



Research on Precise Orbit Determination Time Synchronization and Extended Application of LEO Constellation Based on GNSS

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Abstract. The four major satellite navigation suppliers announced by the International Committee on Global Navigation Satellite Systems (ICG) include GPS of the United States, GLONASS of Russia, BDS of China and Galileo of Europe. The development of the four satellite navigation systems not only changes the trajectory of human activities on the earth's surface, but also provides a cheap and efficient way for the accurate positioning and timing of other aircraft. Due to its low orbit and fast speed, LEO satellite naturally has the characteristics of strong landing power, fast revisit cycle, high ground resolution and fast observation geometry change, which makes it attracted much attention in the fields of mobile communication, remote sensing detection and navigation enhancement, and so on. In addition to the relatively weak irradiation environment, low launch cost and economic advantages, LEO constellation has become a direction that could not be ignored in various fields in recent years. Based on GNSS technology, this paper studied the theoretical method and achievable accuracy of precise orbit determination and time synchronization of LEO constellation. On this basis, the degraded backup effect and application suggestions of LEO constellation under multi-layer architecture including GNSS system were studied.

Keywords: LEO constellation · GNSS technology · Multi-tier architecture · Orbit determination and time synchronization · Expand application

1 Introduction

The four major satellite navigation suppliers announced by the International Committee on Global Navigation Satellite Systems (ICG) include GPS of the United States, GLONASS of Russia, BDS of China and Galileo of Europe. GNSS can provide users with three-dimensional coordinate, velocity and time information at any place on the earth's surface and near earth space [1]. GNSS technology provides a cheap, efficient and high-precision positioning means for Leo vehicles. Due to its low orbit and fast speed, LEO satellite naturally has the characteristics of strong landing power, fast revisit cycle,

high ground resolution and fast observation geometry change, which makes it attracted much attention in the fields of mobile communication, remote sensing detection and navigation enhancement, and so on [2]. In addition to the relatively weak irradiation environment, low launch cost and economic advantages, LEO constellation has become a direction that can not be ignored in various fields in recent years. Based on GNSS technology, this paper studied the theoretical method and achievable accuracy of precise orbit determination and time synchronization of LEO constellation. On this basis, the degraded backup effect and application suggestions of LEO constellation under multi-layer architecture including GNSS system were studied.

2 Present Situation and Principle of Orbit Determination and Time Synchronization of LEO Constellation

2.1 Present Situation of Orbit Determination of LEO Satellites

With the popularity of low earth orbit (LEO) spaceborne GNSS payload and gradually become standard configuration, at present, LEO satellites generally use spaceborne GNSS observation data to obtain their own precision orbit.

The T / P satellite launched in 1992 had obtained orbit accuracy better than 3cm in radial direction based on its dual frequency GPS receiver [3]. The success of T / P satellite has greatly promoted the development of spaceborne GNSS technology. Since then, a series of LEO satellites had been equipped with on-board GNSS receivers, among which Jason series marine satellites [4, 5] and grace / GOCE series gravity satellites [6, 7] were the most successful.

Using spaceborne GNSS technology had obtained radial orbit determination accuracy of 1 cm, which greatly promoted the research of ocean Altimetry and gravity field inversion. China's domestic ocean series satellites, resource satellites and Fengyun series meteorological satellites were also equipped with GNSS receivers for autonomous navigation and precise orbit determination, and had also reached cm level orbit determination accuracy [8–10]. After decades of technology accumulation and development, the development of low earth orbit satellite spaceborne GNSS technology had made rich achievements in orbit determination theory, model method and practical application. At present, the research on LEO satellite borne GNSS technology at home and abroad generally presents the development trend of high precision, real-time and multi system integration.

2.2 Principle of Absolute Orbit Determination for LEO Satellite

The absolute orbit determination of LEO satellite is to determine the optimal estimation of its orbit under a certain adjustment criterion by using the on-board GNSS observations and the orbit dynamics model. Therefore, the determination of the absolute orbit of LEO satellites involves three aspects: Geometric observation, orbital dynamics model and estimation criteria.

