Parametric Study of Average Power from Vibration Energy Harvester

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Abstract Nowadays, renewable energy becomes important due to the increased energy demand and recent limitation of batteries. The technique of vibration energy harvesting has been seen as a promising way to allow self-sufficient wireless sensors and other low-power consumption devices. This is due to the presence of vibration energy in many environments and engineering systems. This research aims to investigate the characteristic of vibration energy harvesting. A single degree of freedom (SDOF) subject to a harmonic base excitation is considered as a harvester. This paper investigates analytically the performance of average power in vibration energy harvester. Results show that higher amplitude will decrease the frequency and increase power density for average power harvested. Furthermore, the amplitude can be higher and give more power density for average power harvesting when the resonance frequency occurs where the applied frequency is equal or closed to the natural frequency. In addition, it is possible to increase the amplitude and power average by tune the parameter of the energy harvester by changing the mass, stiffness, and damping ratio.

Keywords Vibration energy harvesting · Single degree of freedom · Average power

1 Introduction

Over the past two decades, harvesting wasted or underutilized ambient energy has been recognized as a foundation technology for energy-autonomous electronic devices [1]. Energy harvesting is a promising strategy that addressing the global energy crisis without depleting the natural resource [2]. These devices offer a solution by extracting energy from ambient such as sunlight (solar), wind, hydro, vibration (motion), temperature gradient (thermal), and waves of radio frequency (RF) [3].

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A. S. Abdul Sani et al. (eds.), *Enabling Industry 4.0 through Advances in Manufacturing and Materials*, Lecture Notes in Mechanical Engineering,
https://doi.org/10.1007/978-981-19-2890-1_54

Vibration energy harvesting (VEH) technologies have attracted a lot of attention since mechanical vibration energy can be found anywhere [4]. However, the absolute level of achievable performance is still an issue in function implementation. This is because most sources of vibration energy consist of a low-frequency broadband range [5].

VEH systems apply piezoelectric, electromagnetic, or electrostatic elements to convert kinetic energy into usable electrical energy [1]. The idea of VEH is to convert vibration into electrical energy through two conversion steps. First, the use of mechanical transducers between the two elements, such as a comparatively moving mass-spring system. Second, electromechanical transducers such as piezoelectric materials or variable capacitors are used to convert the comparative motion into electrical energy [6].

There was a wide scope of studies carried out on the VEH system. Generally, VEH was represented by an equivalent mass spring damper model. The linear damper was a combination of damping provided by electrical and mechanical systems [7]. Stephen analyzed the maximum of average power from the vibration energy harvester by set up the natural frequency and excitation frequency to be equal [8]. While Ashraf analyzed the dependence of the average power of VEH by considering the role of the damping [9]. Guangxhui Xia et al. also investigated the performance of the output power energy harvesting with different parameters such as load resistance and damping [10].

Since the oscillation energy is distributed on different harmonics, the study about the dependence of the average power on the parametric in vibration is important. Thus, this paper evaluates the dependence of average power on mass, stiffness, and damping of the VEH system. A single-degree-of-freedom (SDOF) mechanical oscillator subject to a harmonic base excitation model was introduced in this paper as a VEH system. The response for each condition was compared.

2 Methodology

A single degree of freedom (SDOF) system has been introduced for the harvester of vibration energy as shown in Fig. 1. Where c represents the damping that provides by mechanical and electrical areas, the mass, m represents the equivalent proof mass of the resonator, and the spring with a constant, k represents the elasticity of the resonator beam.

From Fig. 1, the basic equation of the relative motion of the seismic mass towards the base is defined as,

$$m\ddot{x} + (\dot{x} - \dot{y}) + k(x - y) = 0 \tag{1}$$

When the periodic force acts on the base, the mass oscillates with amplitude z. Rearranging Eq. (1) in terms of the relative displacement of the mass $z = x - y$ yields,

