Model

The aim for traffic flow forecasting is to service the traffic management, so we select the data 16 time intervals from 6:00 to 22:00 each day, and establish the RSDGM(1, 1) models for 16 different cross-sections. In the process of rolling forecasting, we use seven data for model fitting, after one step prediction, remove the oldest data point, and add the most recent one.

We establish the rolling model according to the procedure shown in Fig. 7.4 for 21 cross-section sample data. For $x^{(0)} = (x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(21))$, $q = 7$, $\hat{x}^{(0)}(14), \hat{x}^{(0)}(15), \ldots, \hat{x}^{(0)}(21)$ are the rolling prediction of the Step1–Step8, their MAPEs are measured as a model performance criterion. We also obtain the rolling prediction $\hat{x}^{(0)}(22)$ and calculate its APE for November 4.

The forecasting comparison of the RDGM(1, 1) and RSDGM(1, 1) models is on the Table 7.4 for 16 different cross-sectional data. It is shown that the RSDGM(1, 1) model is better than the DGM(1, 1) model from Fig. 7.8.

Comparing to the DGM(1, 1) model, the RSDGM(1, 1) model is more stable, which is displaying in Table 7.5. There are 14 groups which average relative error is of <6% among 16 groups of cross-sectional data. There are 7 groups in the range of (6%, 10%], 3 groups out 10%, and the maximum relative error was 42.04%. It means the average relative error is more discrete.

The smooth ratios of the above two models for the two sets of cross-sectional data are shown in Fig. 7.9, especially, the 11th group of cross-sectional data in Fig. 7.9a, the 8th group of cross-sectional data in Fig. 7.9b. We can see that both RDGM(1, 1) and RSDGM(1, 1) models meet the quasi-smooth conditions for the 11th set of cross-sectional data, but for the 8th set of data, only RSDGM(1, 1) model fits the quasi-smooth conditions. From the comparing analysis, CTAGO operator in RSDGM(1, 1) model can improve the smooth ratio to meet the modeling conditions for periodic volatile data series.

We display the prediction effect of the above two models for two cross-sectional data sets in Fig. 7.10, the weaker cycle volatility for 1-h data in 10:00–11:00 is in Fig. 7.10a, while at least 14 groups show strong cycle volatility in Fig. 7.10b. For the 8th group, the MAPE reached 42.04%, that is to say, they cannot capture the fluctuation effectively. While the RSDGM(1, 1) model can reflect the fluctuation effectively and its MAPE is only 5.19%. Therefore, the RSDGM(1, 1) model is better than the RDGM(1, 1) model for adaptability and stability.

Table 7.4 The forecast comparison of the RDGM(1, 1) and RSDGM(1, 1) models

Cross-sectional data series	Time interval	RDGM(1, 1)		RSDGM(1, 1)	
		Steps 1–8 Forecast MAPE (%)	*November 4* Forecast APE (%)	Steps 1–8 Forecast MAPE (%)	*November 4* Forecast APE (%)
1	6:00–7:00	0.1207	0.3515	0.1028	0.0379
2	7:00–8:00	0.4204	0.4584	0.0519	0.0292
3	8:00–9:00	0.1682	0.2598	0.0839	0.0829
4	9:00–10:00	0.0842	0.1055	0.0453	0.0581
5	10:00–11:00	0.0271	0.0372	0.0383	0.0526
6	11:00–12:00	0.0576	0.0764	0.0434	0.0714
7	12:00–13:00	0.0992	0.0722	0.0525	0.0101
8	13:00–14:00	0.0848	0.1361	0.0286	0.0294
9	14:00–15:00	0.0453	0.0885	0.0278	0.0165
10	15:00–16:00	0.0411	0.0258	0.0560	0.0458
11	16:00–17:00	0.0671	0.0436	0.0541	0.0357
12	17:00–18:00	0.0629	0.0777	0.0580	0.0537
13	18:00–19:00	0.0591	0.0195	0.0545	0.0275
14	19:00–20:00	0.0500	0.1564	0.0224	0.0568
15	20:00–21:00	0.0723	0.0577	0.0435	0.1108
16	21:00–22:00	0.0779	0.0736	0.0326	0.0074

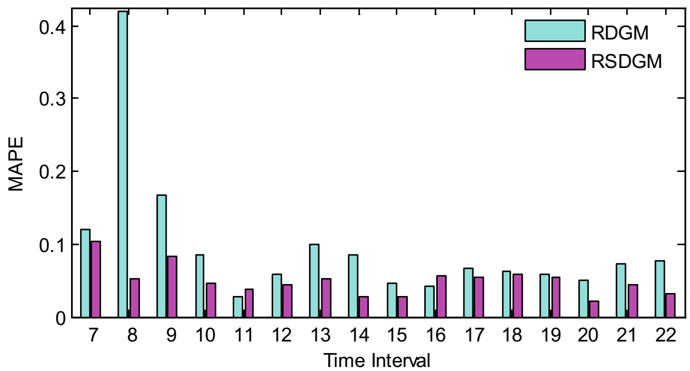

Fig. 7.8 The MAPE values of the RDGM(1, 1) and RSDGM(1, 1) models

Table 7.5 The distribution of the MAPE values for the RDGM(1, 1) and RSDGM(1, 1) models

MAPE	(0, 3%)	[3%, 6%]	(6%, 0%]	(10%, 50%)
RDGM(1, 1)	1	5	7	3
RSDGM(1, 1)	3	11	1	1

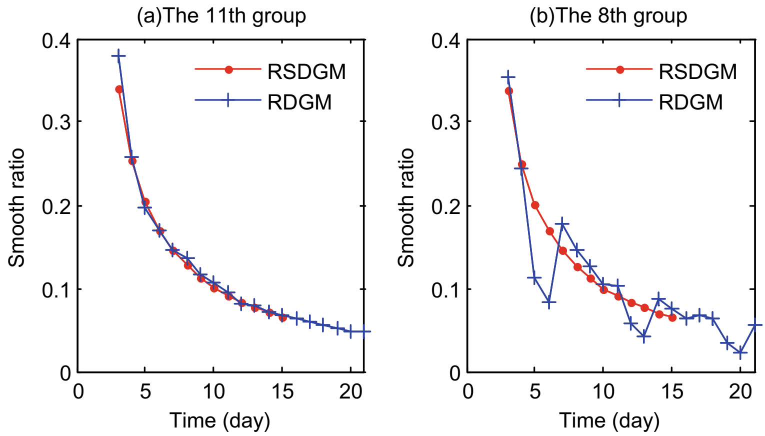

Fig. 7.9 Smooth ratios of the fitting cross-sectional data of two models: **a** the 11th group, **b** the 8th group

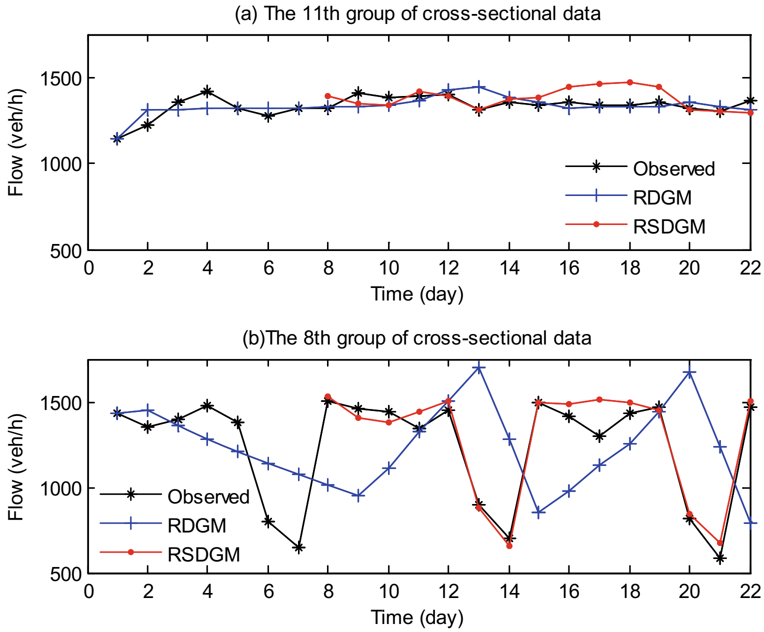

Fig. 7.10 The prediction effects of the cross-sectional data: **a** the 11th group, **b** the 8th group

7.3.7.3 Forecasting Results Analysis of the Coupled Model

The RSDGM(1, 1)-ARIMA model is coupled on the intersection point of the time-series and cross-sectional data. The ARIMA model is used to forecast the time-series data, the RSDGM(1, 1) model is used to forecast the cross-sectional data. The first-order difference data is stable by testing based on 504 time intervals of data in order to establish ARIMA(5, 1, 5) model. Then we construct the coupled model when transverse model and longitudinal models are established according to the rule in Sect. 7.4.

During the modeling, the weights are determined by the nearness grey relational degree in the coupled model. In Fig. 7.11, the weight coefficients of RSDGM(1, 1) model are consistent with the general combination model. The weight coefficients, which reflect the coupled effect, are in [0.45, 0.650]. We can see that the effect of the coupled model is better than the two baseline models in Fig. 7.12. It is obviously that the coupled model improves the forecasting effect of the single model in Fig. 7.13, which shows the error performance of the forecasting results of the three models. For the coupled model, we can obtain a stable output because the average error is only 4.02%, and the maximum error is on more than 10%.

We show the forecasting values of the three coupled models in Table 7.6, which are the coupled model with equal weight, the Bayesian combination model and the proposed coupled model. The relative errors of the three coupled models are all less than that of the single model, and the optimal result is obtained by the proposed coupled model.

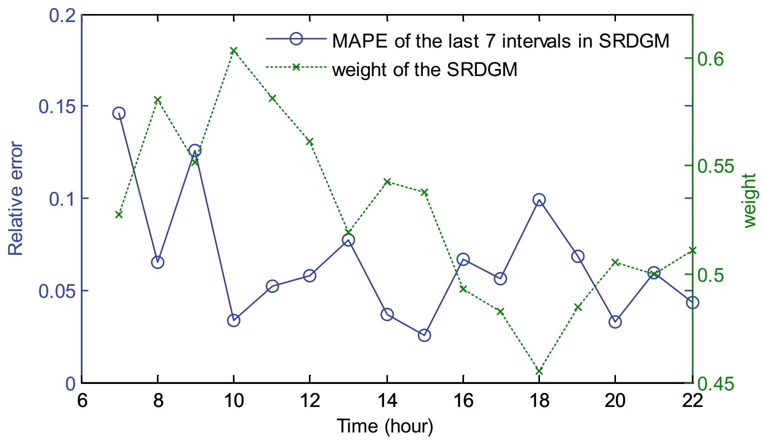

Fig. 7.11 Comparing the weights of the RSDGM and the MAPE of the last 7 intervals

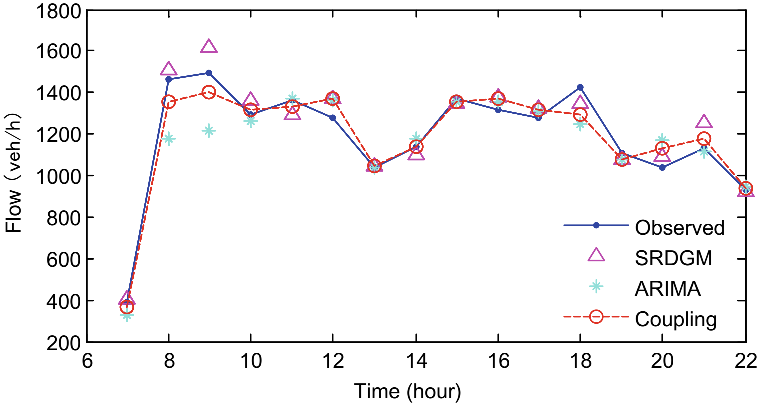

Fig. 7.12 The forecasting effect of the three models for the data from7:00 to22:00 on November 4

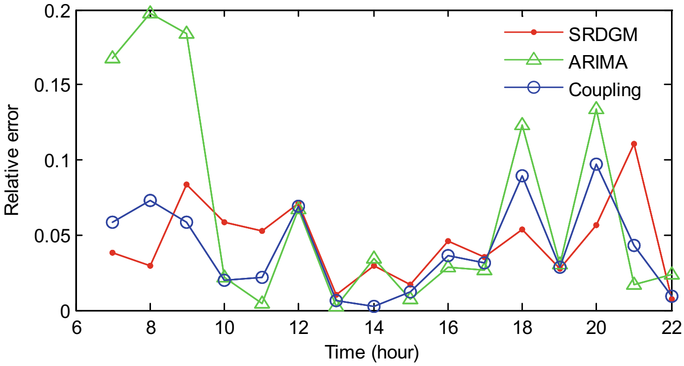

Fig. 7.13 The error performance of the forecasting results of the three models

Table 7.6 Comparing the various model prediction effects

Model	RSDGM(1, 1)	ARIMA	Coupled model with equal weight	Bayesian combination model	The proposed coupled model
MAPE (%)	4.54	6.68	4.17	4.44	**4.02**
EC	0.9732	0.9503	0.9738	0.9722	**0.9743**

7.3.8 Conclusions

For time-series and cross-sectional data of traffic flow, we establish the RSDGM-ARIMA coupled forecasting model and obtain some conclusions as follows: (1) The CTAGO operator can make the cross-sectional traffic flow sequence with the weekly seasonality to satisfy the quasi-smooth condition. This work extends the application of the DGM(1, 1) model and also improves its prediction accuracy. (2) Based on the CTAGO operator, a new RSDGM(1, 1) model is established in this section. CTAGO operator can transform the seasonal fluctuation sequence into a flat sequence in order to construct the DGM(1, 1) rolling model, in which the sequence length in the rolling model is determined by matrix perturbation analysis. (3) By using the nearness grey relational degree to identify the weights of the benchmark model, we establish the new coupled model about the RSDGM(1, 1) and ARIMA models. Numerical examples show that the implementation, availability, and feasibility of the proposed coupled model.

7.4 ITS Dynamic Optimization Design Based on Grey System Theory

By considering the characteristics of traffic systems, we put forward a dynamic optimization system of intelligent traffic signals, based on grey system theory. Firstly, according to the trend of traffic flow, we use grey relational analysis to obtain the most similar sequence (one of the historical sequences in the same interval of various dates), then establish grey forecasting model to get the dynamic prediction value by the trend change of traffic flow and the most similar sequence. Furthermore, for the objective of the smallest delay time, we want to established an optimization model in advance of the next signal cycle and split; Finally, we conduct traffic experiments on the section of Youyi Avenue in order to compare the signal system affection before and after optimization. The results show that the inducted effects of optimized signal are improved significantly, which play the role of real-time controlling.

7.4.1 ITS Optimization Design Based on Grey System System

For controlling the intersection signal, we need to establish a short-time traffic flow model to forecast the future traffic flow data based on the dynamic road traffic flow state data, while the time interval of traffic flow data is limited to 15 min. A signal cycle length generally does not exceed 3 min; we adopt 5 min as an interval in predicting traffic flow for signal optimization control. Obviously, it is feasible to perform signal controlling optimization in one forecast period.

7.4.1.1 The Method for Searching the Most Similar Traffic Flow Historical Sequence

Although the changes of traffic flow stored in the database have a regular pattern, there are many differences between the changes. For dynamic forecasting of traffic flow and the premise of optimization control, it is very important to find the most similar traffic flow sequence which has a similar change trend with the current sequence. Grey relational analysis (GRA) is the right method for dealing with this problem. We use GRA to search for the most similar sequence of historical traffic flow (Xiao et al., 2005).

Suppose that the same day measured traffic flow sequence is $X = (x(1), x(2), \ldots, x(n))$, and $x(1), x(2), \ldots, x(n)$ respectively represents the flow data in the time of $1, 2, \ldots, n$ and n is the current time. Historical traffic flow series collection are $\overline{X} = (X_1, X_2, \ldots, X_m)$, m is the number of historical sequences in $\overline{X}$, and $x_i(k)$ is the measured data of X_i at time k. It is necessary to search for the similar traffic flow sequence to X in $\overline{X}$ and identify which trend is helpful to forecast.

For traffic flow sequences $X, X_1, X_2, \ldots, X_m$, let $\xi \in (0, 1)$, we have

$$\gamma(x(k), x_i(k)) = \frac{\min\limits_i \min\limits_k |x(k) - x_i(k)| + \xi \max\limits_i \max\limits_k |x(k) - x_i(k)|}{|x(k) - x_i(k)| + \xi \max\limits_i \max\limits_k |x(k) - x_i(k)|}$$

$$\gamma(X, X_i) = \frac{1}{n} \sum_{k=1}^{n} \gamma(x(k), x_i(k))$$

$\gamma(X, X_i)$ is the grey relational measure between X and X_i, ξ is the resolution coefficient and generally $\xi = 1/2$. Through the order of grey relational grade, the historical sequence is the most similar to the same day measured traffic flow sequence, which has the largest grey relational grade and is denoted by $X' = (x'(1), x'(2), \ldots, x'(n), \cdots)$.

However, there are no same traffic flows over two days; we can't say that the trends of $X = (x(1), x(2), \ldots, x(n))$ and $X' = (x'(1), x'(2), \ldots, x'(n), \cdots)$ are exactly same, so modifying the historical data based on real-time measured data is very necessary.

7.4.1.2 Dynamic Traffic Flow Information Capturing

By combining with the trend of traffic flow and the most similar historical data $X' = (x'(1), x'(2), \ldots, x'(n))$, we establish a dynamic short-term traffic flow forecasting model for controlling the light optimization by providing real-time traffic information.

Let $X^{(0)}$ be the original traffic flow data and $X^{(0)} = (x^{(0)}(k))$, $k = 1, 2, \ldots, n$, $X^{(1)} = (x^{(1)}(k))$ be the first order accumulated generating operation series of $X^{(0)}$, and

$$x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i),\ \ k = 1, 2, \ldots, n;$$

The differential equation of white form is as follows.

$$\frac{dx^{(1)}(t)}{dt} + ax^{(1)}(t) = b$$

If $X^{(1)}$ is an equal interval subsequence, the grey differential equation on $X^{(1)}$ is

$$x^{(0)}(k) + az^{(1)}(k) = b$$

where $z^{(1)}(k) = 0.5x^{(1)}(k) + 0.5x^{(1)}(k-1)$, a, b are called the developing coefficient and grey input respectively and $P = [a\ b]^T$. By the method of least squares, we have

$$P = \begin{bmatrix} a \\ b \end{bmatrix} = (B^T B)^{-1} B^T y_N$$

where

$$B = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \vdots & \vdots \\ -z^{(1)}(n) & 1 \end{bmatrix}, Y = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix}$$

Then, we have

$$\hat{x}^{(1)}(k+1) = \left(x^{(0)}(1) - \frac{b}{a}\right)e^{-ak} + \frac{b}{a}$$

$$\hat{x}^{(0)}(k+1) = \hat{x}^{(1)}(k+1) - \hat{x}^{(1)}(k)$$

By the above formulas, we can obtain the forecast value $\hat{x}^{(0)}(n+1)$. Combined with the most similar historical sequence and the $x'(n+1)$, the traffic flow on the next occasion can be forecasted in the following.

$$\hat{x}(n+1) = (1-\alpha)\hat{x}^{(0)}(n+1) + \alpha x'(n+1)$$

where $\alpha = \gamma(X, X')$.

7.4.1.3 Intersection Signal Optimization

Since the traffic flow of each time reach intersection is not constant, we need to set the parameters of the signal according to different traffic flows for improving the road capacity. Based on the prediction values, we establish the model to optimize the cycle and split the two phased signal in order to obtain the smallest delay time of traffic flow of reach intersection in the test term.

Suppose that: (1) the traffic flow inside the system is unsaturated; (2) the right-turning vehicles are not constrained by signal; (3) the system is two phased controlling signal, not considering the lost time caused by signal conversion. Considering the delay is caused by even arrival rate and randomness, the signal intersection delay formula is in the following.

$$d = \frac{c(1-\lambda)^2}{2(1-\lambda X)} + \frac{X^2}{2q(1-X)}$$

where d is the average delay of traffic flow in the test term, c is the cycle length of signal; λ is the split which is the ratio of the length of green light and the length of signal; q is the standardized traffic flow (pcu/h); C is the road capacity (pcu/h); X is the saturation and $X = q/C$. Further simplification of delay formula is as follows.

$$d = \frac{c \cdot C \cdot (1-\lambda)^2}{2(C-\lambda q)} + \frac{q}{2C(C-q)}$$

Under certain restricts, the reasonable timing scheme of intersection can minimize delay and increase capacity. Cycle is the key parameter to determine the effect of intersection traffic controlling. The shortest cycle of signal is generally no less than 36 s, otherwise, it cannot ensure that traffic vehicles approaching from several different directions can cross the intersection smoothly. The longest cycle of signal is generally no more than 120 s, otherwise, it will make waiting drivers to be worried or to mistake the delay for a signal control failure. Split is another key parameter when vehicles cross the intersection. We in general use the cycle and split to plan signal time, if the signal time is less, it may not ensure that traffic vehicles can cross safely. Therefore, it is stipulated that the shortest green light phase is no less than 20 s, while the longest green light phase is no more than 50 s. So, we construct the optimization model for signal controlling based on the above objective function and constraint conditions, the model is as follows:

$$\min \quad \frac{c \cdot C \cdot (1-\lambda)^2}{2(C-\lambda q)} + \frac{q}{2C(C-q)}$$

$$s.t. \; 36 \le c \le 120$$

$$20 \leq c \cdot \lambda \leq 50$$

Based on the forecast information of traffic flow, we can solve the optimal cycle and split by applying the optimization model, and control traffic signal to achieve the purpose of real-time inducing. We use the PSO algorithm to solve the problem in the following.

7.4.2 *The Design of an Intelligent Traffic Signal Optimization Control System*

7.4.2.1 Grey Prediction Control Process

Based on system behavior data, grey prediction control can predict the future action of systems and adopt controlling mechanisms. It can effectively prevent accidents and congestion, control road conditions, and improve the ability of roads to operate effectively. Figure 7.14 shows the scheme of grey prediction controlling.

Where, link 1 is the control system, x is the output of link 1, link 2 is the grey prediction controller, $\hat{x}$ is the forecast value, link 3 is the optimization control decision process, d_0 is a quantity given, u is the output of link 3, which is the control parameter optimized by comparing d_0 with $\hat{x}$.

7.4.2.2 The Working Principle of ITS Optimization Design

Based on grey system theory, the working principle of ITS optimization design is as follows: by the trend of traffic flow, the most similar sequence X' is selected by using grey relational analysis in the same interval historical sequences $\overline{X}$; then, based on the change of trend of the traffic flow, the grey prediction model is established by the n measured data sequence X before $n + 1$ time to predict the value $\hat{x}^{(0)}(n + 1)$. By combining $x'(n + 1)$ at $n + 1$ time in historical sequence X', we can obtain the

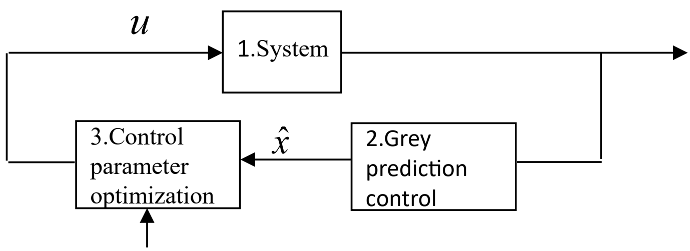

Fig. 7.14 Grey dynamic optimization control

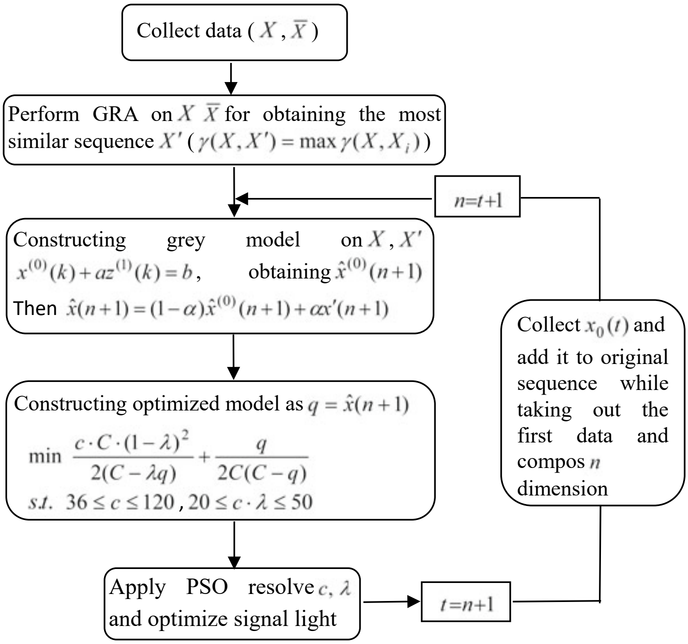

Fig. 7.15 The design flowchart of ITS optimization control

dynamic prediction value $\hat{x}(n+1)$ in order to realize the optimal control at $n+1$ time by the objective of the smallest delay time.

As we all know, the historical information will have a great influence on the prediction of future traffic flow. However, as the traffic flow information can change at any moment, the traffic flow data obtained by more closed information to the prediction time is more valuable. At $n+1$ time, the real traffic flow $x(n+1)$ can be measured. We can then add $x(n+1)$ into sequence X and subtract the farthest data to the current time from the original sequence, and obtain an equal n dimensions sequence. Next we establish a metabolic model to forecast the value $\hat{x}^{(0)}(n+2)$ at time $n+2$, and combine it with the data $x'(n+2)$ at time $n+2$ in historical sequence X' to obtain the dynamic prediction value $\hat{x}(n+2)$ at time $n+2$. The design flowchart of intelligent traffic signal control system is shown in Fig. 7.15.

7.4.3 *Empirical Analysis*

To realize the real-time inducing and optimization controlling of intelligent signals, we conducted experiments using vision trace to photograph the section of Youyi Avenue between Yuanlin Road and Jianshe Road within the Wuchang District of Wuhan. After consulting the city driveway traffic capacity table (Dai & Ma, 2006),

Table 7.7 Measured traffic flow

Measured data	183	209	185	180	219	218

Table 2 Historical flow recorded

Period	X_1	X_2	X_3	X_4	X_5	X_6	X_7
15:00–15:05	138	178	212	157	161	180	198
15:05–15:10	173	185	185	179	191	197	197
15:10–15:15	158	149	320	154	172	167	252
15:15–15:20	197	239	189	218	188	210	185
15:20–15:25	173	185	158	179	195	202	189
15:25–15:30	162	230	144	196	190	224	181
15:30–15:35	188	140	198	164	190	166	195
15:35–15:40	189	162	167	176	193	180	182
15:40–15:45	147	149	180	148	156	157	172
15:45–15:50	165	212	234	189	165	189	200
15:50–15:55	164	176	203	170	173	179	193
15:55–16:00	176	239	243	207	175	207	209

the traffic capacity of the experimental section is $C = 1600$ vehicles/h. Taking 5 min as time intervals and using the Sava video analysis software to cope with the video information collected from the experiment, the observed traffic flows are listed in Table 7.7, in which m (m = 7) historical traffic flow data in the same interval of various dates in the database are selected.

Based on the idea of extracting the most similar sequence and the data between 15:00 and 15:30, we obtained the grey relational degree of real measured data X and historical data $\overline{X} = (X_1, X_2, \ldots, X_m)$. The results are in Fig. 7.16 and Table 7.8, where $\max_i \gamma(X, X_i) = 0.8593$, $i = 5$. That is, the historical sequence X_5 is the closest sequence to the real-time measured traffic flow sequence, which has a high reference value for subsequent predictions. Let $X' = X_5$.

By the idea of dynamically obtaining traffic flow information, based on six measured data sets during 15:00–15:30, we establish the GM(1, 1) model to obtain that $\hat{x}^{(0)}(7) = 218.9907$, $RE = 14.0577\%$ in Table 7.9. As $x'(7) = 190$, $\alpha = 0.8593$, then we get the final prediction value of intersection traffic flow $\hat{x}(7) = 194.0790$, $RE = 1.0828\%$ at time 15:30–15:35 in Table 7.9. The prediction accuracy is improved as expected. At time sequence 15:30, using $q = \hat{x}(7) = 194.0790$, we establish the optimization model to get $c = 95$, $\lambda = 0.5474$ in Table 7.10. Then by optimizing the signal cycle and split in advance for the 15:30–15:35 time period, we obtain that the delay time of this period is 10.4220 s, while the delay time before optimization is 27.91 s.

At 15:35, we can measure that the real traffic flow during the period 15:30–15:35 is $x(7) = 192$, then add $x(7)$ into sequence X and subtract $x(1)$ and get an equal dimension sequence $(x(2), x(3), \ldots, x(7))$ in order to establish a metabolic grey

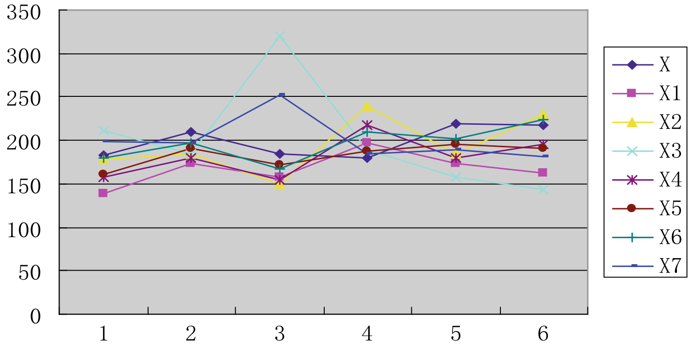

Fig. 7.16 The comparison between real-time measured sequence and historical sequences

Table 7.8 The comparison of grey relational degree

X	X_1	X_2	X_3	X_4	X_5	X_6	X_7
183	138	178	212	157	161	180	198
209	173	185	185	179	191	197	197
185	158	149	320	154	172	167	252
180	197	239	189	218	188	210	185
219	173	185	158	179	195	202	189
218	162	230	144	196	190	224	181
Grey relational degree	0.7456	0.6799	0.5429	0.8185	0.8593	0.7973	0.6604

Table 7.9 Grey dynamic predictions of traffic flow

	Real-Time measured sequence X	Similar sequence X'	GM(1, 1) prediction $\hat{X}^{(0)}$	Relative error (%)	Dynamic prediction $\hat{X}$	Relative error
15:30–15:35	192	190	218.9907	14.0577	194.0790	1.0828
15:35–15:40	196.5	193	214.2043	9.0098	195.9834	0.2629
15:40–15:45	163.5	156	203.2133	24.2895	162.6429	0.5242
15:45–15:50	165	165	162.2170	1.6867	164.6084	0.2373
15:50–15:55	181.5	173	149.9648	17.3748	169.7589	6.4689
15:55–16:00	174	175	164.4336	5.4979	173.5133	0.2797

model to predict one step $\hat{x}^{(0)}(8) = 214.2043$, $RE = 9.0098\%$ in Table 7.9. As $x'(8) = 193$, $\alpha = 0.8593$, we get the final prediction value of intersection traffic flow $\hat{x}(8) = 195.9834$, $RE = 0.2629\%$ at time period 15:35–15:40 in Table 7.9. At 15:35, using $q = \hat{x}(8) = 195.9834$, we establish the optimization model to get $c = 95$, $\lambda = 0.5579$ in Table 7.10. Then by optimizing the signal cycle and split in

Table 7.10 The comparison of the delay time of intersection signal optimization

Period	Measured flow	Prediction flow	Cycle	Split	Optimization delay time	Real delay time
15:30–15:35	192	194.0790	95	0.5474	10.4220	27.91
15:35–15:40	196.5	195.9834	95	0.5579	9.9651	29.98
15:40–15:45	163.5	162.6429	62	0.5000	8.1600	16.98
15:45–15:50	165	164.6084	64	0.5156	7.9303	17.47
15:50–15:55	181.5	169.7589	74	0.5000	9.7690	23.46
15:55–16:00	174	173.5133	74	0.5135	9.2752	20.59

advance for the 15:35–15:40 time period, we obtain the delay time of this period is 9.9651 s, while the delay time before optimization is 29.98 s.

By repeating the above steps, we realize the continuous optimization of signals, which would greatly reduce the delay time of traffic vehicles across the intersection. To some degree it will raise the road utilization rate and alleviate the pressure of urban traffic.

7.4.4 Conclusions

To deal with urban traffic congestion and its related issues, it is necessary to optimize intelligent traffic signal systems. By analyzing the characteristics of traffic flow: trend and change of trend, we propose a dynamical optimization ITS method. For the trend of the traffic flow, the most similar sequence, which is helpful in forecasting, is obtained in the same interval from a variety of dates by GRA. Based on the change of trend of traffic flow, grey prediction model is then established to get traffic flow data. By combining with the most similar sequence, the dynamic prediction value is obtained. According to the prediction value, for the objective of the smallest delay time, the optimization model is constructed in advance of the next signal cycle and split. Finally, we do road traffic experiments on the section of Youyi Avenue, analyze the reality of the situation and perform optimization controlling. Based on real-time data and historical data, we compare the signal system effects before and after optimization. The results show that the inducted effects of optimized signal are improved significantly. To some degree the study helps to raise the utilization rate and realize traffic signal real-time controlling.

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Chapter 8
Yellow River Ice Disaster Risk Management Based on Grey Prediction and Decision Method

Dang Luo

8.1 Research Profile of Yellow River Ice Disaster Risk Management

8.1.1 Profile of Yellow River Ice Flood and Its Disaster

The Yellow River rises in Jogu basin, below Bayan Kara Mountain. It flows through Qinghai, Sichuan, Gansu, Ningxia, Inner Mongolia, Shaanxi, Shanxi, Henan, Shandong from west to east, and is injected into the Bohai Sea in Kenli County, Shandong Province. The total length of the river is 5464 km, and the basin area is 795,000 km^2. The Yellow River Basin to the west belongs to the Qinghai–Tibet Plateau, north adjoins to Gobi desert, south nears the Yangtze River, and the whole river passes through the Huanghuaihai Plain. With the Yellow River being influenced by river, terrain, climate, man-made and other factors, it is easy for ice floods to form, threatening the reach of both sides of the river. All along the two reaches Yellow River ice flood disasters occur frequently and are serious. The two reaches are respectively the upper Ningxia–Inner Mongolia reach from Heshan Gorge to Hekouzhen, and the downstream reach from Huayuankuo to the Yellow River Estuary.

An ice flood is the result of rising river levels, caused by water in the river flow being blocked with ice. This directly affects the operation and maintenance of the water conservancy project, inland shipping, winter water supply, water conservancy and power generation, and adversely affects the areas ecology and environment. The yellow river is the most frequent river to ice flood in China, and the most seriously affected part is the Ningxia–Inner Mongolia reach. The Yellow River channel morphology is curving and changeable. The east–west trend of the Yellow River

D. Luo (✉)
College of Economics & Management, Nanjing University of Aeronautics and Astronautics, 29 Yudao Street, Nanjing 210016, People's Republic of China
e-mail: iamld99@163.com

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channel makes it form a reach that has two north–south trends and crosses the 10 latitudes, that is, the upper Ningxia–Inner Mongolia reach and the lower reach of the Yellow River. The two reaches are from high latitudes to low latitudes. In the winter when the river is frozen, it is traceable. When the river thaws in the spring, it flows from top to bottom. When the upstream thaws into open water and the downstream is still frozen, the ice floods of the upper reaches in the sharp bends, bayonet and other narrow reaches and ice jams can easily form, dam and block the river, threat the embankment safety and even burst and cause a disaster (Chang et al., 2016; Ke et al., 2002).

The Yellow River ice flood is one of the "four floods" (spring flood, summer flood, autumn flood, ice flood). According to the statistics, there were—214 ice jams and ice damming in Ningxia–Inner Mongolia reach from1951 to1968, with 32 disasters and an average of 1.77 disasters per year. In 1969–2010 after the completion of the Liujiaxia Reservoir, there were 132 ice jams and ice damming, 56 disasters, and an average of 1.33 disasters per year. In winter and spring each year, the Yellow River Ningxia–Inner Mongolia reach arises different degrees of ice disasters, and a larger range of disasters occurs on average for two years. 2007–2008 witnessed the most serious ice flood in 40 years, causing great harm. The consequences of the ice disaster were very serious. A variety of housing facilities of floodplain villagers were destroyed and the huge economy suffered losses including traffic and power disruption, etc. The disaster also led to beach/bank areas unable to cultivate food crops. In addition, shore slope and shore vegetation were completely destroyed, resulting in bank collapse; river protection facilities were destroyed; rivers re-routed and ice dams caused new jam points (Wang & Han, 2014).

8.1.2 Profile of Yellow River Ice Disaster Risk Management

Ice disaster research has a large systematization and regionalism. In recent years, with the development of computer technology and the emergence of new mathematical methods, ice flood prevention and control technology and risk management decision methods have been developed rapidly. However, due to its complexity of data and other factor constraints, the research of comprehensive quantitative analysis on ice and flood disaster risk management, the optimization decision theory, model and method are still relatively weak and in the development stage. At home and abroad, there is still a lack of systematic risk management research on "the research of ice disaster mechanisms–risk analysis of ice disaster–prevention and control of ice disasters", the implementation and operability of research results are not strong and the research means and methods are yet to be perfected. To a certain extent, it is a lack of a definite understanding in local problems such as river ice motion (Debele et al., 2007; Fan et al., 2006; Loukas et al., 2002).

At present, there is still little research on Yellow River ice disaster risk management, and even the concept of this risk management is inconclusive. Huang Qiang and the relevant units carry out the research work such as "Survey on the water capacity of Ningxia–Inner Mongolia reach in prevention period and the control leakage program risk analysis of Liujiaxia". Here they put forward the concept of ice disaster risks, and construct a system evaluation method of ice disaster risks in the Ningxia–Inner Mongolia reach (Huang, 2011). Huang discusses the basic theory and model of different types of natural disaster risk assessment, and puts forward the theory and method system of natural disaster risk assessment under incomplete information conditions. The research results can be used to develop and implement natural disaster risk control measures and provide a scientific basis for guidance (Huang, 2005). According to the characteristics of ice evolution and ice disaster, Wu et al. explain the meaning of ice disaster risks in the Ningxia–Inner Mongolia reach of the Yellow River, and put forward the theoretical framework of "risk identification–risk estimation–risk assessment–risk management". Then they construct the comprehensive assessment model of an ice disaster risk based on projection pursuit, fuzzy clustering and accelerating genetic algorithms. Finally, they evaluate the ice disaster risk of the Ningxia–Inner Mongolia reach from 1991 to 2010 (Wu et al., 2015). Luo selects water level, flow and the number of frozen days (three factors of the Ningxia–Inner Mongolia reach of Yellow River) as risk factors, and uses the three-parameter interval grey number rather than the real number to express accurately the complex uncertainty of evaluation information. Then he constructs the optimal phase grey relational decision-making method with three-parameter interval grey number information, and evaluate the ice disaster risk measure values. Finally, he uses the GM(1, 1) model to predict the ice disaster risk (Luo, 2014). Furthermore, Luo et al. analyze and identify seven indexes, including the number of days of closure and peak flow, to construct the ice disaster risk index system, and express the data characteristics of risk index based on the characterization of three-parameter interval grey number. Then they carry out the ice disaster assessment by using the grey interval correlation clustering method, and extract the decision rules reflecting ice information and the development of ice disaster risk degree by using the grey dominant rough set method. The example shows the feasibility and validity of the method (Luo et al., 2017). However, due to its complex influencing factors, changing evolutionary trends, and the commonness of conventional complex system risk assessment and the individuality of other natural disaster risk assessment, the risk of ice disaster in the Ning–Meng reach of the Yellow River is different from that of other natural disaster risk assessment. Only by using advanced uncertainty analysis theories and other multiple disciplinary integrated approaches to research, the theory and method system of risk identification, risk evaluation and risk control are put forward. The calculation results are then compared with the historical data, which can be better applied to practice and guide production.

In the problem of ice analysis and ice prediction, domestic research starts late and began in the 1950s and 1960s. In the tireless efforts of researchers, ice research work has a greater development and has made great achievements. In 1990s, China's ice research introduced advanced foreign experience, and continued to develop in depth and breadth. The current domestic research progress basically synchronizes with the international advanced level. At the same time, combining with the special hydrology, meteorology and geographical conditions of China's ice, domestic researchers not only put forward a number of theories and methods, but also apply these theories and methods to ice analysis and prediction, which greatly expands ice research ideas (Wang & Han, 2014). The results of ice forecasting are encouraging. For example, Zhang establishes the statistical model of ice forecasting in the upper reaches of the Yellow River by using the multiple linear regression analysis method (Zhang, 1997). On the basis of thermodynamics theory and ice water mechanics theory, Ke et al. establish the mathematical model of freeze-up forecasting in the Ningxia–Inner Mongolia reach (Ke et al., 2002). Hu establishes the ice forecasting model of the Ningxia–Inner Mongolia reach, based on the neural network optimized by genetic algorithm and the least squares support vector machines (Hu, 2006). Feng establishes the neural network model, genetic algorithm model and other mathematical models of ice forecasting by analyzing the ice regime characteristics of the Ningxia–Inner Mongolia reach, and discusses the accuracy of the forecast model (Feng, 2009). Han et al. introduce the parameters of the neural network optimized by particle swarm optimization, and the comprehensive forecasting model of the Ningxia–Inner Mongolia reach in different periods (Han et al., 2012). Liu and Huo introduce the medium-short term temperature forecast model, indices and empirical correlation methods, artificial neural network model, hydrology and thermodynamics model, as well as the river ice hydraulic model. It is suggested that the ice hydraulic forecast models need to be built up, based on ice condition observation and research on ice regulation in the river (Liu & Huo, 2015).

The hydrological and water resources systems, such as ice and flood disasters, are a system of the coexistence of randomness, fuzziness and greyness. In order to overcome the shortcomings of single random analysis, fuzzy analysis and grey analysis, there has been a new trend of coupling a variety of uncertainty analysis theory methods to the systematic analysis of hydrological and water resources uncertainty in recent years. For example, under the framework of Bayesian theorem, Huang and Ding consider the prior distribution and likelihood function respectively as grey prior distribution and fuzzy likelihood functions, and then perform the coupled process and deduces the grey fuzzy posteriori distribution. On this basis, they perform decision-making analysis of hydrology and water resources (Huang & Ding, 1994). Li and Chen introduce the cloud model of qualitative and quantitative conversion-representing the information fuzziness and randomness of the qualitative into the grey whitening weight function expression, which improves the traditional whitening weight function and establishes the whitening weight function based on grey cloud model. A grey cloud clustering model is then used to assess flood disaster loss in order to make up for the shortcomings of common methods (Li & Chen, 2013).

In summary, under the tireless efforts of domestic and foreign scientific researchers, the ice disaster risk management and related researches have made some achievements. The using of new technologies, new methods, and new theories opens up a new field for ice disaster research. The essence of ice risk management is how to analyze and deal with various uncertainties with information that exists in the structural and behavior characteristics of ice disaster risk management systems. The existing uncertainty analysis methods of disaster risk management pay more attention to statistical characteristics of the hydrological series. The statistical characteristics analysis only carries out uncertain variables, and is applies usually to uncertainty information of randomness or fuzziness, but not to other types of uncertainty information. In addition, the research on the uncertainty systems of ice disaster risk management is less concerned with the causes of the physical mechanism that drives the change of these uncertain variables. The relative relationship between the characteristic analysis of the risk data and the physical cause is also less. At the same time, the research on the quantitative analysis of ice disaster risk management, comprehensive evaluation, optimization of decision-making models and the effectiveness of various methods is not enough, which makes existing methods difficult to promote and apply in practice. Therefore, the study of ice disaster risk management has received more attention, but it still has a long way to go.