The equation of satellite GNSS pseudo range and carrier phase observations is as follows.

$$\begin{aligned}
 P_{r,j}^s &= \rho_r^s + c(dt_r - dt^s) + I_{r,j}^s + b_{r,j} - b_j^s \\
 L_{r,j}^s &= \rho_r^s + c(dt_r - dt^s) - I_{r,j}^s + \lambda_j(N_{r,j}^s + B_{r,j} - B_j^s) + \lambda_j\omega_r^s
 \end{aligned}
 \tag{1}$$

where, the subscript j represents the frequency f_j ; ρ represents the geometric distance from the phase center of the navigation satellite signal transmission time to the phase center of the receiver signal reception time; c is the speed of light; dt_r, dt^s are the receiver clock difference and the navigation satellite clock difference respectively; I is the ionospheric delay, λ is the signal band, N is the unknown integer ambiguity, ω_r^s is the phase winding effect, b_r, b^s are the pseudo range hardware delay at the receiver and navigation satellite, and B_r, B^s are the phase hardware delay at the receiver and navigation satellite, respectively.

In order to eliminate the ionospheric first-order term delay, ionospheric combined observations was often used in data processing. According to IGS analysis specification, the hardware delay of receiver and satellite were absorbed by receiver clock difference and navigation satellite clock difference respectively. The combined observation equation is:

$$\begin{aligned}
 P_{r,IF}^s &= \rho_r^s + c(\bar{dt}_r - \bar{dt}^s) \\
 L_{r,IF}^s &= \rho_r^s + c(\bar{dt}_r - \bar{dt}^s) + \lambda_1 \bar{N}_{r,IF}^s + \lambda_1 \omega_{r,IF}^s
 \end{aligned}
 \tag{2}$$

where,

$$\begin{aligned}
 \bar{dt}^s &= dt^s + b_{IF}^s/c \\
 \bar{dt}_r &= dt_r + b_{r,IF}/c \\
 \bar{N}_{r,IF}^s &= N_{r,IF}^s + d_{r,IF} - d_{IF}^s \\
 d_{r,IF} &= B_{r,IF} - b_{r,IF}/\lambda_1 \\
 d_{IF}^s &= B_{IF}^s + b_{IF}^s/\lambda_1
 \end{aligned}
 \tag{3}$$

The motion equation of the satellite in the inertial coordinate system was as follows.

$$\ddot{\vec{r}} = \vec{a}_{CF} + \vec{a}_{nCF} + \vec{a}_{emp}
 \tag{4}$$

where, \vec{a}_{CF} represents conservative force, \vec{a}_{nCF} represents non conservative force, and \vec{a}_{emp} is the empirical force added to absorb other unmodeled forces.

Batch processing algorithm was generally used for accurate estimation of track parameters. And its process was shown in the Fig. 1 below.