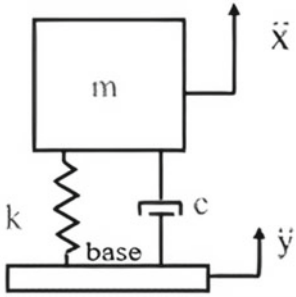

Fig. 1 Mass-spring-damper model for VEH system subjected to base

$$m\ddot{z} + c\dot{z} + kz = m\ddot{y} \tag{2}$$

Substitute the base excitation $y = Y\sin(\omega t)$, Eq. (2) becomes,

$$m\ddot{z} + c\dot{z} + kz = -m\omega^2 Y\sin(\omega t) \tag{3}$$

The steady-state solution of Eq. (3) is,

$$Z = \frac{m\omega^2 Y}{\sqrt{\left(k - m\omega^2\right)^2 + c^2\omega^2}}\sin(\omega t - \phi) \tag{4}$$

The phase difference ϕ is given as,

$$\phi = \tan^{-1}\left(\frac{c\omega}{k - m\omega^2}\right) \tag{5}$$

Equation (4) can be simplified by using $\omega_n = \sqrt{\frac{k}{m}}$ and $\zeta = \frac{c}{2\sqrt{km}}$ to get

$$z = \frac{\left(\frac{\omega}{\omega_n}\right)^2 Y}{\sqrt{\left(1 - \frac{\omega}{\omega_n}^2\right)^2 + \left(2\zeta\frac{\omega}{\omega_n}\right)^2}}\sin(\omega t - \phi) \tag{6}$$

Equation (6) is the amplitude of vibration of the mass respected to the base. This equation is used to find the displacement of the spring-mass-damper system in the time domain, where ω/ω_n is the frequency ratio and ζ is the damping ratio.

The electrical and mechanical dampers are considering the same as velocity dampers. The instantaneous energy absorbed by the dampers can be obtained by multiplying the damping constant with the square of instantaneous velocity. The average power is the integration of instantaneous energy absorbed within a complete cycle of vibration divided by time (period). The equation is shown as,

Table 1 Properties of SDOF system

Parameters	Value
Mass m (kg)	1
Stiffness k (N/mm)	107.5
Damping c (Ns/mm)	0.1
Base excitation amplitude y (m)	0.025

$$Pav = \frac{1}{t}\int_0^t c\dot{z}^2 dt = \frac{m\zeta Y^2\left(\frac{\omega}{\omega_n}\right)^3\omega^3}{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2+\left(2\zeta\frac{\omega}{\omega_n}\right)^2} \tag{7}$$

Equation (7) determine the average power harvest from the system. Then, the maximum average power can be obtained at the frequency ratio by setting the $\partial Pav/\partial\omega = 0$ as shown,

$$\frac{\omega}{\omega_n} = \sqrt{2-4\zeta^2-\sqrt{(4\zeta^2-2)^2-3}} \tag{8}$$

where,

$$\zeta = \frac{c}{2m\omega_n} \tag{9}$$

Equation (8) shows the frequency ratio only has a valid value when the damping ratio, ζ is less than 0.25 [9]. Equation (9) represents the damping ratio of the system.

MATLAB 2020b software was used to find the response for time-domain and frequency-domain of Eq. (6). Fast Fourier Transform (FFT) was used to compute the frequency response. The power average of the VEH was calculated using Eq. (7). Three different tests were carried out to look at the effect of mass, stiffness, and damper towards power average producing by the VEH system. Table 1 shows the properties of the SDOF system.

The mass, stiffness, damper, and frequency ratio have been varied to compare the performance of power average for the VEH system, as shown in Table 2.

3 Result and Discussion

3.1 The Mass Effect

Figure 2 shows the displacement of the VEH system in the time domain. Three different masses, $m_1 = 1\ kg$, $m_2 = 3\ kg$, and $m_3 = 5\ kg$ have been proposed to identify the difference in the responses. The m_3 has a higher amplitude follow by

Table 2 Parameter for the testing

Parameter of mass	
Mass, *m* (kg)	1
	3
	5
Parameter of stiffness of spring	
Stiffness, *k* (N/mm)	107.5
	207.5
	307.5
Parameter of damper	
Damper, *c* (Ns/mm)	0.1
	0.15
	0.25
Frequency ratio	
$\omega < \omega_n$	
$\omega > \omega_n$	
$\omega = \omega_n$	