8.2 Risk Factors of Yellow River Ice Disaster

8.2.1 Analysis of Main Features and Influence Factors of Yellow River Ice Disaster

The Yellow River basin is influenced by the monsoon from Siberian in winter. The climate is dry and cold and the precipitation is scarce. The rivers rely mainly on groundwater recharge, and the flow is small. From west to east, the general trend of temperature distribution of the Yellow River basin is from cold to warm. The temperature gradually becomes cold from south to north, and the temperature change of the east–west gradient is significantly greater than those of the north–south gradient. According to statistics, the average temperature of the whole river basin in January is generally below 0 °C. The average annual extreme low temperature of the upper reach is −53 to −25 °C, the temperature of the middle reach is −40 to −20 °C, and the temperature of the lower reach is −23 to −15 °C. When winter arrives, the Yellow River's main stream and direct current will appears different degrees of ice formation. The ice regime will have negative effects on water transportation, water supply, power generation and hydraulic structures in winter. In the period of break-up, because of the special channel of the Yellow River, the temperature is higher in some upper reaches than the lower reaches, which causes the ice sheet in the upper reach to thaw in advance. The generated ice flow moves to the lower reach along the river, and they form ice jams or ice dam in the frozen river to block the channel and then raise the water level. Eventually when the ice dam ruptures, a large number of ice flows rush downstream and cause flooding.

When the Yellow River is influenced by river, terrain, climate, man–made and other factors, it is easy to form ice floods, which threaten the reach of both sides of the river. All along, the two reaches' Yellow River ice flood disasters occur frequently and are serious. The two reaches are respectively the upper Ningxia–Inner Mongolia reach from Heshan Gorge to Hekouzhen and the downstream reach from Huayuan

Estuary to the Yellow River Estuary. The common characteristics of the two reaches are that stream gradients are small, the flow rate is slow, and the direction of flow is from low latitude to high latitude (from southwest to northeast). The upper reaches of the two sides have a difference of 5° and the lower reaches are 3°. In winter, the temperature of the upper reach is higher than that of the lower reach, and the ice of the upper reach is therefore thinner. The river freezes up from lower reach to upper reach and breaks up from upper reach to lower reach. When the upper reach remains in a freezing condition, a large quantity of defrosted ice water from the upper reach flows down to the lower reach, accumulating more and more, which forms an ice peak.

The date of the Yellow River ice peak is relatively fixed. In the upper reaches of the Ningxia–Inner Mongolia reach, the date is usually in the middle or later March, and in the lower reaches, it is usually in early or middle February. The ice flood peak is low and the duration is short. The peak flow is generally 1500–3000 m^3/s, and the maximum measurement is no more than 4000 m^3/s. The total flood volume of Hekouzhen in the upper reach is between 500 and 800 million m^3, and it is between 600 million and 1,000 million m^3 of Lijin in the lower reach. The main features of ice flood are as follows. On one hand, the flow of flood peak is small and the water level is high. For example, in 1955, the flow rate of Lijin station was only 1960 m^3/s, and the water level reached 15.31 m, which is 1.55 m higher than the water level of 13.76 m of the flood peak 10,400 m^3/s in 1958. On the other hand, when the water drum ice breaks, the flow rate increases along the path, which contrasts to the flood seasons. It is because the river is frozen, the water flow resistance increases and the water level rises, which make the channel-storage increment increased. When this part of channel-storage increment is released with the break-up and follows up, the ice water along the path gathers increasing pace, which forms the downward increasing ice peak.

Since the Sanmenxia reservoir is completed, the challenge of violent break-up has been greatly reduced due to the use of the water storage in the reservoir, greatly altering the ice flood of the Yellow River's lower reach. Determined by particular geographical position, water temperature, meteorological conditions and river characteristics, the upper reaches of the Yellow River from the Black Gap to the estuary in Ningxia–Inner Mongolia reach, have become the most frequent serious reaches of the Yellow River.

In recent years, extensive economic development has damaged the environment, which leads to global warming. The construction of various ice prevention projects, such as reservoir, levee, has caused increasing influences on ice from various aspects. The ice regime of the Ningxia–Inner Mongolia reach of Yellow River has presented some novel features: the dates of ice flow and freeze-up are postponed, the date of break-up comes early, and the freeze-up and break-up of the river are unstable; the capacity of under-ice river flow is decreased, and the channel-storage increment is obviously increased; the water level in the freeze-up and break-up is raised, the peak of ice flood is weakened, and the ice regime trends to be complicated.

According to the change characteristics of the ice regime in the Ningxia–Inner Mongolia reach of the Yellow River in recent years, the main reasons are as follows:

Firstly, affected by warming climate and increasing extreme weather events, the general temperatures of the Ningxia–Inner Mongolia reach are high in winter, and the temperature fluctuates wildly, so the extreme weather events occur frequently.

Secondly, reservoir operation has changed the flow condition of the river. During the period of ice flood, the reservoir increases the discharge flow, the water temperature is raised correspondingly, and the hydrodynamic force is increased, and, thus postponing artificially the dates of the ice flow and freeze up. During the period of break-up, the reservoir reduces the discharge flow, the peak of ice flood is restricted, and the risk of ice disasters is lower. The ice prevention measures change the natural law of the Ningxia–Inner Mongolia reach, and affects the capacity to under–ice river flow, which makes ice present new features.

Thirdly, river sedimentation is serious and the under–ice river flow is reduced. Since 1986, due to the influence of the Longyangxia reservoir operation, climatic conditions and human factors, the peak flow that reaches to the Ningxia–Inner Mongolia reach has decreased. As a result, the amount of water continues to be reduced, the capacity of sediment is decreased, and the sediment volume of the channel is increased significantly.

Finally, the river engineering is increased and the ice transport capacity is weakened. Due to rapid economic development of the Ningxia–Inner Mongolia reach, there are a growing number of Bridges in the river. There are 3 railway Bridges and 10 road ridges between Sanshenggong and Lama Bay, in addition to 12 new cross-river pontoon bridges. Cross-river engineering (including bridges, construction piles, cofferdams, pontoons, etc.) affects the formation and transport of ice jams. Furthermore, with the development of local agricultural economy, large scale agricultural reclamation of the embankment has occurred in the Ningxia–Inner Mongolia reach, and the production in the bottomland has been intertwined, which makes a large amount of ice water stay in the bottomland during the flood period and increases the water storage. In the period of break-up, the channel-storage increment that stays in the river beach releases slowly, which influences the formation process of peak flow, simultaneously increase the difficulty of flood forecasting.

8.2.2 The Main Influencing Factor of Yellow River Ice Disaster Risk

Three conditions for the formation of an ice disaster are as follows:

(1) Cold wave and cold air activity. The temperature intensity, duration and times of cold waves can influence the date, speed of freeze-up and the ice disaster.
(2) Adequate ice flow and ice accretion in the river. When the temperature drops below 0 °C and the water freezes, it will appear multiphase flow interactions including flowing ice, ice sheet and water flow in the river ice, which leads to ice flood problems.

(3) It is necessary to have boundary conditions that impede the leakage of ice. The boundary conditions of the river are mainly the flow direction, the bend and the width of the river, and the influence of any human construction projects of the river banks on the flood.

From the evolution of the ice regime, it can be seen that the formation of ice disasters relies on suitable climatic conditions and geographical location. Therefore, the main factors that affect the ice regime changes in the Ningxia–Inner Mongolia reach of the Yellow River are divided into thermodynamic factor, dynamic factor, river regime factor and human factor (Feng, 2009; Wang & Han, 2014).

(1) Thermodynamic factor. This includes solar radiation, temperature and water temperature. The total radiation of the Inner Mongolia reach is gradually diminishing from the upper reach to the lower reach. The highest level is in June and the least is in December, and the difference is almost three times between them. The maximum fluctuation rate is from October to November and February to March, therefore, the change of ice regime is also the most violent here. The radiation from the sun always heats the water body and covering ice. The sun's angle is the lowest in the winter and has less radiation, the contact between the cold air and water makes the water congeal into ice, while the increase of radiation in spring is very important for the disintegration of ice.
    In the period of break-up, the rising or falling of temperature not only affects the speed of break-up, but also changes the way that the ice breaks, delays or promotes the growth and outburst of ice dams, and has obvious restraining influences on the dynamic action.
(2) Dynamic factor. This includes flow, water level, flow rate, wind force and wave. The power of water flow is mainly reflected in the speed of flow and the mechanical force of the fluctuation of water levels. The flow rate directly influences the ice conditions, the transportation of ice and the submergence. The rise of the water level is closely related to the state of break-up. If the water level/flow is not high, the ice sheet will not be cracked, and it can only be dissolved locally, which is a tranquil break-up and vice versa. However, the change of water level and flow rate often depends on the amount of flow, when the water section is unchanged, the water level and velocity have a function relationship with flow rate. When the flow is increased, the water level is raised and the flow rate is increased, so the ice flow begins with ice and ends up with water. It is essentially the amount of river flow.
    It is the north wind in the winter and spring of the Inner Mongolia reach, with average wind speeds of 4–5 m/s, (maximum up to 34 m/s), which has significant impact on the ice regime. When the direction of the flow is consistent with the direction of the wind, it has an effect on preventing or promoting the freeze-up.
(3) River regime factor. The river regime is the channel morphology, which mainly includes geographical location, the direction of the river and the boundary characteristics of the river. As the temperature decreases with the increase of geographical latitude, the geographical location of the channel and the direction of the river are related to thermodynamic factors. The direction of the river

Table 8.1 The main influencing factor of ice disaster

Factor	Measurable index
Thermodynamic factor	Temperature, the date of freezing up and melting
Dynamic factor	Flow, flow rate, wind power, the density of flowing ice
River regime factor	Width and depth of river, gradient, cannel-storage increment
Human factor	Reservoir operation increment, engineering effect

flows from the southwest to the northeast, and the temperature in the upper reach is higher than that of the lower reach. The characteristics of channel boundary mainly refer to the width, depth, gradient, bend and bifurcate. When the temperature and flow are certain, it is easy to produce an ice dam in the bend, shallows and the split reach. For instance, the river pattern from the Bayangol to Sanhu River reach is plain bending, and the vertical ratio is only 0.139%. In winter, the reach can easily freeze into an ice jam.

(4) Human factor. This refers to build reservoirs, score flood detention, diversion canal and so on. The reservoir operation can not only change the flow distribution process of the original river, but also increase the water temperature. Therefore, the reservoir operation is not only reflected in the hydraulic factors, but also the thermodynamic factors. In the upper reaches of the Yellow River, there are six large reservoirs, including Qingtongxia and Liujiaxia. In particular, the application of Liujiaxia and Longyangxia reservoirs has great influence on the ice regime in the Ningxia–Inner Mongolia reach. However, the reservoir operation also creates new problems, such as the channel is silted up, the bank is scoured, and the curve is increased, which causes a reduction in the capacity to carry water and ice. At the date of freeze-up, the ice is heavily blocked. Generally speaking, after the use of the reservoir, the ice dam in the break-up of the Ningxia–Inner Mongolia reach is alleviated, but the ice plug and ice dam in the freeze-up are serious. In addition, if the reservoir is not properly operated, the discharge will be very large. Not only will the situation not be alleviated, but this also may cause a man–made ice disaster.

To sum up, we divided the ice disaster of the Yellow River into four main factors. In order to facilitate operation, we refine four factors into measurable indexes, as shown in Table 8.1.

8.3 Risk Assessment of Ice Disaster in the Yellow River

Ice disaster of the Yellow River valley has posed a great threat to the rapid economic and social development and the safety of people's lives and property of the Yellow River coastal area. Under such circumstances, the task of reducing these disasters is becoming harder, while the situation becomes more serious. So we urgently need to do well the relevant work of ice disaster risk management to minimize the damage caused by ice disasters.

In order to analyze the possibility of the Yellow River ice disaster causing loss, and reveal the development relationship between the ice data and the disaster risk, we need to enhance the capability of ice disaster risk assessment and emergency management. In this section, the hybrid grey multiple-attribute decision-making method, the grey phase correlation decision-making method and the grey rough combined decision-making method are studied respectively. The above methods are used to solve the risk assessment problem of ice disaster in the Ningxia–Inner Mongolia reach of the Yellow River.

8.3.1 Risk Assessment of Ice Disaster Based on Hybrid Grey Multiple-Attribute Decision-Making Method

In connection with the problem of the coexistence of randomness, fuzziness and greyness in a Yellow River ice disaster, we propose a mixed multi-attribute decision-making problem, in which the attribute values are the "three parameters" interval grey number and fuzzy language. On the basis of carefully studying the connotation of the grey target, the positive and negative bull's-eye is discussed, based on the idea of "clutch" which is separating the different types of information in advance before they are assembled. In order to effectively distinguish a good and bad scheme, the principle of rewarding the good and punishing the bad is used to construct the comprehensive bull's eye distance, and the hybrid grey target method bull's-eye is proposed (Luo & Li, 2016).

Assume that the scheme set of hybrid multi-attribute decision-making problem is $A = \{A_1, A_2, \ldots, A_n\}$, attribute set is $B = \{B_1, B_2\}$, among them $B_1 = \left\{b_1, b_2, \ldots, b_p\right\}$ is the quantitative attribute set, $B_2 = \left\{b_{p+1}, \ldots, b_m\right\}$ is the qualitative attribute set. The effect value of the scheme $A_1 (i = 1, 2, \ldots, n)$ under the quantitative index $b_j (j = 1, 2, \ldots, p)$ is three-parameters interval grey number $x_{ij}(\otimes)$, denoted as $x_{ij}(\otimes) \in \left[\underline{x}_{ij}, \tilde{x}_{ij}, \overline{x}_{ij}\right]$, $U_1 = (a_{ij}(\otimes))_{n \times p}$ is decision makers' quantitative decision matrices. The effect value of the scheme A_i under the qualitative index $b_j (j = p + 1, p + 2, \ldots, m)$ is s_{ij}. s_{ij} is a language evaluation phrase chosen by a decision maker from a pre-defined language phrase, $W_2 = (s_{ij})_{n \times (m-p)}$ is the decision maker's qualitative decision matrix, $w = (w_1, \ldots w_p, w_{p+1}, \ldots w_m)$ is the weight sequence of the attribute set B.

Firstly, according to the correspondence between language phrase and triangular fuzzy number, the language evaluation phrase which is given by the decision maker is converted into the corresponding triangular fuzzy number. The qualitative decision matrix $W_2 = (s_{ij})_{n \times (m-p)}$ is transformed into a quantitative decision matrix $U_2 = (u_{ij})_{n \times (m-p)}$, u_{ij} is a triangular fuzzy number, and $u_{ij} = (u_{ij}^l, u_{ij}^m, u_{ij}^u)$.

Secondly, in order to eliminate the differences between different attributes and increase the comparability in the dimension, the decision matrix should be standardized. With regard to the three-parameters interval grey number, according to the principle of diminishing marginal utility in economics, i.e., the larger the normalized attribute value is, the smaller the change rate should be. The following nonlinear range transformation is constructed:

Let $\overline{x}_j^* = \max_{1\le i\le n}\{\overline{x}_{ij}\}$, $\underline{x}_j^\nabla = \min_{1\le i\le n}\{\underline{x}_{ij}\}$, $j = 1, 2, \ldots, m$, the decision matrix $U_1 = (a_{ij}(\otimes))_{n\times p}$ is normalized, and the normalized decision matrix is $R_1 = (r_{ij}(\otimes))_{n\times p}$. Cost and benefit type is the common type. For the benefit indices:

$$\underline{r}_{ij} = \frac{\left(\underline{x}_{ij} - \underline{x}_j^\nabla\right)^{1/2}}{\left(\overline{x}_j^* - \underline{x}_j^\nabla\right)^{1/2}}, \tilde{r}_{ij} = \frac{\left(\tilde{x}_{ij} - \underline{x}_j^\nabla\right)^{1/2}}{\left(\overline{x}_j^* - \underline{x}_j^\nabla\right)^{1/2}}, \overline{r}_{ij} = \frac{\left(\overline{x}_{ij} - \underline{x}_j^\nabla\right)^{1/2}}{\left(\overline{x}_j^* - \underline{x}_j^\nabla\right)^{1/2}} \tag{8.3.1}$$

For the cost indices:

$$\underline{r}_{ij} = \frac{\left(\overline{x}_j^* - \overline{x}_{ij}\right)^{1/2}}{\left(\overline{x}_j^* - \underline{x}_j^\nabla\right)^{1/2}}, \tilde{r}_{ij} = \frac{\left(\overline{x}_j^* - \tilde{x}_{ij}\right)^{1/2}}{\left(\overline{x}_j^* - \underline{x}_j^\nabla\right)^{1/2}}, \overline{r}_{ij} = \frac{\left(\overline{x}_j^* - \underline{x}_{ij}\right)^{1/2}}{\left(\overline{x}_j^* - \underline{x}_j^\nabla\right)^{1/2}} \tag{8.3.2}$$

For triangular fuzzy numbers, the decision matrix U_2 is normalized based on the method proposed by Xia Yongqi, $R_2 = (r_{ij}(\otimes))_{n\times(m-p)}$ can be obtained (Xia & Wu, 2004).

For the benefit indices:

$$r_{ij}^l = \frac{u_{ij}^l}{\sqrt{\sum_{i=1}^n (u_{ij}^u)^2}}, r_{ij}^m = \frac{u_{ij}^m}{\sqrt{\sum_{i=1}^n (u_{ij}^m)^2}}, r_{ij}^u = \frac{u_{ij}^u}{\sqrt{\sum_{i=1}^n (u_{ij}^l)^2}} \tag{8.3.3}$$

For the cost indices:

$$r_{ij}^l = \frac{(1/u_{ij}^u)}{\sqrt{\sum_{i=1}^n (\frac{1}{u_{ij}^l})^2}}, r_{ij}^m = \frac{(1/u_{ij}^m)}{\sqrt{\sum_{i=1}^n (\frac{1}{u_{ij}^m})^2}}, r_{ij}^u = \frac{(1/u_{ij}^l)}{\sqrt{\sum_{i=1}^n (\frac{1}{u_{ij}^u})^2}} \tag{8.3.4}$$

Then, the positive and negative bull's-eye of the hybrid evaluation vector is determined, and the positive and negative bull's-eye distance and the comprehensive bull's-eye distance of each scheme are calculated, which are as follows.

Definition 8.3.1 Assume that the optimal evaluation value of the scheme A_i under the quantitative attribute b_j is $r_j^+ = \max_{1\le i\le n}\{\tilde{r}_{ij} + (\underline{r}_{ij} + \overline{r}_{ij})/2\}$, denoted as $r_j^+(\otimes) = [\underline{r}_j^+, \tilde{r}_j^+, \overline{r}_j^+]$, $j = 1, 2, \ldots, p$, $r_j^- = \min_{1\le i\le n}\{\tilde{r}_{ij} + (\underline{r}_{ij} + \overline{r}_{ij})/2\}$ is the worst evaluation value, denoted as $r_j^-(\otimes) = [\underline{r}_j^-, \tilde{r}_j^-, \overline{r}_j^-]$. Assume that $r_j^+ = \max_{1\le i\le n}\{r_{ij}^m\}$ and $r_j^- =$

$\min_{1\le i\le n}\{r_{ij}^m\}$ is respectively the optimal evaluation value and the worst evaluation value of the scheme A_i under the qualitative attribute b_j, denoted as $r_j^+ = [r_{ij}^l, r_{ij}^m, r_{ij}^u]$ and $r_j^- = [r_{ij}^l, r_{ij}^m, r_{ij}^u]$, then

$$r^+ = \left[r_1^+(\otimes), \ldots r_p^+(\otimes), r_{p+1}^+(\otimes), \ldots r_m^+(\otimes)\right]$$

and

$$r^- = \left[r_1^-(\otimes), \ldots r_p^-(\otimes), r_{p+1}^-(\otimes), \ldots r_m^-(\otimes)\right]$$

are respectively the positive and negative bull's-eyes.

Definition 8.3.2 Assume that the effect evaluation vector of the scheme A_i under the attribute set B is $r_i = (r_{i1}, \ldots, r_{im})$, and $D_i^+ = D\big(r_i, r^+\big)$ is the positive bull's-eye distance of the vector r_i, $D_i^- = D\big(r_i, r^-\big)$ is the negative bull's-eye distance of the vector r_i, D is the comprehensive bull's-eye distance of the vector r_i, then

$$\begin{aligned} D(r_i, r^+) = w_1 d(r_{i1}(\otimes), r_1^+(\otimes)) + \cdots + w_p d(r_{ip}(\otimes),\\ r_p^+(\otimes)) + w_{p+1} L(r_{i(p+1)}, r_{p+1}^+) + \cdots + w_m L(r_{im}, r_m^+) \end{aligned} \tag{8.3.5}$$

$$\begin{aligned} D(r_i, r^-) = w_1 d(r_{i1}(\otimes), r_1^-(\otimes)) + \cdots + w_p d(r_{ip}(\otimes),\\ r_p^-(\otimes)) + w_{p+1} L(r_{i(p+1)}, r_{p+1}^-) + \cdots + w_m L(r_{im}, r_m^-) \end{aligned} \tag{8.3.6}$$

$$D_i = D_i^+ / D_i^- \tag{8.3.7}$$

If the attribute weight sequence $w = (w_1, \ldots w_p, w_{p+1}, \ldots w_m)$ is known, then directly input into the above formula can calculate the comprehensive bull's-eye distance of the scheme, and the scheme can be sorted based on this. The better the corresponding scheme is, the greater the comprehensive bull's-eye distance.

We consider five factors, including the flow rate, temperature, flow density, river condition and engineering influence, and use grey target decision-making methods with hybrid positive and negative bull's-eye to select data of three river reaches in the period of break up from 2012 to 2014, which are respectively Toudaoguai A_1, Sanhu River A_2 and Bayangol A_3, as is shown in Table 8.2. Please try to identify the river reach that is are more prone to ice dam disaster.

Table 8.2 Risk index data of three river reaches in the period of break up

Index	Toudaoguai	Sanhu River	Bayangol
Flow	[335, 520, 900]	[203, 770, 940]	[126, 650, 820]
Temperature	[−19.2, −10, 4.1]	[−19.5, −9.8, 2.8]	[−22.4, −14, 0.2]
Density of flowing ice	[10%, 10%, 40%]	[5%, 30%, 70%]	[10%, 30%, 60%]
River condition	Poor	Very poor	Poor
Engineering impact	Large	Medium	Medium

The specific steps are as follows.

(1) According to the correspondence between triangular fuzzy numbers and linguistic variables, the qualitative index in the decision matrix is represented by triangular fuzzy numbers, and the quantitative hybrid decision matrix is U:

$$U = \begin{bmatrix} (335, 520, 900) & (-19.2, -10, 4.1) & (10\%, 10\%, 40\%) & (0.2, 0.3, 0.4) & (0.6, 0.7, 0.8) \\ (203, 770, 940) & (-19.5, -9.8, 2.8) & (5\%, 30\%, 70\%) & (0.1, 0.2, 0.3) & (0.4, 0.5, 0.6) \\ (126, 650, 820) & (-22.4, -14, 0.2) & (10\%, 30\%, 60\%) & (0.2, 0.3, 0.4) & (0.4, 0.5, 0.6) \end{bmatrix}$$

(2) The decision matrix is normalized by the formulas (8.3.1)–(8.3.4), and the normalized hybrid decision matrix is R:

$$R = \begin{bmatrix} (0.22, 0.72, 0.86) & (0.35, 0.68, 1) & (0.68, 0.96, 0.96) & (0.20, 0.49, 1.03) & (0.32, 0.45, 0.62) \\ (0, 0.46, 0.95) & (0.33, 0.69, 0.98) & (0, 0.78, 1) & (0.27, 0.73, 2.06) & (0.43, 0.63, 0.94) \\ (0.38, 0.60, 1) & (0, 0.56, 0.92) & (0.39, 0.78, 0.96) & (0.20, 0.49, 1.03) & (0.43, 0.63, 0.94) \end{bmatrix}$$

(3) Determine the positive and negative bull's-eye of the mixed evaluation vector, calculate the positive bull's-eye distance D_i^+, the negative bull's-eye distance D_i^- and the comprehensive bull's-eye distance D_i of the scheme (take $\lambda = 0.5$, which means the importance of the evaluation index is the same):

$$D_1^+ = 0.271,\ D_2^+ = 0.087,\ D_3^+ = 0.272$$
$$D_1^- = 0.129,\ D_2^- = 0.301,\ D_3^- = 0.119$$
$$D_1 = 2.104,\ D_2 = 0.287,\ D_3 = 2.277$$

Table 8.3 The scheme order of the different w and λ values in the bull's-eye distance formula

$w = (0.2, 0.2, 0.2, 0.2, 0.2)$	$\lambda = 0.1$	$A_2 \succ A_1 \succ A_3$
	$\lambda = 0.5$	$A_2 \succ A_1 \succ A_3$
	$\lambda = 0.9$	$A_2 \succ A_3 \succ A_1$
$w = (0.35, 0.1, 0.35, 0.1, 0.1)$	$\lambda = 0.1$	$A_1 \succ A_3 \succ A_2$
	$\lambda = 0.5$	$A_1 \succ A_2 \succ A_3$
	$\lambda = 0.9$	$A_2 \succ A_1 \succ A_3$
$w = (0.2, 0.3, 0.1, 0.3, 0.1)$	$\lambda = 0.1$	$A_2 \succ A_1 \succ A_3$
	$\lambda = 0.5$	$A_2 \succ A_1 \succ A_3$
	$\lambda = 0.9$	$A_2 \succ A_1 \succ A_3$

(4) Based on the rule that the smaller D_i represents the better scheme, we can obtain the order of the scheme, which is $D_2 \succ D_1 \succ D_3$, that is, A_2 is the most impossible to occur ice dam disaster.

In the real-life situation, the evolution law of ice production and disappearance of the Yellow River is complex. In order to effectively prevent and control the occurrence of serious disasters, the staff can analyze the data of previous years, thus they can carry out prevention work on ice floods in advance. In order to observe whether the selection of the attribute weight sequence and the off-target distance formula has an effect on the ordering of the scheme, the different λ values and attribute weight sequences w are selected in step (3), and the scheme is reordered. The results are shown in Table 8.3.

It can be seen from Table 8.3 that when we calculate the bull's-eye distance, the order of the scheme is different based on different λ values and different attribute weight sequences w. If the importance of attribute is different, optimal scheme that we obtain may be different. Therefore, the decision maker can choose different parameters according to the actual situation.

8.3.2 Risk Assessment of Ice Disaster Based on Grey Phase Relation Decision-Making Method

We select the closure river flow, the seal water level and the frozen days as risk indicators, and calculate the risk value of ice disasters at the three reaches in the Ningxia–Inner Mongolia reach of the Yellow River, which are Bayangol, Sanhu River and Toudaoguai. Based on the field investigation, the relevant risk index data in Bayangol, Sanhu River and Toudaoguai are obtained from 2003 to 2012, which are shown in Tables 8.4, 8.5 and 8.6.

The indicator value of frozen days of each year is the real number, which is regarded as the special three-parameters interval grey number, and the risk indicators data of each river reach are standardized by grey range transformation. As shown in Tables 8.7, 8.8 and 8.9.

Table 8.4 Risk indicators data of flowing ice in Bayangol

Years	Flow (m^3/s)	Water level (m)	Frozen days (d)
2003	[710.0, 720.0, 730.0]	[50.51, 51.56, 52.60]	26
2004	[546.0, 598.0, 650.0]	[50.44, 50.79, 51.14]	15
2005	[580.0, 645.0, 710.0]	[50.59, 50.74, 50.88]	16
2006	[680.0, 730.0, 780.0]	[50.40, 51.03, 51.66]	16
2007	[530.0, 615.0, 700.0]	[50.83, 51.89, 52.94]	15
2008	[580.0, 600.0, 620.0]	[50.76, 51.81, 52.85]	17
2009	[420.0, 470.0, 520.0]	[50.87, 50.98, 51.08]	9
2010	[450.0, 535.0, 620.0]	[51.05, 51.22, 51.39]	22
2011	[600.0, 619.0, 638.0]	[51.25, 51.59, 51.92]	28
2012	[610.0, 615.0, 620.0]	[51.34, 51.71, 52.08]	25

Table 8.5 Risk indicators data of flowing ice in Sanhu River

Years	Flow (m^3/s)	Water level (m)	Frozen days (d)
2003	[850.0, 975.0, 1100.0]	[18.19, 18.32, 18.44]	9
2004	[332.0, 581.0, 830.0]	[17.74, 18.15, 18.55]	29
2005	[420.0, 555.0, 690.0]	[17.89, 18.29, 18.68]	39
2006	[350.0, 475.0, 600.0]	[17.71, 18.29, 18.86]	7
2007	[600.0, 720.0, 840.0]	[18.31, 18.41, 18.50]	15
2008	[825.0, 722.5, 620.0]	[17.37, 18.25, 19.12]	29
2009	[410.0, 575.0, 740.0]	[17.84, 18.20, 18.55]	15
2010	[280.0, 330.0, 380.0]	[17.87, 18.07, 18.26]	5
2011	[490.0, 512.5, 535.0]	[18.47, 19.47, 20.46]	25
2012	[150.0, 505.0, 860.0]	[17.60, 18.33, 19.06]	22

Table 8.6 Risk indicators data of flowing ice in Toudaoguai

Years	Flow (m^3/s)	Water level (m)	Frozen days (d)
2003	[520.0, 735.0, 950.0]	[87.03, 87.29, 87.55]	9
2004	[300.0, 495.0, 690.0]	[86.05, 86.74, 87.42]	45
2005	[390.0, 620.0, 850.0]	[86.66, 87.08, 87.50]	36
2006	[320.0, 417.5, 515.0]	[86.56, 86.78, 87.00]	3
2007	[600.0, 705.0, 810.0]	[87.18, 87.29, 87.40]	14
2008	[610.0, 650.0, 690.0]	[87.31, 87.37, 87.42]	6
2009	[320.0, 495.0, 670.0]	[86.41, 86.70, 86.98]	11
2010	[170.0, 335.0, 500.0]	[85.93, 86.49, 87.05]	23
2011	[280.0, 547.5, 815.0]	[86.35, 87.09, 87.83]	53
2012	[590.0, 670.0, 750.0]	[87.27, 87.42, 87.56]	13

Table 8.7 Standardization of the risk indicators data of Bayangol

Years	Flow	Water level	Frozen days
2003	[0.1389, 0.1667, 0.1900]	[0.13, 0.55, 0.96]	0.11
2004	[0.3611, 0.5056, 0.6500]	[0.71, 0.85, 0.98]	0.68
2005	[0.1944, 0.3750, 0.5600]	[0.81, 0.87, 0.93]	0.63
2006	[0.0000, 0.1389, 0.2800]	[0.50, 0.75, 1.00]	0.63
2007	[0.2222, 0.4583, 0.6900]	[0.00, 0.42, 0.83]	0.68
2008	[0.4444, 0.5000, 0.5600]	[0.04, 0.45, 0.86]	0.58
2009	[0.7222, 0.8611, 1.0000]	[0.73, 0.77, 0.81]	1.00
2010	[0.4444, 0.6806, 0.9200]	[0.61, 0.68, 0.74]	0.32
2011	[0.3944, 0.4472, 0.5000]	[0.40, 0.53, 0.67]	0.00
2012	[0.4444, 0.4583, 0.4700]	[0.34, 0.48, 0.63]	0.16

Table 8.8 Standardization of the risk indicators data of Sanhu River

Years	Flow	Water level	Frozen days
2003	[0.0000, 0.1316, 0.2632]	[0.6500, 0.6942, 0.7340]	0.88
2004	[0.2842, 0.5463, 0.8084]	[0.6200, 0.7492, 0.8800]	0.29
2005	[0.4316, 0.5737, 0.7158]	[0.5800, 0.7039, 0.8317]	0.00
2006	[0.5263, 0.6579, 0.7895]	[0.5200, 0.7039, 0.8899]	0.94
2007	[0.2737, 0.4000, 0.5263]	[0.6300, 0.6650, 0.6958]	0.71
2008	[0.5053, 0.3974, 0.2895]	[0.4300, 0.7168, 1.0000]	0.29
2009	[0.3789, 0.5526, 0.7263]	[0.6200, 0.7330, 0.8479]	0.71
2010	[0.7579, 0.8105, 0.8632]	[0.7100, 0.7750, 0.8382]	1.00
2011	[0.5947, 0.6184, 0.6421]	[0.0000, 0.3220, 0.6440]	0.41
2012	[0.2526, 0.6263, 1.0000]	[0.4500, 0.6893, 0.9256]	0.50

Table 8.9 Standardization of the risk indicators data of Toudaoguai

Years	Flow	Water level	Frozen days
2003	[0.0000, 0.2756, 0.5513]	[0.1600, 0.3034, 0.4200]	0.88
2004	[0.3333, 0.5833, 0.8333]	[0.2300, 0.6152, 0.9400]	0.16
2005	[0.1282, 0.4231, 0.7179]	[0.1900, 0.4213, 0.6200]	0.34
2006	[0.5577, 0.6827, 0.8077]	[0.4700, 0.5899, 0.6700]	1.00
2007	[0.1795, 0.3141, 0.4487]	[0.2400, 0.3034, 0.3400]	0.78
2008	[0.3333, 0.3846, 0.4359]	[0.2300, 0.2612, 0.2700]	0.94
2009	[0.3589, 0.5833, 0.8077]	[0.4800, 0.6376, 0.7500]	0.84
2010	[0.5769, 0.7885, 1.0000]	[0.4400, 0.7528, 1.0000]	0.60
2011	[0.1731, 0.5160, 0.8589]	[0.0000, 0.4157, 0.7800]	0.00
2012	[0.2564, 0.3589, 0.4615]	[0.1500, 0.2331, 0.2900]	0.80

The Analytic Hierarchy Process (AHP) is used to calculate the normalized weight vector of the flow, water level and frozen days, which is:

$$w = (w_1, w_2, w_3) = (0.38, 0.42, 0.20)$$

Based on weighted optimal grey phase relation method, let $\partial_1 = \partial_2 = \partial_3 = 1/3$, and $\rho = 10$, then we can calculate the risk value of Bayangol in 2003. Steps are as follows:

(1) Find out the best effect evaluation vector $z^+ = \{z_1^+, z_2^+, z_3^+\}$,

$$z_1^+ = \left\{\underline{x}_{01}^+, \tilde{x}_{01}^+, \overline{x}_{01}^+\right\} = [0.81, 0.87, 0.93],$$

$$z_2^+ = \left\{\underline{x}_{02}^+, \tilde{x}_{02}^+, \overline{x}_{02}^+\right\} = [0.7222, 0.8611, 1.0000],$$

$$z_3^+ = \{x_{02}^+\} = \{1\}.$$

(2) Calculate the weighted optimal grey phase relation factor:
According to

$$\underline{\eta}_{ij}^+ = \frac{1}{1 + \left|\omega_j(\underline{x}_{oj}^+(j) - \underline{x}_{ij}(j))\right|},$$
$$\tilde{\eta}_{ij}^+ = \frac{1}{1 + \left|\omega_j\left(\tilde{x}_{oj}^+(j) - \tilde{x}_{ij}(j)\right)\right|},$$
$$\overline{\eta}_{ij}^+ = \frac{1}{1 + \left|\omega_j\left(\overline{x}_{oj}^+(j) - \overline{x}_{ij}(j)\right)\right|}.$$

We have

$$\underline{\eta}_{11}^+ = \frac{1}{1 + |0.38 \times (0.81 - 0.13)|} = 0.7947,$$
$$\tilde{\eta}_{11}^+ = \frac{1}{1 + |0.38 \times (0.87 - 0.55)|} = 0.8916,$$
$$\overline{\eta}_{11}^+ = \frac{1}{1 + |0.38 \times (0.93 - 0.96)|} = 0.9888.$$

Similarly, we can get

$$\underline{\eta}_{12}^+ = 0.8032, \tilde{\eta}_{12}^+ = 0.7742, \overline{\eta}_{12}^+ = 0.7461,$$

$$\underline{\eta}_{13}^+ = \tilde{\eta}_{13}^+ = \overline{\eta}_{13}^+ = 0.8489.$$

Table 8.10 Risk measure results of ice disaster of three river reaches from 2003 to 2012

Reaches	Years									
	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012
Bayangol	0.2771	0.2525	0.2568	0.2711	0.2709	0.2672	0.2424	0.2491	0.2665	0.2668
Sanhu River	0.2795	0.2541	0.2566	0.2468	0.2607	0.2616	0.2517	0.2373	0.2665	0.2516
Toudaoguai	0.2728	0.2532	0.2658	0.2442	0.2717	0.2693	0.2477	0.2365	0.2646	0.2723

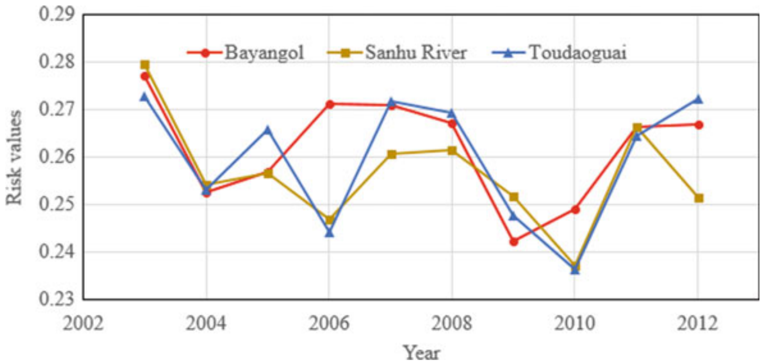

Fig. 8.1 Risk variation trend of ice disaster of three river reaches

(3) Calculate the degree of weighted optimal grey phase relation of Bayangol in 2003

$$\eta^{+}\left(X_0^{+}, X_1\right) = \frac{1}{10}\sum_{j=1}^{3}\frac{\underline{\eta}_{1j}^{+} + \tilde{\eta}_{1j}^{+} + \overline{\eta}_{1j}^{+}}{3} = 0.2771.$$

Similarly, the comprehensive risk measure for each year of Bayangol, Sanhu River and Toudaoguai can be obtained, as shown in Table 8.10 and Fig. 8.1.

In 2010, the risk measure values of ice disasters of the three river reaches are very low. Correspondingly, the flow of this year is also low. In fact, the flow is a significant factor to ice disasters. The decreasing flow will damage the continuity and stability of ice disasters to a certain extent, and further affecting the hydrodynamic action.

8.3.3 *Risk Assessment of Ice Disaster Based on Grey Rough Combined Decision-Making Method*

The grey rough combined decision-making method is carried out in two stages:

The first stage is grey interval relational clustering (GIRC) under grey number information. For the risk assessment problem of ice disaster that index values are three-parameter interval grey numbers, the grey relational coefficient matrix is obtained by calculating the grey relational coefficient between each object and the ideal sequence. Then, the grey interval relational clustering is applied to calculate clustering coefficients that object belongs to each grey class. According to the clustering coefficient, it not only effectively divides the objects into corresponding grey classes, but also determines the ranks in their own class.

The second stage is the decision-making rule extraction based on grey dominance relation rough set (GDRSA). Firstly, the clustering result of the first stage and the disaster data information system are used to construct the disaster risk decision table under the three-parameter interval grey number information. For the three-parameter interval grey number decision table, the resulting decision rules are further simplified by attribute reduction. Secondly, aiming to three-parameters interval grey number decision table, the grey dominant relationship is constructed, the upper and lower approximation sets are defined, and then the decision rules of the decision table are obtained. Finally, the decision-making rules are further simplified by attribute reduction (Luo et al., 2017).

Definition 8.3.3 Assume $GS = (U, AT, V, f)$ is the information system with three-parameter interval grey number,

$$\forall a \in AT, \forall x_i, x_j \in U, f(x_i, a) \in [x_i^l, x_i^m, x_i^u], f(x_j, a) \in [x_j^l, x_j^m, x_j^u].$$

are three-parameter interval grey numbers. For attribute a, the grey dominance degree of object x_i to x_j is defined as

$$D_a(x_i, x_j) = \begin{cases} 0, & x_i^u \le x_j^l \\ 1, & x_i^u \ge x_j^l \ \ or \ \ i = j \\ p(f(x_i, a) \ge f(x_j, a)), & otherwise \end{cases}$$

For the attribute a, if $D_a(x_i, x_j) = 1$, then x_i is completely better than x_j; if $D_a(x_i, x_j) = 0$, then x_i is completely inferior than x_j; if $0 < D_a(x_i, x_j) < 1$, then x_i is better than x_j with grey dominance $D_a(x_i, x_j)$.

Definition 8.3.4 Assume $GS = (U, AT \cup \{d], V, f)$ is a decision table with three-parameter interval grey number, where decision attribute d divides U into a finite subclass with a preference order. Let $Cl = \{Cl_t, t \in T\}$,$T = \{1, 2, \ldots, n\}$ be a set of these ordered classes, that is $\forall t, s \in T$, if $t \le s$, the objects from Cl_s will be preferred than those from Cl_t. The upward cumulative set and downward cumulative set are denoted by $Cl_t^{\ge} = \cup_{t \ge s} Cl_s$, and $Cl_t^{\le} = \cup_{t \le s} Cl_s$ respectively. If $x \in Cl_t^{\ge}$, it means x belongs to a subclass Cl_t at least; if $x \in Cl_t^{\le}$, it means x belongs to a subclass Cl_t at most.

For $\forall Cl_t^{\ge}$, the lower and upper approximations of the upward cumulative set on the attribute set A are denoted by

$$\underline{A^{\theta}}(Cl_t^{\geq}) = \left\{x \in U | S_A^{\theta+}(x) \subseteq Cl_t^{\geq}\right\};$$

$$\overline{A^{\theta}}(Cl_t^{\geq}) = \left\{x \in U | S_A^{\theta-}(x) \cap Cl_t^{\geq} \neq \varnothing\right\}.$$

Similarly, the lower and upper approximations of the downward cumulative set on attribute set A are defined as

$$\underline{A^{\theta}}(Cl_t^{\leq}) = \left\{x \in U | S_A^{\theta-}(x) \subseteq Cl_t^{\leq}\right\};$$

$$\overline{A^{\theta}}(Cl_t^{\leq}) = \left\{x \in U | S_A^{\theta+}(x) \cap Cl_t^{\leq} \neq \varnothing\right\}.$$

Definition 8.3.5 Assume $D_{\{d\}}^{+} = \left\{(x, y) \in U^2 | f(x, d) \geq f(y, d)\right\}$, if $S_A^{\theta} \subseteq D_{\{d\}}^{+}$, the decision table GS is considered as θ-consistent. Otherwise the GS is considered as θ-inconsistent. Thus, if $S_A^{\theta} \subseteq D_{(d)}^{+}$ for $\forall A \subseteq AT$ and $S_A^{\theta} \not\subset D_{(d)}^{+}$ for any $B \subset A$, A is called to a θ-attribute reduction of S.

Hence, we can construct the following identifiable matrices:

$$C_{AT}^{\theta}(x, y) = \begin{cases} \{a \in AT | (x, y) \notin S_a^{\theta}\}, & (x, y) \in D^* \\ \varnothing, & (x, y) \notin D^* \end{cases}$$

where

$$D^* = \left\{(x, y) \in U^2 | f(x, d) < f(y, d)\right\}$$

Then

$$\boldsymbol{C_{AT}^{\theta}} = \left\{C_{AT}^{\theta}(x, y) | x, y \in U\right\}$$

is denoted as a discernible matrix for decision tables.

Correspondingly,

$$L = \wedge\left\{\vee\{a | a \in C_{AT}^{\theta}(x, y)\} | x, y \in U\right\}$$

is denoted as discernible function of decision table.

Risk assessment steps of ice disaster based on grey rough combined decision-making method are as follows:

Step 1: Set the grey class according to demands. Let the year be the object set, set the disaster risk index system be the attribute set, and combine with the collected ice data to build the risk information system of ice disaster.

Step 2: Standardize the effect evaluation matrix by the grey range transformation.

Step 3: Calculate the ideal optimal sequence and the grey relational coefficient, and obtain the grey correlation coefficient matrix.