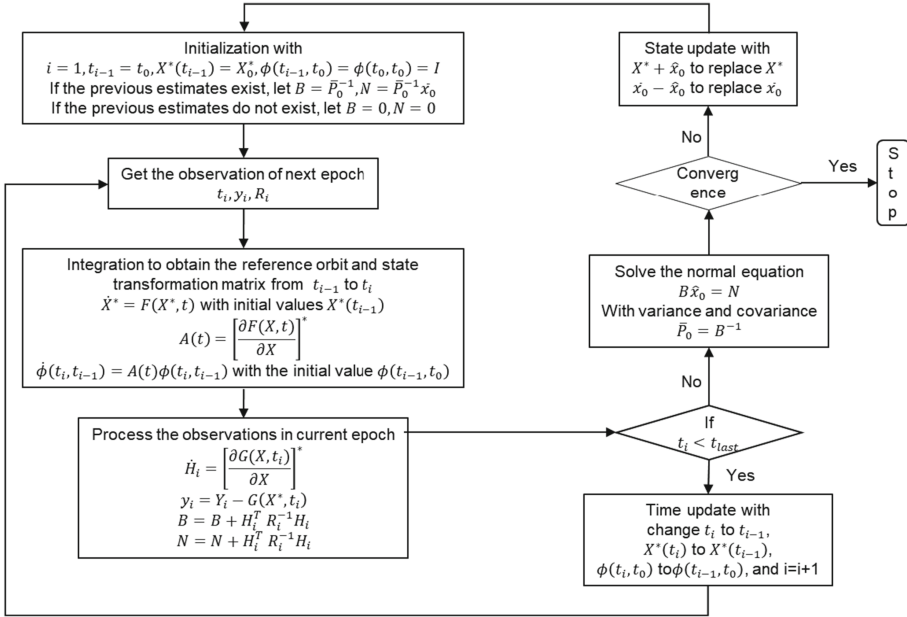


Fig. 1. LEO satellite orbit determination processing flow

2.3 Relative Orbit Determination Principle of LEO Constellation

The key to achieve high-precision inter satellite relative orbit determination was the fixation of GNSS carrier phase ambiguity, including the fixation of single satellite single difference ambiguity (i.e. single station ambiguity) and formation satellite double difference ambiguity. The single difference ambiguity fixation or double difference ambiguity fixation technology could not ensure the optimal absolute and relative orbit determination accuracy. The key to solve this problem was the comprehensive fixation of single satellite single difference ambiguity and inter satellite double difference ambiguity.

Due to space constraints, this section only briefly describes the non-difference ambiguity. In order to fix the ambiguity, the combined ambiguity of non-difference de ionosphere was usually decomposed into integer wide lane ambiguity and floating point narrow lane ambiguity.

$$\bar{N}_{r,IF}^s = \frac{f_1 f_2}{f_1^2 - f_2^2} N_{r,WL}^s + \frac{f_1}{f_1 + f_2} \bar{N}_{r,NL}^s \quad (5)$$

where,

$$\begin{aligned} N_{r,WL}^s &= N_{r,1}^s - N_{r,2}^s \\ \bar{N}_{r,NL}^s &= N_{r,NL}^s + d_{r,NL} - d_{NL}^s \\ d_{r,NL} &= d_{r,IF} (f_1 + f_2) / f_1 \\ d_{NL}^s &= d_{IF}^s (f_1 + f_2) / f_1 \\ N_{r,NL}^s &= N_{r,1}^s \end{aligned} \quad (6)$$

where, $(d_{r,NL}, d_{NL}^s)$ were the narrow lane decimal deviation FCB at the receiver end and navigation satellite end respectively. The ambiguity of wide roadway and narrow roadway could be fixed in two steps. The wide lane ambiguity was fixed by HMW combination. The narrow lane ambiguity could be calculated based on the integer wide lane ambiguity and the ionospheric combination floating-point ambiguity calculated by orbit determination. After the wide lane and narrow lane ambiguity were fixed, the ionospheric combination ambiguity could be fixed also.

Single difference ambiguity fixation was to restore the integer characteristics of single difference wide lane and narrow lane ambiguity by constructing single difference ambiguity cancellation receiver FCB and correcting navigation satellite FCB. And then the single difference ionospheric combined ambiguity was calculated. By constructing double difference ambiguity, the hardware delay at the receiver and navigation satellite could be completely eliminated, that is, the double difference ambiguity itself has integer characteristics. After fixing the ambiguity of double difference wide roadway and narrow roadway in turn, the combined ambiguity of double difference de ionosphere could be calculated. In orbit determination data processing, the single difference ambiguity of the reference satellite and the double difference ambiguity between the reference satellite and other satellites were fixed. That is, the above formula was used to constrain the non-difference de ionospheric combination ambiguity in orbit determination at the same time, so as to realize the comprehensive fixation of single difference double difference ambiguity.

3 Simulation Calculation of Orbit Determination and Time Synchronization of LEO Constellation

In this chapter, taking the cluster composed of six LEO satellites as an example, the orbit determination and time synchronization simulation calculation were carried out. The corresponding orbit settings of 6 satellites were listed in Table 1. The geometric relationship in space was shown in Fig. 2.