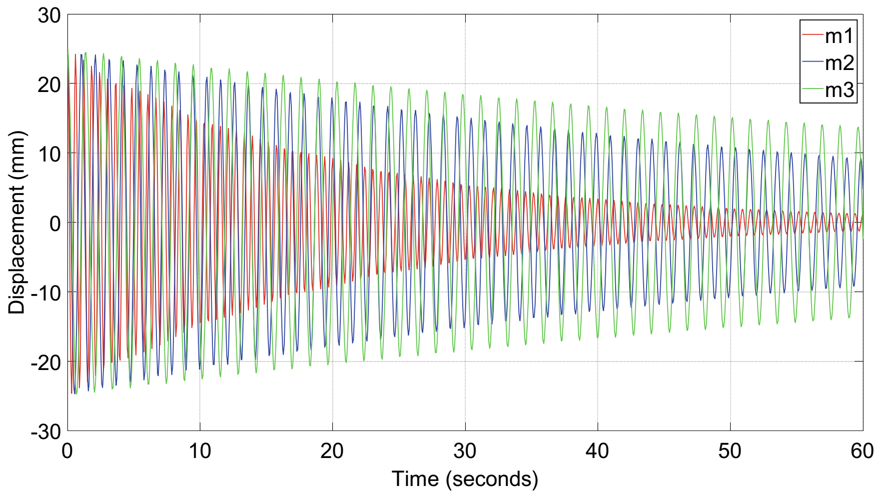

Fig. 2 Time-response for different masses

m_2 and m_1. The higher mass influences the amplitude to enlarge. Heavier mass triggers a larger initial overshoot that causes the system to have a higher amplitude. The displacement for all the masses decreases as time increases. This is due to the damping of the system. However, the displacement of the mass, m_1 decreases drastically compared to other masses. Due to higher energy store in heavier mass, the

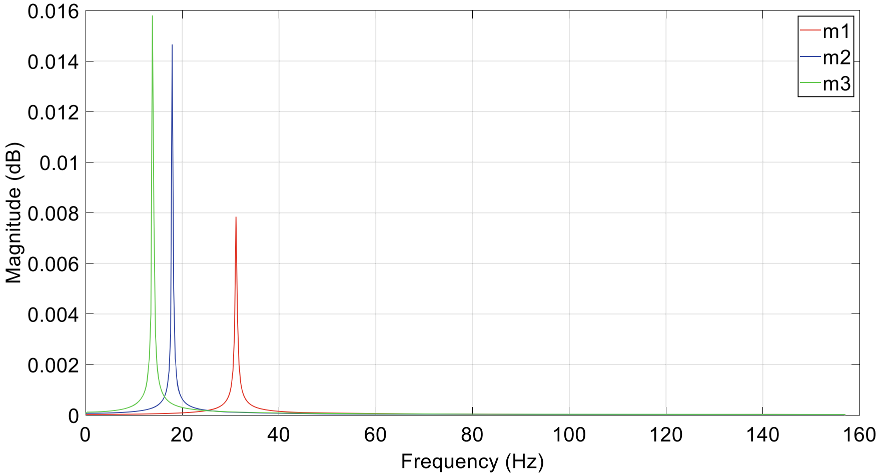

Fig. 3 Frequency–response for different mass

time taken for the energy to dissipate is more compared to lighter mass. This finding is in agreement with the finding of Burchett (2005) who investigated the effect of amplitude in different masses.

The frequency response of the VEH system is shown in Fig. 3. The m_3 shows the lowest value of natural frequency which is $\omega_n = 18$ rad/s but has the highest peak at the magnitude. While m_1 has the highest value of natural frequency, $\omega_n = 36$ rad/s but lowest in amplitude. It can be observed that increasing the mass causing the decrement in the natural frequency of the system. Heavier masses travel more distance to complete one cycle which then causes the system to oscillate slower and have a low natural frequency. This result is in line with the fundamental theory of vibration where the natural frequency is inversely proportional to mass.

To discover the most effective power output measured by the system, the average harvested power has been compared between different masses. Figure 4 shows the variation of average power related to the frequency for different masses. m_3 generates the highest power average, $Pav = 6.5\ mW$. The average power starts to react at 6 Hz by increasing steeply for m_3 and a slight increment for m_2. While m_1 has no power output as the frequency increases. Results show heavier mass produces a large amount of power and increases as frequency going higher. The results are in line with Eq. (7), as the power average is proportional to the square of displacement.