Step 4: Determine the whitening weight function of k-grey class of the *j*-index and the index weight.

Step 5: Calculate the grey clustering coefficients of each object with respect to grey class s, and determine the grey classes that each object belongs to.

Step 6: Construct the decision information table based on the grey class and the risk information system of ice disaster.

Step 7: According to Definition 8.3.3, the grey dominant relationship is built.

Step 8: According to Definition 8.3.4, the corresponding decision rules through the upper and lower approximation sets are defined.

Step 9: Perform attribute reduction through Definition 8.3.5 to further extract the simplest decision rules.

We collected the ice disaster risk index data from 1996 to 2015 in the Ningxia–Inner Mongolia reach of the Yellow River. According to the actual situation, the risk grades corresponding to the three grey categories of low risk, middle risk and high risk are set to grade 1, grade 2 and grade 3. The grey interval correlation clustering method is used to calculate the grey relational degree. Due to the uncertainty of the ice disaster risk index weight, the weight of the index is treated as equal weight to reduce the loss caused by the uncertainty, based on the principle of maximum entropy. The whitening weight functions of low, medium and high grey are given by experts, which are [–, –, 0.4, 0.55], [0.4, 0.6, –, 0.75], [0.65, 0.85, –, –]. We can calculate the grey clustering coefficients for each year relative to the three grey classes, and then determine the risk level for each year, as shown in Table 8.11.

According to the results of Table 8.11, we can see that there are five years of high risk for ice disasters: 1999–2000, 2000–2001, 2004–2005, 2007–2008 and 2009–2010. In these years, the annual ice condition led to a serious disaster, and caused great losses to the cross-strait economic construction. From the clustering coefficient, we can obtain the greatest risk of ice disaster in 2000–2001 and 2007–2008, which is also consistent with the severity of ice disasters in these years. Among them, from 2000 to 2001, the cold air came prematurely, and the ice flow in the Ningxia–Inner Mongolia reach of the Yellow River appeared on November 5. The date of ice flow was 10–20 days earlier than the average year, and the duration of the ice flow was long. It is easy to form ice plug in the Ningxia–Inner Mongolia reach, which made the largest channel capacity reach up to 1.87 billion cubic meters in spring of 2001. When the river opens in late March, the channel storage increment is rapidly released, which caused great losses.

In 2007–2008, the Yellow River Ningxia–Inner Mongolia experienced the most serious ice flow in 40 years. The main reason is: the period of freeze up is too long, and the temperature in winter is abnormally low and continuous. The average temperature of the Yellow River Basin reached about −8.5 °C, which is 3.5 °C lower than the average temperature in the same period of the year. It has been the lowest temperature in the historical period since 1952. Also, the snowfall in the upper reaches of the Yellow River is more than that in the same period of previous years, and the temperature changes fluctuated greatly. The water level in the period of freeze up reached the highest level during the calendar year, which further led to the river channel water increment higher. The time of break up is short, which only

Table 8.11 Clustering results of ice disaster risk grade

Serial number	Years	Comprehensive clustering coefficient			Risk grade
		Low	Middle	High	
1	1995–1996	0.0009	0.6168	0.2179	2
2	1996–1997	0.3346	0.341	0.0907	2
3	1997–1998	0.0598	0.5404	0.2857	2
4	1998–1999	0.3623	0.4169	0.0116	2
5	1999–2000	0.1692	0.3017	0.3976	3
6	2000–2001	0	0.3782	0.5714	3
7	2001–2002	0.5911	0.3067	0	1
8	2002–2003	0.506	0.2634	0.1096	1
9	2003–2004	0.3255	0.3121	0.0657	1
10	2004–2005	0.1689	0.289	0.368	3
11	2005–2006	0.3694	0.442	0	2
12	2006–2007	0.4723	0.3958	0	1
13	2007–2008	0.0634	0.1866	0.4433	3
14	2008–2009	0.5147	0.2568	0.0554	1
15	2009–2010	0.1813	0.286	0.3111	3
16	2010–2011	0.2749	0.4832	0.0724	2
17	2011–2012	0.2876	0.4421	0.1429	2
18	2012–2013	0.2522	0.4564	0.0913	2
19	2013–2014	0.4841	0.4076	0	1
20	2014–2015	0.5226	0.3602	0	1

lasts 15 days. In the period of break up, the temperature rises rapidly. The average daily release of water in the period is about 112 million cubic meters. It has been the year with the shortest breakup and largest daily water release since the beginning of the record.

There are 8 years with middle risk, which are respectively: 1995–1996, 1996–1997, 1997–1998, 1998–1999, 2005–2006, 2010–2011, 2011–2012, 2012–2013. Ice disaster in these years is possible to occur, so the prevention and control work of ice disaster still should not be neglected.

There are 7 years with low risk, which are respectively: 2001–2002, 2002–2003, 2003–2004, 2006–2007, 2008–2009, 2013–2014, 2014–2015. Ice dam would also occur in these years, but it didn't cause a serious ice flood hazard. The reason is that the risk of ice disaster is the results of a combination of multiple factors. For example, in 2008–2009, the max channel storage increment in the Ningxia–Inner Mongolia reach reached up to 1.8 billion cubic meters. However, due to the slow rise of the temperature during the break-up period, the channel storage increment was slowly released, and the flow in the break-up period was relatively small. Thus, the ice disaster risk was greatly reduced, which avoids the flood disaster caused by violent break. Especially, in 2014–2015, the risk of ice disaster was significantly lower. During the period of break-up, the peak flow was 920 cubic meters at per second, and the maximum water of 10 days was 0.56 billion cubic meters, both of which were at the historically lowest levels. On one hand, the reach was less affected by cold air and the temperature was high overall, which results in delayed ice flow date and short frozen days. On January 13, 2015, the first thawed reach is Mahuanggoukou, which is located in the upper of Ningxia–Inner Mongolia reaches. It is about a month earlier than the historical average date. By the middle of March, the whole reaches thawed, and the period of break-up lasted 65 days. The long break-up period made the increment release slowly, which decreases the ice disaster risk. On the other hand, due to the commissioning of the Haibowan Reservoir, which was completed in 2014, the water temperature of the downstream river was increased during the ice flow period, further reducing the ice disaster risk.

The decision table of ice disaster risk under the information of three-parameter interval grey numbers is established, in which the years is taken as the object set, the risk indexed is taken as the attribute set, and the risk grade is taken as the decision attribute set. Take the decision–makers preference coefficient $\lambda = 0.5$, that is, the decision maker takes into account the equilibrium of upper and lower limits of the grey number. Let the dominance threshold $\theta = 0.6$, that is, when a three-parameter interval grey number dominates the other with a possibility degree of more than 0.6, we deem they satisfy the grey dominance relationship. Thus, based on the grey rough set method, the lower approximation with upper and lower cumulative sets (the upper approximation is similar) is obtained, as shown in Table 8.12.

According to the upper and lower approximation set, we can get the corresponding dominance decision-making rule from the decision table. In order to make these rules extracted from the ice disaster risk assessment table more concise, and to accurately reflect the relationship between the risk factors and risk grade, it needs to reduce the

Table 8.12 Low approximation with $Cl_t^{\geq}$ and $Cl_t^{\leq}$

Cumulative set	Low approximation	Objects
$Cl_t^{\geq}$	$\underline{A^{0.6}}(Cl_1^{\geq})$	All years
	$\underline{A^{0.6}}(Cl_2^{\geq})$	1995–1996, 1996–1997, 1997–1998, 1998–1999, 1999–2000, 2000–2001, 2004–2005, 2005–2006, 2007–2008, 2009–2010, 2010–2011, 2011–2012, 2012–2013
	$\underline{A^{0.6}}(Cl_3^{\geq})$	1999–2000, 2000–2001, 2004–2005, 2007–2008, 2009–2010
$Cl_t^{\leq}$	$\underline{A^{0.6}}(Cl_1^{\leq})$	2001–2002, 2002–2003, 2003–2004, 2006–2007, 2008–2009, 2013–2014, 2014–2015
	$\underline{A^{0.6}}(Cl_2^{\leq})$	1995–1996, 1996–1997, 1997–1998, 1998–1999, 2001–2002, 2002–2003, 2003–2004, 2005–2006, 2006–2007, 2008–2009, 2010–2011, 2011–2012, 2012–2013, 2013–2014, 2014–2015
	$\underline{A^{0.6}}(Cl_3^{\leq})$	All years

Table 8.13 Attribute reduction of ice disaster risk assessment decision table

Serial number	Attribute reduction result	Corresponding reduction index
1	T1, H3	Frozen days, maximum water of 10 days
2	T1, T3, H4	Frozen days, cumulative negative temperature, maximum channel-storage increment
3	T1, H1, H4	Frozen days, average flow in freeze-up, maximum channel-storage increment
4	T2, H1, H3	Average temperature, average flow in freeze-up, maximum water of 10 days
5	T1, H2, H4	Frozen days, peak flow, maximum channel-storage increment
6	T3, H1, H3	Cumulative negative temperature, average flow in freeze-up, maximum water of 10 days
7	T2, H2, H4	Average temperature, peak flow, maximum channel-storage increment
8	T2, T3, H1, H4	Average temperature, cumulative negative temperature, average flow in freeze-up, maximum channel-storage increment
9	T3, H2, H3, H4	Cumulative negative temperature, peak flow, maximum water of 10 days, maximum channel-storage increment

attribute of the decision table. Under the premise of ensuring the same classification ability, all the attribute reduction results of the ice disaster risk assessment decision table are obtained by establishing the discernible matrix, which are presented in Table 8.13.

From Table 8.13, all attribute reduction results intuitively reflect that ice disaster risk is caused by the thermodynamic factors, dynamic factors and human factors. Because the annual change of river regime is relatively small, and the influence of human factors on ice regime is mostly reflected in the control of flow, the ice disaster is the combination of thermodynamic and hydraulic factors, which is consistent with the results of Wu et al. (2015). In fact, both the thermal and dynamic factors affecting the ice are mutually constrained, but under certain circumstances, they can be transformed into each other. For example, the rising temperature makes the river ice melt into water, which improves the flow. In the freeze up period, the flow force can promote friction, increase heat, and delay the process of freezing due to the decreasing temperature. During the break-up period, the temperature is decreased, which causes some river to freeze into ice, and thereby weakens the flow power.

According to the attribute reduction result $\{T1, H3\}$, the simplest decision-making rules of ice disaster risk decision table are as follows.

The $\geq^{0.6}$ and $\leq^{0.6}$ decision-making rules are as shown in Table 8.14.

From Table 8.14, the decision information table of ice disaster risk is simplified into the above 14 decision-making rules, which can be utilized for us to judge the corresponding risk grade when the value of risk index reaches into a certain extent.

Table 8.14 Results of the $\geq^{0.6}$ and $\leq^{0.6}$ decision-making rules

Decision-making rules	FD	MW10	Grade
$\geq^{0.6}$	[95, 107, 117]	[1.50, 1.51, 1.52]	3
	[120, 128, 135]	[1.15, 1.18, 1.20]	3
	[120, 122, 132]	[1.25, 1.28, 1.30]	3
	[115, 120, 125]	[1.28, 1.29, 1.30]	3
	[90, 101, 110]	[1.20, 1.23, 1.25]	2 or 3
	[98, 108, 118]	[0.98, 1.00, 1.02]	2 or 3
	[110, 118, 128]	[0.97, 0.98, 1.00]	2 or 3
$\leq^{0.6}$	[100, 108, 114]	[0.80, 0.82, 0.85]	1
	[90, 98, 108]	[1.30, 1.31, 1.32]	1
	[100, 109, 119]	[1.08, 1.10, 1.12]	1
	[102, 112, 120]	[1.15, 1.19, 1.22]	1 or 2
	[110, 121, 130]	[1.10, 1.12, 1.15]	1 or 2
	[110, 115, 120]	[1.15, 1.18, 1.20]	1 or 2
	[90, 101, 110]	[1.20, 1.23, 1.25]	1 or 2

For instance, the R5 of the $\geq^{0.6}$ decision-making rule indicates that if the frozen day dominates [90, 101, 110] with the grey dominates degree of not less than 0.6 and the maximum water in 10 days dominates [1.2, 1.23, 1.25] with the grey dominance degree of not less than 0.6, we can judge the risk grade of ice disaster is at least 2. Moreover, the decision-making rules with three-parameter interval grey numbers is applied to the condition that the ice disaster data obtained are real numbers. This is because real number can be taken as a special three-parameter interval grey number, in which the lower, middle and upper bound are equal. Therefore, the above decision-making rules of ice disaster risk are more comprehensive and universal.

Through the above mentioned decision-making rules, the warning line of ice disaster risk grade can be determined. When the disaster risk index data are close to the warning line, the ice prevention and reduction work can be conducted in advance, which is contributed to reduce the loss caused by ice disasters.

8.4 Risk Prediction of Yellow River Ice Disaster

The effective prediction for the Yellow River ice disaster can facilitate us to grasp the development trend of ice risk, assess the risk of ice disaster, and do well the disaster prevention and mitigation work in advance to reduce the loss of people's life and property. In this section, based on the monitoring data of the Yellow River basin, the model GM(1, 1) and GMP(1, 1, N) are established respectively to predict the development trend of the Yellow River ice disaster, which provides the research method for the risk prediction for natural disaster.

8.4.1 Risk Prediction of Yellow River Ice Disaster Based on Model GM(1, 1)

In the third section, we select flow, water level and frozen days as the risk indicators to estimate the disaster risks in Bayangol, Sanhu River and Toudaoguai in the Ningxia–Inner Mongolia Reach of the Yellow River. The measure results of the ice disaster risk in the three reaches are obtained from 2003 to 2012, which is presented in Tables 8.10. Here, we take the measure result as original data to construct the corresponding GM(1, 1) models, which are used to predict the ice regime development trend of the three reaches.

The modeling process of risk measure values in the Bayangol reach is as follows:

(1) The risk measure sequence is

$$X^{(0)} = (0.2771, 0.2525, 0.2568, 0.2711, 0.2709, 0.2672, 0.2424, 0.2491, 0.2665, 0.2668).$$

$X^{(1)}$ is the 1–AGO sequence of $X^{(0)}$:

$$X^{(1)} = (0.2771, 0.5296, 0.7864, 1.0575, 1.3284, 1.5956, 1.8380, 2.0871, 2.3536, 2.6204).$$

$Z^{(1)}$ is called the background value of $X^{(0)}$:

$$Z^{(1)} = (0.40335, 0.65800, 0.92195, 1.19295, 1.46200, 1.71680, 1.96255, 2.22035, 2.48700).$$

(2) According to

$$Y = [0.2525\ 0.2568\ 0.2711\ 0.2709\ 0.2672\ 0.2424\ 0.2491\ 0.2665\ 0.2668]^T,$$

and

$$B = \begin{bmatrix} -0.40335 & -0.65800 & -0.92195 & -1.19295 & -1.46200 & -1.71680 & -1.96255 & -2.22035 & -2.48700 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}^T,$$

The parameter estimate of GM(1, 1) model is

$$\hat{a} = (a, b)^{\mathrm{T}} = \left(B^{\mathrm{T}}B\right)^{-1}B^{\mathrm{T}}Y = (0.000894, 0.259081)^{\mathrm{T}}.$$

(3) The resulting time response sequence of Bayangol reach is given by

$$\begin{cases} \hat{x}^{(1)}(k) = 290.1532e^{0.0008937(k-1)} - 289.8761 \\ \hat{x}^{(0)}(k) = \hat{x}^{(1)}(k) - \hat{x}^{(1)}(k-1) \end{cases},$$

Similarly, the resulting time response sequence of Sanhu River is given by

$$\begin{cases} \hat{x}^{(1)}(k) = -474.8202e^{0.0005364(k-1)} + 475.0998 \\ \hat{x}^{(0)}(k) = \hat{x}^{(1)}(k) - \hat{x}^{(1)}(k-1) \end{cases},$$

and the resulting time response sequence of Toudaoguai is given by

$$\begin{cases} \hat{x}^{(1)}(k) = 118.4832e^{-0.002160(k-1)} - 118.2105 \\ \hat{x}^{(0)}(k) = \hat{x}^{(1)}(k) - \hat{x}^{(1)}(k-1) \end{cases},$$

The assessing accuracy of GM(1, 1) model used for the three reaches is as shown in Table 8.15.

From Table 8.15, the development coefficients of risk sequences in Bayangol, Sanhuhe River and Toudaoguai are approximately 0, and the grey actions are maintained at 0.25, which indicates that the sequence is quite smooth. The average percentage error of the model is less than 5%. It belongs to the grey model of first-order precision. Since the average percentage error of the three models are less than 5%, belonged to the accuracy grade A, it is theoretically appropriate to use the model for short-term prediction. Based on which, the forecasted values and trend of ice disaster risk in the three reaches are shown in Table 8.16.

The above analysis shows that the ice disaster risk of the Ningxia–Inner Mongolia reach has certain fluctuation from 2003 to 2012, but the general risk trend keeps stable. It is estimated that the risk of ice jam disaster in Bayangol and Toudaoguai will present decline trend from 2013 to 2015, while it will be rising trend in Sanhu River.

(1) Affected by climate warming and reservoir regulation, the average flow and freeze up dates are universally later. But the flow ice date of three reaches in the

Table 8.15 Assessing accuracy of GM(1, 1) used for the three reaches

Item	a	b	APE (%)	Accuracy grade
Bayangol	0.000894	0.259081	3.43	A
Sanhu River	−0.000536	0.254862	2.45	A
Toudaoguai	0.002160	0.255277	4.38	A

Table 8.16 Forecasted values and trend of ice disaster risk in the three reaches

Item	Forecasted values			Trend
	2013	2014	2015	
Bayangol	0.2615	0.2618	0.2620	Decline
Sanhu River	0.2534	0.2533	0.2531	Rise
Toudaoguai	0.2612	0.2617	0.2623	Decline

Ningxia–Inner Mongolia Reach is earlier, especially in the Toudaoguai station which has been advanced nearly 9 days. The break-up dates of Bayangol and Toudaoguai are relatively late, especially the Toudaoguai which is postponed for about 4 days.

(2) Affected by reservoir regulation for reducing ice-jam disasters in the upper and lower reaches of the Ningxia–Inner Mongolia, the average flow of the three reaches during the breakup period was decreased to 118, 68 and 81 m^3/s respectively, which causes ice jam disasters to be at high risk.

(3) The ice temperature data of the Ningxia–Inner Mongolia Reach shows that the average temperature from November to next March of 2001–2002 and 2010–2011 is −4 °C. The average temperature of the Sanhu River is −5.5 °C at the lowest, while the average temperature of Bayangol and Toudaoguai is about −4 °C.

In order to alleviate the ice disaster risk in the Ningxia–Inner Mongolia Reach, we can regulate reservoir, increase the ice overcurrent capacity and reduce the channel storage increment so as to cut the peak flow in break up period. In addition, according to the relationship between hydraulic factors and the evolution of ice regime, the flow rate in winter can be adjusted and the control effect of reservoirs on ice disasters is fully exerted.

8.4.2 Risk Prediction of Yellow River Ice Disaster Based on Model GMP(1, 1, N)

Model GM(1, 1) and its improved model group have good effect on the data sequence with homogeneous exponential law. However, the evolution of the actual data sequence cannot simply be described by exponential law. In this section, the grey prediction model (GMP(l, l, N)) with time polynomial is constructed to forecast ice disaster risk of the Yellow River, and its modeling mechanism and properties are studied. Then the relationship with other grey prediction models is analyzed to expand the grey theoretical system and application range.

Definition 8.4.1 The following differential equation

$$x^{(0)}(k) + \alpha z^{(1)}(k) = \beta_0 + \frac{2k-1}{2}\beta_1 + \cdots + \frac{k^{N+1} - (k-1)^{N+1}}{N+1}\beta_N \quad (8.4.1)$$

is called the basic form of model GMP(1, 1, N);

The following differential equation

$$\frac{dx^{(1)}(t)}{dt} + \alpha x^{(1)}(t) = \beta_0 + \beta_1 t + \cdots + \beta_N t^N \quad (8.4.2)$$

is called the whitening equation of model GMP(1, 1, N) (Luo & Wei, 2017).

Similar to the idea of the model GM(1, 1), the whitening equation of model GMP(1, 1, N) is a continuous approximation of its basic form. Base on the integrated whitening equation on the interval $[k-1, k]$, we can get the right side of the equation, which is given by

$$\int_{k-1}^{k} (\beta_0 + \beta_1 t + \cdots + \beta_N t^N)dt = \beta_0 + \frac{2k-1}{2}\beta_1 + \cdots + \frac{k^{N+1} - (k-1)^{N+1}}{N+1}\beta_N.$$

The left side of the equation is given by

$$x^{(1)}(k) - x^{(1)}(k-1) + \alpha\int_{k-1}^{k} x^{(1)}(t)dt = x^{(0)}(k) + \alpha\int_{k-1}^{k} x^{(1)}(t)dt.$$

where, the integral term $\int_{k-1}^{k} x^{(1)}(t)dt$ expresses the area between the function curve $x^{(1)}(t)$ and the horizontal axis. We use the method of "replace the curve with straight line" to substitute $\overline{AB}$ for AB, then we have

$$\int_{k-1}^{k} x^{(1)}(t)dt = \frac{1}{2}(x^{(1)}(k-1) + x^{(1)}(k)).$$

Theorem 8.4.1 *Let the original non-negative sequence be* $X^{(0)}$, *the 1–AGO and the mean generation sequence be* $X^{(1)}$ *and* $Z^{(1)}$ *respectively, then the parameter vector of model GMP(1, 1, N) is*

$$\hat{\boldsymbol{\kappa}} = [\ \hat{\alpha}\ \hat{\beta}_0\ \hat{\beta}_1\ \cdots\ \hat{\beta}_N\ ]^T = (\boldsymbol{B}^T\boldsymbol{B})^{-1}\boldsymbol{B}^T\boldsymbol{Y} \tag{8.4.3}$$

where the samples number and the polynomial order satisfy $n \geq N+4$,

$$\boldsymbol{Y} = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix}, \boldsymbol{B} = \begin{bmatrix} -z^{(1)}(2) & 1 & \frac{3}{2} & \cdots & \frac{2^{N+1}-1}{N+1} \\ -z^{(1)}(3) & 1 & \frac{5}{2} & \cdots & \frac{3^{N+1}-2^{N+1}}{N+1} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ -z^{(1)}(n) & 1 & \frac{2n-1}{2} & \cdots & \frac{n^{N+1}-(n-1)^{N+1}}{N+1} \end{bmatrix}.$$

Proof Substituting the data into Eq. (8.4.1), the equation set is obtained as

$$\begin{cases} x^{(0)}(2) + z^{(1)}(2)\alpha = \beta_0 + \frac{3}{2}\beta_1 + \cdots + \frac{2^{N+1}-1^{N+1}}{N+1}\beta_N \\ x^{(0)}(3) + z^{(1)}(3)\alpha = \beta_0 + \frac{5}{2}\beta_1 + \cdots + \frac{3^{N+1}-2^{N+1}}{N+1}\beta_N \\ \vdots \\ x^{(0)}(n) + z^{(1)}(n)\alpha = \beta_0 + \frac{2n-1}{2}\beta_1 + \cdots + \frac{n^{N+1}-(n-1)^{N+1}}{N+1}\beta_N \end{cases},$$

and the matrix form is denote as

$$\boldsymbol{B\kappa} = \boldsymbol{Y}.$$

According to $n \geq N + 4$, we know that the matrix equation is an over determined system, and it has no exact solution. By the least square method, the least squares estimate of parameter $\boldsymbol{\kappa}$ is

$$\hat{\boldsymbol{\kappa}} = \min_{\kappa}\{L(\boldsymbol{\kappa}) = (\boldsymbol{Y} - \boldsymbol{B\kappa})^T(\boldsymbol{Y} - \boldsymbol{B\kappa})\}.$$

Because the loss function $L(\boldsymbol{\kappa})$ is a quadratic convex function, according to the existence of the extreme value conditions, we have

$$\frac{dL(\boldsymbol{\kappa})}{d\boldsymbol{\kappa}} = 2\boldsymbol{B}^T\boldsymbol{B\kappa} - 2\boldsymbol{B}^T\boldsymbol{Y} = 0,$$

that is

$$\boldsymbol{B}^T\boldsymbol{B\kappa} = \boldsymbol{B}^T\boldsymbol{Y}.$$

In general, the $\boldsymbol{B}$ is a matrix of column full rank and the matrix $\boldsymbol{B}^T\boldsymbol{B}$ is non-singular, then the parameter estimate is obtained as

$$\hat{\boldsymbol{\kappa}} = (\boldsymbol{B}^T\boldsymbol{B})^{-1}\boldsymbol{B}^T\boldsymbol{Y}.$$

Theorem 4.2 *Let $\hat{x}^{(1)}(1) = x^{(1)}(1)$, then the time response sequence is denoted as follows:*

$$\begin{cases} \hat{x}^{(1)}(k) = (x^{(1)}(1) - \sum_{j=0}^{N}\gamma_j)e^{-\alpha(k-1)} + \gamma_0 + \gamma_1 k + \cdots + \gamma_N k^N \\ \hat{x}^{(0)}(k) = \hat{x}^{(1)}(k) - \hat{x}^{(1)}(k-1), k = 2, 3, \ldots, n, n+1, \ldots, n+p \end{cases} \tag{8.4.4}$$

where p is the prediction step size,

$$\boldsymbol{\gamma} = \boldsymbol{M}^{-1}\boldsymbol{\beta},$$

$$\boldsymbol{\gamma} = \begin{bmatrix} \gamma_0 \\ \gamma_1 \\ \vdots \\ \gamma_N \end{bmatrix}, \boldsymbol{\beta} = \begin{bmatrix} \hat{\beta}_0 \\ \hat{\beta}_1 \\ \vdots \\ \hat{\beta}_N \end{bmatrix}, \boldsymbol{M} = \begin{bmatrix} \hat{\alpha} & 1 & \cdots & 0 & 0 \\ 0 & \hat{\alpha} & \ddots & 0 & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & \hat{\alpha} & N \\ 0 & 0 & \cdots & 0 & \hat{\alpha} \end{bmatrix}.$$

Proof Allowing the parameter estimate into whitening equation, we can obtain

$$\frac{d\hat{x}^{(1)}(t)}{dt}+\hat{\alpha}\hat{x}^{(1)}(t)=\hat{\beta}_0+\hat{\beta}_1 t+\cdots+\hat{\beta}_N t^N.$$

Let the general solution of difference equation be

$$\hat{x}^{(1)}(t)=x_h^{(1)}(t)+x_p^{(1)}(t),$$

where $x_h^{(1)}(t)=ce^{-\alpha t}$ is the general solution of homogeneous differential equation, which is presented by

$$\frac{d\hat{x}^{(1)}(t)}{dt}+\hat{\alpha}\hat{x}^{(1)}(t)=0,$$

$x_p^{(1)}(t)=\gamma_0+\gamma_1 t+\cdots+\gamma_N t^N$ is the special solution of differential equation, which is denoted as

$$\frac{dx_p^{(1)}(t)}{dt}+\hat{\alpha}x_p^{(1)}(t)=\hat{\beta}_0+\hat{\beta}_1 t+\cdots+\hat{\beta}_N t^N.$$

The equations can be obtained as

$$\boldsymbol{M}\boldsymbol{\gamma}=\boldsymbol{\beta}.$$

According to the development coefficient $\widehat{\alpha}\neq 0$, we know that $\boldsymbol{M}$ is a non-singular matrix, then

$$\boldsymbol{\gamma}=\boldsymbol{M}^{-1}\boldsymbol{\beta}.$$

Allowing the initial condition $\hat{x}^{(1)}(1)=x^{(1)}(1)$ into the equation, which is obtained as

$$c=\boldsymbol{e}^{\hat{\alpha}}\left(x^{(1)}(1)-\sum_{j=0}^{N}\gamma_j\right),$$

and the time response sequence is obtained as follows:

$$\hat{x}^{(1)}(k)=\left(x^{(1)}(1)-\sum_{j=0}^{N}\gamma_j\right)\boldsymbol{e}^{-\hat{\alpha}(k-1)}+\gamma_0+\gamma_1 k+\cdots+\gamma_N k^N.$$

Based on the 1-IAGO operator, we know that the conclusion is true.

Definition 8.4.2 Let the *r*-order differential sequence of the original sequence

$$X^{(0)}=\left(x^{(0)}(1),x^{(0)}(2),\ldots,x^{(0)}(n)\right)$$

be

$$D^{(r)} = \left\{d^{(r)}(r+1), d^{(r)}(r+2), \ldots, d^{(r)}(n)\right\}.$$

The r-order difference ratio sequence is

$$\delta^{(r)} = \left\{\delta^{(r)}(r+2), \delta^{(r)}(r+3), \ldots, \delta^{(r)}(n)\right\},$$

where

$$\delta^{(r)}(k) = \frac{d^{(r)}(k)}{d^{(r)}(k-1)}, k = r+2, r+3, \ldots, n. \tag{8.4.5}$$

Definition 4.3 Assume that

$$\delta_{\max}^{(r)} = \max_{r+2<k<n} \delta^{(r)}(k),\ \delta_{\min}^{(r)} = \min_{r+2<k<n} \delta^{(r)}(k).$$

If $\vartheta^{(r)} = 0$, the original sequence has r-order homogeneous exponential law; if $\vartheta^{(r)} = \delta_{\max}^{(r)} - \delta_{\min}^{(r)} > 0$, the original sequence has r-order grey exponential law. We know that $\vartheta^{(r)}$ can measure the grey exponential law of the original sequence to a certain extent. Considering the volatility characteristics of the high order polynomial and the generalization capability of models, the empirical criterion of identifying the polynomial order is

$$N = \min \arg \min_{r\in\Re} \vartheta^{(r)} = \min \arg \min_{r\in\Re}\left(\delta_{\max}^{(r)} - \delta_{\min}^{(r)}\right),\ \Re = \{0, 1, 2, 3\}.$$

Based on this, it can preliminary determine the alternative value of polynomial order number. However, since the GMP(1, 1, N) model on the N-order homogeneous exponential law sequence is partial biased, the final value of polynomial order number should be determined by the debugging method.

Corollary 8.4.1 *According to the polynomial order N or the difference parameter of GMP(1, 1, N) model, the following conclusions can be obtained:*

(1) *If $N = 0$, the GMP(1, 1, N) model degenerates into GM(1, 1) model, and its basic form and whitening equation are respectively*

$$x^{(0)}(k) + z^{(1)}(k)\alpha = \beta_0 \text{ and } \frac{\mathrm{d}x^{(1)}(t)}{\mathrm{d}t} + \alpha x^{(1)}(t) = \beta_0.$$

The estimated parameter vector is

$$\hat{\boldsymbol{\kappa}} = [\hat{\alpha}\ \hat{\beta}_0]^T = \left(\boldsymbol{B}_1^T \boldsymbol{B}_1\right)^{-1} \boldsymbol{B}_1^T \boldsymbol{Y},$$

where

$$\boldsymbol{B}_1 = \begin{bmatrix} -z^{(1)}(2) & -z^{(1)}(3) & \cdots & -z^{(1)}(n) \\ 1 & 1 & \cdots & 1 \end{bmatrix}^{\boldsymbol{T}} = \boldsymbol{B}\begin{bmatrix} \boldsymbol{e}_1 & \boldsymbol{e}_2 \end{bmatrix}.$$

(2) *If* $N = 1$, *the GMP (1, 1, N) model degenerates into NGM(1, 1, k) model, its basic form and whitening equations are respectively*

$$x^{(0)}(k) + z^{(1)}(k)\alpha = \beta_0 + \frac{2k-1}{2}\beta_1 \text{ and}$$
$$\frac{\mathrm{d}x^{(1)}(t)}{\mathrm{d}t} + \alpha x^{(1)}(t) = \beta_0 + \beta_1 t,$$

the estimated parameter vector is

$$\hat{\boldsymbol{\kappa}} = \begin{bmatrix} \hat{\alpha} & \hat{\beta}_0 & \hat{\beta}_1 \end{bmatrix}^{\boldsymbol{T}} = \left(\boldsymbol{B}_2^{\boldsymbol{T}}\boldsymbol{B}_2\right)^{-1}\boldsymbol{B}_2^{\boldsymbol{T}}\boldsymbol{Y},$$

where

$$\boldsymbol{B}_2 = \begin{bmatrix} -z^{(1)}(2) & -z^{(1)}(3) & \cdots & -z^{(1)}(n) \\ 1 & 1 & \cdots & 1 \\ \frac{3}{2} & \frac{5}{2} & \cdots & \frac{2n-1}{2} \end{bmatrix}^{T} = \boldsymbol{B}\begin{bmatrix} \boldsymbol{e}_1 & \boldsymbol{e}_2 & \boldsymbol{e}_3 \end{bmatrix}.$$

(3) *If* $\beta_1 = \beta_2 = \cdots = \beta_{N-1} = 0$, *the GMP(1, 1, N) model degenerates into GM(1, 1,* t^{α}*) model, its basic form and whitening equations are respectively*

$$x^{(0)}(k) + z^{(1)}(k)\alpha = \beta_0 + \frac{k^{N+1} - (k-1)^{N+1}}{N+1}\beta_N \text{ and } \frac{\mathrm{d}x^{(1)}(t)}{\mathrm{d}t} + \alpha x^{(1)}(t) = \beta_0 + \beta_N t^N,$$

the estimated parameter vector is

$$\hat{\boldsymbol{\kappa}} = \begin{bmatrix} \hat{\alpha} & \hat{\beta}_0 & \hat{\beta}_N \end{bmatrix}^{\boldsymbol{T}} = \left(\boldsymbol{B}_3^{\boldsymbol{T}}\boldsymbol{B}_3\right)^{-1}\boldsymbol{B}_3^{\boldsymbol{T}}\boldsymbol{Y},$$

where

$$\boldsymbol{B}_3 = \begin{bmatrix} -z^{(1)}(2) & -z^{(1)}(3) & \cdots & -z^{(1)}(n) \\ 1 & 1 & \cdots & 1 \\ \frac{2^{N+1}-1}{N+1} & \frac{3^{N+1}-2^{N+1}}{N+1} & \cdots & \frac{n^{N+1}-(n-1)^{N+1}}{N+1} \end{bmatrix}^{T} = \boldsymbol{B}\,[\boldsymbol{e}_1,\ \boldsymbol{e}_2,\ \boldsymbol{e}_{N+2}].$$

Corollary 8.4.1 reveals that model GM(1, 1), NGM(1, 1, k) and GM(1, 1, t^{α}) are special forms of model GMP(1, 1, N), hence different grey prediction models can be constructed by selecting the parameters values.

The steps to establish GMP(1, 1, N) model are as follows:

Step 1: Model Ordering: analyze the difference ratio of the original sequence, and initially determine the candidate values of the polynomial order.

Step 2: Parameter identification: identify the model parameters, and give the initial conditions to calculate the fitting values corresponding to the original sequence.

Step 3: Model Test: analyze the model accuracy according to the residual test, posterior error test and relational degree test method.

Step 4: Prediction: given the prediction step size, the model that pass the test is used to predict the value and analyze the development trend of the original sequence.

According to Table 8.17, we can obtain that the average percentage error of the GMP(1, 1, 3) model is smaller than the GM(1, 1), DGM(1, 1) and Verhulst model. In addition, the maximum and minimum absolute percentage errors of the model GM(1, 1) are 7.6733 and 1.6355% respectively, the maximum and minimum absolute percentage errors of the model DGM(1, 1)are 8.5396 and 1.0105%, while the maximum and minimum absolute percentage errors of the GMP(1, 1, 3) model are only 5.2843 and 0.8166%.

Figure 8.2 shows that the accuracy of GMP(1, 1, 3) model is higher than those of GM(1, 1), DGM(1, 1) and Verhulst model. The cause is that the GM(1, 1), DGM(1, 1) and the Verhulst model cannot identify the fluctuation of sequences. However, the ice disaster risk shows the fluctuation characteristics in the Ningxia–Inner Mongolia Reach, which conforms to the modeling sequence of the GMP(1, 1, 3) model. Therefore, the GMP(1, 1, 3) model has high accuracy. From 2015 to 2016, the risk value of ice disaster in the Ningxia–Inner Mongolia Reach was 0.2928, the highest in the calendar year, which is caused by the increasing instability of meteorological and other factors effected by the strong El Nino phenomenon in 2015 (One of the strongest El Nino events on record).

By comparing the prediction results, we conclude that the GMP(1, 1, 3) model can identify the fluctuation characteristics of the disaster risk data sequence in the Ningxia-Inner Mongolia Reach, and it achieves a higher prediction accuracy. Moreover, the prediction results also show that the risk value of ice disaster in the Ningxia-Inner Mongolia Reach in 2016 reaches the highest value in recent years, which is consistent with the actual situation. Thus, it has a certain engineering application value.

8.5 Summary

In this chapter, we analyze the risk characteristics of Yellow River ice disasters and attempt to provide the risk management process for the disasters. This paper focuses on several risk assessment and prediction method for Yellow River ice disasters, and gives the paradigm for ice disaster risk management based on grey prediction and decision-making methods.

Firstly, we outline the basic situation of the Yellow River and its ice disasters. In view of the incomplete historical data and the limited human cognition, the ice disaster system shows grey uncertainty that contains partially known information with small samples and poor information. Base on which, from the perspective of grey system theory and application, we try to analyze systematically, quantify uniformly

Table 8.17 Actual value of ice disaster risk and grey model fitting prediction results in the Ningxia–Inner Mongolia reach of the Yellow River

Index	Actual values	GM(1, 1)		DGM(1, 1)		Verhulst		GMP(1, 1, 3)	
		Forecasted value	APE (%)	Forecasted value	APE (%)	Forecasted value	APE (%)	Forecasted value	APE (%)
1	0.2771	0.2771	–	0.2771	–	0.2771	–	0.2771	–
2	0.2525	0.2609	3.3321	0.2610	3.3663	0.2736	8.3564	0.2493	1.2710
3	0.2568	0.2609	1.6135	0.2610	1.6355	0.2707	5.4128	0.2666	3.8157
4	0.2771	0.2610	5.8198	0.2610	5.8102	0.2683	3.1757	0.2716	1.9807
5	0.2709	0.2610	3.6532	0.2610	3.6545	0.2663	1.6980	0.2687	0.8166
6	0.2672	0.2610	2.3079	0.2610	2.3204	0.2645	1.0105	0.2619	1.9714
7	0.2424	0.2611	7.6994	0.2610	7.6733	0.2631	8.5396	0.2552	5.2843
8	0.2491	0.2611	4.8146	0.2610	4.7772	0.2618	5.0984	0.2522	1.2270
9	0.2665	0.2611	2.0176	0.2610	2.0638	0.2608	2.1388	0.2562	3.8651
10	0.2668	0.2612	2.1165	0.2610	2.1739	0.2599	2.5862	0.2706	1.4107
MAPE(%)			3.7083		3.7149		4.2241		2.4047
11		0.2612		0.2610		0.2585		0.2982	

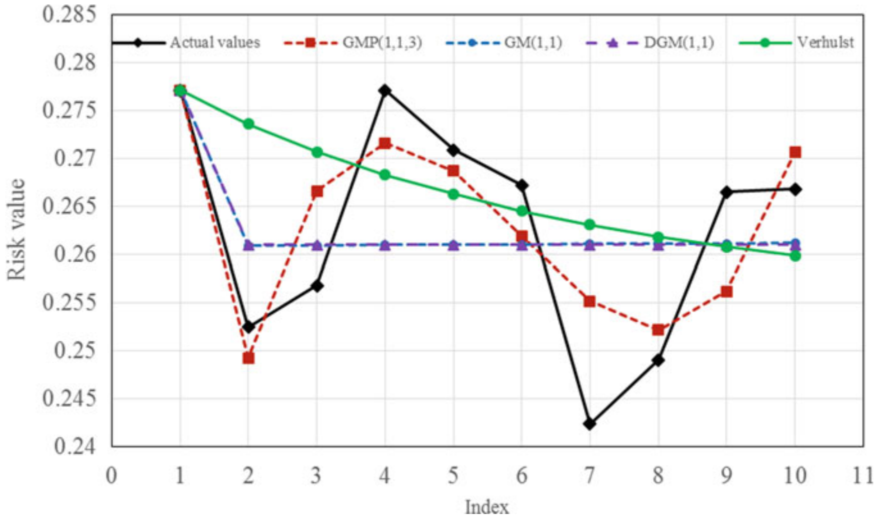

Fig. 8.2 Risk curve of ice disaster in the Ningxia-Inner Mongolia reach of the Yellow River

and deal with effectively the grey uncertainty characteristics of the Yellow River ice disaster system, and construct the Yellow River ice disaster risk management application model based on grey prediction and decision-making methods.

Secondly, according to the characteristics of climate, geography and other and historical data of the Yellow River basin, the causes of disaster are analyzed. Especially in recent years, extensive economic development has destroyed the environment. The ice regime in the Ningxia–Inner Mongolia reach of the Yellow River also presents some new features, which mainly include the delay of flow and break up date, the postponed first freezing up place, the unstable nature of freezing up and break up, the reduction of ice flow capacity, the significant increase of channel storage increment, the high water level during the freezing up and break up period, the decreased flood peak and the complex ice regime. The main factors that identify the Yellow River ice disaster are thermal factors, dynamic factors, river regime factors and human factors, which can be refined into measurable factors.

Thirdly, we study respectively the hybrid grey multiple-attribute decision-making method, the grey phase relation decision-making method and the grey rough combined decision-making method to solve the ice disaster risk assessment problem in the Ningxia–Inner Mongolia Reach.

Finally, we predict the risk development of the Yellow River ice disaster based on the GM(1, 1) model and the GMP(1, 1, N) model.

The research of this chapter not only promotes the theoretical research of grey systems, but also provides a solid method basis for the study of Yellow River ice disaster risk management under complex uncertain environment, which has an important application prospect in natural disaster risk management.

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Chapter 9
Millennial Generation: Enthusiastic Versus Stressed Consumers. How Different Is Their Behaviour and Their Opinion on Companies' Image in Online Social Networks?

Camelia Delcea and Ioana-Alexandra Bradea

9.1 Introduction

Several studies from the behavioural science have underlined the so-called "generational effect" (Gauzente & Roy, 2012) in online consumer behaviour (Horrigan, 2008), finding some general lines through which a certain group of people (corresponding to a specific period depending on their year of birth) can be characterized with respect to their behaviour in the online social networks.

As part of four main generations that can be encountered: Millennial (age 18–34), Generation X (age 35–50), Boomer (51–69), Silent (70–87), Millennial generation is "forging a distinctive path into adulthood" (PewResearchCenter, 2014), presenting a series of distinct characteristics, such as: they are relatively unattached to organized politics and religion (PewResearchCenter, 2014), they have a unique mind-set when it comes to their expectations in the marketplace (Wells et al., 2015), they are less mature with regard to their online behaviour than the Generation X (Hargittai, 2010), they are more involved in the products' creation process, passing from simple consumers to content-creators, more attentive to their online right; attracted by internet commerce; social conscious; "selfie-conscious" meaning that they live with the idea that the "world they live in should look perfect to outsiders" (Wells et al., 2015); with a high intention to consume organic food (Sa'ari & Koe, 2014); focused on health and fitness (Wells et al., 2015); with a high supremacy of brand experience dimensions on brand equity (Qader & Omar, 2016); having lower levels of social trust than the previous generations (PewResearchCenter, 2014); with a specific information search behaviour in online environments (Taylor, 2012); having, in general, great expectations (Ng et al., 2010); etc.

C. Delcea (✉) · I.-A. Bradea
Bucharest University of Economic Studies, Bucharest, Romania
e-mail: camelia.delcea@csie.ase.ro

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Johri et al. (2014) divided the attributes of the Millennials in five main categories: immediacy—they prefer to receive information in a very short amount of time; visual—they are very comfortable with interactive environments; multitasking—they are able to handle multiple tasks simultaneously; highly connected—they are comfortable with extensive networking, online collaboration and socializing; information seeking—they are active participants in online activities and very open to seeking information online, learning by trial and error.