Table 1. LEO satellite orbit setting

Satellite	Semimajor axis	Eccentricity	Inclination [°]	Perigee angular distance [°]	True proximal angle [°]	Ascending node right ascension [°]
L01	7503.14	0.0	89	0	9	0
L02	7503.14	0.0	89	0	6	0
L03	7503.14	0.0	89	0	9	3
L04	7503.14	0.0	89	0	6	3
L05	7503.14	0.0	89	0	9	6
L06	7503.14	0.0	89	0	6	6

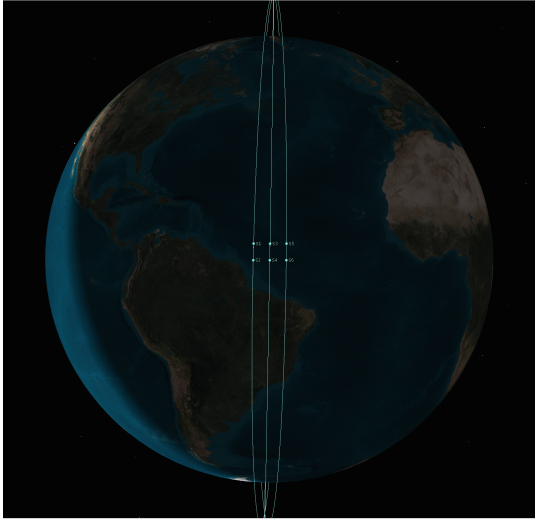


Fig. 2. Space distribution of the LEO constellation consisted of 6 satellites

According to the above orbital elements and orbital dynamics model and geometric observation model shown in Table 2, the simulation orbit was generated, and the on-board GPS, GLONASS, Galileo and BDS observations were further simulated. The dynamic orbits of the above six satellites were calculated by using the satellite GNSS observations of GPS, GLONASS, Galileo and BDS, and compared with the reference orbit to evaluate their orbit determination accuracy. The specific evaluation results were given in Table 3. Taking L01 satellite as an example, the absolute orbit determination error in the simulation period was shown in Fig. 3.

Table 2. Dynamic and observational model for simulation

Orbital dynamics model	
Earth’s Gravity	EIGEN_6C 120 order, including time-varying terms
Earth solid tide	IERS Conventions 2010
Earth solid polar tide	IERS Conventions 2010
Ocean tide	EOT13a, 30 order
Ocean polar tide	IERS Conventions 2010
Three-body perturbation	Sun, Moon, Mercury, Venus, Mars, Jupiter Saturn, Uranus, Neptune, Pluto Planetary ephemeris: JPL DE405

(continued)

Table 2. (continued)

Orbital dynamics model	
Solar radiation pressure	Box-wing model Including earth shadow and moon shadow
Atmospheric drag	Atmospheric density: DTM94
Earth albedo radiation	Non
Relativistic effect	IERS Conventions 2010
Geometric observation model	
Receiver clock error	Non
Observed value	GPS: L1, L2; GLONASS: G1, G2; Galileo: E1, E5a; BDS: B1I, B3I Phase noise: 2 mm Pseudo range noise: 20 cm
Orbit and clock error of GNSS	WUM product
Ionosphere	Non
Ambiguity	Non

Table 3. Absolute orbit determination accuracy of 6 LEO satellites

Satellite	Tangential (mm)	Normal (mm)	Radial (mm)	3D (mm)
L01	24.5	7.3	8.6	27
L02	24.8	7.3	8.8	27
L03	23.7	8.2	8.0	26
L04	23.1	8.3	7.7	27
L05	19.5	7.2	6.8	22
L06	19.6	6.9	7.1	22

The relative orbit determination accuracy of 6 satellites was simulated and evaluated. The statistical result was shown in Table 4. Compared with L01-L02, the relative orbit determination error sequence was shown in Fig. 4.