3.2 The Stiffness Effect

Figure 5 shows the time response of the VEH in different stiffness of spring, $k_1 = 107.5$, $k_2 = 207.5$ and $k_3 = 307.5$ N/mm. The system has a similar mass and

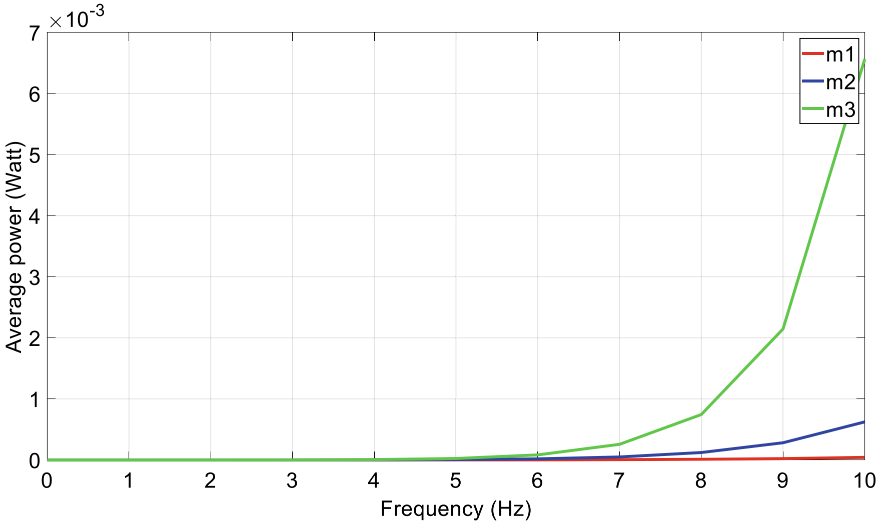

Fig. 4 Average power–frequency for different mass

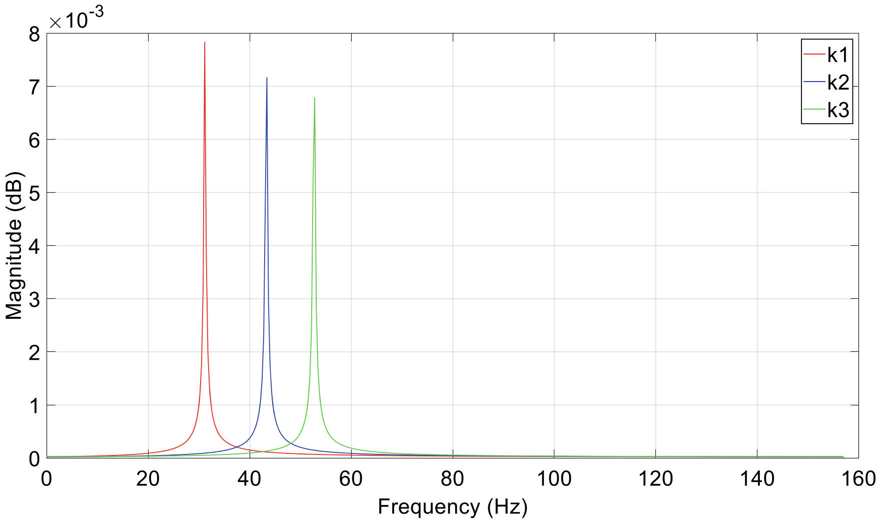

Fig. 5 Time-response for different stiffness

damping. Results show that k_3 has the smallest time cycle and amplitude. While k_1 shows the highest time cycle and amplitude of oscillation. This is because a larger value of stiffness, k, has the larger force constant to resist the system with a similar mass from oscillating at high amplitude and time cycle. These results are in line with the previous work by Casiesz [11].