As for the stress level, Jayson (2013) believe that it is higher than the one of the previous generations and "it is not letting up" as the individual failure is hard to accept. In a survey conducted on American Millennials, Jayson (2013) discovered that 39% of the respondents said that their stress level has increased in the past year, while 52% admitted that the stress have kept them awake in the night in the past month.

In addition to these characteristics, Gauzente and Roy (2012) found that the members of this generation are more homogenous in education, skills and behaviour than the past generations, which make their behaviour easier to be modelled, while the obtained results on a sample can be extended to the whole population.

9.2 Researches on Online Social Networks

Since the launch of the first online social network 'SixDegree.com' in 1997, a number of social networks were developed over time, many of them small, with a quite low number of people (members), focusing on niche community type such as: religious communities (MyChurch), business/professional (Xinhua, LinkedIn), centred on certain activities (Couchsurfing), exclusivist (aSmallWorld), etc. (Heidemann et al., 2012).

On the other hand, other online social networks took into account a much broader audience, which they have managed to attract: Facebook, Google+, Flixter, Yahoo!360, etc. Among them, Facebook is the largest online social network with 1.3 billion active users each month.

Worldwide, it was determined that a total of 3.61 billion people own a mobile phone, representing approximately 50% of the world population (7.18 billion in 2014), which led that in 2014, 77% of the online social networks users accessed these networks also via mobile phone (Yang et al., 2010). The motivations to connect on such online social network can vary from one user to another, based on the need for communication, obtaining information on areas of interest, identifying more convenient opportunities to travel to, contact more easily with various companies to purchase their products, facilitating to find a job (Cotfas et al., 2016).

In the qualitative studies conducted over the time, the researchers have shown different aspects related to the phenomena that take place in online social networks. For example, Sakas et al. (2014) have taken into account in their questionnaire aspects regarding the affiliation to a particular brand, conversations, interactive elements (games), the search for opportunities and information on online social networks, Dijkmans et al. (2015) focus on the intensive use of social media, the online social involvement of the company's activities and reputation, while Kim et al. (2014), by referring to Twitter, analysed users' identification with the brand, the feeling of belonging to a certain community, the intention to be part of an online community and the frequency of using Twitter for shaping the consumer behaviour. Moreover, Lien and Cao (2014) focused on the qualitative aspects of using social networks such as: the reliability of the comments on these networks, how easy they can be used, the time needed to access information, etc.

Other recent studies in the area of consumers' behaviour and risks in online social networks (OSN) are: Shibchurn and Yan (2015) is taking into consideration the risks that are faced while using online social networks, Eisingerich et al. (2015) is discussing the social risk generated by the users' recommendations on social networks regarding products of certain companies, Labrecque (2014) is studying the para-social interaction on these networks and the degree to which users are eager to share information, Bright et al. (2015) is examining the trust in social networks and the fact that consumers can feel overwhelmed by the abundance of information that companies transmit through them, Wang et al. (2012) is showing how online communication is made on social networks and their impact on purchase intent, while Hollebeek et al. (2014) is analysing the affiliation with a certain brand, etc.

There are also some studies which focus on the gender differences by analysing the sexualized behaviour on Facebook (Sarabia & Estévez, 2016; Tortajada-Giménez et al., 2013) or how men and women communicate in these networks (Åström & Karlsson, 2016), showing that there can be a difference in how men and women want to be perceived by their virtual friends and how they are building their profile and expressing in OSN in order to achieve it.

Considering the previous studies in the literature and the characteristics of the now-a-days economic era, in which the social an economic pressure is highly re-felt in our every-day life, the present study tries to see whether it could be useful for companies to analyse their customers by knowing not only their opinion on the company's products, reputation, how they are influenced, if the influence of online social networks' usage can be put in connection with both consumers' decisions and companies' image, but also their inner wellbeing state.

Starting from this idea, the current study use a standard general affective measure PANAS (Positive and Negative Affect Schedule) analysis in order to split the Millennial customers and online social networks users into two main categories, generally referred as: "Enthusiastic" and "Stressed". Therefore, the questionnaire proposed in this study tries to capture and combine as many as possible of the issues raised individually in each of the studies mentioned above, in order to see how these elements are combining in a more complex view over the online social networks' users. To these, a PANAS analysis is added (Eisingerich et al., 2015), which is facilitating the extraction of the respondents' feelings at the moment they complete the questionnaire.

Table 9.1 The basic components of the questionnaire used and the related questions

Category	Sub-category
The social networks usage (SNU)	Qualitative aspects regarding the usage of online social networks (QSNU)
	The perceived social risk in the context of active participation in online social networks (RSNU)
	Attachment and affiliation with companies activities on online social networks (ASNU)
The consumer's behaviour (CB)	The consumer attitudes in the online environment (ACB)
	The intention to use and to purchase goods from a company (UPCB)
The company's image (CI)	Expectations about the company (ECI)
	Attitude and confidence in company (ACCI)

9.3 Research Methodology

The analysis of the Millennial consumers' behaviour in OSN started from a questionnaire composed of three main categories: the social networks usage (SNU); the consumer's behaviour (CB); the company's image (CI), each of them formed by two to three sub-components (see Table 9.1).

For each sub-category, a series of questions have been created, with the aim of better understanding the opinions of the consumers over the considered aspects, which will be explained in Sect. 9.4 of the present paper.

The received answers have been individually evaluated through a 5-point Likert scale, with respect to the structural equation modelling. As known, the structural equation modelling (SEM) is a statistical methodology by which a confirmatory approach is realized in order to test the model assumptions by taking into account both the observed and latent variables (unnoticed) (Spanos & Lioukas, 2001). The factor analysis can be done in an exploratory (EFA) or confirmatory (CFA) manner, by considering the awareness of the phenomenon under investigation (Delcea et al., 2014).

Based on the literature review on the use of online social networks, mainly on their structure and influence they can exert on consumer behaviour and company's image, it was decided to carry out a confirmatory factor analysis (Koeske et al., 1994).

The questionnaire has been structured and validated using AMOS 22.0.0,[1] while the proposed model was tested in a statistical analysis that takes into account the system's variables as a whole and tries to determine the extent to which this is consistent with the collected data.

[1] http://www-03.ibm.com/software/products/en/spss-amos.

9.4 Confirmatory Factor Analysis with AMOS 22.0.0

The proposed model was tested in a statistical analysis that takes into account the system's variables as a whole and tries to determine the extent to which this is consistent with the collected data.

The initial structure to be tested is presented in Fig. 9.1.

Using AMOS 22.0.0, the following aspects related to the considered construction have been tested and validated: uni-dimensionality, construct validity, content validity, similarity validity, convergent validity and feasibility (Spanos & Lioukas, 2001). Some of the indicators that sustain the construct's validation are presented below, taken from the AMOS 22.0.0 model fit summary (see Tables 9.2, 9.3 and 9.4).

Considering Table 9.2 it can be seen that the minimum discrepancy function, CMIN/DF (minimum discrepancy/degrees of freedom), is 3.156 and, according to Spanos and Lioukas (2001) a value below 5 is considered acceptable in order to test the model fit, which make us conclude that the model is well fit. This is also proved

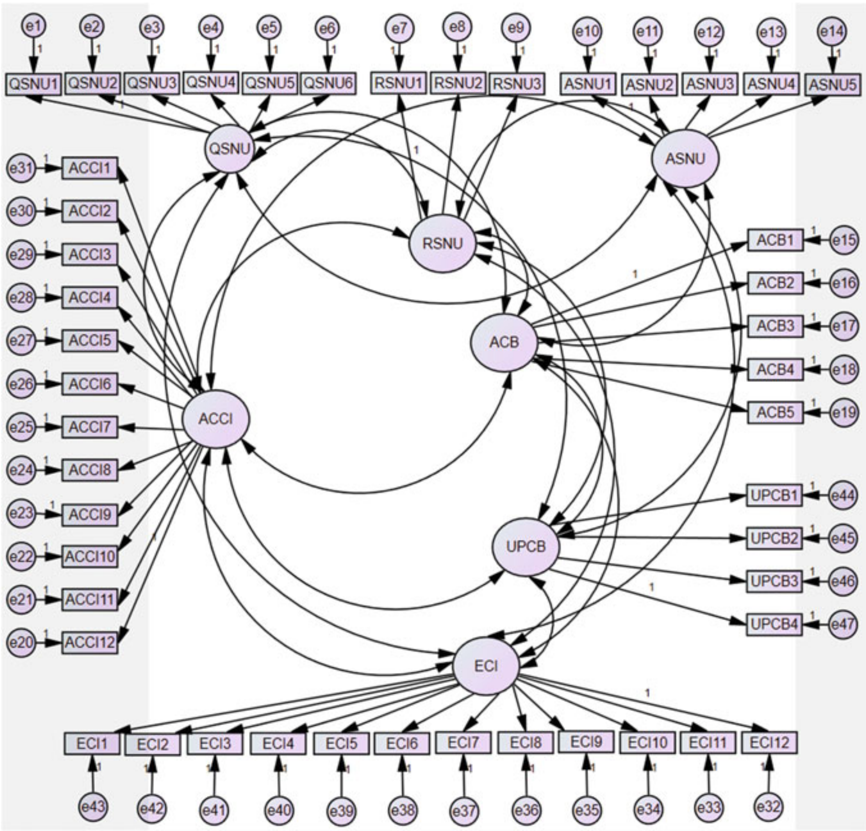


Fig. 9.1 The structure needed to be tested with AMOS 22.0.0

Table 9.2 Model fit summary—CMIN

CMIN					
Model	NPAR	CMIN	DF	P	CMIN/DF
Default model	115	3197.221	1013	0.000	3.156
Saturated model	1128	0.000	0		
Independence model	47	9996.249	1081	0.000	9.247

Table 9.3 Model fit summary—RMR, GFI

RMR, GFI				
Model	RMR	GFI	AGFI	PGFI
Default model	0.070	0.750	0.722	0.674
Saturated model	0.000	1.000		
Independence model	0.194	0.302	0.272	0.289

Table 9.4 Model fit summary—RMSEA

RMSEA				
Model	RMSEA	LO 90	HI 90	PCLOSE
Default model	0.072	0.069	0.075	0.000
Independence model	0.141	0.138	0.144	0.000

by the value of RMR (root mean square residual) in Table 9.3. RMR represents an average residual value which results from the comparison of the variance–covariance matrix of the considered model to the variance–covariance matrix of the sampling. The standardised RMR value range from 0 to 1 and it is smaller than the non-standardised RMR value returned by AMOS 22.0.0. In a well-specified model, this will take values as small as possible. In our model, the non-standardised RMR is 0.070, taking a very small value, which can bring us to the conclusion that the model is well specified.

The third indicator to consider is presented in Table 9.4, namely RMSEA (root mean square error of approximation). This indicator, initially proposed in 1980, has begun to have more and more importance in the covariance's structural modelling and to become one of the most important criterion in the structural equations models. RMSEA takes into account the approximation error within the population and gives an answer to the question: how well the model with the unknown parameters values, but optimally chosen, would fit the covariance matrix for the population from which this sample has been extracted, in the case this population would be available for the study? This discrepancy is measured by RMSEA and depends on the degrees of freedom, which makes it sensitive both to the model's number of parameters and its complexity. Byrne (2016) is showing that the RMSEA value within 0.08 and 0.10 is indicating a mediocre fit, while the ones above 0.10 can be obtained only in the case of a very poor model fit. Therefore, the value of 0.072 is considered acceptable and it indicates a good model fit.

Following this confirmatory factor analysis performed using AMOS 22.0.0, it can be concluded that the degree of "fit" is appropriate, which leads us to the conclusion that the alleged relationship between the proposed variables are plausible.

Having the validated questionnaire, a grey analysis will be performed on the collected data in order to better shape and analyse the considered situation. For this purpose, the theory related to the grey systems theory methods is presented below and it will be used in Sect. 9.4.

9.5 The Degrees of Grey Incidence

Grey systems theory is one of the newest theories in the field of artificial intelligence and it is based on the idea that, in general, in our every-day life we have different levels of knowledge on the things and events that surrounds us, succeeding of knowing almost everything about them, nothing about them or any other intermediate level of knowledge between these two extremities. Making a parallel with the control theory where the observed things could be either "black", case in which the investigator new nothing about the object under investigation, or "white" when everything was known about the object, the grey theory is overlapping on the gap between these two states ("black" and "white") and affirms that, on a certain moment of time, we can only know something about an object (defined as the "degree of greyness"), having this represented through a grey tone (Delcea, 2015a, 2015b).

Over the years, the methods proposed by the grey systems theory have been successfully used on their own or as a result of hybridization with other intelligent artificial methods such as: fuzzy, neural networks, genetic algorithms, rough sets theory, etc., in different social and economic area, physics, neuroscience, telecommunications, agriculture, geology, etc. (Delcea, 2015a, 2015b; Liu et al., 2016). From these methods, two of them were often used in applications: the grey model when making predictions and the grey analysis when conducting diagnosis or calculating the incidence of a phenomenon to another phenomenon/phenomena.

As in our case the purpose was to measure the impact of the OSN usage on the consumers' behaviour, a grey analysis is required. Before presenting the needed steps , it should be stated that the synthetic degree of grey incidence, determined as explained in the following, is characterised by a value between 0 and 1. As close this value is to 1, as high the impact of the selected variable on the other considered variables is.

The Absolute Degree of Grey Incidence

There are considered two sequences of data with non-zero initial values and with the same length, data X0 and Xj, j = 1…n, with t = time period and n = variables (Liu et al., 2016; Liu & Lin, 2011):

$$X_0 = (x_{1,0}, x_{2,0}, x_{3,0}, x_{4,0}, \ldots, x_{t,0}), \tag{9.1}$$

$$X_j = (x_{1,j}, x_{2,j}, x_{3,j}, x_{4,j}, \ldots, x_{t,j}), \tag{9.2}$$

The zero-start points' images are:

$$X_j^0 = (x_{1,j} - x_{1,j}, x_{2,j} - x_{1,j}, \ldots, x_{t,j} - x_{1,j}) = (x_{1,j}^0, x_{2,j}^0, \ldots, x_{t,j}^0) \tag{9.3}$$

The absolute degree of grey incidence is:

$$\varepsilon_{0j} = \frac{1 + |s_0| + |s_j|}{1 + |s_0| + |s_j| + |s_0 - s_j|} \tag{9.4}$$

with $|s_0|$ and $|s_j|$ computed as follows:

$$|s_0| = \left| \sum_{k=2}^{t-1} x_{k,0}^0 + \frac{1}{2} x_{t,0}^0 \right| \tag{9.5}$$

$$|s_j| = \left| \sum_{k=2}^{t-1} x_{k,j}^0 + \frac{1}{2} x_{t,j}^0 \right| \tag{9.6}$$

The Relative Degree of Grey Incidence

There are two sequences of data with non-zero initial values and with the same length, X0 and Xj, j = 1…n, with t = time period and n = variables (Liu & Lin, 2011):

$$X_0 = (x_{1,0}, x_{2,0}, x_{3,0}, x_{4,0}, \ldots, x_{t,0}), \tag{9.7}$$

$$X_j = (x_{1,j}, x_{2,j}, x_{3,j}, x_{4,j}, \ldots, x_{t,j}), \tag{9.8}$$

The initial values images of X_0 and X_j are:

$$X_0' = (x_{1,0}', x_{2,0}', \ldots, x_{t,0}') = \left(\frac{x_{1,0}}{x_{1,0}}, \frac{x_{2,0}}{x_{1,0}}, \ldots, \frac{x_{t,0}}{x_{1,0}} \right) \tag{9.9}$$

$$X_j' = (x_{1,j}', x_{2,j}', \ldots, x_{t,j}') = \left(\frac{x_{1,j}}{x_{1,j}}, \frac{x_{2,j}}{x_{1,j}}, \ldots, \frac{x_{t,j}}{x_{1,j}} \right) \tag{9.10}$$

The zero-start points' images calculated based on (9.9) and (9.10) for X_0 and X_j are:

$$X_0^{0'} = (x_{1,0}' - x_{1,0}', x_{2,0}' - x_{1,0}', \ldots, x'_{t,0} - x_{1,0}') = (x_{1,0}^{'0}, x_{2,0}^{'0}, \ldots, x_{t,0}^{'0}) \tag{9.11}$$

$$X_j^{0'} = (x'_{1,j} - x'_{1,j}, x'_{2,j} - x'_{1,j}, \ldots, x'_{t,j} - x'_{1,j}) = (x'^{0}_{1,j}, x'^{0}_{2,j}, \ldots, x'^{0}_{t,j}) \quad (9.12)$$

The relative degree of grey incidence is computed as:

$$r_{0j} = \frac{1 + |s'_0| + |s'_j|}{1 + |s'_0| + |s'_j| + |s'_0 - s'_j|} \quad (9.13)$$

with $|s'_0|$ and $|s'_j|$:

$$|s'_0| = \left| \sum_{k=2}^{t-1} x'^{0}_{k,0} + \frac{1}{2} x'^{0}_{t,0} \right| \quad (9.14)$$

$$|s'_j| = \left| \sum_{k=2}^{t-1} x'^{0}_{k,j} + \frac{1}{2} x'^{0}_{t,j} \right| \quad (9.15)$$

The Synthetic Degree of Grey Incidence

The synthetic degree of grey incidence is based on both the absolute and the relative degrees of grey incidence (Lian et al., 2009; Liu & Lin, 2011):

$$\rho_{0j} = \theta \varepsilon_{0j} + (1 - \theta) r_{0j}, \quad (9.16)$$

with j = 2, …, n, $\theta \in [0, 1]$ and $0 < \rho_{0j} \leq 1$.

Grey incidence analysis is increasingly more used in economic and social sciences due to the fact that focuses on the closeness of relations between factors, being based on the similarity level of the geometrical patterns of sequence curves (Liu et al., 2016). The size of the degree of incidence is directly proportional to the degree of similarity between the considered variables' curves.

9.6 Case Study: Enthusiastic Versus Stressed Consumers

The starting hypothesis is that the online social networks' users are, in general, young persons with ages between 18 and 34 years old, corresponding to the Millennial generation, which, due to the information access that they have and due to the intense usage of internet, try to pay a more convenient price for the goods they need. According to Smith (2016) the percentage of Millennials that use Facebook is 91%.

Moreover, due to the characteristics of this group, they are considered more oriented to the social media participation and more willing to freely express their feelings in comparison to other age categories, which due to education or different moral standards, are not so open to expressing truly the information on their true inner feelings.

9.7 Questionnaire and Data Analysis

The questionnaire has been posted on the Google Forms platform in period 15.05.2015–01.07.2015 and it have been distributed through different groups available on the online social networks. A number of 416 questionnaires have been kept in the analysis, for all the persons with the age in the above interval.

Analysing the received answers (evaluated through a 5 point Likert scale), it has been observed that the respondents have said that that Facebook represents an important online social network (4.039 points on Likert scale), being easy to use (4.555 points on Likert scale) and that on this online social network the information they need on their every-day life can be found very easy (4.224 points on Likert scale).

Even though the respondents have stated that, on average, they are connecting daily on this online social network (4.507 points on Likert scale), they consider that they don't spend long period of time on this network.

As for the number of "friends" on this network, based on the answers received to the question "In present, the number of my friends on Facebook is:…" it has been calculated an average number of 964 friends (greater than the average number of friends, only 338, found among adult Facebook users (Smith, 2014)), 528 of them being, on average, considered friends or acquaintances in the every-day life.

More, the respondents believe that the face-to-face conversation is happening more frequently (a 4.075 average on Likert scale) than the communication made through the online social networks (with an average of 3.729 points on Likert scale), but without a significant difference regarding the communication quality among the two types of communication.

As for the company's image, each respondent was asked to think of a brand he/she has been using lately and to indicate the name of the company. Based on their answers, six main categories have been set up as they result from Fig. 9.2: personal care, clothes, food, electronics, services and other. It can be seen that most of the brands belong to companies from the personal care area which was one of the expected answers due to the fact that the Millennials have been characterised as a group very focused on health and fitness and very "selfie-conscious" (Wells et al., 2015), followed by the companies in the electronics and food domain. Even the selection of the companies from the electronics and food domain is not a surprise as Millennials have been characterised over the time as being highly interested in technology and new devices (Johri et al., 2014) and the adopters of organic food (Sa'ari & Koe, 2014).

Related to the mentioned companies, the respondents have evaluated their expectations (ECI) and their attitude and trust towards these companies (ACCI). In the same time, their attitude towards a company as a result of that company' activity on

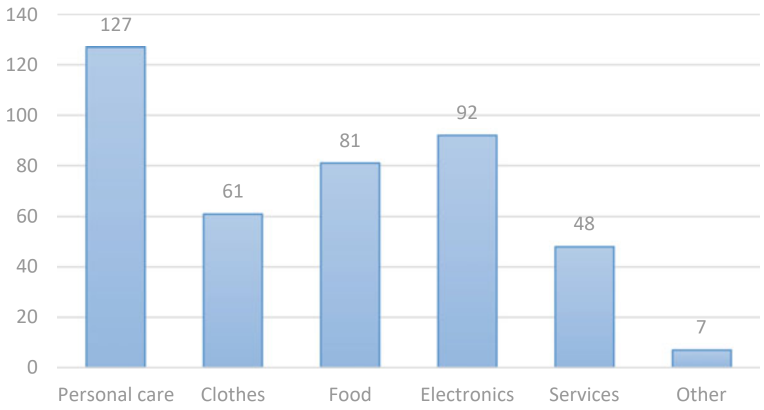


Fig. 9.2 Main categories for the mentioned companies

online social networks has been evaluated (ACB) and also the future acquisition and usage intention has been assessed (UPCB) through the applied questionnaire.

More, considering Eisingerich et al. (2015) study which underlines the importance of a PANAS analysis in order to better shape the consumers' attitude towards companies on online social networks, the respondents' feelings have been extracted by asking them to evaluate on a 5 point Likert scale how they were feeling at that specific moment of time in which they were filling the questionnaire. Two main situations have been taken into account, each one of them formed by ten elements:

- Positive Affect (PA): Enthusiastic/strong/excited/interested/proud/inspired/ determined/attentive/active/alert—this group has been called in the following as the "Enthusiastic";
- Negative Affect (NA): Distressed/upset/guilty/irritable/scared/ashamed/hostile /nervous/jitter/afraid—referred in the following as "Stressed".

Eisingerich et al. (2015) believe that this type of analysis can be very useful mainly for separating the data set of the stressed respondents from the initial set as it is believed that their answers can negatively impact the analysis' results. Therefore, one of the goals of this study was to see whether the two groups are distinct one to another in such a manner that will determine the exclusion of the "stressed" group from the analysis' dataset.

From the whole set of respondents, based on these twenty elements' evaluation (both PA and NA) on a 5-point Likert scale, 344 have been marked as being "Enthusiastic", representing 82.69%, while 72 of have been retained in the "Stressed" group, a 17.31%.

As we are interested in the similarities and differences between the two considered groups, in the following, the received answers will to be analysed in parallel and simultaneously for both groups.

The first similarity was encountered when analysing the answers gathered for the question regarding the communication with friends in OSN: it can easily be seen that both of the groups are saying, on average, that they are having this activity a couple of times a day (59.88% of the “Enthusiastic” vs. 56.94% of the “Stressed”—see Fig. 9.3), just a small amount of respondents are communicating just once a couple of weeks (2.03% of “Enthusiastic” vs. 5.56% of “Stressed”).

Beside the fact that the respondents have been asked to evaluate their inner state and, based on their evaluation, they have been divided into one of the two considered groups (“Enthusiastic” vs. “Stressed”), three additional questions have been added to the questionnaire, through which we wanted to better extract their general state of being. The answers received are depicted in Fig. 9.4. Based on them, we can conclude that there is a significant difference between the respondents’ wellbeing, the “Enthusiastic” group marking, on average, smaller values on the Likert scale in comparison to the “Stressed” group.

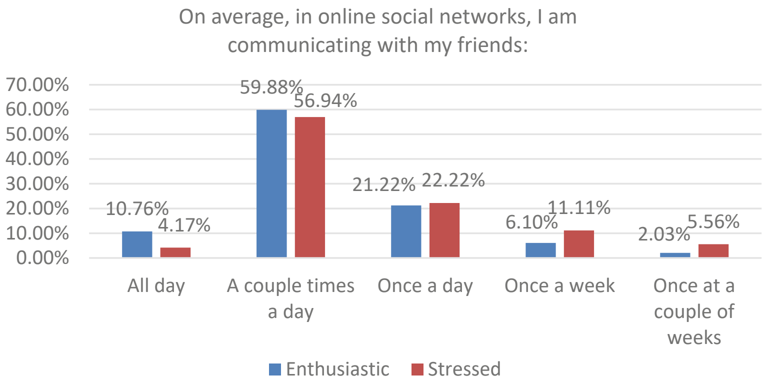


Fig. 9.3 Communication with friends in online social networks

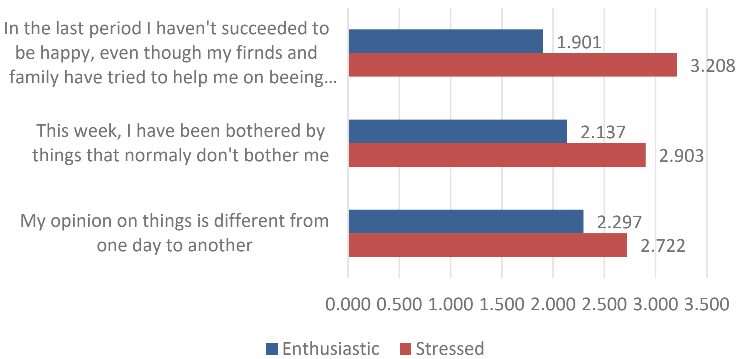


Fig. 9.4 Respondents’ general state of being

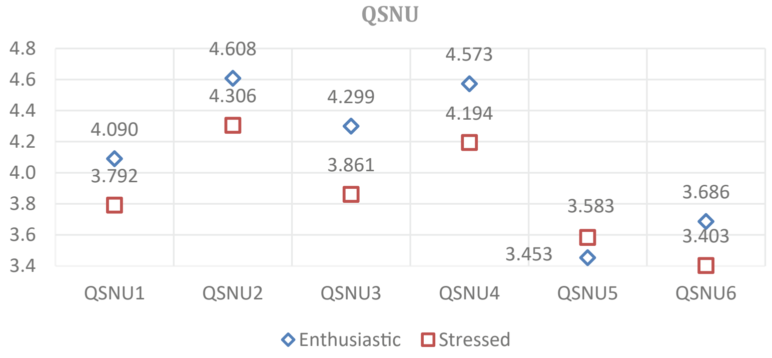


Fig. 9.5 Average answers received for QSNU

More, on average, the number of friends on the online social networks is almost the same for both groups: 1033 for "Enthusiastic" and 748 friends for "Stressed", while the communication with their friends on these networks is similar among these groups—see Fig. 9.3.

The only significant differences are given by the answers received to the questions related to the general state of being for each respondent in the last week and for how often they are changing their opinion over different things—see Fig. 9.4.

More, it can be observed that, on average, the respondents from the "Stressed" set have the tendency to be undecided both regarding their happiness state from the last week and their opinion changing (most of the answers being, on average, almost 3 on Likert scale), while the respondents from the "Enthusiastic" set are rejecting the first two questions and consider that, in general, their opinion over the things stays the same in time (values close to 2 are gathered, on average, in Likert scale).

Continuing the analysis for the seven variables (QSNU, RSNU, ASNU, ACB, UPCB, ECI and ACCI) the following average values have been obtained for the both "Enthusiastic" and "Stressed" set—Figs. 9.5, 9.6, 9.7, 9.8, 9.9, 9.10 and 9.11.

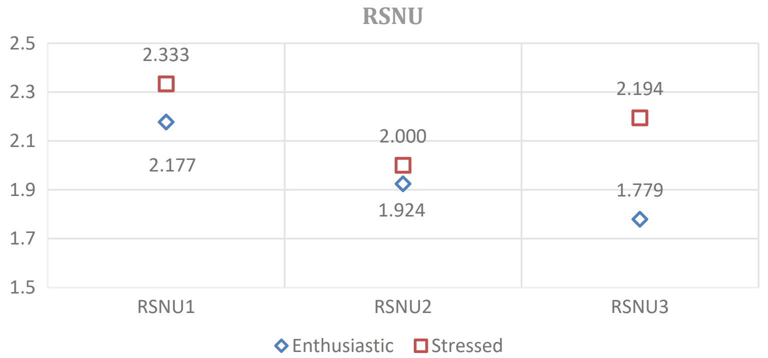


Fig. 9.6 Average answers received for RSNU

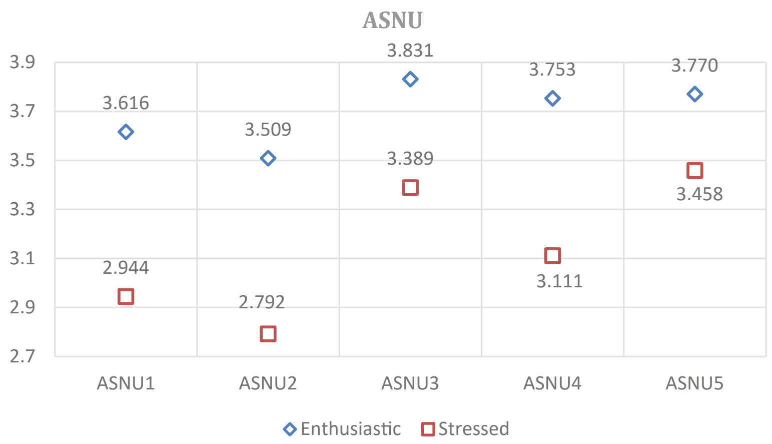


Fig. 9.7 Average answers received for ASNU

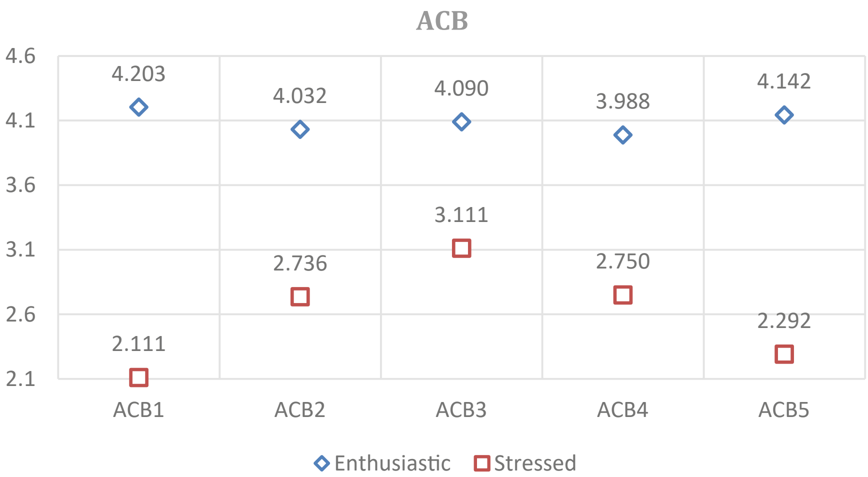


Fig. 9.8 Average answers received for ACB

The qualitative aspects related to the usage of social networks (QSNU) have underlined the fact that Facebook is considered an important online social network (QSNU1), easy to used (QSNU2), on which one ca easily obtain the needed information (QSNU3) and on which the respondents are connecting every day (QSNU4), even though they are spending relatively long periods of time on this network (QSNU5), and, when they do, they are active users (QSNU6). The answers are following the same trend within the two groups, the "Enthusiastic" giving higher marks than the "Stressed" group (Fig. 9.5). A small difference is encountered in the answers received

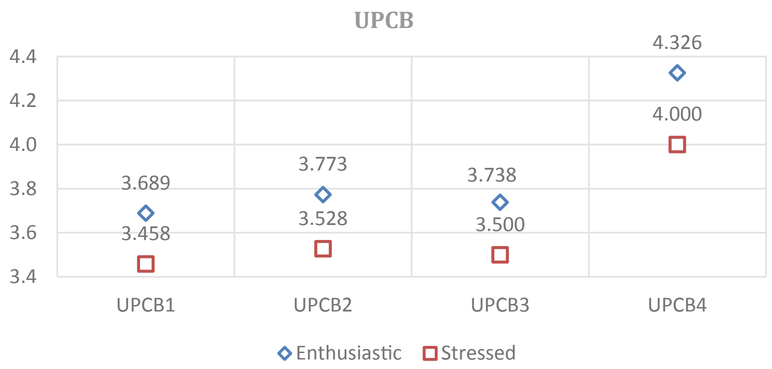


Fig. 9.9 Average answers received for UPCB

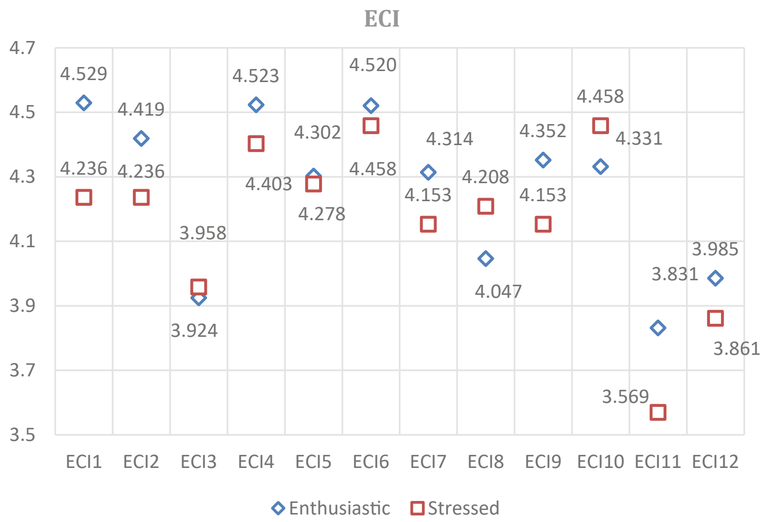


Fig. 9.10 Average answers received for ECI

for QSNU5 which is underlying the fact that both groups consider that they are spending relatively long period of time on this network.

The same behaviour regarding the answers is identified even when analysing the answers received for the perceived social risk on these networks (RSNU): the "Enthusiastic" are giving, on average, greater marks than the "Stressed" group (Fig. 9.6). Based on the average results, it can be concluded that the perceived risk is moderately low as the values obtained are below 2.5. The respondents do not find risky to affirm positive things about a company's products/services in OSN (RSNU1), they are not afraid that the others are going to disapprove their opinion regarding the

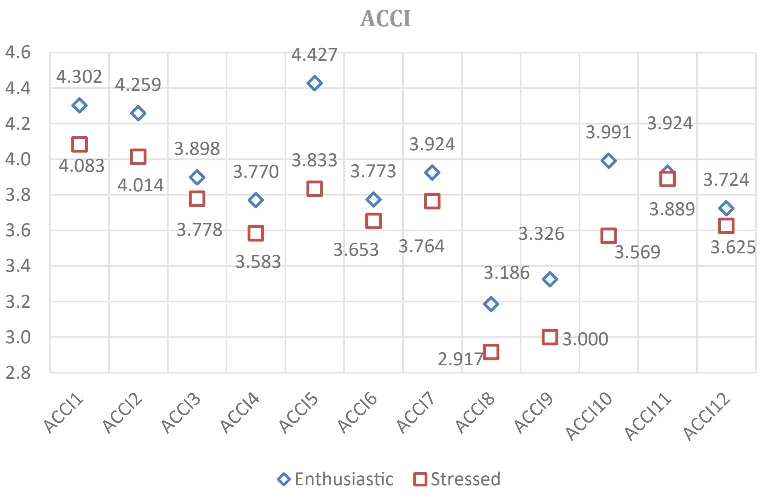


Fig. 9.11 Average answers received for ACCI

products/services of a company (RSNU2) and they are not ashamed of the fact that they are recommending that company's products/services in OSN (RSNU3).

The attachment on the companies' activity in OSN (ASNU), measured through five questions is presenting the fact that both groups are aware of the mentioned company's activity (ASNU1) and that they are following the mentioned company's activity on OSN (ASNU2), but at the same time they are following the companies related to the respondent's lifestyle (ASNU3), the companies from where they plan to buy in the future (ASNU4) and the other companies they are buying from (ASNU5). Even in this case, the answers received from the "Stressed" group have relatively smaller values in comparison to the "Enthusiastic" answers (Fig. 9.7).

Both the consumers' attitude (ACB) and buying intention (UPCB) are respecting the same rule encountered in the previous cases: they are receiving smaller values on the Likert scale from the "Stressed" group (Figs. 9.8 and 9.9). While the buying intention is on a positive trend: both groups are using the services/products of the mentioned company to the detriment of other companies with similar quality products (UPCB1), similar price products (UPCB2), similar characteristics products (UPCB3) and it is very probable to continue to buy them (UPCB4), the consumers' attitude in OSN can be disjoint: while the "Enthusiastic" are speaking with their friends in OSN about the company's activity (4.203 points for ACB1), are willing to affirm positive things about that company (4.032 on ACB2), they would recommend the company in OSN (3.988 points for ACB4) and they are willing to encourage their family/friends to buy from that company (4.142 points on ACB5), the "Stressed" have little to medium interest in conducting such activities. A similarity among the two groups is only on the fact that both of them are taking into account the products recommended by their friends in OSN (4.090 points vs. 3.111 points for ACB3—Fig. 9.8). Even in this case, the "Stressed" group is scoring a small value for this indicator, proving that the group is considering the products to a lesser extent than the "Enthusiastic" do.

Following the literature, it is already known that the Millennials have great expectations (Ng et al., 2010), so the next question to answer is: are really these expectations so great and are they applying to both groups? Figure 9.10 is presenting the values obtained for the expectations (ECI) and, as the great majority of the average values are around 4 or more for the following issues: the companies in OSN should: offer more information on their products/services (ECI1), offer promotions (ECI2), offer free products (ECI3), present the news related to their products/services and other news considered to be important for the company (ECI4), invite the customers to participate to products improvement or design (ECI5), announce events (ECI6), share ideas and offer additional information on the future products/services to be offered by the company (ECI7), explain how the feedback received from their customers was used (ECI8), surprise their customers (ECI9), offer information related to their customers lifestyle (ECI10), offer exclusivist information (ECI11), carry a dialogue with their customers (ECI12), it can be concluded that they are indeed great expectations. As for the group differences, the "Stressed" are more interested than the "Enthusiastic" in receiving information related to their lifestyle (4.458 points) and less likely to appreciate the exclusivist information offered by their mentioned company (3.569 points).

The attitude and confidence in the mentioned company (ACCI) marked good scores for both groups. Besides the relatively small values given by received by the company from the "Stressed" group on the fact that the company is not supporting the humanitarian causes (2.917 points for ACCI8) not it is social responsible (ACCI9), the other components score relatively high values: opinion on the company (ACCI1; 4.083 points "Stressed" vs. 4.302 points given by the "Enthusiastic"), the last experience regarding the company (ACCI2; 4.014 vs. 4.259), the company is giving a safety feeling to its customers (ACCI3), is respected and admired by its customers (ACCI4), is offering qualitative products/services (ACCI5), the company's employees are trustworthy (ACCI6), the company has: a high-quality management (ACCI7), high standards regarding the relationship with the people (ACCI10), good financial performance and low bankruptcy risk (ACCI11), the company takes into account the consumers' opinion when making a decision (ACCI12).

Considering all the variables and their average values, it can be concluded that trend of the received answers from the "Stressed" set is following, in general, the trend of the answers received from the "Enthusiastic" set, but, they have, on average, smaller values.

9.8 Grey Incidence Analysis

In addition, a grey incidence analysis has been conducted on these indicators in order to see the influence of online social networks usage on both consumers' decisions and companies' image. The values for the synthetic degree of grey incidence have been determined using Grey System Theory Modelling Software 6.0 (Bo & Liu, 2009). The results are summarized in Tables 9.5 and 9.6.

By referring to the two categories of respondents ("Enthusiastic" and "Stressed"), it can be seen that there have been obtained almost the same values for all the analysed components, less for ACB (Fig. 9.12), underlining the fact that this variable is less influenced by the consumers' attachment on the company's activity on online social networks, while other two components are taking into account the online social network usage (QSNU and RSNU) are influencing in a small measure the consumers' behaviour.

Therefore, the synthetic degree of grey incidence analysis, is underlying the following: first of all, for the consumers' attitude (ACB) it can be observed that it is influenced basically by the respondents' attachment over the company's activity on the online social networks (ASNU), and by the fact that they are not experiencing a social risk while expressing their attitude and opinions on these networks. A high influence is also manifesting from the fact that the respondents are using these social networks.

On the other hand, the using and buying intention (UPCB) seems to be mostly influenced by the users' attachment to the company's activity on the online social network (ASNU). These users are choosing, in most of the cases, the products of the mentioned company instead of the products of another company with the same characteristics, price or quality. The absence of the social risk is not neutral in the buying intention, while the online social networks usage degree seems to have a minimal positive influence on consumers' behaviour.

Also, the absence of the perceived social risk on the online social network is not significantly influencing the expectations regarding the company (ECI). Instead, these are connected both to the degree of online social networks usage and to the fact that the respondents are following the social pages of the companies they are buying

Table 9.5 Synthetic degree of grey incidence—"Enthusiastic"

Synthetic degree of grey incidence	QSNU	RSNU	ASNU
ACB	0.7618	0.8793	0.9247
UPCB	0.6431	0.5188	0.9172
ECI	0.7717	0.5290	0.8639
ACCI	0.6410	0.5815	0.7406

Table 9.6 Synthetic degree of grey incidence—"Stressed"

Synthetic degree of grey incidence	QSNU	RSNU	ASNU
ACB	0.5155	0.6004	0.6612
UPCB	0.6106	0.5002	0.8933
ECI	0.6945	0.5014	0.8307
ACCI	0.6001	0.5445	0.6931

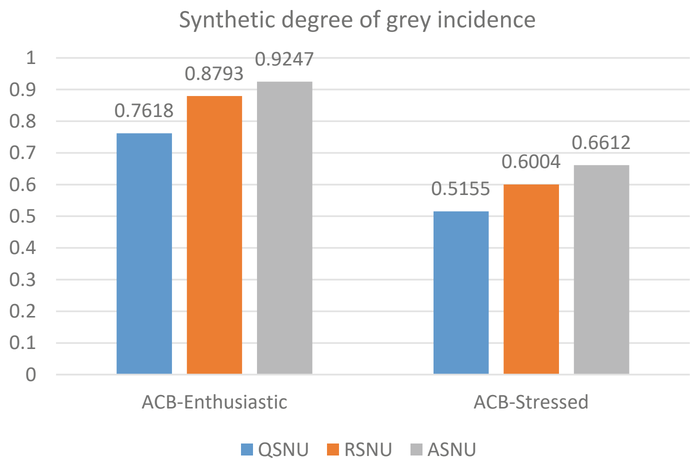


Fig. 9.12 Synthetic degree of grey incidence of QSNU, RSNU and ASNU on ACB

from or they are wishing to buy from in the future, which conducts to the growth of the expectations regarding these companies.

As for the consumers' attitude and trust in a company (ACCI), the factor with the great influence is the fact that the respondents are searching the information regarding these companies' online pages, followed by the degree of these networks' usage and, to a lesser extent to the absence of the perceived social risk.

9.9 Millennials Characteristics—A Summary

Therefore, by considering the answers from above and the personal answers received from the respondents, related to age, sex, studies, etc., a profile for the most representative characteristics and attitudes of each group has been created and presented in Table 9.7.