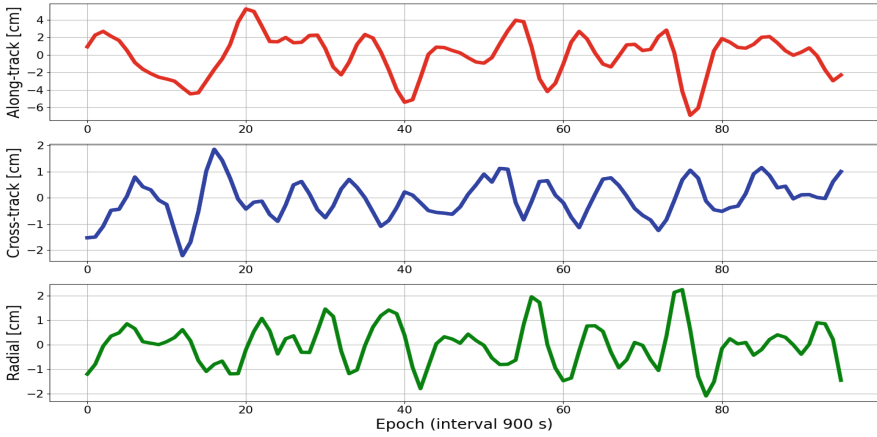


Fig. 3. L01 satellite absolute orbit determination error sequence

Table 4. Relative orbit determination accuracy of 6 satellites

Satellite	Relative orbit determination accuracy (mm)
L01-L02	2.67
L01-L03	12.56
L01-L04	32.45
L01-L05	12.71
L01-L06	27.90

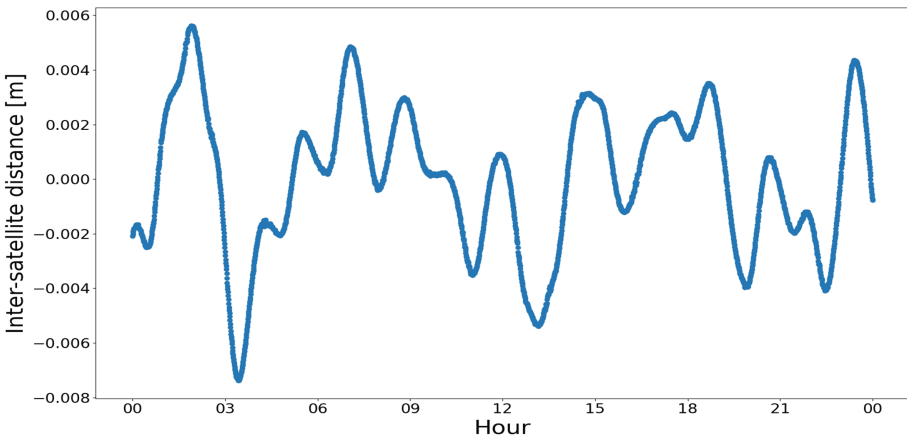


Fig. 4. Relative orbit determination error sequence of L01 and L02 satellites

The relative time synchronization of 6 satellites was simulated and evaluated. And the mean value of 6 satellites was taken as the benchmark. The statistical results were shown in Table 5, and the L01 time synchronization error sequence was showed in Fig. 5.

Table 5. Time synchronization results of 6 satellites

Satellite	Time synchronization accuracy (ns)
L01	0.185
L02	0.184
L03	0.194
L04	0.187
L05	0.165
L06	0.162