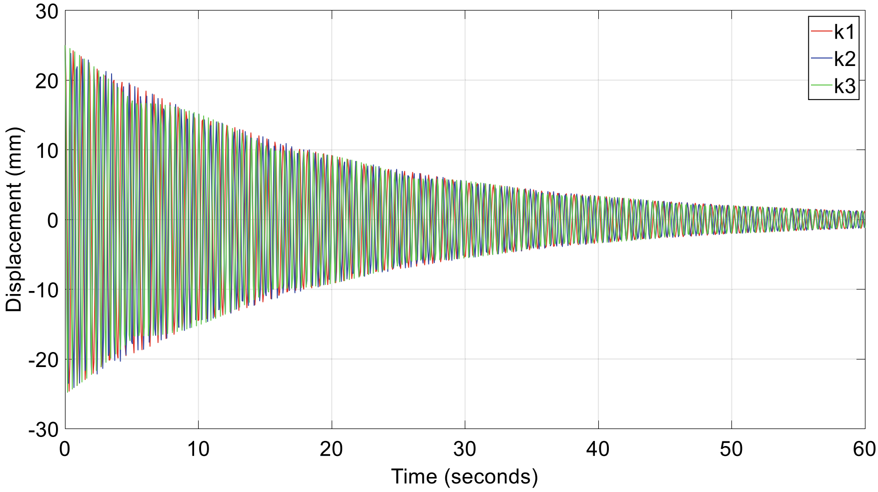

Fig. 6 Frequency–response for different stiffness

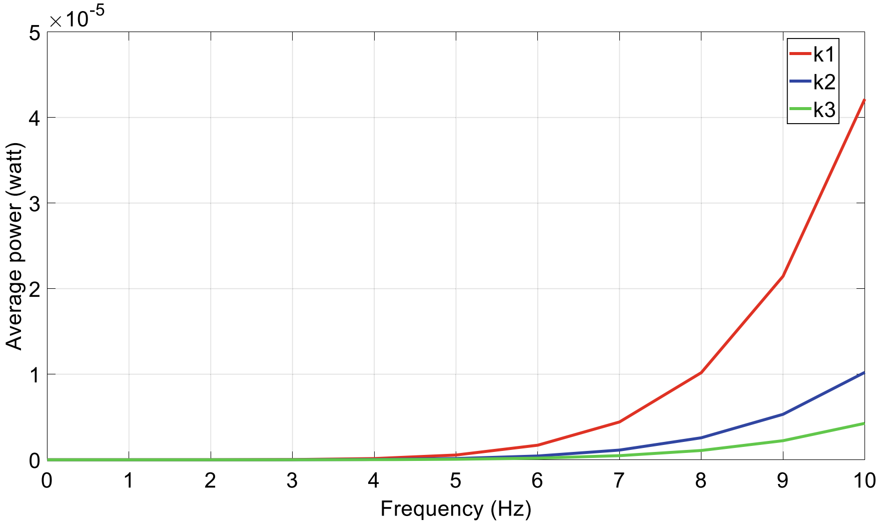

Fig. 7 Average power–frequency for different stiffness

The natural frequency increases with the growth of the stiffness, as revealed in Fig. 6. The stiffness spring constant k_3 has the highest value of natural frequency, compared to k_1 where the value of natural frequency is $= 53$ rad/s and $\omega_n = 31$ rad/s respectively. This result is in line with Ashraf et al. where the stiffness of spring is proportional with the natural frequency of the system [9].