In addition to Table 9.7, it can be said that, in general, the persons from the "Stressed" set have the tendency to offer smaller values for all the answers than the persons from the "Enthusiastic" set (with some exceptions underlined above) and, as their percentage is not negligible in the Millennials group, their exclusion from the analysis should no longer be considered an option, as suggested in previous studies (Eisingerich et al., 2015). As a solution, the "Stressed" group should be retained in the analysis both to the fact that even their opinions should be heard and taken into account and that they are a group of customers that can increase in numbers due to the socio-economic characteristics of the every-day-life and, in time, this group can be determinant in the companies' survival on the market. More, depending on the study's characteristics, one can decide whether there can be needed or necessary a further analysis through which it can determined if the persons from the "Stressed"

Table 9.7 "Enthusiastic" versus "Stressed" Millennials—a general overview

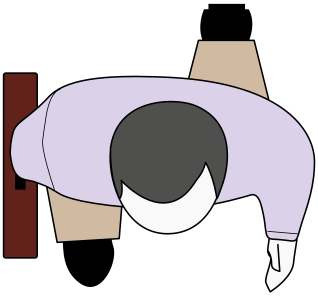
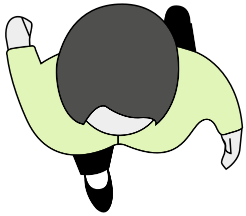
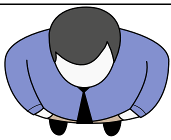
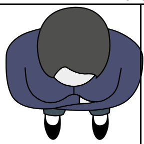
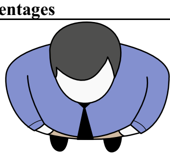
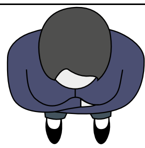
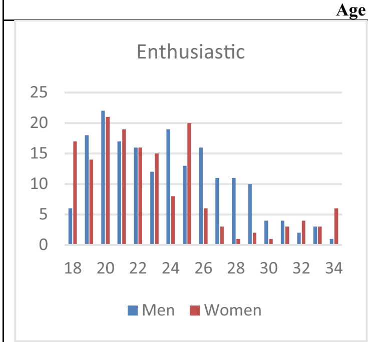
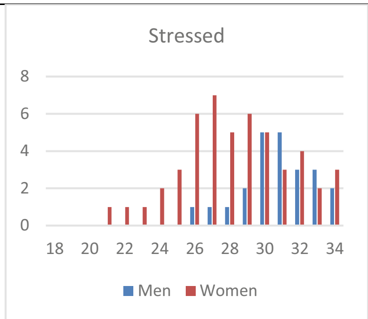
<table>
<tr><th>Issue</th><th colspan="2">"Enthusiastic"</th><th colspan="2">"Stressed"</th></tr>
<tr><td rowspan="10">General Profile</td><td colspan="4">Sex
(based on the dominant percentage)</td></tr>
<tr><td colspan="2"></td><td colspan="2"></td></tr>
<tr><td colspan="4">Percentages</td></tr>
<tr><td>
53.78%</td><td>
46.22%</td><td>
31.94%</td><td>
68.06%</td></tr>
<tr><td colspan="4">Average age</td></tr>
<tr><td>23.8</td><td>23</td><td>30.7</td><td>28.3</td></tr>
<tr><td colspan="4">Age distribution</td></tr>
<tr><td colspan="2">
</td><td colspan="2">
</td></tr>
<tr><td colspan="4">General opinion on things</td></tr>
<tr><td colspan="2">Stays the same in time</td><td colspan="2">Changeable in time</td></tr>
</table>

Table 9.7 (continued)

Attitude in OSN	**Communication with friends**	
	A couple of times a day	A couple of times a day
	Time spend in OSN	
	Relatively long periods of time	Relatively long periods of time
	Social risk in OSN	
	• Moderately low perceived risk; • Not risky to affirm positive things about a company's products/ service in OSN.	• Moderately low perceived risk; • Not risky to affirm positive things about a company's products/ service in OSN.
	Attachment to the company's activity in OSN	
	Moderate to high attachment	Relatively moderate attachment
	Buying intention	
	Use the company's products/ services in detriment of other companies with similar products	Use the company's products/ services in detriment of other companies with similar products
	Behaviour and attitude in OSN	
	• Speaking with friends in OSN about company's activity; • Are willing to affirm positive things about the company; • Are willing to encourage their family / friends to buy from a specific company; • Are recommending the company in OSN.	• Little to medium interest in conducting such activities.
	• Are taking into account their friends' recommendations.	• Are taking into account their friends' recommendations.
	Expectations	
	Interested in obtaining information on company's products and events.	Interesting in receiving information related to their lifestyle and events. Modarately interested in obtaining exclusivist information.
OSN influence	**OSN influence on consumers' decisions and companies' image (resulted from the grey analysis)**	
	High influence of OSN usage and perceived risk on consumers' attitude.	Medium influence of OSN usage and perceived risk on consumers' attitude.
	High influence of buyers' attachment on companies' activity in OSN on: future buying decisions, consumers' expectations and attitude toward a company.	High influence of buyers' attachment on companies' activity in OSN on: future buying decisions, consumers' expectations and attitude toward a company.
	High influence of the personal characteristics on consumers' expectations.	Medium to high influence of the personal characteristics on consumers' expectations.

group are facing a temporary or a permanent period of stress and whether this two new groups should be both kept in the analysis or just the one characterized through the existence of a permanent stress feeling, as this group will continue to take its decisions all the time in stressful conditions.

9.10 Conclusions

Recent studies have shown that based on informal communication, done from one user to another, certain opinions regarding the activity of a company can be formed with long-term influence on perception of consumers about its image.

In this context, this research examines the consumer's behaviour on online social networks and how his/hers perception towards certain companies may change due to communications on these networks with other users. Due to their group characteristics, Millennial have been chosen in order to extract the data needed in this study and a PANAS was used in order to see whether a difference can be identified between the two considered sub-groups: "Enthusiastic" and "Stressed". Based on the latest studies in the literature, a questionnaire was created, containing the most of the issues identified in the publications related directly to consumer behaviour and company's image. After its validation using AMOS SPSS 22.0.0, a first qualitative analysis of the collected data was carried out and a series of statistics results were presented.

From the grey analysis on the whole set of respondents (including the "Enthusiastic" and "Stressed" sets) it has been observed that both the consumer attitude and the use and purchase intent are influenced mostly by the respondent's attachment to the company's activity on online social networks, and that they do not feel exposed to the social risk generated by the fact that they are expressing their attitudes on these networks. The attitude and confidence in the firm are influenced by: seeking the information about these companies on their pages, followed by the degree of utilization of the network and in a lesser way by the absence of the perceived social risk.

More, on average and with small differences, the two groups of persons have almost the same answers for the considered variables (the "Stressed" group is giving smaller values on the Likert scale, but having the same trend as the answers received from the "Enthusiastic" group, for the most of the questions), which makes us believe that there is no need to exclude the "Stressed" persons from the entire group gathered in a future research as these persons represent an important part of the consumers that a firm can have. A possible solution can be to conduct a parallel study for the two groups and to interpret the results accordingly or to weight the answers received from the "Stressed" group.

Therefore, based on the analysis we can conclude that the online social networks can influence consumer behaviour (either that we are referring to an enthusiastic or to a stressed consumer) regarding the purchase of products/services and their image about companies, the social pages of companies playing an important role in forming opinions. Firms can benefit from this by raising awareness and efforts in the ownership, maintenance and training an online social community around the promoted brand, that facilitates the consumer access to information and their active participation in creating content on those pages for distributing it in their online networks.

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Chapter 10
Grey Models and Its Application in Energy-Economy System

Chaoqing Yuan

10.1 Introduction

10.1.1 The Greyness of Energy-Economy System Analysis

Many things about the macroeconomic principles, such as Business Cycle (Cooley & Dwyer, 1998; Stock & Watson, 2005), the stages of economic development (Panayotou, 1993), will change the relationship between energy consumption and economic growth. Changes in the environmental attitudes of residents (Dunlap et al., 2000; Harris, 2006) and the energy and environmental policies (Helm, 2002; Yuan et al., 2009), will improve energy efficiency and develop renewable energy. Energy price volatility also has effect on energy consumption and macro-economy (Hooker, 1996; Pindyck & Rotemberg, 1983; Yuan et al., 2010a).

These make energy-economy systems extremely complex, dynamic and uncertain. Using the econometric model, we can analyze the long-term, stable and balanced relationship of energy-economy system. While, grey system theory can be used to better grasp the characteristics of the stage and the latest trends.

10.1.2 Literature Review

Grey systems theory has widely been used in Energy-Economy system analysis, especially in energy consumption prediction as shown in Table 10.1 and the relation between energy consumption and economic growth as shown in Table 10.2.

C. Yuan (✉)
College of Economics & Management, Nanjing University of Aeronautics and Astronautics, 29 Yudao Street, Nanjing 210016, People's Republic of China
e-mail: yuanchaoqing@126.com

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Table 10.1 Energy consumption prediction by using grey models

No.	Authors	Methods	Prediction	Year
1	Hsu and Chen (2003)	An improved grey GM(1, 1) model, using a technique that combines residual modification with artificial neural network sign estimation	Power demand forecasting of Taiwan	2003
2	You-Xin et al. (2004)	Grey GM(1, 1) model with function-transfer method	The oil consumption per unit output analysis	2004
3	Yao and Chi (2004)	Taguchi–Grey based predictor	The demand value of electricity on line	2004
4	Zhou et al. (2006)	Combining the traditional grey model GM(1, 1) with the trigonometric residual modification technique	Electricity demand	2006
5	El-Fouly et al. (2006)	GM(1, 1)	Wind speed forecasting and wind power prediction	2006
6	Akay and Atak (2007)	Grey-Model with rolling mechanism	Turkey's total and industrial electricity consumption	2007
7	Kumar and Jain (2010)	Grey-Model with rolling mechanism	Coal, electricity (in utilities) consumption in India	2010
8	Lee and Tong (2011)	An improved grey forecasting model, which combines residual modification with genetic programming sign estimation	Chinese energy consumption	2011
9	Pao et al. (2012)	Nonlinear grey Bernoulli model (NGBM)	Carbon emissions, energy consumption and real outputs	2012
10	Hamzacebi and Huseyin Avni (2014)	Optimized Grey Modeling (1, 1)	Total electric energy demand of Turkey	2014
11	Xie et al. (2015)	An optimized single variable discrete grey forecasting model	The developing trends of China's energy production and consumption	2015
12	Yuan et al. (2016)	GM(1, 1), ARIMA	China's primary energy consumption forecasting	2016

(continued)

Table 10.1 (continued)

No.	Authors	Methods	Prediction	Year
13	Yuan et al. (2017)	GM(1, 1) cluster	Global oil consumption forecasting	2017

Table 10.2 Grey incidence analysis of energy consumption and economic growth

No.	Authors	Methods	Relationship	Year
1	Chang and Lin (1999)	Grey relation analysis	Energy-induced CO2emissions from 34 industries	1999
2	Lin et al. (2007)	Grey relation analysis	INTER-relationships among economy, energy and CO_2 emissions of 37 industrial sectors in Taiwan	2007
3	Feng et al. (2009)	Grey relational analysis and econometric approach	Relationship between Chinese energy consumption and the GDP, industrial structure, technology progress, and opening degree	2009
4	Yuan et al. (2010b)	Grey incidence analysis	The relation between Chinese energy consumption and economic growth	2010
5	Feng et al. (2011)	Grey relational analysis	The relationship between energy consumption, consumption expenditure and CO_2 emissions for different lifestyles	2011
6	Kose et al. (2013)	Grey relational analysis	The relationship between energy consumption and economic growth in turkey	2013
7	Liu (2013)	Grey relational analysis	The interaction between ecosystem and behavior system, per-capita energy ecological footprint	2013

From Table 10.1, it is found that grey prediction model, improved grey prediction model, and those combined with the other models are used to forecast oil consumption, coal consumption, primary energy consumption and so on. The prediction results show that grey prediction models work well in this field.

From Table 10.2, grey relational analysis or grey incidence analysis is used to measure the relationship between economic growth and energy consumption in many cases. It can reflect the relation between the variables in various stages of the economic development or business cycles, while the long term relation is revealed by statistical methods and econometric models.

10.2 Energy Consumption Predictions with Grey Models

It is pointed out that the better accuracy can be achieved when the energy consumption is predicted with socio-economic indicators and demand side management (DSM) data (Ardakani & Ardehali, 2014). But it is quite evident that predictions of these socio-economic and DSM indicators are as difficult as that of energy consumption. A rough estimate of these independent variables will obviously reduce the accuracy of energy consumption forecasting. While the univariate methods including GM(1, 1) model and ARIMA (the autoregressive integrated moving average) model are used in this section, in which the independent variables need not be predicted. And An ARIMA model, a commonly used econometric model, is generally denoted ARIMA(p, q, d), where p is the order of the AR model, d is the degree of differencing, and q is the order of the MA model (Hamilton, 2015), which is compared with GM(1, 1) in this section.

10.2.1 China's Primary Energy Consumption Predictions

China is the world's largest energy consumer, which has important influence on international energy market. So China's energy consumption forecasting, can not only provide some important references for China's economic and energy policy-making, but also some evidence for understanding the international energy market trends.

10.2.1.1 Forecasting with ARIMA(2, 1, 1) Model

The time series of China's primary energy consumption is shown as Fig. 10.1. It is not difficult to find that there is a turning point in 2001. After 2001, China's primary energy consumption rise at a larger growth rate. As it is known, China joined World Trade Organization (WTO) in 2001. The international market is more and more open to China. And China's exports have sharply grown since 2001 due to China's accession to WTO. In fact, China's development has entered a new stage since then. The sustained and rapid economic growth has stimulated rapid increase in energy consumption. In econometric time series analysis, a dummy variable may be used to indicate the occurrence of wars, major strikes or other incidents, and it takes the value 0 or 1. And a dummy variable F is used to indicate China's accession to WTO in this section. If China is a WTO member, F is assigned the value of 1; otherwise it

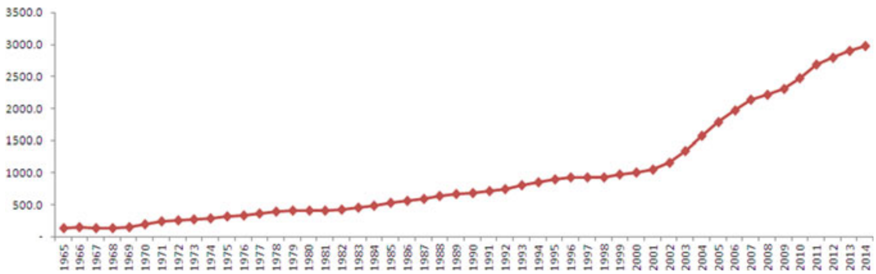

Fig. 10.1 China's primary energy consumption (million barrel barrels of oil equivalent). *Source* BP Statistical Review of World Energy 2015

would get the value of 0. The coefficient of F can be interpreted as the influence on energy consumption of China's WTO accession or not.

And it can also be found that there is a time trend in China's primary energy consumption in Fig. 10.1.

In order to verify the time trend and sudden change mentioned above, the following model is constructed:

$$E = c + b_1 t + b_2 F + u \tag{10.1}$$

where E is China's primary energy consumption;

c is a constant;

t is time;

u is white noise.

F is a dummy variable, and $F = \begin{cases} 0, t \leq 2000 \\ 1, t \geq 2001 \end{cases}$, represents China's accession to WTO which lead to a sudden change of China's primary energy consumption.

b_1, b_2 are the coefficients.

Equation (10.1) is estimated, and the results are shown in Table 10.3.

It is shown that the coefficients of the t and F are statistically significant at 1% significance level in Table 10.3. And the assumptions of time trend and sudden change of China's primary energy consumption is confirmed by this result.

Due to the time trend and sudden change, the times series of China's primary energy consumption will be detrended before the stationary test. And the following equation is obtained according to Eq. (10.1) and Table 10.3:

$$\hat{u} = E + 101.0223 - 33.13844t - 756.1929F \tag{10.2}$$

where $\hat{u}$ is the residual series of Eq. (10.2), and it represents the detrended series of China's primary energy consumption. And the stationarity of $\hat{u}$ is tested by ADF, PP and KPSS methods, and the results are shown in Table 10.4.

Table 10.3 The estimation of Eq. (10.11)[1]

Variable	Coefficient	Std. error	t-Statistic	Prob.
C	−101.0223	91.60366	−1.10282	0.2757
F	756.1929	137.7991	5.487649	0.0000
t	33.13844	4.287450	7.729174	0.0000
R-squared	0.893562	Mean dependent var		955.7420
Adjusted R-squared	0.889033	S.D. dependent var		825.3805
S.E. of regression	274.9489	Akaike info criterion		14.12917
Sum squared resid	3,553,054	Schwarz criterion		14.24389
Log likelihood	−350.2293	Hannan-Quinn criter		14.17286
F-statistic	197.2855	Durbin-Watson stat		0.213071
Prob(F-statistic)	0.000000			

Table 10.4 Stationary tests of the residual series of Eq. (10.2)

		t-Statistic	Prob.
Level	ADF	−0.638553	0.9720
	PP	−0.843320	0.9541
	KPSS	0.133030**	
First difference	ADF	−5.875086*	0.0001
	PP	−5.872409*	0.0001
	KPSS	0.097434	

* Significant at 1% significance level
** Significant at 10% significance level

Although KPSS tests show that $\hat{u}$ is stationary at 10% significance level and the first order difference is non-stationary, the ADF and PP tests indicate that $\hat{u}$ is non-stationary and the first order difference is stationary at 1% significance level. It is more reasonable that the first order difference of $\hat{u}$ is stationary according to these results, namely $\hat{u} \sim I(1)$. In further, the following equation is assumed:

$$D(E) = c + b_1 t + b_2 F + u \tag{10.3}$$

where $D(E)$ is the first order of China's primary energy consumption;

c, t, F, b_1, b_2, u are defined as Eq. (10.1).

Then, Eq. (10.3) is estimated, and the results are shown in Table 10.5.

The autocorrelation and partial correlation of the residuals (u) of Eq. (10.3) are shown in Table 10.6. It is shown that AR(1), AR(2), MA(1) and MA(3) should be induced to Eq. (10.3) by trailing or censored of the autocorrelation and partial correlation. But MA will be irreversible when AR(1), AR(2), MA(1) and MA(3) are included. And MA(3) is excluded. According to the form of ARIMA model in

[1] All the estimations related to ARIMA model are made with Eviews 6.0.

Table 10.5 Estimation of Eq. (10.3)

Variable	Coefficient	Std. error	t-Statistic	Prob.
c	17.80857	11.98227	1.486243	0.1440
F	106.7771	17.31149	6.167992	0.0000
t	0.371429	0.552995	0.671667	0.5052
R-squared	0.716184	Mean dependent var		57.97347
Adjusted R-squared	0.703844	S.D. dependent var		62.61961
S.E. of regression	34.07770	Akaike info criterion		9.954434
Sum squared resid	53,419.33	Schwarz criterion		10.07026
Log likelihood	−240.8836	Hannan-Quinn criter		9.998378
F-statistic	58.03849	Durbin-Watson stat		1.040166
Prob(F-statistic)	0.000000			

Table 10.6 The autocorrelation and partial correlation of the residuals of Eq. (10.3)

Autocorrelation	Partial correlation		AC	PAC	Q-Stat	Prob.
. |*** |	. |*** |	1	0.435	0.435	9.8367	0.002
.*|. |	***|. |	2	−0.143	−0.409	10.918	0.004
***|. |	.*|. |	3	−0.395	−0.192	19.374	0.000
.*|. |	. |*. |	4	−0.180	0.112	21.165	0.000
.*|. |	**|. |	5	−0.076	−0.254	21.494	0.001
. |. |	. |. |	6	0.014	0.036	21.506	0.001
. |. |	. |. |	7	0.050	0.018	21.653	0.003
.*|. |	**|. |	8	−0.085	−0.325	22.092	0.005
**|. |	.*|. |	9	−0.270	−0.181	26.630	0.002
.*|. |	. |. |	10	−0.175	0.039	28.589	0.001
. |*. |	. |. |	11	0.134	0.002	29.769	0.002
. |*. |	.*|. |	12	0.189	−0.194	32.191	0.001

Eq. (10.3), AR(1), AR(2) and MA(1) will be added to the right side of Eq. (10.3). And then an ARIMA(2, 1, 1) is obtained as follows.

$$D(E) = c + b_1 t + b_2 F + a_1 AR(1) + a_2 AR(2) + mMA(1) \tag{10.4}$$

Then Eq. (10.4) is estimated, and the results are shown in Table 10.7. The t tests show that all the coefficients are significant at 1% significance level. And the adjusted R-squared is 0.823305, which also indicates that the model is good.

The related tests are shown in Table 10.8. All the tests except LJB, show that the model of Eq. (10.4) is stable and in line with requirements. LJB test shows that the residuals are not normally distributed. And ARCH test shows no conditional

Table 10.7 Estimation of ARIMA model of China's Primary Energy Consumption

Variable	Coefficient	Std. error	t-Statistic	Prob.
C	14.06834	5.673192	2.479793	0.0173
F	103.6168	8.526556	12.15225	0.0000
T	0.612647	0.264409	2.317042	0.0256
AR(1)	1.281217	0.128431	9.975931	0.0000
AR(2)	−0.695122	0.126146	−5.510462	0.0000
MA(1)	−0.950779	0.040622	−23.40576	0.0000
R-squared	0.842511	Mean dependent var		60.50426
Adjusted R-squared	0.823305	S.D. dependent var		62.64294
S.E. of regression	26.33200	Akaike info criterion		9.498191
Sum squared resid	28,428.34	Schwarz criterion		9.734380
Log likelihood	−217.2075	Hannan-Quinn criter		9.587070
F-statistic	43.86721	Durbin-Watson stat		1.766658
Prob(F-statistic)	0.000000			
Inverted AR roots	0.64 − 0.53i	0.64 + 0.53i		
Inverted MA roots	0.95			

Table 10.8 Tests for ARIMA model of China's Primary Energy Consumption

Tests	Q_{20}	Q^*_{20}	LM2	Heteroskedasticity test: ARCH	Ramsey RESET test	LJB
Statistic	21.822	12.759	0.343401	0.578516	0.595640	35.64274
P value	0.192	0.752	0.7115	0.4510	0.6217	0.00000

heteroskedasticity. So the non-normal distribution of the residuals is probably caused by ignoring the nonlinear characteristics. In a word, the Eq. (10.4) is acceptable.

And the fitted values of ARIMA(2, 1, 1) are shown in Fig. 10.2 by the green polyline. It is shown that it is a good fitting. The MAPE (Mean Absolute Percent Error) is defined to reflect the accuracy of the fittings:

$$\overline{\Delta} = \frac{1}{n}\sum_{t=1}^{n}\frac{\left|\hat{y}_t - y_t\right|}{y_t} \tag{10.5}$$

where $\overline{\Delta}$ is MAPE;

$\hat{y}_t$ is the fitted value at time t;

y_t is the actual value at time t.

And MAPE of ARIMA(2, 1, 1) calculated according to Eq. (10.5) is 4.62%.

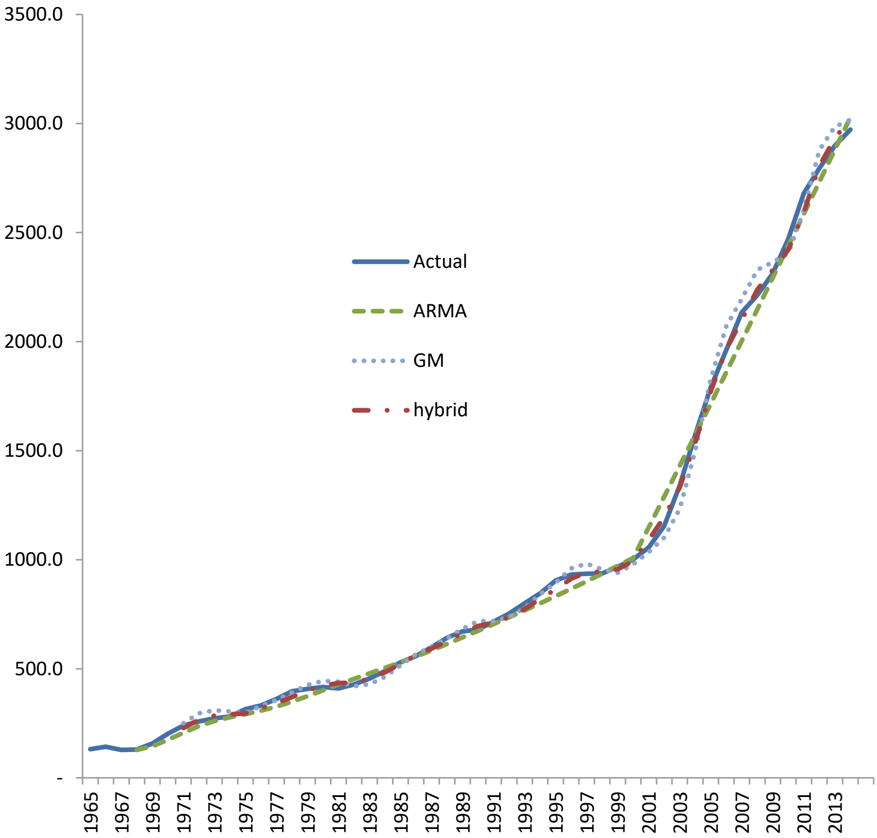

Fig. 10.2 The fittings of the three models

10.2.1.2 Forecasting with GM(1, 1) Model

GM(1, 1) model can predict with four data. And the data of four consecutive years are used to forecast the next one by GM(1, 1) to get the fitted value of the next year. The smooth ratio should be less than 0.5 (Deng, 2002), and the value of 1966 can not pass this test. Thus, the first fitted value of GM(1, 1) model is that of 1971. Then, roll forward until the fitted value of 2014 is obtained. And all the fitted values are shown as the light blue polyline in Fig. 10.2. And the It is shown that it also is a good fitting. And MAPE of GM(1, 1) model calculated according to Eq. (10.5) is 3.75%.

10.2.1.3 Forecasting with GM-ARIMA Model

It can be found that the residual of ARIMA model and GM(1, 1) model have opposite signs in many points in Fig. 10.2. It is very interesting. The fitted values of ARIMA model, influenced by long-term trend, respond less to the fluctuations of the original data; while GM(1, 1) model predicts with the latest four data, and the corresponding fitted values respond sensitively to the fluctuations. In further, this assumption will be tested with Wilcoxon signed rank test, because the residuals of ARIMA model are not normally distributed mentioned in Sect. 10.2.1.1.

A Wilcoxon signed rank test has a following null hypothesis and Alternative Hypothesis:

H0: difference between the pairs follows a symmetric distribution around zero.

H1: difference between the pairs does not follow a symmetric distribution around zero.

While we try to test that the residuals have opposite signs. Equivalently, we can test the relation between the residuals of ARIMA model and the opposite numbers of the residuals of GM(1, 1) model. And they are paired for Wilcoxon signed rank test, and the results are shown in Table 10.9. It accepts the null hypothesis, which indicates that the residuals of the two models are opposite in the statistical sense.

So, the combination of these two models is a good idea to improve the accuracy of prediction. And the following GM-ARIMA hybrid model is defined to forecast China's primary energy consumption:

$$Hybrid = 0.5GM(1, 1) + 0.5ARIMA(2, 1, 1) \tag{10.6}$$

Both the weights of the two univariate models are equal to 0.5, because their residuals are mutually opposite.

The fitted values of the hybrid model of GM-ARIMA are shown in Fig. 10.2 by the red polyline. It is shown that it is a better fitting. And MAPE of the hybrid model calculated according to Eq. (10.5) is 2.30%.

The MAPEs of the three methods are shown in Table 10.10, and a smaller MAPE indicates that hybrid model is better than ARIMA and GM(1, 1) model.

Table 10.9 Wilcoxon signed rank test

	−GM(1, 1) residuals ARIMA residuals
Z	−1.435[a]
Asymp. Sig. (2-tailed)	0.151

[a] Based on negative ranks

Table 10.10 MAPEs of the three methods

	ARIMA	GM(1, 1)	Hybrid
MAPE (%)	4.62	3.75	2.30

10.2.1.4 Results and Conclusions

China's primary energy consumption is predicted by the three models, and the results are shown in Table 10.11. China's economic growth rate drops to about 7%, about two percentage less than the rates in previous years. It will lead to a slower growth of China's primary energy consumption. It is a real change. All the models reflect the newly trend. According to the above results, ARIMA model will react less while GM(1, 1) model will respond more. That is, GM(1, 1) model will give a smaller prediction than the actual ones and the ARIMA model will give a bigger one. And the hybrid model will give a moderate and better one. It is worth noting that, the average annual growth rate of China's primary energy consumption reaches 8% since the new century, which induced the massive investment in the energy sector, while the slowdown of China's primary energy consumption in the next years could result in overcapacity of China's energy industry.

10.2.2 Global Primary Energy Consumption Predictions

The issue of energy consumption is frequently talked about. Primary energy consumption has increased year by year, which led to the constant rise of carbon dioxide emissions, the critical issues of energy security, fluctuations of energy prices, and so on. Global primary energy consumption prediction can provide many important evidences to understand the trends of the above-mentioned issues. The data about global primary energy consumption are collected from *BP Statistical Review of World Energy 2015* and *BP Energy Outlook 2035*. It is found that the logarithmic sequence of the global primary energy consumption, denoted as *LnE*, has a linear growth trend, shown as the green polyline in Fig. 10.3.

And its linear trend is shown as the black polyline with an R squared of 0.985, which indicates that the linear trend is significant. In further, *LnE* is used to predict with GM(1, 1) model because it is a Linear growth sequence (LGS). Similar with the method in Sect. 10.2.1, the data of four consecutive years are used to forecast the next one by GM(1, 1) model. Thus, the first predicted value of GM(1, 1) model is that of 1987. Then, roll forward until the predicted value of 2016 is obtained. And

Table 10.11 The prediction of China's primary energy consumption

	2015	2016	2017	2018	2019	2020	Average annual growth rate (%)
ARIMA	3177.448	3326.995	3477.154	3627.923	3779.304	3931.296	4.77
GM	3069.8579	3165.5343	3264.1926	3365.9256	3470.8294	3579.0026	3.15
Hybrid	3123.653	3246.265	3370.673	3496.924	3625.067	3755.149	3.97

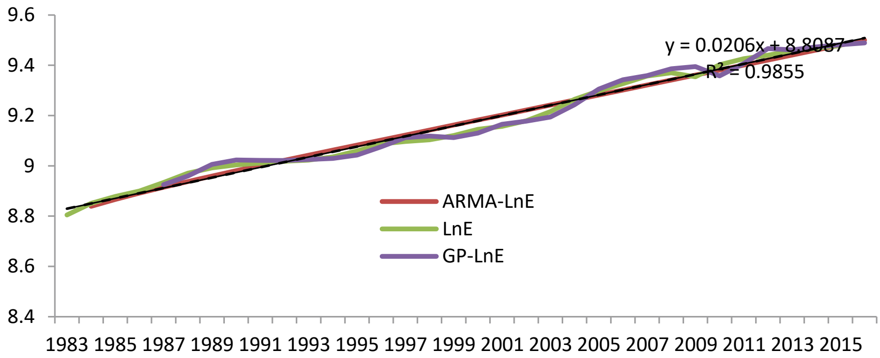

Fig. 10.3 Prediction of global primary energy consumption

the computations of GM(1, 1) modeling are committed by the software supplied by Institute for Grey System Studies, Nanjing University of Aeronautics and Astronautics.[2] All the sequences passed class ratio tests and smooth ratio tests, and have rather small MAPEs. And all the predicted values (*GP-LnE*) are shown as the blue polyline in Fig. 10.3. And it is shown that it is quite good. And the prediction percentage errors of GM(1, 1) model are quite small with an average of 0.15%.

In order to illustrate the effectiveness of GM(1, 1) model for LGS, ARIMA model is also used as the reference. An ARIMA model for *LnE* can be derived according to Tables 10.12, 10.13 and 10.14, as follows:

$$LnE = 8.844460 + 0.019871@trend(1983) + 0.701787AR(1) + 0.466115MA(1) \tag{10.7}$$

And dynamic prediction of ARIMA is shown as the red polyline in Fig. 10.9. It describes the linear trend perfectly with a average of prediction percentage errors of 0.25%, which is larger than that of GM(1, 1) model. It indicates that GM(1, 1) model is better than ARIMA model when *LnE*, a LGS, is predicted. GM(1, 1) can reflect both the linear trend and the fluctuations, while ARMIA mainly reveals the long-term linear trend, as shown in Fig. 10.9.

GM(1, 1) model can also give an convincible prediction. Table 10.15 shows the results by using GM(1, 1) model and AMIRA model. The predicted value of GM(1, 1) indicates a growth of 1.03% while that of ARIMA gives a growth of 2.15%. Taking into account the causal relationship between energy consumption and economic growth, global energy consumption growth will slow down as the global economy is still in trouble. Global primary energy consumption increased by 1.18% in 2015. So the prediction by using GM(1, 1) model is more reasonable and acceptable.

[2] http://igss.nuaa.edu.cn/.

Table 10.12 Estimation of ARIMA model of global primary energy consumption

Variable	Coefficient	Std. error	t-Statistic	Prob.
C	8.844460	0.026664	331.7020	0.0000
@TREND(1983)	0.019871	0.001241	16.00587	0.0000
AR(1)	0.701787	0.142850	4.912770	0.0000
MA(1)	0.466115	0.183564	2.539252	0.0169
R-squared	0.996021	Mean dependent var		9.169463
Adjusted R-squared	0.995594	S.D. dependent var		0.193078
S.E. of regression	0.012816	Akaike info criterion		−5.759830
Sum squared resid	0.004599	Schwarz criterion		−5.576613
Log likelihood	96.15728	Hannan-Quinn criter		−5.699099
F-statistic	2336.097	Durbin-Watson stat		1.944986
Prob(F-statistic)	0.000000			
Inverted AR roots	0.70			
Inverted MA roots	−0.47			

Table 10.13 Correlations of residuals of Eq. (10.7)

Autocorrelation	Partial correlation		AC	PAC	Q-stat	Prob.
. | .|	. | . |	1	0.010	0.010	0.0038	
. |*. |	. |*. |	2	0.114	0.114	0.4727	
. |*. |	. |*. |	3	0.122	0.121	1.0284	0.311
. | . |	.| . |	4	0.055	0.043	1.1454	0.564
. *| . |	. *| . |	5	−0.136	−0.168	1.8948	0.595
. | . |	. | . |	6	0.024	−0.003	1.9190	0.751
. | . |	. | . |	7	−0.038	−0.014	1.9804	0.852
. | . |	. |*. |	8	0.050	0.090	2.0953	0.911
.**| . |	.**| . |	9	−0.248	−0.245	5.0092	0.659
. *| . |	. *| . |	10	−0.143	−0.196	6.0249	0.644
. *| . |	. *| . |	11	−0.150	−0.126	7.1953	0.617
. *| . |	. | . |	12	−0.135	−0.051	8.1849	0.611
.**| . |	.**| . |	13	−0.283	−0.207	12.760	0.309
. *| . |	. *| . |	14	−0.114	−0.191	13.552	0.330
. | . |	. | . |	15	−0.006	−0.001	13.555	0.406
. |*. |	. |*. |	16	0.094	0.171	14.154	0.438

Table 10.14 Tests for ARIMA model of global primary energy consumption

Tests	LM2	Heteroskedasticity test: ARCH	Ramsey RESET test	LJB
Statistic	0.713654	1.776184	1.811206	5.934400
P value	0.4992	0.1930	0.0812	0.051447

Table 10.15 Prediction of global primary energy consumption (2016)

	GM(1, 1)	ARIMA
LnE	9.4891	9.500195
Global primary energy consumption (million tonnes oil equivalent)	13,214.9	13,362.33
Change 2016 over 2015 (%)	1.03	2.15

In this section, GM(1, 1) and ARIMA are compared, and GM(1, 1) is as good as ARIMA at least. And GM(1, 1) model can be used for an Approximate exponential sequence in Fig. 10.2, and also suitable for a linear growth sequence in Fig. 10.3.

10.3 Grey Incidence Analysis of Energy-Economy System

10.3.1 The Relationship Between China's Energy Consumption and Economic Growth

10.3.1.1 The Stages of China's Economic Development

The relation between energy consumption and economic growth in a long period has been investigated by econometric methods. But China's economy has different characters in different periods. Correspondingly, the relation between energy consumption and economic growth may be different in different periods. The history of China's economy can be divided into several periods from the 1980s: (1) Period I: 1980s–1992; (2) Period II: 1993–1996; (3) Period III: 1997–2000; (4) Period IV: 2001–present.

(1) Period I: 1980s–1992

Since 1978, China began to make major reforms to its economy. The China's leadership reformed the economic policies to raise personal income and consumption and to increase productivity by introducing new management systems. In 1981, the government began to dismantle the collectively farmed land, and established the household responsibility system that these fields were contracted out to the private families to cultivate. Resource allocation by state planning was reduced and enterprises were made ultimately responsible for their own profits and losses. The private sector was allowed to compete with state-owned firms in a number of service sectors, and then in the sectors such as construction. The government encouraged foreign trade to increase economic growth and permitted foreign direct investment in several small "special economic zones" along the coast. In the 1980s, the government created the Dual-Track Price System which tried to combine central planning with market-oriented reforms to increase productivity and living standards. During the 1980s, these reforms led to average annual growth rates of 10% in agricultural and industrial output.

(2) Period II: 1993–1996

Deng Xiaoping, China's leader at the time, made a series of political pronouncements to give new impetus to the process of economic reform when he visited the southern China in early 1992. The 14th National Communist Party Congress held later in the year backed up Deng's pronouncements, stating that China's key task in the 1990s was to create a "socialist market economy". It was approved of establishment of more than 2000 special economic zones, and lots of foreign investment in the special economic zones. These accelerated China's economic growth in the period. The government approved the long-term reforms which try to give more space to market-oriented institutions and to strengthen central control over the financial system. State-owned enterprises would continue to dominate many key industries. The growth rate was thus tempered, and the inflation rate dropped.

(3) Period III: 1997–2000

China's economy growth was slowed down by the Asian financial crisis. The Asian financial crisis affected China by decreasing foreign direct investment and a sharp drop in the growth of its exports. But the impact on China's economy of the Asian financial crisis was rather slight. The reasons were that China had huge reserves, a currency that was not freely convertible, and capital inflows that consisted overwhelmingly of long-term investment. However, China had to face slowing growth and rising unemployment.

In the Asian financial crisis, over half of China's state-owned enterprises were reporting losses. The inefficiency of the state-owned enterprises made China's financial system burdened by huge amounts of bad loans. And the reform of state-owned enterprises brought massive layoffs. During the 15th National Communist Party Congress held in September 1997, President Jiang Zemin announced plans to sell, merge, or close the most of state-owned enterprises. In 2000, China claimed that the majority of large state-owned enterprises were profitable.

(4) Period IV: 2001–2007

China already entered into the heavy chemical industry stage. The rapid development of heavy industry and chemical industry accelerates China's economic growth. The growth rate of China's economy was above 10% before the Global Financial Crisis. Based on the development path of the developed countries, China's total energy consumption will increase abruptly.

10.3.1.2 Data Used

Indices of GDP and Composition of GDP are collected as shown in Table 10.16. Real GDP is computed according to the indices of GDP. And values added of the

Table 10.16 Indices of GDP and composition of GDP

Year	Indices of gross domestic product	Composition of gross domestic product			
		Gross domestic product	Primary industry	Secondary industry	Tertiary industry
1985	192.9	100.0	28.4	42.9	28.7
1986	210.0	100.0	27.2	43.7	29.1
1987	234.3	100.0	26.8	43.6	29.6
1988	260.7	100.0	25.7	43.8	30.5
1989	271.3	100.0	25.1	42.8	32.1
1990	281.7	100.0	27.1	41.3	31.6
1991	307.6	100.0	24.5	41.8	33.7
1992	351.4	100.0	21.8	43.4	34.8
1993	400.4	100.0	19.7	46.6	33.7
1994	452.8	100.0	19.8	46.6	33.6
1995	502.3	100.0	19.9	47.2	32.9
1996	552.6	100.0	19.7	47.5	32.8
1997	603.9	100.0	18.3	47.5	34.2
1998	651.2	100.0	17.6	46.2	36.2
1999	700.9	100.0	16.5	45.8	37.7
2000	759.9	100.0	15.1	45.9	39.0
2001	823.0	100.0	14.4	45.1	40.5
2002	897.8	100.0	13.7	44.8	41.5
2003	987.8	100.0	12.8	46.0	41.2
2004	1087.4	100.0	13.4	46.2	40.4
2005	1200.8	100.0	12.2	47.7	40.1
2006	1340.7	100.0	11.3	48.7	40.0
2007	1500.7	100.0	11.3	48.6	40.1

Source China Statistical Yearbook 2008

industry = Real GDP × the proportion of the industry. Real GDP and Values added of each industry are shown in Table 10.17.

This section will study the relationship between economic development and energy consumption. Gross Domestic Product (GDP) and values added of primary industry, secondary industry and tertiary industry are used to describe Chinese economic development. And total energy consumption and consumption of Coal, Crude Oil, Natural Gas and Hydro-power, Nuclear Power and Wind Power are used to represent Chinese energy consumption.

Total energy consumption and its composition are collected, as shown in Table 10.18. Consumption of the energy product = total energy consumption × the percentage of total energy consumption of the energy product. Consumption of Coal,

Table 10.17 Real GDP and values added of each industry

Year	GDP	Primary industry	Secondary industry	Tertiary industry
1985	32,074.58	9122.85	13,755.44	9196.287
1986	34,917.89	9480.232	15,267.43	10,173.72
1987	38,958.39	10,445.15	16,966.62	11,546.62
1988	43,348.07	11,138.6	18,981.97	13,227.5
1989	45,110.59	11,325.02	19,321.37	14,464.2
1990	46,839.86	12,701.18	19,363.9	14,779.46
1991	51,146.4	12,544.33	21,373.4	17,228.67
1992	58,429.27	12,731.68	25,384.39	20,307.36
1993	66,576.78	13,121.24	31,003.72	22,451.82
1994	75,289.62	14,878.17	35,061.84	25,274.32
1995	83,520.27	16,589.03	39,400.71	27,447.02
1996	91,883.94	18,092.88	43,678.54	30,112.53
1997	100,413.9	18,362.8	47,735.78	34,315.32
1998	108,278.7	19,009.36	50,038.02	39,231.32
1999	116,542.6	19,194.85	53,326.46	43,904.74
2000	126,352.9	19,032.58	58,016.86	49,303.46
2001	136,844.9	19,694.34	61,652.35	55,361.37
2002	149,282.3	20,515.49	66,863.24	61,903.57
2003	164,247.1	21,019.28	75,502.69	67,725.13
2004	180,808.2	24,215.9	83,579.21	73,013.09
2005	199,663.8	24,358.98	95,239.63	80,065.18
2006	222,925.8	25,190.62	108,564.9	89,170.32
2007	249,529.9	28,196.88	121,271.5	100,061.5

Crude Oil, Natural Gas and Hydro-power, Nuclear Power and Wind Power are shown in Table 10.19.

10.3.1.3 Results and Discussion

Grey incidence analysis can be used to study the issues with small samples. In each period of China's economy's history, only several data can be obtained. Grey incidence analysis is applied to examine the relation between GDP and total energy consumption at aggregated levels and at disaggregated levels. The relations can be studied in the four periods, so that the dynamic of the relation can be revealed.

(1) The Relation between GDP and Total Energy Consumption

The fundamental idea of the grey incidence analysis is that the closeness of a relation is judged based on the similarity level of the geometrical patterns of sequence curves.