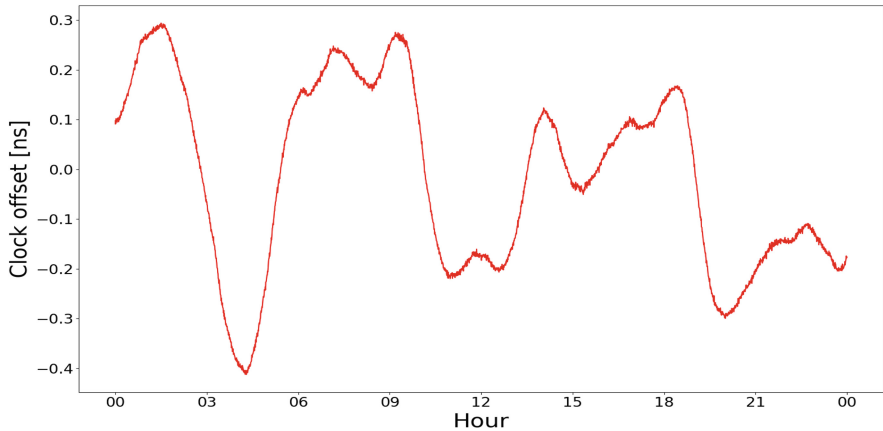


Fig. 5. Time synchronization error of L01 satellite

To sum up, based on the spaceborne GNSS receiver, LEO satellites could achieve centimeter level absolute orbit determination and relative orbit determination accuracy. And the time synchronization was less than 0.2ns, which provided a good orbit and time basis for LEO satellites.

4 Extended Application of Navigation and Positioning Based on LEO Constellation

As mentioned above, under the multi-layer architecture of GNSS system, LEO constellation could better determine its own orbit and clock error. This provided good space-time basic information for LEO constellation to realize reconnaissance, detection, communication and other mission objectives. At the same time, in view of the characteristics of

strong landing power and fast movement speed of LEO satellites, it was also possible for them to provide navigation and positioning expansion services to ground users.

Taking the satellite cluster composed of six LEO satellites as an example, based on the absolute orbit determination and time synchronization results completed in Sect. 3, and considering the error factors of transmission section and receiver, this chapter studied the degraded backup navigation service performance that LEO satellites can provide.

It is assumed that LEO satellites provided pseudo range and pseudo range rate observation data. The pseudo range measurement accuracy (including space segment, transmission segment and receiver) was 4m, and the pseudo range rate measurement accuracy was 0.05m/s (including space segment, transmission segment and receiver).

The positioning equation of pseudo range and pseudo range rate combined measurement was as follows.

$$\begin{cases} \rho_i = \sqrt{(x - x_{si})^2 + (y - y_{si})^2 + (z - z_{si})^2} + c\delta t_u \\ \dot{\rho}_i = \frac{(x - x_{si})(\dot{x} - \dot{x}_{si}) + (y - y_{si})(\dot{y} - \dot{y}_{si}) + (z - z_{si})(\dot{z} - \dot{z}_{si})}{\sqrt{(x - x_{si})^2 + (y - y_{si})^2 + (z - z_{si})^2}} + c\delta t_f \end{cases} \quad (7)$$

where, ρ_i , $\dot{\rho}_i$ was the pseudo range measurement value and pseudo range rate measurement value received by the user, (x_{si}, y_{si}, z_{si}) was the satellite position, $(\dot{x}_{si}, \dot{y}_{si}, \dot{z}_{si})$ was the satellite speed, (x, y, z) was the user receiver position, $(\dot{x}, \dot{y}, \dot{z})$ was the user receiver speed, static user is 0, δt_u was the receiver clock difference, and δt_f was the user frequency difference.

One of the six LEO satellites was selected for navigation positioning expansion simulation. The available time for satellite transit was 8 min. The static points at different latitudes of 0–90° were taken to filter and locate. The available arc segments of the satellite were shown in Fig. 6.