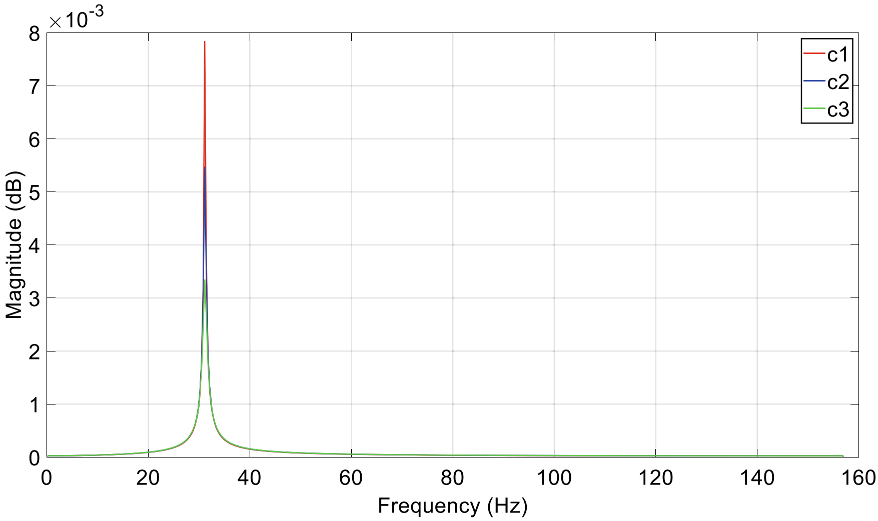

Fig. 8 Time-response different damping

The comparison of average power between the stiffness is shown in Fig. 7. The power average starts to react at 5 Hz for lower stiffness k_1, earlier than higher stiffness k_2 and k_3. The stiffness k_1 produces average power higher than k_2 and $k_3 as$ frequency increases. These results have a good agreement with work by Zhou et al. [4], who stated the increase in stiffness of VEH resulted in decreased bandwidth and significantly reduced the power average.

3.3 *The Damping Effect*

The VEH system has been tested by varying the damping coefficient value, $c_1 = 0.1$, $c_2 = 0.15$, and $c_3 = 0.25$ Ns/mm. The rest parameters remain unchanging, as indicated in Fig. 8. The lowest damping coefficient value, c_1, shows the highest displacement compared to the highest damping coefficient c_3. Friction or damping condition reduces the mechanical energy of the system. Thus, the damping coefficient gradually reduces the amplitude of the oscillating motion.

The frequency response in Fig. 9 shows the effect of damping towards natural frequency. The graph shows the lowest damping, c_1 has the highest amplitude at 8×10^{-3} dB. While higher damping, c_3 has the lowest amplitude 3×10^{-3} dB at the resonance frequency. These results, agree with the work by Stephen [8] that reported by reducing the damping coefficient, the amplitude of the system can increase to the maximum value. In this condition, changes in damping only affected the peak area of the displacement response.