Table 10.18 Total consumption of energy and its composition

Year	Total energy consumption (10,000 tce)	As percentage of total energy consumption			
		Coal	Crude oil	Natural gas	Hydro-power, nuclear power, wind power
1985	76,682	75.8	17.1	2.2	4.9
1986	80,850	75.8	17.2	2.3	4.7
1987	86,632	76.2	17	2.1	4.7
1988	92,997	76.2	17	2.1	4.7
1989	96,934	76	17.1	2	4.9
1990	98,703	76.2	16.6	2.1	5.1
1991	103,783	76.1	17.1	2.0	4.8
1992	109,170	75.7	17.5	1.9	4.9
1993	115,993	74.7	18.2	1.9	5.2
1994	122,737	75.0	17.4	1.9	5.7
1995	131,176	74.6	17.5	1.8	6.1
1996	138,948	74.7	18.0	1.8	5.5
1997	137,798	71.7	20.4	1.7	6.2
1998	132,214	69.6	21.5	2.2	6.7
1999	133,831	69.1	22.6	2.1	6.2
2000	138,553	67.8	23.2	2.4	6.7
2001	143,199	66.7	22.9	2.6	7.9
2002	151,797	66.3	23.4	2.6	7.7
2003	174,990	68.4	22.2	2.6	6.8
2004	203,227	68.0	22.3	2.6	7.1
2005	224,682	69.1	21.0	2.8	7.1
2006	246,270	69.4	20.4	3.0	7.2
2007	265,583	69.5	19.7	3.5	7.3

Source China Statistical Yearbook 2008

The curves of the GDP and total energy consumption are shown as Fig. 10.4. From the geometrical patterns of the curves, it can be found that the relations between total energy consumption and GDP in different periods are not the same.

The absolute degree of grey incidences, relative degree of grey incidences and synthetic degree of grey incidences of China's total energy consumption and GDP in the four periods are computed respectively. The results are shown in Table 10.20.

During the period I, the absolute degree of grey incidences between China's total energy consumption and GDP is 0.8492 which shows that total energy consumption and GDP are related to each other in the volumes; The relative degree of grey incidences between China's total energy consumption and GDP is 0.8311, which shows the growth rates of total energy consumption and GDP are related to each other; the synthetic degree of grey incidences between total energy consumption and GDP is

Table 10.19 Consumption of coal, crude oil, natural gas and hydro-power, nuclear power, wind power

Year	Coal	Crude oil	Natural gas	Hydro-power, nuclear power, wind power
1985	58,125	13,113	1687	3757
1986	61,284	13,906	1860	3800
1987	66,014	14,727	1819	4072
1988	70,864	15,809	1953	4371
1989	73,670	16,576	1939	4750
1990	75,212	16,385	2073	5034
1991	78,979	17,747	2076	4982
1992	82,642	19,105	2074	5349
1993	86,647	21,111	2204	6032
1994	92,053	21,356	2332	6996
1995	97,857	22,956	2361	8002
1996	103,794	25,011	2501	7642
1997	98,801	28,111	2343	8543
1998	92,021	28,426	2909	8858
1999	92,464	30,206	2864	8298
2000	93,869	32,158	3256	9269
2001	95,485	32,750	3652	11,313
2002	100,672	35,536	3886	11,704
2003	119,658	38,865	4515	11,952
2004	138,174	45,381	5284	14,388
2005	155,255	47,183	6291	15,952
2006	170,911	50,239	7462	17,731
2007	184,580	52,320	9295	19,388

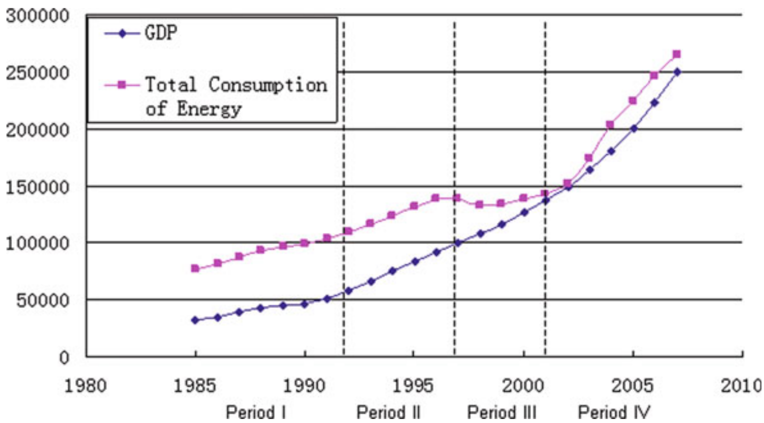

Fig. 10.4 GDP and total energy consumption

Table 10.20 The degree of grey incidences of GDP and total energy consumption

Period	Absolute degree of grey incidences	Relative degree of grey incidences	Synthetic degree of grey incidences
Period I	0.8492	0.8311	0.84015
Period II	0.9360	0.8664	0.9012
Period III	0.6241	0.8263	0.7252
Period IV	0.9175	0.9477	0.9326

0.840125 which shows total energy consumption and GDP overall related to each other.

During the period II, the absolute degree of grey incidences between China's total energy consumption and GDP is 0.9360 which shows that total energy consumption and GDP are related to each other in the volumes; The relative degree of grey incidences between China's total energy consumption and GDP is 0.8664, which shows that the growth rates of total energy consumption and GDP are related to each other; the synthetic degree of grey incidences between total energy consumption and GDP is 0.9012 which shows that total energy consumption and GDP are overall related to each other.

During the period III, the absolute degree of grey incidences between China's total energy consumption and GDP is 0.6241 which shows low correlation between total energy consumption and GDP in the volumes; The relative degree of grey incidences between China's total energy consumption and GDP is 0.8263, which shows that the growth rates of total energy consumption and GDP are related to each other; the synthetic degree of grey incidences between total energy consumption and GDP is 0.7252 which shows low overall correlation between total energy consumption and GDP.

During the period IV, the absolute degree of grey incidences between China's total energy consumption and GDP is 0.9175 which shows that total energy consumption and GDP are related to each other in the volumes; The relative degree of grey incidences between China's total energy consumption and GDP is 0.9477, which shows that the growth rates of total energy consumption and GDP are related to each other; the synthetic degree of grey incidences between total energy consumption and GDP is 0.9326 which show that total energy consumption and GDP are overall related to each other closely.

In the period I and period II, the reform of economic system increased the motivation of China's people and improved the productivity. The extensive growth mode led to a rapid economic growth with a sharp increase of energy consumption. So in these periods, total energy consumption and GDP are related.

In period III, because of the impact of the Asian financial crisis, many factories bankrupted. The development of China's industry was slowed down, and less energy was consumed. So in the period, total energy consumption and GDP are less relevant.

In period IV, the heavy industry and chemical industry, such as steel, automobile, Oil refinery and so on, increased very rapidly. These industries accelerate economic

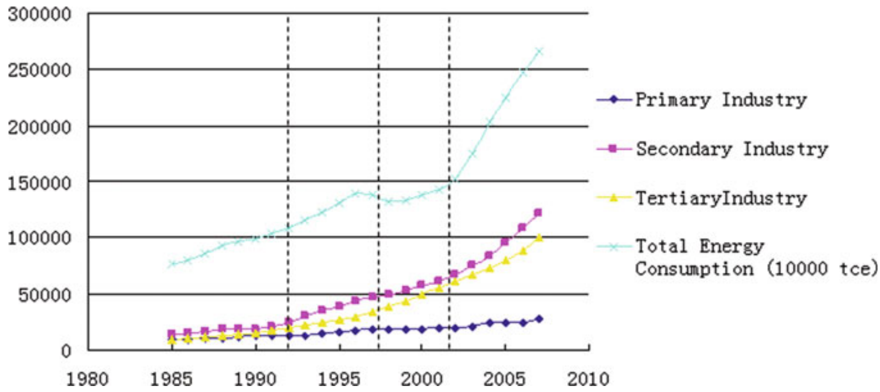

Fig. 10.5 Values added and total energy consumption

growth, meanwhile consume much more energy. So in this period, total energy consumption and GDP are closely related.

(2) The Relation between Values Added and Total Energy Consumption

The curves of total energy consumption and values added of primary industry, secondary industry and tertiary industry are shown in Fig. 10.5.

The absolute degree of grey incidences, relative degree of grey incidences and synthetic degree of grey incidences between China's total energy consumption and values added of primary industry in the four periods are computed respectively. The results are shown in Table 10.21.

During the period I, the absolute degree of grey incidences between China's total energy consumption and values added of primary industry is 0.5633 which shows low correlation between total energy consumption and values added of primary industry in the volumes; The relative degree of grey incidences between China's total energy consumption and values added of primary industry is 0.9739, which shows that the growth rates of total energy consumption and values added of primary industry are related to each other; the synthetic degree of grey incidences between total energy consumption and values added of primary industry is 0.7686 which shows low overall correlation between total energy consumption and values added of primary industry.

Table 10.21 The degree of grey incidences between values added of primary industry and total energy consumption

Period	Absolute degree of grey incidences	Relative degree of grey incidences	Synthetic degree of grey incidences
Period I	0.5633	0.9739	0.7686
Period II	0.6154	0.8657	0.74055
Period III	0.5989	0.9631	0.781
Period IV	0.5304	0.7702	0.6503

During the period II, the absolute degree of grey incidences between China's total energy consumption and values added of primary industry is 0.6154 which shows low correlation between total energy consumption and values added of primary industry in the volumes; The relative degree of grey incidences between China's total energy consumption and values added of primary industry is 0.8657, which shows the growth rates of total energy consumption and values added of primary industry are related to each other; the synthetic degree of grey incidences between total energy consumption and values added of primary industry is 0.74055 which shows low overall correlation between total energy consumption and values added of primary industry.

During the period III, the absolute degree of grey incidences between China's total energy consumption and values added of primary industry is 0.5989 which shows low correlation between total energy consumption and values added of primary industry in the volumes; The relative degree of grey incidences between China's total energy consumption and values added of primary industry is 0.9631, which shows the growth rates of total energy consumption and values added of primary industry are related to each other; the synthetic degree of grey incidences between total energy consumption and values added of primary industry is 0.781 which shows low overall correlation between total energy consumption and values added of primary industry.

During the period IV, the absolute degree of grey incidences between China's total energy consumption and values added of primary industry is 0.5304 which shows low correlation between total energy consumption and values added of primary industry in the volumes; The relative degree of grey incidences between China's total energy consumption and values added of primary industry is 0.7702, which shows low correlation between the growth rates of total energy consumption and values added of primary industry; the synthetic degree of grey incidences between total energy consumption and values added of primary industry is 0.6503 which shows low overall correlation between total energy consumption and values added of primary industry.

Because the primary industry consumes little energy, total energy consumption and values added of primary industry are low relational. Especially in the period IV industry consumed more and more energy so that the degrees of grey incidences are much smaller.

The absolute degree of grey incidences, relative degree of grey incidences and synthetic degree of grey incidences between China's total energy consumption and values added of secondary industry in the four periods are computed respectively. The results are shown in Table 10.22.

During the period I, the absolute degree of grey incidences between China's total energy consumption and values added of secondary industry is 0.6489 which shows low correlation between total energy consumption and values added of secondary industry in the volumes; The relative degree of grey incidences between China's total energy consumption and values added of secondary industry is 0.8328, which shows that the growth rates of total energy consumption and values added of secondary industry are related to each other; the synthetic degree of grey incidences between total energy consumption and values added of secondary industry is 0.74085 which shows low overall correlation between total energy consumption and values added of secondary industry.

Table 10.22 The degree of grey incidences between values added of secondary industry and total energy consumption

Period	Absolute degree of grey incidences	Relative degree of grey incidences	Synthetic degree of grey incidences
Period I	0.6489	0.8328	0.74085
Period II	0.7813	0.8575	0.8194
Period III	0.8519	0.8622	0.85705
Period IV	0.7185	0.9949	0.8567

During the period II, the absolute degree of grey incidences between China's total energy consumption and values added of secondary industry is 0.7813 which shows low correlation between total energy consumption and values added of secondary industry in the volumes; The relative degree of grey incidences between China's total energy consumption and values added of secondary industry is 0.8575, which shows that the growth rates of total energy consumption and values added of secondary industry are related to each other; the synthetic degree of grey incidences between total energy consumption and values added of secondary industry is 0.8194 which shows that total energy consumption and values added of secondary industry are overall related to each other.

During the period III, the absolute degree of grey incidences between China's total energy consumption and values added of secondary industry is 0.8519 which shows that China's total energy consumption and values added of secondary industry are related to each other in the volumes; The relative degree of grey incidences between China's total energy consumption and values added of secondary industry is 0.8622, which shows that the growth rates of total energy consumption and values added of secondary industry are related to each other; the synthetic degree of grey incidences between total energy consumption and values added of secondary industry is 0.85705 which shows that total energy consumption and values added of secondary industry are overall related to each other.

During the period IV, the absolute degree of grey incidences between China's total energy consumption and values added of secondary industry is 0.7185 which shows low correlation between total energy consumption and values added of secondary industry in the volumes; The relative degree of grey incidences between China's total energy consumption and values added of secondary industry is 0.9949, which shows that the growth rates of total energy consumption and values added of secondary industry are related to each other; the synthetic degree of grey incidences between total energy consumption and values added of secondary industry is 0.8567 which shows that total energy consumption and values added of secondary industry are overall related to each other.

In the period I, China was an agriculture country, So China's energy consumption and values added of secondary industry is not closely related. After that, Chinese energy consumption and values added of secondary industry is related. The relative degree of grey incidences increased continuously from period I to period IV. It

Table 10.23 The degree of grey incidences between values added of tertiary industry and total energy consumption

Period	Absolute degree of grey incidences	Relative degree of grey incidences	Synthetic degree of grey incidences
Period I	0.6370	0.7522	0.6946
Period II	0.6744	0.8873	0.78085
Period III	0.7085	0.7478	0.72815
Period IV	0.6696	0.9503	0.80995

indicates that the development of China's secondary industry increasingly relies on energy consumption.

The absolute degree of grey incidences, relative degree of grey incidences and synthetic degree of grey incidences between China's total energy consumption and values added of tertiary industry in the four periods are computed respectively. The results are shown in Table 10.23.

During the period I, the absolute degree of grey incidences between China's total energy consumption and values added of tertiary industry is 0.6370 which shows low correlation between total energy consumption and values added of tertiary industry in the volumes; The relative degree of grey incidences between China's total energy consumption and values added of tertiary industry is 0.7522, which shows low correlation between the growth rates of total energy consumption and values added of tertiary industry; the synthetic degree of grey incidences between total energy consumption and values added of tertiary industry is 0.6946 which shows low overall correlation between total energy consumption and values added of tertiary industry.

During the period II, the absolute degree of grey incidences between China's total energy consumption and values added of tertiary industry is 0.6744 which shows low correlation between total energy consumption and values added of tertiary industry in the volumes; The relative degree of grey incidences between China's total energy consumption and values added of tertiary industry is 0.8873, which shows that the growth rates of total energy consumption and values added of tertiary industry are related to each other; the synthetic degree of grey incidences between total energy consumption and values added of tertiary industry is 0.78085 which shows low overall correlation between total energy consumption and values added of tertiary industry.

During the period III, the absolute degree of grey incidences between China's total energy consumption and values added of tertiary industry is 0.7085 which shows low correlation between total energy consumption and values added of tertiary industry in the volumes; The relative degree of grey incidences between China's total energy consumption and values added of tertiary industry is 0.7478, which shows the low correlation between the growth rates of total energy consumption and values added of tertiary industry; the synthetic degree of grey incidences between total energy consumption and values added of tertiary industry is 0.72815 which shows low overall correlation between total energy consumption and values added of tertiary industry.

During the period IV, the absolute degree of grey incidences between China's total energy consumption and values added of tertiary industry is 0.6696 which shows low correlation between total energy consumption and values added of tertiary industry in the volumes; The relative degree of grey incidences between China's total energy consumption and values added of tertiary industry is 0.9503, which shows that the growth rates of total energy consumption and values added of tertiary industry are related to each other; the synthetic degree of grey incidences between total energy consumption and values added of tertiary industry is 0.80995 which shows that total energy consumption and values added of tertiary industry are overall related to each other.

The share of China's tertiary industry in the country's GDP is small. So the absolute degree of grey incidences between total energy consumption and values added of tertiary industry is small. It is found that the relative degrees of grey incidences are larger when the tertiary developed faster in period II and period IV, So the development of tertiary industry relies on energy consumption too.

(3) The Relation between GDP and Consumption of Energy Product

The curves of GDP and consumption of coal, crude oil, natural gas and hydro-power, nuclear power and wind power are shown in Fig. 10.6.

The absolute degree of grey incidences, relative degree of grey incidences and synthetic degree of grey incidences between China's consumption of coal and GDP in the four periods are computed respectively. The results are shown in Table 10.24.

During the period I, the absolute degree of grey incidences between China's consumption of coal and GDP is 0.9526 which shows that consumption of coal and GDP are related to each other in the volumes; The relative degree of grey incidences between China's consumption of coal and GDP is 0.8377, which shows that the growth rates of consumption of coal and GDP are related to each other; the synthetic

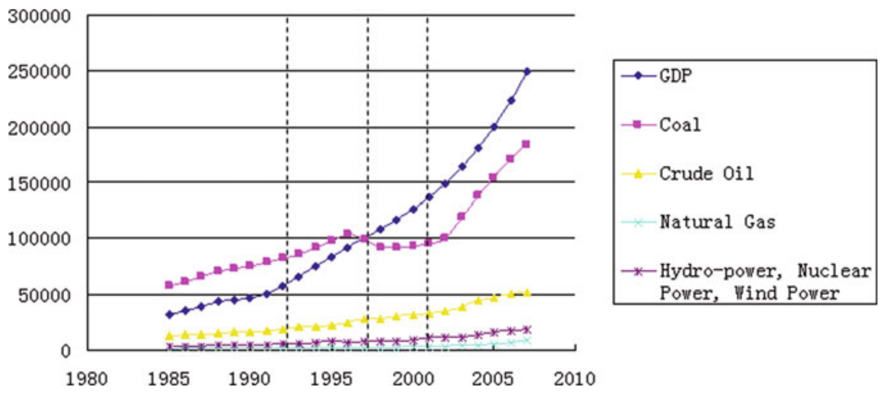

Fig. 10.6 GDP and consumption of energy product

Table 10.24 The degree of grey incidences between GDP and consumption of coal

Period	Absolute degree of grey incidences	Relative degree of grey incidences	Synthetic degree of grey incidences
Period I	0.9526	0.8377	0.89515
Period II	0.8288	0.8692	0.849
Period III	0.7108	0.8764	0.7936
Period IV	0.9356	0.9163	0.92595

degree of grey incidences between consumption of coal and GDP is 0.89515 which shows that consumption of coal and GDP overall related to each other.

During the period II, the absolute degree of grey incidences between China's consumption of coal and GDP is 0.8288 which shows that consumption of coal and GDP are related to each other in the volumes; The relative degree of grey incidences between China's consumption of coal and GDP is 0.8692, which shows that the growth rates of consumption of coal and GDP are related to each other; the synthetic degree of grey incidences between consumption of coal and GDP is 0.849 which shows that consumption of coal and GDP overall related to each other.

During the period III, the absolute degree of grey incidences between China's consumption of coal and GDP is 0.7108 which shows low correlation between consumption of coal and GDP in the volumes; The relative degree of grey incidences between China's total energy consumption and GDP is 0.8764, which shows that the growth rates of consumption of coal and GDP are related to each other; the synthetic degree of grey incidences between consumption of coal and GDP is 0.7963 which shows low overall correlation between consumption of coal and GDP.

During the period IV, the absolute degree of grey incidences between China's consumption of coal and GDP is 0.9356 which shows that consumption of coal and GDP are related to each other in the volumes; The relative degree of grey incidences between China's consumption of coal and GDP is 0.9163, which shows that the growth rates of consumption of coal and GDP are related to each other; the synthetic degree of grey incidences between consumption of coal and GDP is 0.92595 which shows that consumption of coal and GDP are overall related to each other.

In period III, because of the impact of the Asian financial crisis, the degree of grey incidences between GDP and consumption of coal is small. In the other three periods, consumption of coal and GDP are related to each other.

The absolute degrees of grey incidences, relative degrees of grey incidences and synthetic degrees of grey incidences between China's GDP and consumption of Crude Oil, Natural Gas, and Hydro-power, Nuclear Power, Wind Power in the four periods are computed respectively. The results are shown in Table 10.25.

The absolute degrees of grey incidences are small because the shares of these energy products are much less than coal. The relative degrees of grey incidences are large, because the consumption of these energy products increased sharply with economic growth.

Table 10.25 The degree of grey incidences between GDP and consumption of crude oil, natural gas, and hydro-power, nuclear power, wind power

Consumption of energy product	Period	Absolute degree of grey incidences	Relative degree of grey incidences	Synthetic degree of grey incidences
Consumption of crude oil	Period I	0.6201	0.8278	0.72395
	Period II	0.5527	0.8224	0.68755
	Period III	0.5600	0.8736	0.7168
	Period IV	0.6094	0.9674	0.7884
Consumption of natural gas	Period I	0.5111	0.7591	0.6351
	Period II	0.5057	0.8248	0.66525
	Period III	0.5209	0.8766	0.69875
	Period IV	0.5208	0.8448	0.6828
Consumption of hydro-power, nuclear power, wind power	Period I	0.5325	0.8129	0.6727
	Period II	0.5488	0.9756	0.7622
	Period III	0.5059	0.8161	0.661
	Period IV	0.5332	0.9205	0.72685

In the four periods, the synthetic degrees of grey incidences between GDP and consumption of coal are larger than those between GDP and consumption of Crude Oil, Natural Gas, and Hydro-power, Nuclear Power, Wind Power, because Coal supplied the vast majority (about 70%) of China's total energy consumption requirements.

Some researchers found the unidirectional granger causality running from economic growth to energy consumption and thought that energy conservation will not affect economic growth. However, it is found that the relations between China's energy consumption and economic growth in different periods are not the same from the grey incidence analysis. The relation between energy consumption and economic growth in the future would be not the same as those in the past. It is very important to the policy-makers. The effect on economic growth of energy conservation can't be simply judged by the relation between energy consumption and economic growth in the past. Some energy-saving policies maybe will slow down economic growth.

The degrees of grey incidences between values added of secondary industry and total energy consumption are much larger. Nowadays, secondary industry consumes about 50% of the total energy consumption. Therefore, it is very important to improve the energy efficiency of secondary industry for China/s energy conservation.

The degrees of grey incidences between GDP and consumption of coal are much larger. The shares of coal in total energy consumption are very large from 1980s. Many researchers suggested that China should optimize the structure of energy consumption and consume less and less coal. But in fact, coal is the energy that China can obtain rather easily and cheaply. The degrees of grey incidences have revealed the importance of coal on Chinese economic growth. So the efficiency of coal mining and using should be focused on rather than the energy consumption structure in the short term.

But large amount of coal is burned which leads to increasingly serious environment pollution. And the fossil fuels including coal would be used up in the next century. So China must develop new energy sources such as Hydro-power, Nuclear Power and Wind Power. And China has abundant resources to develop these new powers.

China has abundant hydro electricity resources nearly 1/6 of the world. Among the reserves and the exploitable hydro electricity resources, china ranks first in the world. And with the completion of Three Gorges Hydro electricity station at Yangtze River, the total installed capacity of hydro-power of China will rank first in the world. But it is a long way for China to make full use of hydro electricity resources. Nuclear power is an import energy source in the developed countries. USA is the world's largest producer of nuclear power, accounting for more than 30% of worldwide nuclear generation of electricity. The country's 104 nuclear reactors produced 809 billion kWh in 2008, almost 20% of total electrical output. And France derives over 75% of its electricity from nuclear energy. While the share of nuclear power in total power generation capacity is currently less than 2% in China. With its large land mass and long coastline, China has exceptional wind resources. China became the third largest wind energy provider worldwide (behind USA and Germany).

The consumption of Hydro-power, Nuclear Power and Wind Power increased rapidly with the economic growth. But the development of the Hydro-power, Nuclear Power and Wind Power in China is just getting starting compared with the developed countries. So China should accelerate the development of Hydro-power, Nuclear Power and Wind Power to generate enough electricity instead to establish lots of thermal power plants.

10.3.2 The Relation Between China's Gasoline Prices and Crude Oil Price

10.3.2.1 An Improved Grey Incidence Model

Different from the US, UK and other countries, China's refined oil prices are not entirely determined by the market and still regulated by government, although China's refined oil pricing mechanism has been reformed for several times. The government's regulation will destroy the random or stochastic distribution rules of the data which were strictly required by time series analysis. For example, even with the latest China's refined oil pricing mechanism, the anchored oil prices are required to meet rising or declining 50 RMB per ton in 10 sequential workdays before an adjustment of China's gasoline prices. In most cases, when international oil prices change, China's gasoline prices remain unchanged. In such cases, it may be hard to reveal the true relationship between the China's gasoline prices and international crude oil prices with time series analysis. And it is proved by the results of the unit root tests. Year 2003–2013 is divided into five stages according to the reform of China's refined oil pricing mechanism, as shown in Table 10.26. In all the periods, the variables, including international oil prices and China's gasoline prices, will be tested for stationarity by ADF tests and PP tests. The results of these tests are shown in Table 10.26, in levels and after one differentiation (prefixed by Δ). It is shown that international crude oil prices and China's gasoline prices are co-integrated in the 2nd and 4th period, while not co-integrated in the 1st, 3rd, and 5th period.

And China's refined oil pricing mechanism is not transparent, because it only sets the price adjustment conditions, but how to adjust is not specified. This means that China's refined oil pricing has strong grey characteristics. The grey incidence models based on similarity and nearness provide a workable solution for studying the relationship between China's gasoline prices and international crude oil prices, which include the similitude degree of grey relation model and the close degree of grey relation model. The similitude degree of grey relation model is used to measure the similar degree of geometry between sequence X_i and sequence X_j; the close

Table 10.26 ADF and PP tests for stationary of the variables

Period	Variables	Methods			
		ADF		PP	
		t-Statistic	Prob.	t-Statistic	Prob.
1st: 2003.1.1–2006.3.24	COP_1	−3.656305[b]	0.0260	−3.640797[b]	0.0271
	GP_1	−2.813129	0.1929	−2.823836	0.1891
	$\Delta\ GP_1$	−27.00305[a]	0.0000	−26.98581[a]	0.0000
2nd: 2004.3.27–2008.12.31	COP_2	−0.505261	0.8847	−0.601373	0.8675
	$\Delta\ COP_2$	−28.88488[a]	0.0000	−28.96651[a]	0.0000
	GP_2	−1.953969	0.6247	−1.970206	0.6159
	$\Delta\ GP_2$	−25.35015[a]	0.0000	−25.35015[a]	0.0000
3rd: 2009.1.5–2009.5.6	COP_3	−3.732607[b]	0.0259	−3.668917[b]	0.0306
	GP_3	−2.221985	0.4707	−2.221985	0.4707
	$\Delta\ GP_3$	−8.701980[a]	0.0000	−8.701974[a]	0.0000
4th: 2009.5.7–2013.3.26	COP_4	−2.364017	0.1524	−2.331471	0.1623
	$\Delta\ COP_4$	−31.19567[a]	0.0000	−31.19601[a]	0.0000
	GP_4	−2.952027	0.1466	−2.954123	0.1460
	$\Delta\ GP_4$	−30.44744[a]	0.0000	−30.44789[a]	0.0000
5th: 2013.3.27–2013.9.12	COP_5	−3.245202[c]	0.0812	−3.266613[c]	0.0774
	GP_5	−1.185465	0.6789	−1.227230	0.6670
	$\Delta\ GP_5$	−10.35285[a]	0.0000	−10.35285[a]	0.0000

Note COP means Crude Oil Price; *GP* means Gasoline Price; one differentiation is prefixed by Δ
[a] Denotes significance at the 1% level
[b] Denotes significance at the 5% level
[c] Denotes significance at the 10% level

degree of grey relation model is used to measure the close degree in space between sequence X_i and sequence X_j (Liu et al., 2011). And the grey incidence models based on similarity and nearness can reflect the relation of fluctuation ranges between sequences, not measure time response of the sequence (time series), because of the basis on shapes of the sequences.

The close degree of grey relation model of X_i and X_j, is defined as below:

$$\rho_{ij} = \frac{1}{1 + |S_i - S_j|} \tag{10.8}$$

And the similitude degree of grey relation model of X_i and X_j, is defined as below:

$$\varepsilon_{ij} = \frac{1}{1 + |s_i - s_j|} \tag{10.9}$$

where S_i, $S_j s_i$, s_j, $s_i - s_j$, $S_i - S_j$ were defined respectively (Liu et al., 2011).

However Eqs. (10.8) and (10.9) ignore the real background of the study issue, which will lead to the failure of reflecting the relation between two variables changes in some cases. For example, when it comes to height comparison, it is a very common phenomenon that adult heights vary 5 cm; but for three year old children, 5 cm difference in height is a very big gap. So, the real background of study issue should be reflected in the equations. And the models should be improved further as follows (Yuan et al., 2014).

Definition 10.1 Assume sequence X_i and sequence X_j have the same length, then call

$$\varepsilon_{ij} = 1 - \frac{2|S_i - S_j|}{|S_i| + \left|S_j\right|} \tag{10.10}$$

the proximity of X_i and X_j.

Definition 10.2 Assume sequence X_i and sequence X_j have the same length, then call

$$\varepsilon'_{ij} = 1 - \frac{2|s_i - s_j|}{|s_i| + \left|s_j\right|} \tag{10.11}$$

the similarity of X_i and X_j.

In Eqs. (10.10) and (10.11), $|S_i - S_j|$ and $|s_i - s_j|$ represent the difference between system behavior sequences; while $|S_i| + \left|S_j\right|$, $|s_i| + \left|s_j\right|$ represent the real background of study issue. Correspondingly, the proximity of X_i and X_j reflects the closeness of the two sequences, and the similarity of X_i and X_j reflects the similarity of the two sequences.

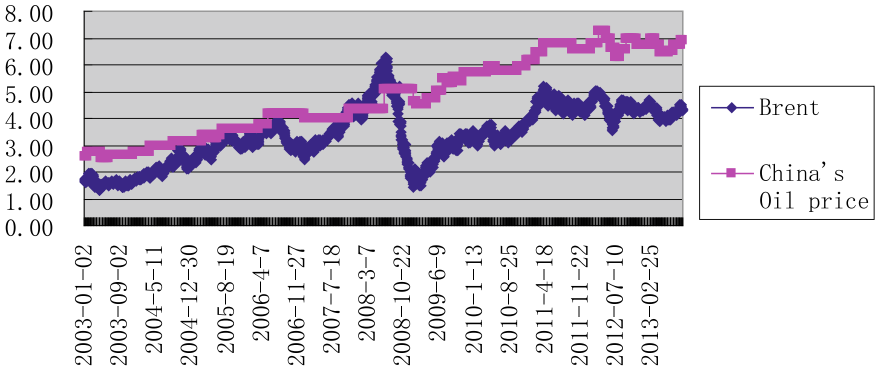

Fig. 10.7 China's gasoline prices and Brent crude oil prices from 2003 to 2013

10.3.2.2 Data

China's gasoline retail prices are not available because of lack of authorized investigation. So the ex-factory price of No. 93 gasoline is used because No. 93 gasoline is most consumed in China. According to the National Development and Reform Commission's reined oil price adjustment notice, related materials from the oil companies and some news reports, the China's gasoline prices sequences are obtained. Overall, there are cumulative 48 adjustments from January 1, 2003 to September 12, 2013. in recent years, the adjustment frequency is relatively high.

Before calculation, the Brent crude oil prices are converted into those of RMB per liter. And then, China's gasoline prices have same meaning, unit and magnitude with Brent crude oil prices. The sequences of China's gasoline price and Brent crude oil price can be used to calculate similarity and proximity. Excluding holidays without price or exchange rate data, a total of 2617 valid data is obtained, as shown in Fig. 10.7. Figure 10.7 shows that the relations between the China's gasoline prices and Brent crude oil prices are not the same in the different periods.

10.3.2.3 Results and Discussion

Taking calculation method similar to that Liu et al. (2011) proposed, S_i, S_j, $S_i - S_j$, s_i, s_j, and $s_i - s_j$ are computed, then similarity and proximity of China's gasoline prices and Brent crude oil prices in the five stages are calculated according to Eqs. (10.10) and (10.11), as shown in Table 10.2.

In the first two stages, the Brent crude oil prices and China' a gasoline prices both rose, and $s_i - s_j < 0$, indicating that the increase of China's gasoline prices is less than that of Brent crude oil prices. The similarity in the first stage is 0.685038, indicating that the increase of China's gasoline prices is rather less than that of Brent crude oil prices in the first stage. The similarity in the second stage is 0.981229, almost equal to 1. Although China's gasoline prices grew stably in this period, with the Brent crude oil prices shocking, their overall changes are very close.

In the third stage, there is a declining trend in Brent crude oil prices, and $s_i - s_j > 0$, indicating that the decline of China's gasoline prices is less than that Brent crude oil prices. In fact, the Brent crude oil prices declined in this stage while the China's gasoline prices even rose slightly, which leads to a small similarity of them, 0.228211.

In the last two stages, Brent crude oil prices have shown a rising trend, and $s_i - s_j > 0$ indicating that the increase of China's gasoline prices is more than that of Brent crude oil prices. In the fifth stage, the similarity is 0.653118, a little bit larger than 0.553436 in the fourth stage, which means that in the fifth stage China's gasoline prices changed more consistently with Brent crude oil than the fourth stage.

If similarity degree is less than 1, it indicates that the changes of China's gasoline prices and Brent crude oil prices are inconsistent, which will certainly change the proximity. In the first two stages, the Brent crude oil prices rose, while the similar degree is less than 1 and $s_i - s_j < 0$, which narrows down the gap between them and intensifies the proximity. But after 2009, the similarity actually shows that when international oil prices declined, there was only a small decline in China's gasoline oil prices, and when international oil prices rose, there was a larger rising in China's gasoline prices, which caused a significant decline in the proximity of China's gasoline prices and Brent crude oil prices.

From these results, some important findings are obtained as follows:

First, the Brent crude oil prices change according to the international market situation, while China's gasoline prices change under the regulation of the government. So the changes between them are inconsistent, which causes the different similarity and proximity of Brent crude oil prices and China's gasoline prices in the different periods.

Second, the relation between China's gasoline prices and Brent crude oil prices are influenced by China's refined oil pricing mechanism. Before 2009, China's gasoline prices raised less than the Brent crude oil prices; But after 2009, this situation was reversed. As a result, even excluding the influence of refined oil consumption tax, the proximity between the China's gasoline prices and the Brent crude oil prices has been greatly reduced. The notion that domestic gasoline prices react quickly and more to international crude oil price increases while slowly and less to international crude oil price reductions is largely accepted among consumers, although China's refined oil pricing mechanism implies that china's gasoline prices are highly related to international crude oil prices. Chinese consumers' perceptions to the China's refined oil prices indeed reflect the inconsistency between China's gasoline prices changes and international crude oil prices changes.

Third, the similarity and proximity of Brent crude oil prices and China's gasoline prices vary, indicating that refining enterprises have to bear not only the price fluctuation risk of international crude oil but also the policy risk.

Table 10.27 The proximity and similarity of China's gasoline price and Brent crude oil price in different periods

Period	Similarity	Sign of $S_i - S_j$	Proximity	
2003.1.1–2006.3.24	0.685038	−	0.722711	
2004.3.27–2008.12.31	0.981229	−	0.864281	
2009.1.5–2009.5.6	0.228211	+	0.189618	0.357428[a]
2009.5.7–2013.3.26	0.553436	+	0.547787	0.679337[a]
2013.3.27–2013.9.12	0.653118	+	0.537712	0.662196[a]

[a] The proximity excluding the influence of gasoline consumption tax

Forth, the government can set gasoline prices so well that changes of China's gasoline prices and those of international crude oil prices are very consistent sometimes, for example, the similarity in the second stage is 0.981229. But more often, the government can not do it well (Table 10.27).

10.4 Grey Programming Models for a Multi-factors Energy-Environment-Economy System

It is self-evident that there is a relationship between economic growth and CO_2 emissions. However, some fundamental problems remain open in this field.

Firstly, the relation between GDP and CO_2 emissions has been widely investigated, but what is the impact of economic growth rate on the CO_2 emission intensity?

Secondly, what can be significantly affected by the reduction of CO_2 emissions during the economic development except the economic growth, such as unemployment rate, water resource consumption and fixed capital usage?

Thirdly, the methods of time series are used to test the long term historical relationship between CO_2 emission intensity and economic growth. Here, a programming model is proposed, and the parameters are predicted, which is a future perspective on the corresponding relation.

10.4.1 The Grey Programming Model

Economic growth is accompanied by the adjustment of industrial structure, which means that the proportion of primary industry, secondary industry and tertiary industry will change. Such a change will have an impact on CO_2 emissions due to their different CO_2 emission intensities of the three industries. In this section, the CO_2 emission intensities of the three industries will be calculated, and the relationship between the secondary industry and tertiary industry will be analyzed. And then, a liner programming model is built to analyze the relation between China's economic development and CO_2 emission.

10.4.1.1 The CO_2 Emission Intensities of the Three Industries

The CO_2 emission intensities of the three industries can be calculated according to the following equation:

$$CE_i = \sum EN_{ij\times} c_j \tag{10.12}$$

where

CE_i represents the CO_2 emission intensity of industry *i*;

EN_{ij} refers to the *j*th type of energy consumed in industry *i*;

c_j indicates the energy emission factor of the *j*th type of energy.

Emission factors of energy are determined according to IPCC (Intergovernmental Panel on Climate Change) default CO_2 emission factors (http://acm.eionet.europa.eu/docs/ETCACC_TechnPaper_2003_10_CO2_EF_fuels.pdf). As a secondary energy, Electricity's emission factor is converted based on the proportion of electricity-generating resources and the data collected by Wu (2010) from State Electricity Regulatory Commission (SERC) (Wu, 2010). As for the heat consumption, it is assumed that all the heat is generated by the combustion of raw coal; and its emission factor is considered to be equal to that of raw coal. According to the description of *Energy Statistics Report* published by National Bureau of Statistics, "other petroleum products" mainly refers to the oil, grease, naphtha, paraffin, petroleum asphalt that of non-fuel uses, and "other coking products" mainly refers to the coking products such as benzene. Since these two are not primarily used for fuel, they are not included in the energy consumption. And they are not included when the amount of CO_2 emissions is estimated. The amount of CO_2 emissions from cement production is relatively small and relatively stable; therefore it is not included in this paper in that the change of CO_2 emission is the major focus here. The CO_2 emission factors of different types of energies are showed in Table 10.28.

The final energy consumption of the three industries in 2012 is shown in Table 10.29. The emissions of the three industries are calculated with Eq. (10.12). Then the emission intensities of the three industries are calculated with added values of the three industries, as shown in Table 10.30.

10.4.1.2 The Relationship Between the Secondary Industry and Tertiary Industry

China's secondary industry and tertiary industry are highly related, as shown in Fig. 10.8, which must be taken into consideration during modeling. In the process of economic development, the secondary industry provides the tertiary industry the

Table 10.28 The CO_2 emission factors of different energies

Energy types	CO_2 emission factor tCO_2/tce	Energy types	CO_2 emission factor tCO_2/tce
Raw coal	2.7725	Diesel fuel	2.1707
Cleaned coal	2.7725	Fuel oil	2.2674
Coke	3.1379	LPG	1.8483
COG	1.3003	Refinery gas	1.6871
Other gas	1.3003	Natural gas	1.6442
Crude	2.1492	Heat	2.7725
Gasoline	2.0525	Electricity[a] (2006)	0.7756 kg/kwh
Kerosene	2.1062	Electricity[b] (2012)	0.72098 kg/kwh

[a] Data of year 2005 is applied
[b] The average of the predicted values in 2010 and 2015

Table 10.29 Energy consumption of three industries in 2012 (10,000 tec)

	Primary industry	Secondary industry	Tertiary industry
Coal	1297.91	49,723.44	3830.63
Coke	55.84	38,092.9	8.44
Gas		5190.39	33.14
Crude		793.57301	7965.5619
Gasoline	283.77	1276.9305	2780.8013
Kerosene	1.75	58.757652	18,069.133
Diesel fuel	1945.95	3206.1031	
Fuel oil	2.81	940.44	2018.01
LPG	10.97	805.78	357.01
Refinery gas		2019.08	
Natural gas	8.37	8646.9	2775.75
Heat	3.87	8323.1	541.75
Electricity (TWh)	1012.57	33,944.45	5690.50

Data Source China Energy Statistical Yearbook 2013

necessary foundation and conditions, and at the same time the tertiary industry provides the secondary industry services and support. To further analyze the interrelation between Chinese secondary industry and tertiary industry, a linear regression equation is established with the added value of secondary industry as the independent variable and the added value of tertiary industry as the dependent variable, namely:

$$Y_3 = \alpha + \beta Y_2 \tag{10.13}$$

where

Table 10.30 The added values and CO_2 emissions of China's three industries

	Added value[a] (100 million Yuan)	Industrial structure (%)	CO_2 emission (10,000 tons)	Emission intensity (t CO^2/10,000 Yuan)
Primary industry	30,472	10.1	16,488.5	0.541103
Secondary industry[b]	184,444.8	45.3	583,143	3.161613
Tertiary industry[c]	155,672.4	44.6	127,011.3	0.815888

[a] The added values of industries in 2012 are converted according to the GDP index, calculated with the constant prices in 2010

[b] Primary industry refers to agriculture, forestry, animal husbandry, fishery and water conservancy; secondary industry refers to the industry and construction

[c] Tertiary industry refers to the transportation, storage and postal services, wholesale and retail trade and accommodation, catering industry, other industries

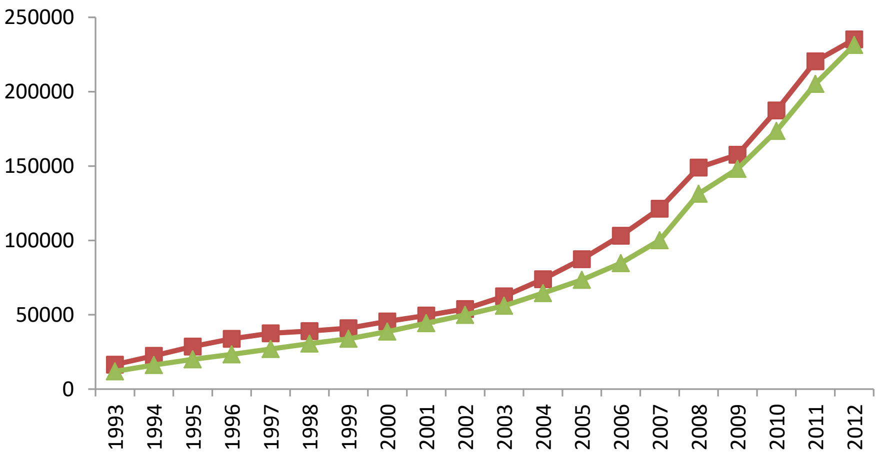

Fig. 10.8 The added value of China's secondary industry and tertiary industry (100 million Yuan)

Y_3 refers to the added value of the tertiary industry.

Y_2 is the added value of the secondary industry.