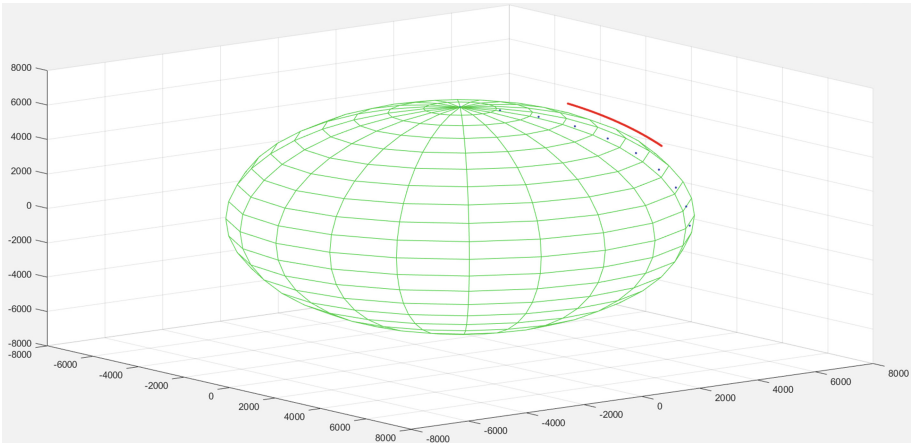


Fig. 6. Visible arc trajectory of single satellite

Considering the influence of rounding and overflow errors, the indirect filtering state vector was adopted as follows.

$$X = [\Delta x, \Delta y, \Delta z, \delta t_u, \delta t_f]^T \tag{8}$$

The initial position error was taken as 10 km. The process noise of the user receiver was the noise of the constant temperature crystal oscillator, as shown below. The position noise was simplified to 0.

$$Q = \begin{bmatrix} 2h_0 \cdot \Delta t + h_{-2} \frac{8\pi^2 \cdot \Delta t^3}{3} & 4\pi^2 h_{-2} \cdot \Delta t^2 \\ 4\pi^2 h_{-2} \cdot \Delta t^2 & 8\pi^2 h_{-2} \cdot \Delta t \end{bmatrix} \tag{9}$$

where, $h_0 = 8 \times 10^{-20}$, $h_{-1} = 2 \times 10^{-21}$, $h_{-2} = 4 \times 10^{-23}$, Δt was the filtering interval, taken as 1s.

Only pseudo range was used for positioning. Only pseudo range rate was used for positioning. Pseudo range and pseudo range rate were used for simulation positioning. The positioning error evaluation results of different latitude points were shown in Fig. 7, 8 and 9.

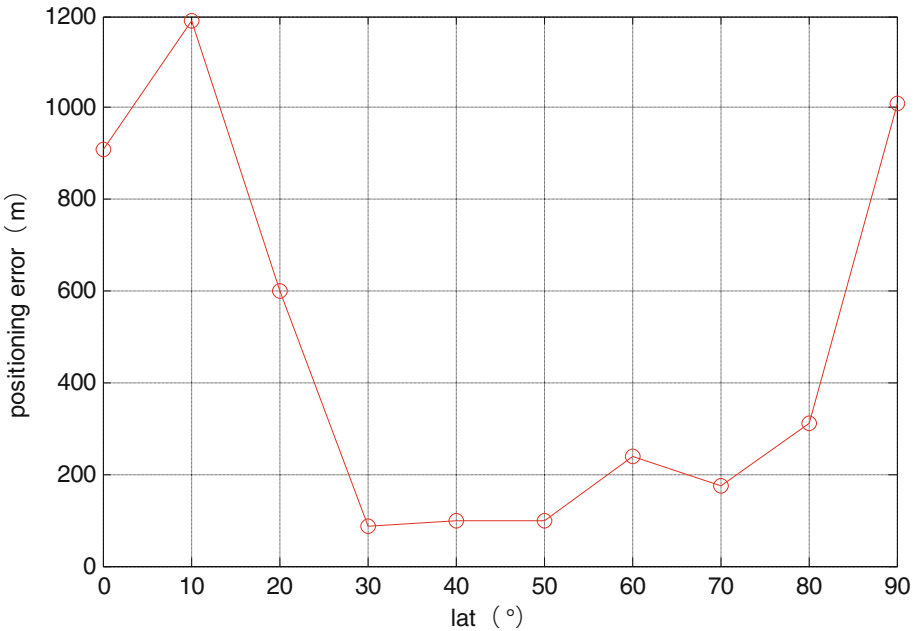


Fig. 7. Single satellite positioning results using pseudo range only

The simulation results show that only the pseudo range positioning error was 87 m–1160 m. Only the pseudo range rate positioning error was 120 m–1250 m. And the pseudo range and pseudo range rate positioning error was 2 m–350 m. When the number of satellites available for navigation and positioning was small, the combination of pseudo range and pseudo range rate could significantly improve the positioning results.