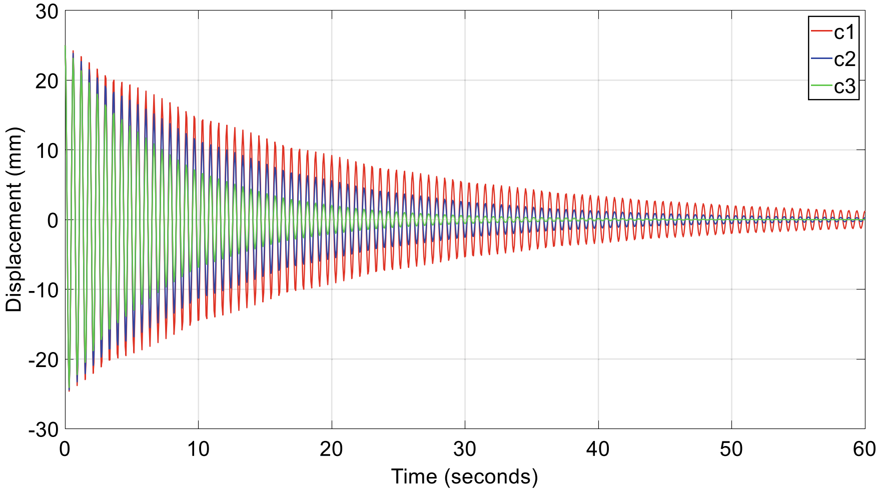

Fig. 9 Frequency–response different damping

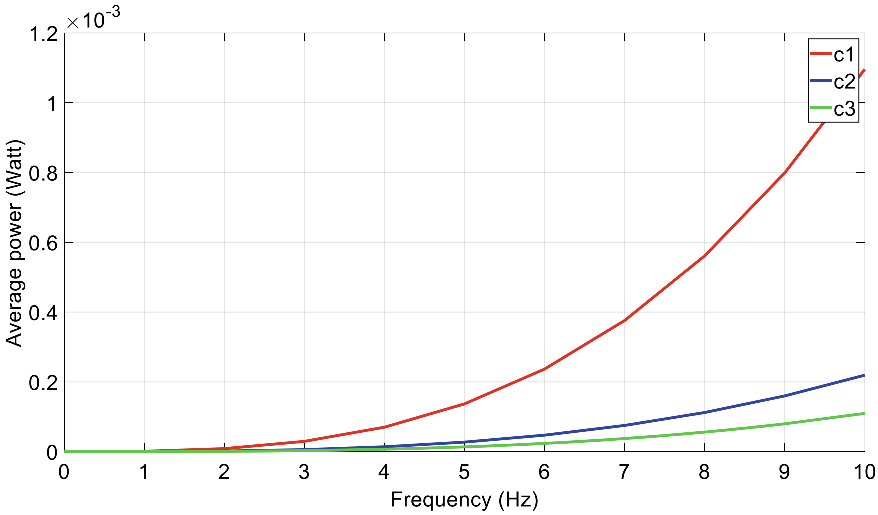

Fig. 10 Average power–frequency for different damping

The average power related to frequency contours is computed and shown in Fig. 10. The average power for damping c_1 shows the highest value of $Pav = 1.1$ mW. While c_2 and c_3 both show the lower value, which is $Pav = 0.21$ mW and $Pav = 0.1$ mW respectively as frequency increases. This finding was in line with the finding of Lei et al. where increasing the mechanical damping reduced the output voltage of the VEH system [12]. Besides being involved with the harvesting and dissipation of

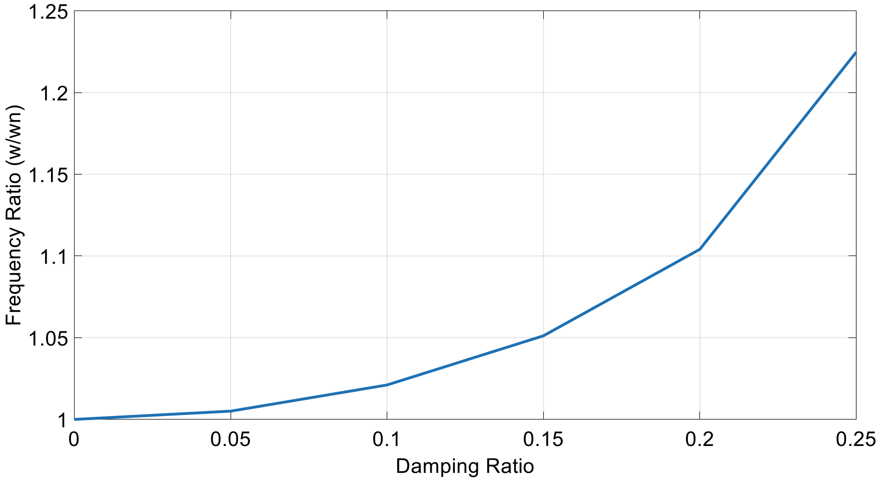

Fig. 11 Variation of optimal frequency ratio with damping ratio

Table 3 Average power with different frequency excitation

$\omega_n = 31$ rad/s	$\omega < \omega_n$ ($\omega = \mathbf{10}$ **rad/s**)	$\omega > \omega_n(\omega = \mathbf{50}$ **rad/s**)	$\omega = \omega_n(\omega = \mathbf{31}$ **rad/s**)
Power (μW)	0.000421544	0.2062	288.6

energy, the damping coefficient is important in the VEH system as it transfers the energy from the vibrating base into the system [8].

Figure 11 shows the variation of optimal frequency with damping ratio. The small damping ratio contributes to the frequency ratio close to unity. The production of average power is high at a frequency ratio equal or close to unity due to resonance at a low damping ratio. Table 3 compares the average power harvested for three conditions of excitation frequency based on the properties of VEH in Table1. Results show the highest power average can be produced once the excitation frequency is equal to or close to the natural frequency of the VEH system that so-called resonance condition. This result is in line with the work by Ashraf [9].

4 Conclusion and Recommendations

The average power performance of the SDOF model as a VEH system was investigated. The performance of average power was compared for different parameters. The output of the power was dependable on the amplitude of the vibration. Thus, the heavier mass, the less stiff, the low damping and resonance produced more power since this condition contributed to high amplitude. However, to scavenge more power from unused vibration, the implementation of broader bandwidth in the system needs to be considered for the VEH system.

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