Data in Fig. 10.8 cover the duration 1993–2012. Using SPSS to solve the regression model, it is obtained:

$$Y_3 = -7468.987 + 0.970Y_2 \tag{10.14}$$

Regression parameters and corresponding tests are shown in Tables 10.31 and 10.32. Adjusted R^2 is 0.995 and the significance of F test is 0.000, which indicates a

Table 10.31 R test

R	R square	Adjusted R square	Change statistics	
			F change	Sig. F change
0.997	0.995	0.995	4813.41	0.000

Table 10.32 Regression coefficients and T test

Model		Unstandardized coefficients		t	Sig.
		B	Std. error		
1	(Constant)	−7468.987	1796.457	−4.158	0.001
	Y2	0.970	0.016	59.567	0.000

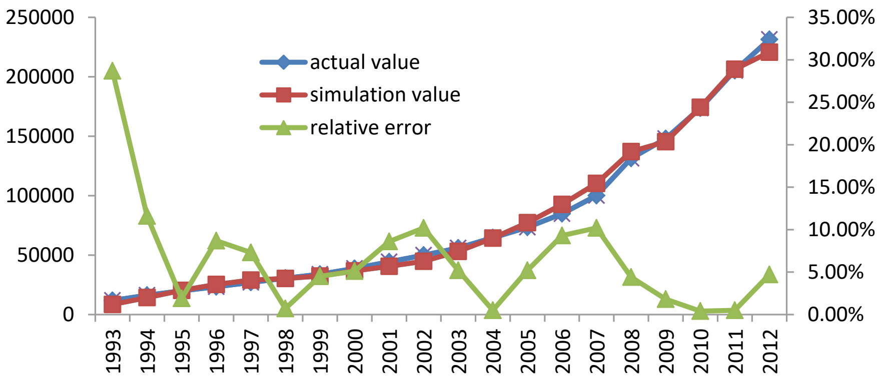

Fig. 10.9 The relative error of regression model

significant regression equation. T test also shows that the relation between Y_2 and Y_3 are statistically significant. The results of the regression equation strongly prove that China's secondary industry and tertiary industry are highly related, which should be included in the economic development modeling.

Through the analysis of residual error and relative error of regression Eq. (10.14) shown in Fig. 10.9, it can be found that for the data during 1993–2012, except the first two years, the following relationship between the secondary industry and the tertiary industry generally exists as follows:

$$0.9Y_3 < -7468.987 + 0.970Y_2 < 1.1Y_3 \tag{10.15}$$

Two constraint conditions can be obtained from Eq. (10.15), shown as follows:

$$-7468.987 + 0.970Y_2 < 1.1Y_3 \tag{10.16}$$

$$0.9Y_3 < -7468.987 + 0.970Y_2 \tag{10.17}$$

While the development of primary industry is seriously constrained by arable land, and its long-term growth rate is much smaller than the second industry and tertiary industries. The share of GDP of the primary industry has continued to decline. Therefore, in this study the added value of primary industry will not be directly related to those of secondary industry and tertiary industry.

10.4.1.3 The Linear Programming Model

Economic development needs inputs of capital, labor, water resource. Therefore, when establishing the programming model, these factors should be taken into account. And the following model can be built:

$$Min\ 0.5411033Y_1 + 3.1616126Y_2 + 0.815888\,Y_3$$

$$\begin{cases} a_{11}Y_1 + a_{12}Y_2 + a_{13}Y_3 \le b_1 \\ a_{21}Y_1 + a_{22}Y_2 + a_{23}Y_3 \le b_2 \\ a_{31}Y_1 + a_{32}Y_2 + a_{33}Y_3 \le b_3 \\ Y_1 + Y_2 + Y_3 \ge Y_A \\ -7468.987 + 0.970Y_2 < 1.1Y_3 \\ 0.9Y_3 < -7468.987 + 0.970Y_2 \\ Y_1 > 0 \\ Y_2 > 0 \\ Y_3 > 0 \end{cases} \tag{10.18}$$

where, the following words are used to expain the variabls in Eq. (10.18)

The objective function is built as minimizing CO_2 emissions, and the coefficients are the CO_2 emission factors obtained in Sect. 10.4.1.1 and shown in the last column of Table 10.30. Both Eqs. (10.16) and (10.17) are included in the programming model.

Y_1, Y_2 and Y_3 are the added values of the primary industry, secondary industry and tertiary industry respectively;

a_{11}, a_{12} and a_{13} indicate the input coefficients of fixed assets of the primary industry, secondary industry and tertiary industry respectively, namely the fixed assets needed per added value;

a_{21}, a_{22} and a_{23} represent the labor input coefficients of the primary industry, secondary industry and tertiary industry respectively, namely the labor needed per added value;

a_{31}, a_{32} and a_{33} mean the water consumption coefficient of the primary industry, secondary industry and tertiary industry respectively, namely the water consumption per added value;

b_1 denotes the total fixed assets;

b_2 refers to the total number of labor;

b_3 represents the total water consumption;

Y_A is for the economic development goals.

The various inputs or consumption coefficients ($a_{ij}, i, j = 1, 2, 3$) and total resources (b_1, b_2 and b_3) of the programming model of Eq. (10.18) should be forecasted to analyze the future situation.

10.4.2 China's Relation of Economic Development and CO_2 Emission

10.4.2.1 Estimation of Fixed Assets Stock

Although the capital stock is one of the important macroeconomic variables in the study of economic growth, in China's current national economic accounting system, there is no relevant data of the capital stock. In this situation, the perpetual inventory method is used to estimate China's capital stock (Xu et al., 2007). There is a need to investigate the feasible method to estimate China's capital stock. Here we propose a new way to estimate the depreciation rate σ, which is more objective.

$$FA_{t-1} = DFA_t/\sigma \tag{10.19}$$

where FA_{t-1}: the fixed assets stock in year t − 1;

DFA_t: the amount of depreciation of fixed assets in year t;

σ: the depreciation rate of the fixed assets.

As for the fixed assets, new fixed assets will be formed in addition to depreciation. According to this relationship, the following equation can be obtained:

$$FA_t = FA_{t-1} \times (1 - \sigma) + IV_t \tag{10.20}$$

where,

FA_t, FA_{t-1} and σ are the same as Eq. (10.19).

IV_t refers to the newly increased fixed assets. From Eqs. (10.19) and (10.20), we have

$$DFA_t - DFA_{t-1} = (DFA_t + IV_t)\sigma \tag{10.21}$$

The whole country's depreciation of fixed assets is obtained by gathering the depreciation of fixed assets in each province published by *China Statistical Yearbook*. Meanwhile, newly increased fixed assets during 1995–2012 were gathered, relevant data as shown in Table 10.33. σ is estimated by using linear regression with the intercept equal to 0 according to Eq. (10.21). Results of model test is shown in Table 10.34, R^2 means that it's significant.

Table 10.33 The depreciation of fixed assets and newly increased fixed assets in China (100 million Yuan)

Year	Depreciation of fixed assets	Newly added fixed assets
1997	10,486.41	25,698
1998	11,981.24	28,181
1999	13,209.04	29,476
2000	14,972.42	32,624
2001	16,779.28	37,213.5
2002	18,426.45	43,499.9
2003	21,551.46	51,303.9
2004	25,536.725	62,351.4
2005	29,521.99	77,304.8
2006	33,641.84	90,150.8
2007	39,018.85[a]	105,435.9
2008	44,194.245	126,209.5
2009	49,369.64	156,679.8
2010	56,227.58	182,340.4
2011	67,344.51	215,682.0
2012	74,132.87	241,756.8

Data Source China Statistical Yearbook 1996–2013

[a] Due to data missing, data in is the average of data in years before and after

Table 10.34 σ estimation model

Model	R	R square[b]	Adjusted R square	Standard error
1	0.973[a]	0.946	0.942	1196.25101

[a] Predictive variable: VAR00002

[b] For regression through the origin (model without intercept), R square can measure the variability ratio among the dependent variables near the origin (the regression explained). For the model which contains the intercept cannot be compared with R square

The coefficient of this model is the estimated value of σ and T test is significant, as shown in Table 10.35. The results show that the depreciation rate of China's fixed asset is about 3%.

Xu et al. (2007) estimated the fixed asset stock of the three industries in China's provinces. Because of data limitations, the estimated results didn't cover the data after 2002 and the capital stock was calculated in constant prices in 1978 (Xu et al., 2007). By sorting out the results, the proportion of fixed asset stock of the three industries in 2002 can be obtained as 0.04259: 0.454471: 0.502939. According to the depreciation of fixed assets in 2003 in Table 10.33, the total fixed assets of 2002 can be estimated based on Eq. (10.20) and divided according to the three industries with the above proportion.

Table 10.35 σ estimation coefficient[a,b]

Model		Unstandardized coefficients		Standardized coefficients	t	Sig.
		B	Std. error			
1	VAR00002	0.030	0.002	0.973	15.650	0.000

[a] Dependent variable: VAR00001
[b] The linear regression through the origin

Table 10.36 Estimation of fixed assets of China's three industries (100 million Yuan)

Year	Newly increased fixed assets[a]			Fixed assets		
	Primary industry	Secondary industry	Tertiary industry	Primary industry	Secondary industry	Tertiary industry
2002				30,595.89	326,483.8	361,302.3
2003	375.0	10,402.47	17,886.29	30,053.06	327,091.7	368,349.5
2004	458.9	13,895.91	20,376.54	29,610.41	331,174.9	377,675.6
2005	608.8	19,096.45	25,501.38	29,330.92	340,336.1	391,846.7
2006	807.0	25,280.86	30,203.09	29,257.94	355,406.9	410,294.4
2007	1094.8	31,828.73	34,443.94	29,475.03	376,573.4	432,429.5
2008	1694.1	39,147.97	43,703.15	30,284.92	404,424.2	463,159.8
2009	2529.3	54,063.45	57,351.17	31,905.64	446,354.9	506,616.1
2010	2865.0	64,398.51	69,706.79	33,813.43	497,362.7	561,124.4
2011	5381.6	90,133.72	88,838.59	38,180.62	572,575.6	633,129.3
2012	6731.5	108,904.2	106,764.1	43,766.67	664,302.5	720,899.5

Data Source China Statistical Yearbook 2013
[a] Peasant households are excluded

The newly increased fixed assets of the three industries published by *China Statistical Yearbook* are used to estimate the fixed assets of the three industries in Table 10.33. On the basis of Eq. (10.21) and set σ as 3%, the fixed assets of the three industries can be estimated, as shown in Table 10.36.

10.4.2.2 The Prediction of Consumption or Input Coefficient and Total Resources

In this study, GDP at constant price (the price in 2010) of each year is calculated with GDP index and GDP of 2010. China's GDP index is shown in Table 10.37. GDP 2010 is shown in Table 10.38.

$$Y_t = \frac{Y_{2010}}{I_{2010}} \times I_t \tag{10.22}$$

Table 10.37 GDP index (year 1978 = 100)

Year	GDP	Primary industry	Secondary industry	Tertiary industry
2006	1363.8	352.8	2050.0	1797.3
2007	1557.0	366.0	2358.8	2084.6
2008	1707.0	385.6	2591.8	2301.4
2009	1864.3	401.8	2849.4	2521.5
2010	2059.0	418.9	3198.4	2767.5
2011	2250.5	436.8	3527.4	3028.0
2012	2422.7	456.6	3806.6	3272.0

Data Source China Statistical Yearbook 2013

Table 10.38 GDP 2010 (100 million Yuan)

GDP	Primary industry	Secondary industry	Tertiary industry
401,512.8	40,533.6	187,383.2	173,596.0

Data Source China Statistical Yearbook 2013

where Y_t: the GDP or added value of year t;

Y_{2010}: the GDP or added value of year 2010;

I_{2010}: the GDP index or added value index of year 2010;

I_t: the GDP index or added value index of year t.

The calculated GDP constant prices in 2010 are shown in Table 10.39. The total employment and employment in China's three industries are shown in Table 10.40. Total water consumption and water consumption in China's three industries are shown in Table 10.41.

Based on the total resource consumption, resource consumption or input coefficient of the three industries can be calculated as follows:

$$a_{ij} = b_{ij}/Y_i \quad (i, j = 1, 2, 3) \tag{10.23}$$

Table 10.39 GDP (100 million 2010 Yuan)

Year	GDP	Primary industry	Secondary industry	Tertiary industry
2006	265,946.2	34,137.6	120,102.4	112,738.6
2007	303,620.9	35,414.9	138,193.9	130,760.0
2008	332,871.5	37,311.4	151,844.6	144,359.1
2009	363,545.6	38,879.0	166,936.5	158,165.2
2010	401,512.8	40,533.6	187,383.2	173,596.0
2011	438,856.0	42,265.6	206,658.2	189,936.3
2012	472,435.7	44,181.5	223,015.5	205,241.6

Table 10.40 Employment in China's three industries (10,000 people)

Year	Primary industry	Secondary industry	Tertiary industry	Employment
2006	31,941	18,894	24,143	74,978
2007	30,731	20,186	24,404	75,321
2008	29,923	20,553	25,087	75,564
2009	28,890	21,080	25,857	75,828
2010	27,931	21,842	26,332	76,105
2011	26,594	22,544	27,282	76,420
2012	25,773	23,241	27,690	76,704

Data Source China Statistical Yearbook 2013

Table 10.41 Water consumption in China's three industries (100 million cubic meters)

Year	Total water consumption	Primary industry	Secondary industry	Tertiary industry
2006	5795.0	3664.4	1343.8	693.8
2007	5818.7	3599.5	1403.0	710.4
2008	5910.0	3663.5	1397.1	729.3
2009	5965.2	3723.1	1390.9	748.2
2010	6022.0	3689.1	1447.3	765.8
2011	6107.2	3743.6	1461.8	789.9
2012	6141.8	3880.3	1423.9	728.8

Data Source China Statistical Yearbook 2013

where a_{ij} represents the consumption or input coefficient;

b_j indicates the resource input or consumed such as labor, water resource and fixed assets in industry i;

Y_i denotes the added value of industry i.

Labor input coefficient is shown in Table 10.42, the consumption coefficient of water resources is shown in Table 10.43 and the input coefficient of fixed capital is shown in Table 10.44.

Table 10.42 Labor consumption coefficient (10,000 people/100 million Yuan)

Year	Primary industry	Secondary industry	Tertiary industry
2006	0.935655	0.157316	0.21415
2007	0.867742	0.14607	0.186632
2008	0.80198	0.135355	0.173782
2009	0.743075	0.126276	0.163481
2010	0.689083	0.116563	0.151686
2011	0.629211	0.109088	0.143638
2012	0.583344	0.104212	0.134914

Table 10.43 The consumption coefficient of water resources (100 million cubic meters/100 million billion Yuan)

Year	Primary industry	Secondary industry	Tertiary industry
2006	0.107342	0.011189	0.006154
2007	0.101638	0.010152	0.005433
2008	0.098187	0.009201	0.005052
2009	0.095761	0.008332	0.00473
2010	0.091013	0.007724	0.004411
2011	0.088573	0.007074	0.004159
2012	0.087826	0.006385	0.003551

Table 10.44 The input coefficient of fixed capital (100 million Yuan/100 million Yuan)

Year	Primary industry	Secondary industry	Tertiary industry
2006	0.857059	2.959199	3.639343
2007	0.832278	2.724964	3.307047
2008	0.81168	2.663409	3.208387
2009	0.820639	2.673801	3.203082
2010	0.834207	2.654254	3.232358
2011	0.90335	2.770641	3.333377
2012	0.990611	2.978728	3.512443

From the beginning of the "Eleventh Five-Year", China begun to transform its growth mode, and these consumption coefficients or input coefficients have changed significantly, so a prediction using historical data before the "Eleventh Five-Year" cannot reflect this new trends. Here, we adopt GM(1, 1) model to establish meaningful prediction model using small recent data only. GM(1, 1) model is the primary method to predict with small sample in grey system theory, which is applied to predict the total amount and consumption or input coefficients of different resources. MAPEs are very low, and it is sufficient to be used for prediction. Results of the predicted total resources are shown in Table 10.45 and the consumption or input coefficients are shown in Table 10.46.

Table 10.45 The prediction of total amount of resources

Year	Simulation relative error (%)	Predicted value	
		2010	2015
Total employment	0.0231	77,538.3274	78,974.4346
Total water supply	0.1870	6359.1210	6711.2538
Fixed assets	1.9278	1,951,283.0084	3,399,963.6378

Table 10.46 Consumption coefficient or input coefficient prediction

			Primary industry	Secondary industry	Tertiary industry
Labor consumption coefficient	Relative error (%)		0.2794	0.6978	0.3884
	Predicted value	2015	0.4603	0.0831	0.1104
		2020	0.3095	0.0586	0.0797
Water resources consumption coefficient	Relative error (%)		0.7584	0.5698	0.2986
	Predicted value	2015	0.0788	0.0054	0.0034
		2020	0.0674	0.0034	0.0024
Fixed assets input coefficient		Year	Primary industry	Secondary industry	Tertiary industry
	Relative error (%)		3.3890	2.3715	2.2113
	Predicted value	2015	1.0578	3.0065	3.5364
		2020	1.2720	3.2562	3.7674

China's economy is transiting from labor-intensive to capital-intensive and knowledge-intensive. In perspective of the changes of consumption or input coefficients, it is shown that labor consumption coefficient decreases while fixed assets input coefficient increases, which matches the characteristics of China's economic development stage. And the changes of the coefficients also reflect the influence of technical progress.

10.4.3 Results and Discussions

According to Eq. (10.18), the programming model in 2015, shown as Eq. (10.24), and the programming model in 2020, shown as Eq. (10.25), can be obtained, whose parameters are obtained in Sect. 10.4.2.

$$Min\ 0.5411033Y^1_{2015} + 3.1616126Y^2_{2015} + 0.815888\, Y^3_{2015}$$

$$\begin{cases} 0.4603Y^1_{2015} + 0.0831Y^2_{2015} + 0.1104Y^3_{2015} \le 77538.3274 \\ 0.0788Y^1_{2015} + 0.0054Y^2_{2015} + 0.0034Y^3_{2015} \le 6359.1210 \\ 1.0578Y^1_{2015} + 3.0065\, Y^2_{2015} + 3.5364Y^3_{2015} \le 1951283.0084 \\ Y^1_{2015} + Y^2_{2015} + Y^3_{2015} \ge Y^D_{2015} \\ -7468.987 + 0970Y^2_{2015} < 1.1Y^3_{2015} \\ 0.9Y^3_{2015} < -7468.987 + 0970Y^2_{2015} \\ Y^1_{2015} > 0 \\ Y^2_{2015} > 0 \\ Y^3_{2015} > 0 \end{cases} \tag{10.24}$$

and

$$Min\ 0.5411033Y_{2012}^1 + 3.1616126Y_{2012}^2 + 0.815888\,Y_{2012}^3$$

$$\begin{cases} 0.3095Y_{2020}^1 + 0.0586Y_{2015}^2 + 0.0797Y_{2015}^3 \le 78974.4346 \\ 0.0674Y_{2020}^1 + 0.0034Y_{2020}^2 + 0.0024Y_{2020}^3 \le 6711.2538 \\ 1.2720Y_{2020}^1 + 3.2562Y_{2020}^2 + 3.7674Y_{2020}^3 \le 3399963.6378 \\ Y_{2020}^1 + Y_{2020}^2 + Y_{2020}^3 \ge Y_{2020}^D \\ -7468.987 + 0970Y_{2020}^2 < 1.1Y_{2020}^3 \\ 0.9Y_{2020}^3 < -7468.987 + 0970Y_{2020}^2 \\ Y_{2020}^1 > 0 \\ Y_{2020}^2 > 0 \\ Y_{2020}^3 > 0 \end{cases} \tag{10.25}$$

where $Y_t^j\,(j = 1, 2, 3;\ t = 2010, 2015)$ represents the added value of the industry j of year t.

The solutions to Eqs. (10.24) and (10.25) of these two programming models show the added value and resource input or consumption of the three industries in 2015 and 2020. According to the results, situations like CO_2 emissions and CO_2 emissions intensity can be calculated. In the analysis of the results, the surplus of labor is called unemployment, and the unemployment accounts for the proportion of the total labor is unemployment rate; Similarly, the surplus of fixed assets is called overcapability, and the overcapability accounts for the proportion of the total fixed assets is defined as overcapability rate.

Analysing the results shown in Table 10.47, we can draw the following conclusions:

(1) The faster the economy grows, the lower the unemployment rate is. In cases when the economic growth rate is 6.5%, the unemployment rate in 2015 is 4.27 and 10.28% in 2020. In cases when the economic growth rate is 7.5%, the unemployment rate in 2015 is 2.68 and 7.76% in 2020. In cases when the economic growth rate is 8.5%, the unemployment rate in 2015 is 1.06 and 0.93% in 2020.

(2) The faster the economy grows, the higher utilization proportion the fixed assets have. In cases when the economic growth rate is 6.5%, the overcapacity in 2015, which means some fixed assets will not be in operation, is 10.05 and 23.65% in 2020. In cases when the economic growth rate is 7.5%, the overcapacity in 2015 is 7.21 and 17.28% in 2020. In cases when the economic growth rate is 8.5%, the overcapacity in 2015 is 4.23 and 10.28% in 2020. Whatever the economic growth rate is, China is facing overcapacity situation according to the current investment.

(3) The remaining of water resources is 0%, which means that water resources have become an important constraint on China's development.

Table 10.47 Solutions to programming models

Economic growth rate	Index		2015	2020
6.5%	Added value[a] (100 million Yuan)	Primary industry	51,875.91	69,041.12
		Secondary industry	253,685.2	347,071.2
		Tertiary industry	265,117.4	365,766.7
	Industrial structure (%)	Primary industry (%)	9.1	8.8
		Secondary industry (%)	44.5	44.4
		Tertiary industry (%)	46.5	46.8
	CO_2 emission intensity[b]	t CO_2/10,000 Yuan	−0.0620	−0.0643
		Compared with 2012 (%)	−3.27	−3.39
	Fixed assets	Overcapacity (%)	10.05	23.65
	Water resources	Water resources remain (%)	0.00	0.00
	Employment	Unemployment rate (%)	4.27	10.28
7.5%	Added value[a] (100 million Yuan)	Primary industry	50,924.9	66,330.39
		Secondary industry	261,952.6	377,589.5
		Tertiary industry	274,027.9	398,658.7
	Industrial structure (%)	Primary industry (%)	8.7	7.9
		Secondary industry (%)	44.6	44.8
		Tertiary industry (%)	46.7	47.3
	CO_2 emission intensity[b]	t CO_2/10,000 Yuan	0.0050	−0.0524
		Compared with 2012 (%)	0.27	−2.86
	Fixed assets	Overcapacity (%)	7.21	17.18
	Water resources	Water resources remain (%)	0.00	0.00

(continued)

Table 10.47 (continued)

Economic growth rate	Index		2015	2020
	Employment	Unemployment rate (%)	2.68	5.76
8.5%	Added value[a] (100 million Yuan)	Primary industry	49,956.03	63,437.29
		Secondary industry	270,375.3	410,161.1
		Tertiary industry	283,105.6	433,763.7
	Industrial structure (%)	Primary industry (%)	8.3	7.0
		Secondary industry (%)	44.8	45.2
		Tertiary industry (%)	46.9	47.8
	CO_2 emission intensity[b]	t CO_2/10,000 Yuan	0.0108	0.0237
		Compared with 2012 (%)	0.59	1.29
	Fixed assets	Overcapacity (%)	4.32	10.28
	Water resources	Water resources remain (%)	0.00	0.00
	Employment	Unemployment rate (%)	1.06	0.93

[a] Added value is calculated according to 2010 constant prices
[b] The calculation of emission reduction set 2012 as the base year

(4) The faster the economy grows, the larger the CO_2 emission intensity becomes. In cases when the economic growth rate is 6.5%, the CO_2 emission intensity will reduce 3.27% in 2015 and 3.39% in 2020. In cases when the economic growth rate is 7.5%, the CO_2 emission intensity will rise 0.27% in 2015 and reduce 2.86% in 2020. In cases when the economic growth rate is 8.5%, the CO_2 emission intensity will rise 0.29% in 2015 and 1.29% in 2020.

The solutions of above programming model not only reveal certain interrelation between China's economic development and CO_2 emissions, but also expose some problems during China's economic growth.

Firstly, the reverse relationship between economic growth rate and CO_2 emission reduction can be easily found. To reduce CO_2 emissions, economic growth should be slower, but it will bring problems like overcapacity and unemployment. So there is a certain contradiction between CO_2 emission reduction and economic development. In the recent two years, China's government has set its economic growth target as 7.5%. Such a modest growth rate maintains not only a stable economic and social development, but also a reasonable CO_2 emission reduction. Therefore, such a growth rate should be maintained.

Secondly, China's overcapacity problem is around the corner. Even with the growth rate 7.5%, there will be 7% overcapacity in 2015 and 17% in 2020. Currently, the dissemination and sharing of investment information should be strengthened across various regions and investments in different regions should be coordinated to avoid repetitive investments. Meanwhile, investment threshold should be established, such as setting energy consumption and carbon emissions as an important investment criterion, so as to improve the quality of investment. Low-level redundant construction should be minimized and over-investment and waste of resources should be avoided.

Thirdly, unemployment will gradually appear. In addition to creating employment opportunities as much as possible and providing unemployment insurance, the development of new energy technologies and low-carbon technologies should also be taken into consideration because it can create so many jobs, which can play a significant role in easing the employment pressure and reducing CO_2 emissions.

Fourthly, water resources have become an important constraint on China's development. Greater efforts must be made to protect water resources, especially to prevent it from heavy chemical pollution.

Fifth, with economic growth comes the rapid growth of living energy consumption and carbon emissions, which will bring more pressure on emission reduction. Thus, while pursuing economic development, low-carbon concept and low-carbon consumption must be promoted.

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Chapter 11
Selection of the Company's Strategy Using Grey Stratified Decisions Model

Rafał Mierzwiak

11.1 Introduction

Strategic management is a complex information and decision-making process which aims at adjusting the organization to changing market conditions. On the one hand, this adjustment must take into consideration the characteristics of the environment in which the organization functions, on the other hand, its own potential and ability to act. The central element of this approach to management is the category of strategy. The concept of strategy was originally related to military and political action. Not until later did it began to be associated with business as a result of social and economic changes.

In the classical business approach, strategy can be understood as the direction and goal set by an organization in the long term, together with a plan for allocating resources that allow to implement these activities and goals efficiently. In such an approach to strategy, the issue of strategic management includes areas such as: the formulation of the major goals, values and visions of the organization, the determination of the ways to identify and gain competitive advantages, planning the use and allocation of the most important resources, recognizing the relationships and changes which take place in the environment, in particular, relations with stakeholders of the organization.

Strategies can have different degrees of specialization. According to this criterion, strategies may apply to (Hunger & Wheelen, 2003): the entire enterprise (corporate strategies), particular types of activities (business strategies) or individual company functions (functional strategies). Regardless, however, at what level of specialization

R. Mierzwiak (✉)
College of Management and Economics, Nanjing University of Aeronautics and Astronautics, Nanjing, People's Republic of China
e-mail: rafal.mierzwiak@put.poznan.pl

Faculty of Engineering Management, Poznan University of Technology, Poznań, Poland

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the strategy is formulated, it must be characterized by several important features. Firstly, it must indicate the domain and scope in which the company intends to concentrate its activities which aim at gaining a competitive advantage. Secondly, what concrete results are expected as a result of strategy implementation. Thirdly, what the schedule and structure of activities needed to implement the adopted strategy is.

Adjusting to the changing environment and gaining a competitive advantage is probably not only the goal of strategic management, but also overall of every business activity. Hence, what are the specific features of strategic management that distinguishes them from other management concepts? Is a question worth asking. The answer is the approach to the strategy formulation process. The process of formulating and choosing strategy should include such elements as (Miller & Dess, 1996, pp. 33–32):

- Focusing on the most important goals of the organization and their proper communication in the organizational structure,
- Focusing on key organizational resources and their efficient use in achieving the most important goals and objectives,
- Focusing on integration of individual management processes in order to facilitate their coordination and enforce consistency with the assumptions of the adopted central element strategy,
- Focusing on a comprehensive and holistic approach to the analysis of internal and external factors for the organization,
- Focusing on stability of operations and on long-term goals at the expense of short-term benefits,
- Focusing on balancing the expectations and goals of the stakeholders by building compromises.

When analyzing these features of the process of formulating and choosing strategy, it can be concluded that complexity, holism and multidimensionality are one of the most important features of a strategic approach in management. Therefore, the basic analytical tools in strategic management have to deal with the analysis of uncertain information. In the classical approach, this uncertainty is modeled by using qualitative analysis. However, attempts are being made to quantify this process using mathematical methods related to uncertainty modeling. One of them is the grey systems theory, whose application area is related to small sets of uncertain data. The possibility of analyzing small sets of uncertain data, in the context of strategic management, is of such great importance that usually these methods are based on the knowledge of a small number of experts that is difficult to quantify. Therefore, the integration of the grey systems theory and methods derived from strategic management has a large application potential for real problems.

The purpose of this chapter is, thus, to present the integration of the SWOT analysis and Grey Stratified Decisions Model (GSDM) (Liu & Lin, 2006, pp. 358–366; Zuoyong et al., 1992) in order to develop a grey quantitative model of the enterprise strategy choice. The chapter is organized in the following way: firstly, on the basis of critical analysis of literature, the analysis of current solutions in the field of GST in strategic management is presented. Subsequently, a formally developed model combining the SWOT and GSDM analysis is shown. Finally, a case study showing the choice of strategy for a production company using the developed model which integrates SWOT with GSDM is presented.

11.2 Current State of Research in the Field GST Applications in Strategic Management

Due to its high complexity and uncertainty, strategic management is a difficult issue for mathematical modeling. Therefore, currently used methodical tools in the field of strategic analysis and planning are qualitative. The information which is obtained in the strategic analytical process is, however, uncertain. The issue is further complicated by the fact that the availability of reliable data for strategic management is limited, which makes difficult the use of methods: data mining, big data, or classical statistics.

In the grey systems theory, the concept of grey information is taken as the basis. In the mathematical sense, it is possible to axiomatically determine it using the category of grey space. This space is analogous and similar in its construction to the probability space (Mierzwiak et al., 2018). The forms of expression that grey information may take are grey numbers, whitening functions and greyness understood in the distributive approach. These forms are an essential element of each GST method. Thanks to them, GST methods have specific properties that may be useful in modeling comprehensive systems with a small number of data for analysis. Furthermore, these methods allow for elimination of any initial assumptions about the nature of the distribution of the data. Therefore, it is not surprising that GST can be successfully used as a tool to support strategic management.

When analyzing the literature on the GST integration with the strategic approach, the concept that combines GST with the Decision Making Trial and Evaluation Laboratory (DAMETEL) comes to the fore. This method allows to analyze the relationship between any qualitative factors based on expert judgment. Thus, it has many uses in research and real applications (Si et al., 2018). In strategic management, it was used to evaluate the strategy evaluation criteria in combination with the SWOT analysis (Mostamand et al., 2017). Another possible application of the Grey DAMETEL approach is the critical analysis of success factors of the company (Bai & Sarkis, 2013). This approach also works in the analysis of barriers in a given industry (Wu, 2014). Apart from the approach based on the DAMETEL method, an interesting example of the use of GST methods is the concept of building a model of interaction between factors influencing strategic positioning of the enterprise. This model uses the SWOT analysis and grey number arithmetic (Zakeri et al., 2018). GST methods can also be successfully integrated into the balanced scorecard (Jahantigh et al., 2018; Kung et al., 2009). In addition, the use of the grey approach may apply to the selection of a company's strategic vision (Moghadasian et al., 2011). It can also be used in conjunction with the DEA method to select strategic alliances (Nguyen et al., 2015). An important research direction where strategic thinking is intertwined with GST are studies related to the selection of suppliers (Rajesh et al., 2015), as well as supply chain management and selection of production systems (JiShukla et al., 2014). There are also many studies on the strategic approach to services and public policy (Du et al., 2014).

The issue of the application of GST methods in strategic management is currently being developed in a multi-threaded manner. The main areas of application are related to the selection of functional strategies. Research is often embedded in the specific context of specific organizational functions. Many studies refer to strategic thinking. However, this is the context of the undertaken issue. Nevertheless, this does not alter the fact that the specific features of GST allow for alternative methodical approach to strategic management in regard to fuzzy logic, rough set theory or statistical methods.

11.3 Analysis of the Potential and Environment in Strategic Management

Every strategic analysis of an organization must refer to the analysis of factors present in the environment and factors relating to the internal attributes of the organization. This approach is the initial assumption of a very polar analytical approach which is the Analysis of Strengths and Weaknesses and Opportunities and Threats (SWOT). In the SWOT analysis, in relation to the organization's environment, factors that may be opportunities or threats for the organization are identified. In relation to the analysis of internal elements of an organization, the SWOT analysis identifies strengths and weaknesses that determine the organization's potential. The idea of the SWOT analysis can be presented in an illustrative format as in Table 11.1.

As can be seen in Table 11.1, the analysis of various combinations of internal and external factors makes it possible to formulate the following scenarios of strategic actions:

Table 11.1 The idea of SWOT analysis

	Strengths	Weaknesses
Opportunities	*Expansion strategy* How to take advantage of the organization's strengths?	*Competitive strategy* How to take advantage of opportunities despite limitations resulting from weaknesses?
Threats	*Conservative strategy* How to avoid threats thanks to your strengths?	*Defensive strategy* How to survive in an unfavorable external environment and problems within the organization?

- focusing on taking advantage of the organization's opportunities and strengths as part of an expansive strategy,
- focusing on taking advantage of opportunities and eliminating weaknesses of the organization (competitive strategy),
- focusing on eliminating threats by taking advantage of the organization's strengths (conservative strategy),
- focusing on avoiding threats and eliminating weaknesses, the purpose is the survival of organization through its deep restructuring (defensive strategy).

The choice of one of the presented strategies should depend on a number of criteria. One of them should include taking into consideration various groups of organization's stakeholders in the decision-making process (Freeman & Reed, 1983). Another important criterion includes the balancing of strategic accents with regard to results in the financial area, customer service, the functioning of processes or the development of personnel (Kaplan & Norton, 1995). Regardless of which set of criteria will be taken into consideration, the process of strategy selection can be formalized as a multi-layer grey decision model of Grey Stratified Decisions Model (GSDM). On the one hand, it can take into account the influence of different groups and their perspectives on making decisions on strategic activities. On the other hand, it can take into account substantive and discipline decision criteria.

11.4 Grey Stratified Decisions Model as a Tool for Choosing Corporate Strategies

In the process of potential and environment analysis with the use of the SWOT analysis, potential possibilities of strategic actions are generated. The set of these actions will be marked as below (11.1):

$$S = \left\{ s_q | q \in N \right\} \tag{11.1}$$

where s_q—generated strategy; as a result of the SWOT analysis, the following four strategies are usually obtained: s_1—expansive strategy, s_2—competitive strategy, s_3—conservative strategy, s_4—defensive strategy.

The selection of predefined strategy will be carried out using Grey Stratified Decisions Model (GSDM). In this model, the number of decision layers is determined first. The set of these layers will be denoted as below (11.2):

$$L = \{ L_i | i \in N \} \tag{11.2}$$

where L_i—a layer of decision makers; the following layers of decision makers are usually used in the decision-making process: L_1—a mass layer, i.e. a group of stakeholders with a relatively small impact on the organization, lacking a wide range of formal competences and authority in the organization (implementation staff), L_2—an expert layer, or middle management in an organization with expert knowledge of the main processes that are implemented by the organization, L_3—a management layer, i.e. a group of managers who have a real influence on the strategic orientation of a company. In the further description, only the three-tier division is taken into account.

The division of decision-makers into the decision-making layers results from the fact that if in the decision-making process one wants to achieve a balance between different perspectives, it is necessary for decision-makers to gain various knowledge, experience, education or position in the organizational structure. This division enables to adjust methodological tools that best reflect the character of information provided in the selection process by individual decision-making layers.

With the L decision layers and the generated S strategies, the procedure for choosing the optimal strategy is as described below.

1. Determining the statistical decision coefficient for each potential strategy s_q

The determination of the statistical decision coefficient is made by the mass layer L_1. This indicator has the following analytical form (11.3):

$$\sigma_q^k = \frac{\sum_{i=1}^n f^k\left(x_{iq}\right) \cdot \eta_i}{\sum_{k=1}^s \sum_{i=1}^n f^k\left(x_{iq}\right) \cdot \eta_i} \tag{11.3}$$

where

σ_q^k—is the statistical decision coefficient designated for the s_q strategy for the grey class $k = 1, \ldots, s$ where the grey class k is arbitrary defined by a class into which it is possible to classify individual strategies, example classes may be as follows: $k = 1$—distinguishing strategy, $k = 2$—used strategy, $k = 3$—conditional strategy, $k = 3$—strategy to implement,

$f^k\left(x_{iq}\right)$—the value of the whitening weight function f^k determined for the assessment of individual strategies s_q by the evaluation groups $i = 1, \ldots, n$ found in the mass layer L_1,

f^k—the whitening weight function for the class $k = 1, \ldots, s$, this function is determined arbitrarily; however, it usually has a triangular form in practical applications,

x_{iq}—average evaluation for individual strategies s_q given by group members $i = 1, \ldots, n$,

η_i—the weight determined for individual groups $i = 1, \ldots, n$, an important assumption is that: $\sum_{i=1}^n \eta_i = 1$.

The basic element in determining the statistical decision coefficient is the estimation of the value x_{iq}. The proposed method of estimation is to determine the average evaluation of all group members i. However, the estimation process carried out by group members should use a uniform estimation scale. An example of such a scale shows Table 11.2.

Table 11.2 Linguistic and numerical scale of evaluations of individual strategies s_q

Linguistic evaluation	Numerical evaluation
Very low (VL)	0.2
Low (L)	0.4
Medium (M)	0.6
High (H)	0.8
Very high (VH)	1.0

As a result of designating the indicator σ_q^k in accordance with the principles described above, the following vector will be obtained (11.4):

$$\sigma_q = (\sigma_q^1, \sigma_q^1 \ldots \sigma_q^s) \tag{11.4}$$

The vector σ_q is the carrier of information about the preferences of individual strategies in the decision layer L_1. It will be an important element of the integrated decision and final evaluation of individual strategies.

2. Determining the development coefficient—a_q using the grey forecasting model $GM(1, 1)$

In this stage of the decision procedure, firstly, the set of assessment criteria should be chosen $j = 1, \ldots, m$. When adopting the concepts of the balanced scorecard, in which the following four perspectives are dimensions: finances, operational processes, client and staff, the proposition of the criteria j is as follows: $j = 1$—financial condition of the company, $j = 2$—perfection of business processes, $j = 3$—level of customer satisfaction, $j = 4$—the quality of the company's staff. In the next step, decision makers from the expert layer generate the following vector data:

$$X_q^{(0)} = [x_q^{(0)}(1), x_q^{(0)}(2), \ldots, x_q^{(0)}(t)] \tag{11.5}$$

where

$x_q^{(0)}(t)$—evaluation of the strategy by decision-makers from the expert layer related to time t, where it is assumed that: $x_q^{(0)}(1)$—evaluation taking into account the current state of the environment, $x_q^{(0)}(2)$—evaluation resulting from the forecast of the environment within the next 1 year, $x_q^{(0)}(3)$—evaluation resulting from the forecast of the environment within the next 2 years etc. Evaluation $x_q^{(0)}(t)$ is a unified evaluation, i.e.: decision-makers evaluate each strategy s_q in relation to the adopted set of criteria j and give an assessment, which is the average evaluation of all decision makers. Each assessment is made on the scale presented in Table 11.2.

In the next step, the vector of subtotals is determined as follows (11.1):

$$X_q^{(1)}(t) = \sum_{i=1}^{n} x_q^{(0)}(t), \quad t = 1, 2, \ldots, n \tag{11.6}$$

With vectors data expressed by the formula (11.5) and (11.6) the $GM(1, 1)$ model can be presented as follows (11.7):

$$x_q^{(0)}(t) = -\frac{a_q}{2}\left[x_q^{(1)}(t) + x_q^{(1)}(t+1)\right] + u \quad (11.7)$$

The estimation of the parameter a_q can be made using the method of least squares using the following formula (11.8) and (11.9):

$$[a, u]^T = \left(B^T B\right)^{-1} B^T \left[X_q^{(0)}\right]^T \quad (11.8)$$

$$B = \begin{bmatrix} -\frac{1}{2}\left[x^{(1)}(1) + x^{(1)}(2)\right] & 1 \\ -\frac{1}{2}\left[x^{(1)}(2) + x^{(1)}(3)\right] & 1 \\ \vdots & \vdots \\ -\frac{1}{2}\left[x^{(1)}(n-1) + x^{(1)}(n)\right] & 1 \end{bmatrix} \quad (11.9)$$

The final element in this stage of the procedure is the determination of the vector A defined by the formula (11.10):

$$A = \begin{bmatrix} -a_1 \\ -a_2 \\ \vdots \\ -a_q \end{bmatrix} \quad (11.10)$$

The vector A is the carrier of information about the preferences of individual strategies in the decision layer L_2. It will be an important element of the integrated decision and final evaluation of individual strategies.

3. The calculation of the grey cluster decision coefficient δ_q^k

In this stage of the decision procedure, the grey cluster decision coefficient δ_q^k is determined using the formula (11.11):

$$\delta_q^k = \sum_{j=1}^{m} f_j^k\left(x_{qj}\right) \cdot \eta_j \quad (11.11)$$

where

f_j^k—the whitening weight function for the class $k = 1, \ldots, s$ for the criterion j, where the example classes should be identical to those described using the formula (11.3), whereas the proposed criteria j should be identical to those described using the formula (11.5); the form of the whitening weight function may be different, however, the most indicated solution due to its simplicity is the adoption of triangular functions.

x_{qj}—the value of the evaluation s_q of the strategy in relation to criterion j by decision makers in the layer L_3; the estimation is the average evaluation of the decision makers and can be made on the scale from Table 11.2.

η_j—the validity of each criterion j.

Finally, the result of this stage of the decision-making process for the layer L_3 is the vector δ_q (11.12).

$$\delta_q = (\delta_q^1, \delta_q^2, \ldots, \delta_q^s) \tag{11.12}$$

The indicator δ_q is the carrier of information about the preferences of individual strategies in the decision layer L_3. It will be an important element of the integrated decision and final evaluation of individual strategies.

4. Determining the final integrated evaluation and the choice of the best strategy

At this stage, firstly, the results of evaluation of individual strategies are to be integrated by decision-makers from layers L_1 and L_2 in accordance with the formula (11.13)

$$\Delta = (\Delta^1, \ldots, \Delta^s) = \begin{bmatrix} -a_1 \\ -a_2 \\ \vdots \\ -a_q \end{bmatrix}^T \begin{bmatrix} \sigma_1^1 & \sigma_1^2 & \ldots & \sigma_1^s \\ \sigma_2^1 & \sigma_2^2 & \ldots & \sigma_1^s \\ \ldots & \ldots & \ldots & \ldots \\ \sigma_q^1 & \sigma_q^2 & \cdots & \sigma_q^s \end{bmatrix} \tag{11.13}$$

In the next step, using the vector Δ, the integrated decisions should be determined between L_1, L_2 and L_3. This integration is carried out using the Grey Relational Analysis (GRA) procedure. The GRA model is the basic GST model. This model and its theoretical elaboration is easily available in the literature on the subject (Liu & Lin, 2006, pp. 87–90; Liu et al., 2017, pp. 67–72; Xie & Liu, 2009). Using GRA, the compatibility indicator between the vector Δ and vector δ_q will be calculated. This indicator is $\gamma(\Delta, \delta_q)$ and is calculated for every $q = 1, \ldots, s$. With this indicator, the following decision-making rule is used: if $\max\{\gamma(\Delta, \delta_q) = \gamma(\Delta, \delta_{q^*})\} \Rightarrow s_{q^*}$ is the best strategy from the set of potential S strategies.

11.5 Case Study

The method of strategy selection, developed in the previous section, was applied to a company manufacturing wood joinery. Firstly, the SWOT analysis was carried out and four potential strategies s_q were formulated. The results of this analysis are shown in Table 11.3.

Having the potential strategies, in the first selection step, the statistical decision coefficient is calculated for each potential strategy s_q. In order to determine this coefficient, three groups of decision-makers were created from the layer L_1. These

Table 11.3 SWOT analysis for a production enterprise

	Strengths 1. Modern technological production line 2. Ability to perform non-standard orders 3. Diversified foreign markets 4. High level of language competence of employees in the foreign sales department 5. High level of service 6. High level of knowledge of medium technical personnel 7. Very good financial ratios (no significant debts)	*Weaknesses* 1. Not very stable production personnel—high level of fluctuation 2. Lack of a comprehensively implemented ERP system 3. Poorly developed quality services—lack of quality systems and professional production supervision 4. Organizational problems related to the flow of information between the sales department and the production department 5. Unclear organizational structure—unclear hierarchical relations
Opportunities 1. Intense economic growth and the development of the construction industry 2. Increase in demand for individualized window joinery designs 3. Possibilities of obtaining subsidies for innovations within R&D funds (national and EU) 4. Strong price competition between suppliers of production components 5. The increase of pro-ecological trends among customers	*Expansion strategy* s_1 1. Sales mobilization on foreign markets by creating local branches of the company 2. Expanding the possibilities of the technological line aimed at fast refitting for individual orders (implementation of lean management elements) 3. Implementation of a formalized supplier evaluation system 4. Obtaining subsidies for R&D research in the scope of recycling of PVC production waste 5. Implementation of an integrated quality and environmental management system	*Competitive strategy* $\boldsymbol{s}_2$ 1. In order to ensure the stability of employment in case of an increasing number of orders, cooperation with a temporary employment agency should be established 2. To improve the management, the quality service responsible for implementing the factory production control system and ultimately implementing the formalized ISO 9001 quality system should be appointed 3. Description and analysis of business processes to implement a comprehensive ERP system—obtaining grants for innovative business services in regional support programs 4. Adjustment of the unused production line (instead of utilization) to the production unit and atypical production

(continued)

Table 11.3 (continued)

Threats	*Conservative strategy s_3*	*Defensive strategy s_4*
1. Vague provisions in the area of conformity assessment of products and, consequently, high costs of certification tests 2. High degree of price competition between manufacturers in the typical window and door segment, where dominate large enterprises with low production cost, which results from returns to scale 3. Increasing labor costs and difficulties in obtaining suitable staff 4. Unclear tax and customs regulations 5. Decreasing the popularity of plastic joinery and growing popularity of wooden and aluminum joinery	1. Establishing a quality department responsible for monitoring the regulations and quality requirements 2. Establishing an aluminum joinery department producing products for individual customer requirements 3. Forming a strategic alliance with a larger market operator in order to subcontract atypical production orders 4. Establishing constant cooperation with a law firm specializing in taxes and customs law	1. Restructuring the product range and focusing it on the production of cheap plastic windows and doors for the domestic market 2. Creating an aluminum joinery department which produces windows for individual orders 3. Establishing cooperation for subcontracting with a large entity on the market—taking over excess production 4. Reducing the number of target foreign markets only to those that bring the highest profits 5. Reducing the number of employees working full-time. Hiring temporary employees instead

groups were represented by the implementation personnel in the following areas: production and logistics, sales and marketing, warranty service. Each of these groups had the following weights: $\eta_1 = 0.4$ (production and logistics), $\eta_2 = 0.3$ (sales and marketing), $\eta_3 = 0.3$ (post-warranty service). The results of the strategy evaluation show the matrix below (11.12). The estimation was made using the scale from Table 11.2.

$$A = \begin{bmatrix} 0.60\ 0.86\ 0.58\ 0.20 \\ 0.66\ 0.73\ 0.66\ 0.26 \\ 0.43\ 0.93\ 0.86\ 0.33 \end{bmatrix} \tag{11.14}$$

The whitening function for the calculation of the statistical decision coefficient takes the form according to the formula (11.15)–(11.18). The form of these functions is determined in such a way that they are triangular functions whose lower and upper limits correspond to the lower and upper limits of the variability range of the estimation scale (Golińska et al., 2015).

$k = 1$—distinguishing strategy

$$f^1(x) = \begin{cases} \frac{x}{0.25} dla\ x \leq 0.25 \\ \frac{1-x}{0.75} dla\ x > 0.25 \end{cases} \tag{11.15}$$

$k = 2$—used strategy

$$f^2(x) = \begin{cases} \frac{x}{0.5} dla\, x \leq 0.5 \\ \frac{1-x}{0.5} dla\, x > 0.5 \end{cases} \tag{11.16}$$

$k = 3$—conditional strategy

$$f^3(x) = \begin{cases} \frac{x}{0.75} dla\, x \leq 0.75 \\ \frac{1-x}{0.25} dla\, x > 0.75 \end{cases} \tag{11.17}$$

$k = 4$—strategy to implement

$$f^4(x) = x \tag{11.18}$$

Using the formula for statistical decision coefficient (11.3) the following results were obtained:

$$\sigma_1 = (0.215; 0.291; 0.281; 0.211)$$

$$\sigma_2 = (0.107; 0.160; 0.304; 0.427)$$

$$\sigma_3 = (0.168; 0.252; 0.300; 0.278)$$

$$\sigma_4 = (0.442; 0.257; 0.171; 0.128)$$

In the next step, decision-makers from the L_2 layer made a uniformed prognostic assessment of each strategy in relation to the adopted set of criteria $j = 1, 2, 3, 4$. The results of this assessment are as below.

$$X_1^{(0)} = [0.66; 0.6; 0.66; 0.73]$$

$$X_2^{(0)} = [0.66; 0.73; 0.86; 0.93]$$

$$X_3^{(0)} = [1.0; 0.8; 0.86; 0.93]$$

$$X_4^{(0)} = [0.2; 0.26; 0.33; 0.33]$$

Having obtained the data presented above, the parameters of the relevant $GM(1, 1)$ models were estimated and the vector A, composed of parameters a_q, was obtained.

$$A = \begin{bmatrix} 0.098 \\ 0.117 \\ 0.075 \\ 0.109 \end{bmatrix}$$

In the next step, the results for the L_1 and L_2 layers were integrated by determining the vector $\Delta = (\Delta^1, \Delta^2, \Delta^3, \Delta^4)$. The determination of this vector was made according to the formula (11.13) as shown below.

$$\Delta = \left(\Delta^1, \Delta^2, \Delta^3, \Delta^4\right) = \begin{bmatrix} 0.098 \\ 0.117 \\ 0.075 \\ 0.109 \end{bmatrix}^T \begin{bmatrix} 0.215 & 0.291 & 0.281 & 0.211 \\ 0.107 & 0.160 & 0.304 & 0.427 \\ 0.168 & 0.252 & 0.300 & 0.278 \\ 0.442 & 0.257 & 0.171 & 0.128 \end{bmatrix} = [0.094; 0.094; 0.104; 0.105]$$

In the next step, the individual strategies were assessed by the decision-makers of the L_3 layer. For this purpose, the following whitening weight functions were adopted as defined by formulas (11.19)–(11.22). The form of these functions is determined analogously to the functions described in formulas (11.15)–(11.18).

$$f^1_{j=1,2,3,4} = \begin{cases} \frac{x}{0.25} dla\ x \le 0.25 \\ \frac{1-x}{0.75} dla\ x > 0.25 \end{cases} \tag{11.19}$$

$$f^2_{j=1,2,3,4} = \begin{cases} \frac{x}{0.5} dla\ x \le 0.5 \\ \frac{1-x}{0.5} dla\ x > 0.5 \end{cases} \tag{11.20}$$

$$f^3_{j=1,2,3,4} = \begin{cases} \frac{x}{0.75} dla\ x \le 0.75 \\ \frac{1-x}{0.25} dla\ x > 0.75 \end{cases} \tag{11.21}$$

$$f^4_{j=1,2,3,4}(x) = x \tag{11.22}$$

The whitening weight functions are necessary to evaluate individual strategies s_q in relation to adopted criteria $j = 1, 2, 3, 4$. The criteria were considered equivalent. For each criterion, the same weight η_j was used. Therefore, in the next steps of the determination of the grey cluster decision coefficient δ^k_q, this element was omitted.

The estimation of decision-makers from the L_3 layer for individual strategies in relation to the adopted criteria is presented below in the matrix B.

$$B = \begin{bmatrix} 0.75 & 0.7 & 0.75 & 0.9 \\ 0.75 & 1 & 0.9 & 0.9 \\ 0.60 & 0.65 & 0.55 & 0.65 \\ 0.40 & 0.30 & 0.30 & 0.20 \end{bmatrix}$$

On the basis of the table, the value of the indicator δ_q was determined. The results are shown below.

$$\delta_1 = (0.30; 0.45; 0.83; 0.78)$$

$$\delta_2 = (0.15; 0.23; 0.45; 0.89)$$

$$\delta_3 = (0.52; 0.78; 0.82; 0.61)$$

$$\delta_4 = (0.87; 0.60; 0.40; 0.30)$$

The final step of the strategy selection procedure is the unification of the estimation for decision-makers from all decision layers L_1, L_2, L_3 by determining the indicator $\gamma(\Delta, \delta_q)$ as shown below.

$$\gamma(\Delta, \delta_1) = 0.541$$

$$\gamma(\Delta, \delta_2) = 0.710$$

$$\gamma(\Delta, \delta_3) = 0.467$$

$$\gamma(\Delta, \delta_4) = 0.573$$

Due to the fact that $\max\{\gamma(\Delta, \delta_q) = \gamma(\Delta, \delta_2)\}$, the s_2 strategy (competitive strategy) is the best strategy from the S strategy set for the analyzed case of the production company.

11.6 Conclusion

The chapter presents how to integrate the SWOT method with Grey Stratified Decisions Model (GSDM). Such a combination enables to choose management strategy. This approach gives the possibility to take into account different aspects resulting from the different specializations and positions of decision-makers in the organizational structure. Furthermore, the proposed method of strategy selection may include criteria and aspects that are taken into consideration in the context of the balanced scorecard. In further considerations on the use of GST methods in strategic management, it should be important to focus on the methodical aspects of not only choosing the strategy itself, but also on developing the problems of the potential analysis and the company's environment. For this purpose, the grey relational analysis and models integrating GST with Fuzzy Cognitive Maps (FCM) can be useful. The implementation of these methods in the context of strategic management, however, requires further in-depth research.

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Chapter 12
A Cost Level Analysis for the Components of the Smartphones Using Greyness Based Quality Function Deployment

Erdal Aydemir, Yusuf Sahin, and Kenan Karagul

12.1 Introduction: Background and Driving Forces

The changes in technology are rapidly increasing and affecting life of humankind. Many useful solutions are presented by the manufacturers to provide an easy life for the society. One critical part of the emerging technologies is communication. The structural and cultural types of communication are changing rapidly over the last two decades. Today, GSM communication has started to leave with wide bandwidth ranges in 3G, 4G, 4.5G, and 5G technologies. The mobile technology supports us in managing meetings, messages, work-plans, reminders, notes, spreadsheets. With those various features and the video calls, the mobile phones today are much more than the conventional ones.

With the development of technology, mobile phones have come to have advanced computing and communication capabilities, including internet access and geo-positioning systems, as well as standard features such as voice and text communications (Boulos et al., 2011). The emerging communication technologies require high power devices in order to use these platforms. By 2017, there is over 40 international manufacturers' brands with over 3000 different models (IDC, 2017). So, the production of smartphones has become a strategic issue in production management for

E. Aydemir (✉)
Engineering Faculty, Department of Industrial Engineering, Suleyman Demirel University, 32260 Isparta, Turkey
e-mail: erdalaydemir@sdu.edu.tr

Y. Sahin
Faculty of Economics and Administrative Sciences, Department of Business Administration, Mehmet Akif Ersoy University, 15100 Burdur, Turkey

K. Karagul
Department of Logistics, Honaz MYO, Pamukkale University, 20330 Denizli, Turkey

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following this emerging high technology know-how. Because, it is known that smartphones can work with different types of applications that serve different individual, commercial or development purposes. Thus, it can be mentioned that there is a perfect competition market and the companies have to increase their level of customer satisfaction to enlarge their market share by taking into account the customer requirements with innovative solutions. At this point, production and customer service management have become very important issues in business management, and companies also have to raise production and cost performance too. From the customer point of view, they prefer to select the best smartphone to their own requirements, expectations and budget. There is not a single factor that influences the decision to buy a smartphone. Selection decisions are influenced by different factors such as product features, brand name, product price and social influences (Mohd Suki, 2013).

In this study, the cost difference and/or surplus between the target cost levels achieved by assessing customer expectations based on 12 different criteria with the current cost levels of the manufacturers to the smartphone components were analyzed using quality function deployment and Grey System Theory. At first, the expectations of the consumers from a smartphone were determined with the help of a survey and the degrees of greyness levels are presented in order to reflect the uncertainties of the customer expectations. Two different degrees of greyness values are proposed using the degree of greyness level. One is calculated by using the degree of greyness level for each customer requirement indicator. Also, another white number is calculated by using the mean total of the degree of greyness levels for each customer requirement as a whole system degree of greyness. Then, the relation between technical measurements and the customer requirements are given by determining the intensity of importance levels for the house of quality matrix. The degree of greyness values based model and the technical measurements related to the customer requirement are summarized in the house of quality matrix. Later, in order to analyze these features, quality function deployment model that is based on the degree of greyness values is proposed to determine customer needs.

About the rest of the paper, a brief technical analysis about smart phones is made in Sect. 12.2 and customer requirements for smart phones are defined in the Sect. 12.3. QFD and the greyness-based quality function deployment model approach are defined in Sect. 12.4. A case study for smartphones is presented to demonstrate the effectiveness and feasibility of the proposed approach in Sect. 12.5. Finally, Sect. 12.6 discusses the conclusions and further research.

12.2 A Brief Technical Analysis of Smartphones

In general, the features of the products should satisfy the customer wishes and requirements (Armstrong & Kotler, 2013). The smartphone industry requires a high-tech production and innovation composed of hardware and software components. The hardware is defined with the physical features such as size, shape, weight, color, design etc. The software is defined as operating systems and applications (Lay-Yee et al., 2013).

A smartphone production process can be categorized in four steps: integrated circuit, printed circuit board assembly (PCBA), manufacturing and assembly. In the manufacturing phase, concept designs are transformed to final products considering the customer feedbacks. This process requires an efficient management schedule for supply chain key factors in order to provide the best possible costs and production times. Then, the casing design is the next stage which considers the primary design of PCBA. The ergonomic features impose limitations for the casing design. However, considering the patent laws, all casing designs must be different with aesthetical care. In the hardware development, a touch panel, screen, communication modules (antenna-GSM-LTE), some sensors, speaker, battery, and accessories (adaptor and earphone) are considered with various configurations.

The final stage is the assembly process which is important for the smartphone manufacturers' compatibility. Assembly process ends with the testing phase in general. Testing starts with physical tests and then it continues with the software tests using a known mobile software platform. In addition, during this period the manufacturer must have an efficient supply chain management (SCM) organization. All components may have separate suppliers, different volumes and contracts. The manufacturers are able to coordinate all these suppliers and deliver the components just-in-time. In fact, the SCM is also related to components and production costs (Hassan et al., 2013; Zink et al., 2014).

In this study, the twelve technical components (TCs) are evaluated within the nine technical measurements (TMs) on the smartphone's specifications which are encoding as follows in Table 12.1.

Table 12.1 Technical components and measures on the smartphones

Code	Components	Measurements
TC1	Display	TM-1 [display]
TC2	Mobile processor	TM-2 [mobile CPU]
TC3	RAM module	TM-3 [memory]
TC4	Storage	
TC5	PCBA	TM-4 [mainboard]
TC6	Camera module	TM-5 [camera]
TC7	Casing	TM-6 [other components]
TC8	GPS-NFC etc.	
TC9	Mobile modules (3G-LTE/4G-4.5G)	
TC10	Battery	TM-7 [battery]
TC11	Production assembly process	TM-8 [production cost]
TC12	Packaging with accessories	TM-9 [packaging]

To determine the technical measurements, the technical components of the RAM and storage sizes are associated with memory measurements (TM3). For TC codes rear-and-front cameras are evaluated together and taken as the camera module. The casing, GPS, NFC, mobile modules are associated with TM6 measurement. The product cost is generally defined by the sum of purchasing, production and distribution costs. While the purchasing, distribution processes, and their costs may be similar for firms, the production process shows cost differences for the firms. For this reason, the production assembly process (TC11) is associated with production cost (TM8) as a technical measurement.

12.3 Customer Requirements on Smartphones

Smartphones are mobile phones with more advanced computing capabilities and connectivity, including functions such as portable media players, low-compact digital cameras, pocket video cameras, and global positioning system (GPS) navigation units (Mohd Suki, 2013). The smartphones are changing rapidly and are designed with the addition of extra features of PDAs. Therefore, smartphones are starting to work as assistants. With the internet connection feature, it is now possible for users to access the information they need at any time and place. People start to use smartphones more intensively in their daily business. Consumers are perceived to be dependent on their smartphones when they view them as a necessity and have strong propensity for continuous high usage, being engaged and unwilling to be apart from them, because they use them when traveling from home to work or traveling overseas and resting at home (Genova, 2010; Jena et al., 2016; Kwok & Lee, 2015; Mallinson, 2015; Mohd Suki, 2013; Takahashi et al., 2016; Tian et al., 2009; Zink et al., 2014). Then, the aim of using a smartphone will be transformed to improve the network connectivity and throughput in the Internet of Things (IoTs) age for smart cities (Ning et al., 2017). So, Internet connectivity properties will gain importance in the selection criteria of customers in the future.

According to IDC (International Data Corporation), first time, the sales of smartphones exceeded 1 billion items at the end of the year of 2013 and phone companies shipped a total of 344.3 million smartphones worldwide in the first quarter of 2017 (IDC, 2017). Customers should follow a process to include perceptions of customer needs, information research, evaluation of alternatives, buying decision and post-purchase behavior when they are purchasing any product. At the end of the process, the customer will select a product or brand from various options in the market. At this point, it should not be forgotten that consumers purchase decision is strongly influenced by cultural, social, personal and psychological characteristics (Singh et al., 2014). In addition to these characteristics, brand, price, quality, the need for valuable activities to enjoy their leisure time, and innovation awareness can affect customer decisions (Leo et al., 2005). Customers expect a variety of features from a smartphone. This situation causes diversity in the product range and causes producers to try to reach the optimum smartphone. In addition to personal preferences, users are also entering into expectations for ease of life when they pick up the phone.

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Smartphones have mobile operating systems which can be used more actively for many different purposes, and an application can be found for almost every job. Operating systems such as iOS (Apple), Android (Google), Symbian (Nokia), Windows (Microsoft), Bada (Samsung) and BlackBerry OS (RIM) are widely used on smartphones (Lay-Yee et al., 2013). According to IDC (International Data Corporation), the Android operating system has 85% of the global smartphone operating system market in the first quarter of 2017 (IDC, 2017). In addition to the operating system, many factors such as price, CPU power, screen size, storage capacity, RAM capacity, battery life, camera resolution affect consumers' smartphone purchasing decision (Osman et al., 2012).

Today, the intensive usage of internet and its applications has caused downloading and processing of data at high speeds, and many applications currently in use require high CPU capacity. The CPU, internal storage, and RAM capacity are important factors for the device to perform multiple tasks at the same time and to store processed data. High-resolution pictures and videos, as well as the storage of many files (mail, documents, etc.) on these devices, cause the amount of internal storage space to be one of the main factors affecting the purchase decision. In this study, 13 factors are determined as the customer requirements on the smartphones and the encoding of the determined factors is shown in Table 12.2.

While choosing smart devices, smartphone users consider different features such as price (CR12), design (CR11) and brand perception (CR13). The performance of smartphone applications that make everyday life much easier is directly affected by features such as the phone's CPU power (CR1), operating system (CR8) and RAM Capacity (CR3). On the other hand, the above-mentioned applications can be used more comfortably in large screen (CR7) phones. Increased use of Internet and location information makes the high-speed connection (CR9) and internal GPS module a necessity. In addition to a voice call, devices that provide a high-quality sound (CR6) for music and video are able to stand out in the competition. Camera resolution (CR5) is another important choice factor especially for users who like to take photos and videos. The features listed here make high capacity memory (CR2)

Table 12.2 Customer requirements on the smartphones

Code	Components
CR1	CPU-power
CR2	Storage capacity
CR3	RAM capacity
CR4	Battery time
CR5	Camera resolution
CR6	Sound quality
CR7	Screen size
CR8	Operating system
CR9	Mobile networks (3G-LTE/4G/5G)
CR10	Integrated GPS module
CR11	Design
CR12	Selling price
CR13	Brand perception

and a battery (CR4) that can meet the energy requirements for a long time, a standard for the customers.

12.4 Material and Methods

12.4.1 The Basic Concept of Quality Function Deployment (QFD)

QFD is defined as a method to add customer satisfaction, requirements and expectations into the design with some technical measures and components of the product by a developed quality design for the final product at the production phase (Akao & Mizuno, 1994). As an extended description, the QFD focuses and coordinates the organizational skills, design, manufacture, and goods by formatting the customers' requirements (CRs). So, why do customers prefer to purchase and continually purchase your goods? (Cristiano et al., 2001; Hauser & Clausing, 1988). The purpose of QFD is accepted to incorporate the customer choice into various periods of the product lifecycle for responding to the customer wishes. For an enterprise's product development team, analyzing CRs and responding to their needs have now become an important and inevitable task (Amy et al., 2016; Ayag et al., 2013; Chaudha et al., 2011; Kahraman et al., 2006; Karsak et al., 2002; Liu et al., 2017; Nahm et al., 2013; Sousa-Zomer & Miguel, 2017).

A four-stage QFD model is widely used in most of the studies in the literature, as shown in Fig. 12.1, which includes product planning, part placement, process planning, and production planning (Wu, 2006). The basic idea of QFD is to translate the customer requirements into product design or engineering specifications and then convert them into features of manufacturing parts, process plans, and production requirements. At the first stage, the customer's wishes are translated into relevant engineering features. Then in the second stage, the engineering features are transformed into the features of the parts, and one step further back in the design process is achieved. The third stage is the phase at which critical process parameters and operations are identified. At the last stage, detailed production requirements are defined (Kim et al., 2007).

The QFD is carried out with a set of measurements called "House of Quality (HOQ)" which provides functional communication paths (Hauser & Clausing, 1988). HOQ, as shown in Fig. 12.2, is the basic structure of the QFD and created by the QFD team. HOQ is the set of matrices used to relate customer requirements and quality characteristics to meet them, to compare product characteristics based on perceptions, to compare quality characteristics based on objective measures, and to identify positive or negative correlations between them (Chen et al., 2013; Morris & Morris, 1999; Wu et al., 2005).

It consists of the main table to compare the customer requirements and quality features and a table to compare the quality features with the help of structure as a roof at the top of this table. HOQ is a kind of conceptual map that provides tools for planning and communicating between departments in general, it is a kind of

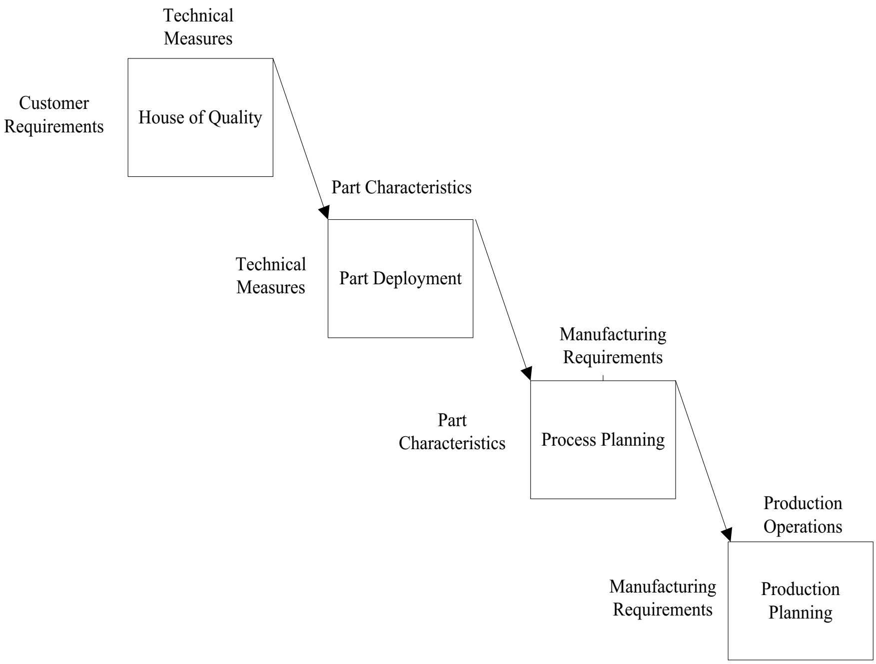

Fig. 12.1 The generic framework of quality function deployment (*Source* Kim et al., 2000)

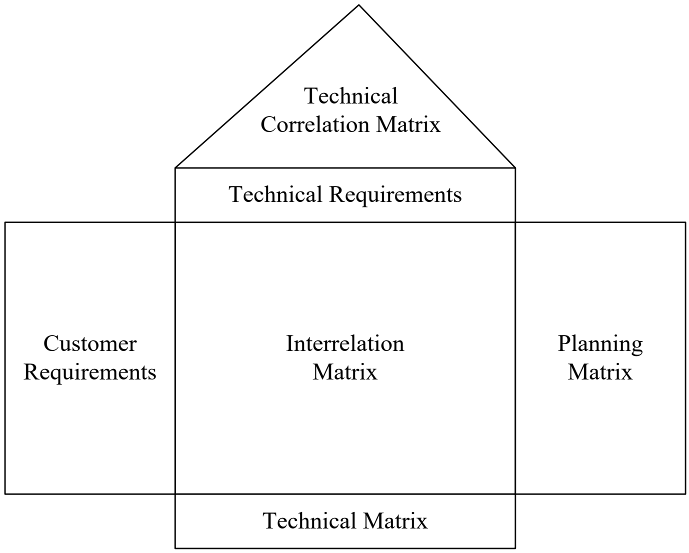

Fig. 12.2 Structure of house of quality (*Source* Wu et al., 2005)

conceptual map that provides tools for inter-departmental planning and communication (Chaudha et al., 2011; Hauser & Clausing, 1988; Nahm et al., 2013). On the other hand, the frequent occurrence of uncertainty situations in the product design and development phase in the QFD implementation process indicates that the uncertainty state cannot be neglected in QFD processes. All of this uncertainty and variability needs to be integrated into the QFD method (Deng, 1989; Dror, 2016).

12.4.2 Grey System Theory: Degree of Greyness

In 1982, Professor Deng, proposed grey system theory that is used by decision makers or researchers with partially known and partially unknown, uncertain or incomplete data. The Grey Systems Theory (GST) covered and applied in the systems with grey numbers which represent a number with incomplete information by clustering, modeling, prediction, programming, decision making and I/O control (Deng, 1989; Liu & Lin, 2010; Li et al., 2010; Yin, 2013). The concept of the grey system is illustrated by Xie (2017) is given as Fig. 12.3.

The system is categorized with black, grey and white subsystems. A black (box) system is the relation between inputs and outputs that are known but the behavior of it is unknown. Some developing theories like chaos theory etc. are related to this concept. If the system behavior is completely known, the system is also called as a white (box) system. The white systems are presented with exact value. And also, if the system behavior is partially known and/or partially unknown, the system would

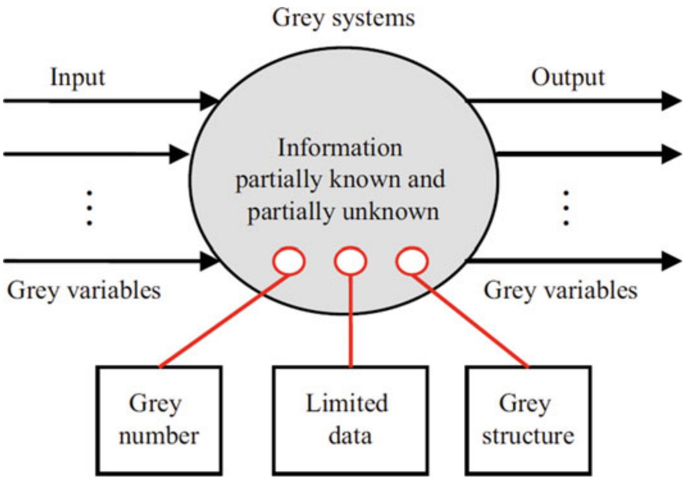

Fig. 12.3 The concept of the grey system (*Source* Xie, 2017)

be called as a grey (box) system (Aydemir et al., 2015). The fundamental definitions of interval grey numbers are given as follows (Jiang et al., 2017; Liu & Lin, 2010).

Definition 12.1 A grey number is such a number whose exact value is unknown but a range within that the value lies is known.

Definition 12.2 A grey number with both a lower limit $\underline{a}$ and an upper $\bar{a}$ is an interval grey number, denoted as $\otimes \in [\underline{a}, \bar{a}]$.

Definition 12.3 A grey number with both a lower limit $\underline{a}$ and an upper $\bar{a}$ is an interval grey number, denoted as $\otimes \in [\underline{a}, \bar{a}]$. The whitenization of the form $\tilde{\otimes} = \alpha\underline{a} + (1-\alpha)\bar{a}$, $\alpha \in [0, 1]$, is called equal weight whitenization.

Definition 12.4 In an equal weight whitenization, the whitenization value, obtained when taking $\alpha = ½$, is called an equal weight mean whitenization.

The degree of greyness reflects the unknown degree of the grey number and the unknown degree of the behavior characteristics of a grey system. On the other hand, the degree of the greyness of the grey numbers would mainly be related to the length of the defined information range and the basic value of the grey numbers. Also, for a grey number, a known whitenization weight function is defined by Deng in 1985 (Liu & Lin, 2006).

Definition 12.5 A degree of greyness g^o is obtained from the typical whitenization weight function f(x) in Fig. 12.4a and given as follows (Liu & Lin, 2005)

$$g^o = \frac{2|b_1 - b_2|}{b_1 + b_2} + \max\left\{\frac{|a_1 - b_1|}{b_1}, \frac{|a_2 - b_2|}{b_2}\right\} \tag{12.1}$$

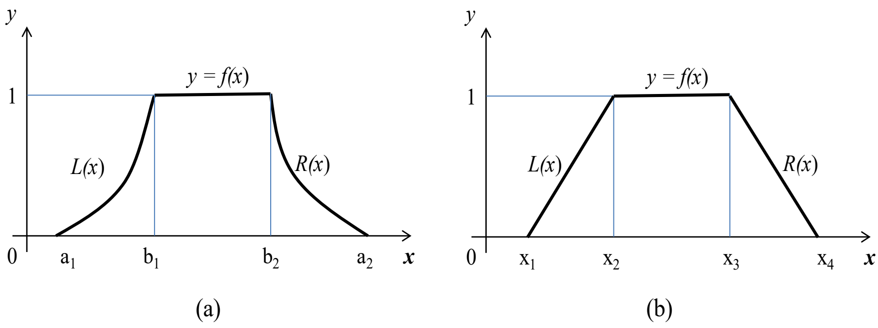

Fig. 12.4 The typical whitenization weight functions (*Source* Liu & Lin, 2006)

The equation g^o is the sum of two parts, the first one is the peak area and the second one is $L(x)$ and $R(x)$ functions (see Fig. 12.4b).

$$\max\left\{\frac{|a_1 - b_1|}{b_1}, \frac{|a_2 - b_2|}{b_2}\right\} = 0,\ g^o = \frac{2|b_1 - b_2|}{b_1 + b_2} \tag{12.2}$$

In this paper, the degree of greyness-based quality function model approach to determine the customer requirements. The results are obtained in a case study and evaluated with cost advantages on the smartphone's components.

12.5 Case Study

Today, there are many smartphone models that are produced by many companies and have different features. Smartphone manufacturers have to design products that will be produced according to customer requirements. The lack of a device that can address all customer wishes in the same way and the multiplicity of the alternatives make it difficult to make decisions on this issue. It is important to consider the properties that will make the most use of the phone when setting the criteria as the most important selection criteria. At this point, the person who will buy the phone is expected to choose a model that will be useful and will respond exactly to their needs. Table 12.3 summarizes the key criteria to consider when purchasing a smartphone. In this table, there are 13 selection criteria, 12 of which are objective and 1 of which is subjective.

After the criteria were determined, a questionnaire was prepared using criteria shown in Table 12.3. The survey is applied to 320 participants under these customer requirements factors. This questionnaire, which is applied to a randomly selected group of 320 participants, is required to score the customers' needs between 1 and

Table 12.3 Customer requirements on the smartphones

Objective criteria	Subjective criteria
Processor power	Brand perception
Storage capacity	
Ram capacity	
Battery time	
Camera resolution	
Sound quality	
Screen size	
Operating system	
Mobile capacity	
GPS module	
Design	
Selling price	

10 points (Saaty, 1980). At this point, 320-person client group is partitioned into 40-person groups and thus a total of eight periods (P1–P8) is obtained. The participants who have the technical information about the smartphones are selected for the survey.

In fact, the survey sample size is small, so, the criteria weight for the customers is determined by using the grey system modeling approach. A grey-based QFD model is established for each criterion. The survey results and the degrees of greyness are shown in Table 12.4.

According to Table 12.4, the white number w_1 is calculated by using the degree of greyness level (g°) for each customer requirement indicator. Also, w_2 is calculated by using the mean total of the degree of greyness levels for each customer requirement as a whole system degree of greyness (g°_{avg}) in order to reflect the uncertainties of the customer voice.

Then, the relation between technical measurements and the customer requirements are given with determining the intensity of importance levels (Saaty, 1980) for the HOQ matrix. The degree of greyness values (w_1 and w_2) based model and the technical measurements related to the customer requirement are summarized in the HOQ matrix shown in Table 12.5.

The zero cells in technical measurements columns in Table 12.5 mean that there is no relationship between the measurements and the corresponding customer requirement. The relation degrees given in this part are based on the technical experts' opinions from a smartphone production company. In this table, the total weights are obtained by multiplying the importance grades of technical measurements by the w_1 and w_2 values respectively.

After determining the HOQ matrix and weights, the total production costs of the five different smartphone models belonging to the firm, which have a significant share in the smartphone market, and the costs related to the technical specifications of the phones, are obtained from gsmarena web site (www.gsmarena.com) and summarized in Table 12.6. In this table, relevant information about the cost of the technical

Table 12.4 Results of the survey and the degree of greyness values

	CR1	CR2	CR3	CR4	CR5	CR6	CR7	CR8	CR9	CR10	CR11	CR12	CR13
P1	7.8447	6.6394	6.6359	4.9012	5.8619	6.2948	5.8729	8.8827	4.7852	6.4735	5.1849	4.0659	6.4906
P2	8.0446	5.3047	6.7451	4.5235	4.3514	5.7916	4.3412	4.9548	6.4853	6.6772	8.6531	4.2005	6.3217
P3	7.1520	7.0491	4.2651	8.7178	7.3562	6.0843	7.6165	5.2562	5.2659	4.6398	8.8288	5.7271	5.1833
P4	7.2216	7.9388	5.7452	5.1113	6.6992	5.9607	8.0923	4.1005	5.6003	6.6300	4.8646	4.2312	5.7098
P5	5.3973	5.4126	5.4185	6.4956	6.7761	5.0336	4.6886	5.7787	8.2005	7.8086	7.8369	4.4218	4.6646
P6	7.4749	6.2729	4.8151	7.2785	8.8784	4.8812	5.2572	5.1211	7.3309	5.5484	4.8617	4.7853	5.7626
P7	7.0754	8.8079	4.4452	5.4016	4.8129	4.9463	8.3812	6.3197	4.9480	5.8219	6.9485	4.5903	6.7730
P8	7.4464	8.9679	5.3770	6.3312	4.3792	5.4466	6.8569	6.2891	4.9405	7.6480	4.5304	6.3364	4.2501
avg	7.2071	7.0492	5.4309	6.0951	6.1394	5.5549	6.3884	5.8379	5.9446	6.4059	6.4636	4.7948	5.6445
Min	5.3973	5.3047	4.2651	4.5235	4.3514	4.8812	4.3412	4.1005	4.7852	4.6398	4.5304	4.0659	4.2501
Max	8.0446	8.9679	6.7451	8.7178	8.8784	6.2948	8.3812	8.8827	8.2005	7.8086	8.8288	6.3364	6.7730
g^0	0.3939	0.5133	0.4505	0.6335	0.6844	0.2530	0.6351	0.7367	0.5260	0.5091	0.6435	0.4365	0.4577
g^0_{avg}	0.5287												
w_1	7.0019	7.0875	5.6279	6.0606	5.7803	5.9372	5.8155	5.3598	6.4040	6.1953	6.0628	5.3452	5.6182
w_2	6.6293	7.0095	5.4192	6.4755	6.4582	5.5391	6.2214	6.3261	6.3746	6.1145	6.5308	5.1225	5.4391

Table 12.5 HOQ matrix

	w_1	w_2	TM1	TM2	TM3	TM4	TM5	TM6	TM7	TM8	TM9
CR1	7.0019	6.6449	9	4	5	3	5	3	3	5	1
CR2	7.0875	7.0311	4	9	3	3	3	1	3	3	1
CR3	5.6279	5.4339	4	9	3	3	3	3	5	3	1
CR4	6.0606	6.5002	5	3	9	5	3	7	3	3	0
CR5	5.7803	6.4849	3	3	5	9	3	7	1	5	0
CR6	5.9372	5.5474	9	0	3	0	5	0	1	1	0
CR7	5.8155	6.2452	3	3	7	7	3	9	3	5	1
CR8	5.3598	6.3543	5	7	5	3	5	3	5	5	0
CR9	6.404	6.3948	4	3	5	0	5	0	5	3	1
CR10	6.1953	6.1332	3	1	3	0	5	0	3	1	1
CR11	6.0628	6.5562	1	0	5	5	5	7	3	5	7
CR12	5.3452	5.1360	9	7	5	9	3	9	3	9	3
CR13	5.6182	5.4391	3	3	3	7	1	7	1	5	5
Model w_1		Total weight	374.429	312.6123	366.4222	316.0137	297.5742	326.1559	235.0006	313.9718	124.6983
		Relative (%)	14.04	11.72	13.74	11.85	11.16	12.23	8.81	11.77	4.68
Model w_2		Total weight	375.1303	318.5227	378.8283	328.0530	304.0874	339.6241	241.1270	322.6077	126.3799
		Relative (%)	13.72	11.65	13.85	12.00	11.12	12.42	8.82	11.80	4.62

Table 12.6 Component costs of smartphone models ($—%)

Model		TM1	TM2	TM3	TM4	TM5	TM6	TM7	TM8	TM9	
Model A	$	30.00	10.00	10.00	14.00	28.00	75.00	28.00	244.50	8.50	448
	Rel.%	6.70	2.20	2.20	3.10	6.30	16.70	6.30	54.60	1.90	
Model B	$	20.00	6.72	11.00	11.00	20.00	45.00	22.00	200.00	7.00	342.7
	Rel.%	5.80	2.00	3.20	3.20	5.80	13.10	6.40	58.40	2.00	
Model C	$	18.00	10.40	4.50	18.00	34.00	44.00	37.50	207.00	7.00	380.4
	Rel.%	4.70	2.70	1.20	4.70	8.90	11.60	9.90	54.40	1.80	
Model D	$	32.00	11.00	3.60	13.00	19.00	41.00	54.70	199.00	7.00	380.3
	Rel.%	8.40	2.90	0.90	3.40	5.00	10.80	14.40	52.30	1.80	
Model E	$	22.00	37.12	3.95	31.08	22.00	39.25	21.30	196.97	9.50	383.2
	Rel.%	5.70	9.70	1.00	8.10	5.70	10.20	5.60	51.40	2.50	
Mean (%)		6.28	3.9	1.72	4.52	6.35	12.49	8.49	54.22	2.02	

specifications and the percentage of these costs according to the total cost of the smartphone models are given.

Information obtained by the degree of greyness-based quality function deployment model provides a significant advantage in meeting the future needs of the customers. The differences between the percentages of actual component costs of the phones and the percentage of costs obtained using this model are shown in Table 12.7. This table contains useful information to analyze customer needs and to determine the technical specifications that can be taken into consideration in planning studies in the near future. When compared to customer requirements, some features can be provided at lower cost, while some features are added to the smartphone at a cost over expectations.

The difference between the actual cost percentages of producers and the expected costs calculated by grey-based QFD is shown in Table 12.7 and Fig. 12.5. These differences were calculated with Eq. (12.3).

$$Difference\,(\%) = Actual\,Cost\,Rate\,(\%) - Expectation\,Cost\,(\%) \quad (12.3)$$

Table 12.7 The differences between actual cost and grey based QFD

	TM1	TM2	TM3	TM4	TM5	TM6	TM7	TM8	TM9
Actual cost rate (%)	6.28	3.90	1.72	4.52	6.35	12.49	8.49	54.22	2.02
Model w_1	14.04	11.72	13.74	11.85	11.16	12.23	8.81	11.77	4.68
Difference (%)	−7.76	−7.82	−12.02	−7.33	−4.81	0.26	−0.32	42.45	−2.66
Model w_2	13.72	11.65	13.85	12.00	11.12	12.42	8.82	11.80	4.62
Difference (%)	−7.44	−7.75	−12.13	−7.48	−4.77	0.07	−0.33	42.42	−2.60

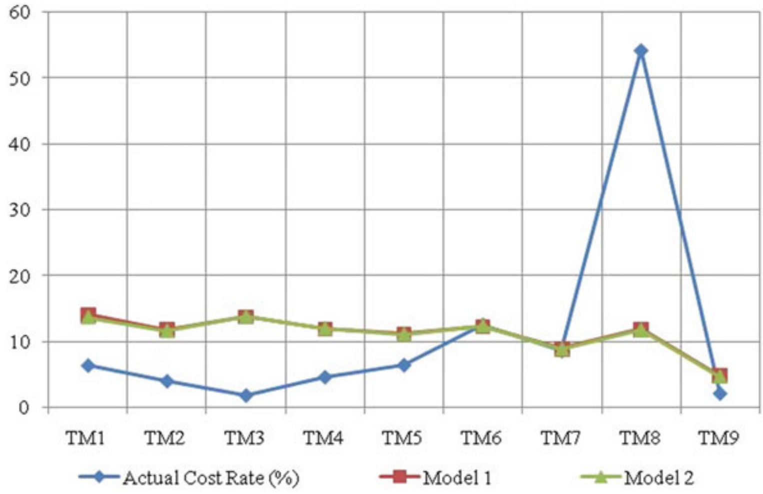

Fig. 12.5 Cost opportunity levels

For example, in the current situation for TM1 (mobile CPU), manufacturers allocate 6.28% of the product cost. However, for customers, the value below the significance level was calculated as 14.04% for Model w_1 and 13.72 for Model w_2. It can be said that the manufacturers have a surplus on TM1. On the other hand, when the value obtained for TM8 (production cost) is examined, it is seen that the share of production costs of the manufacturers in the unit cost is high (54.22). For this technical measurement, the importance level given by customers is 11.77 for Model w_1 and 11.80 for Model w_2. This difference clearly shows that the customers are not interested in the production costs of the manufacturers.

12.6 Conclusion

The goal of applying QFD model is to be able to provide the level of quality that the consumer expects in the product, process, and system development phases, by including customer expectations in various stages of the production. Also, the GST, which has been widely used in recent years such as clustering, modeling, forecasting, programming, decision making, is a method applied in systems with grey numbers that indicate a number with incomplete information.

In this study, the cost difference and/or surplus between the target cost levels achieved by assessing customer expectations based on twelve different criteria with the current cost levels of the manufacturers to the nine smartphone technical measurement components were analyzed. In this respect, it has been tried to determine which components should be important to the manufacturers considering their product costs, and which components they should first decide to reduce costs. On the other hand, prioritized customer expectations have been identified by this process to identify the points that manufacturers need to take into account in their product or service development work toward customer expectations in the future. Comparisons were made in terms of the values that the producers would be willing to pay for the specified specifications in a way that would be appropriate for customer expectations. As a result, the technical specifications that must be considered in order to reduce the costs of the manufacturers have been determined. This is directly related to the production process as well as the product design process. So, the proposed model has many advantages for product-cost analysis in the advanced manufacturing systems. The ergonomic characteristics of smartphones can also be considered as an additional criterion within customer requirements and technical measures of ergonomic factors can be planned as further research.

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