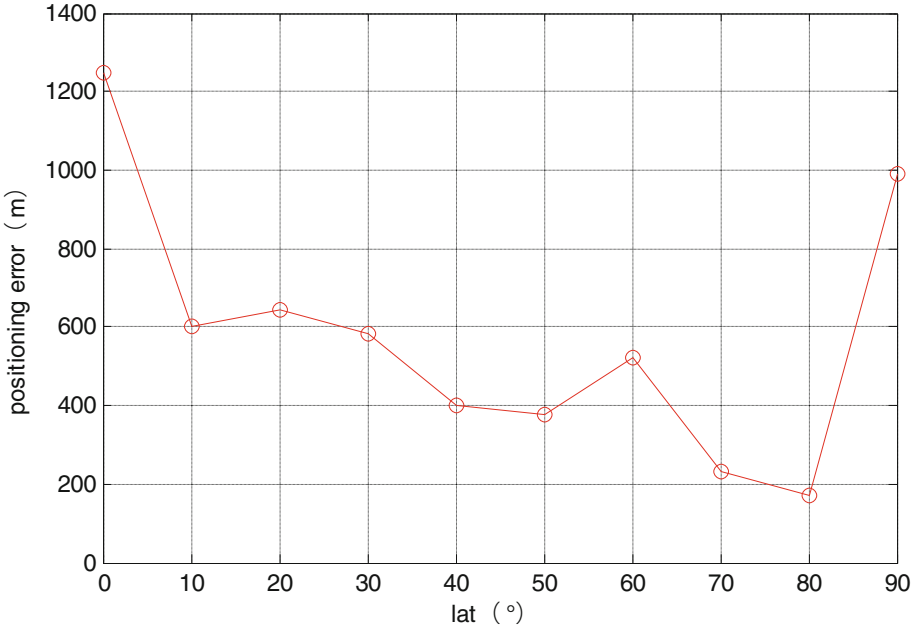


Fig. 8. Single satellite positioning results using only pseudo range rate

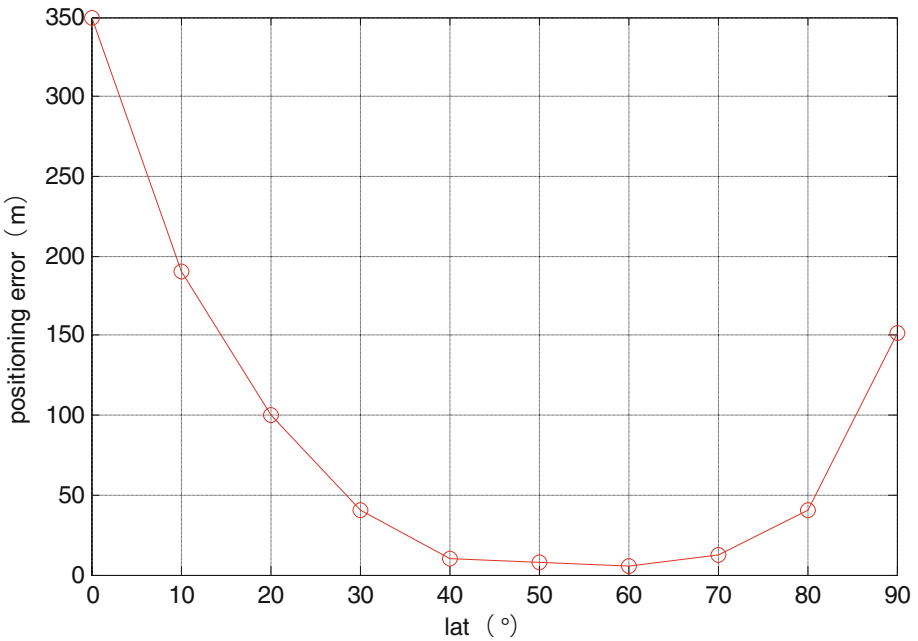


Fig. 9. Positioning results of single satellite using pseudo range and pseudo range rate

5 Conclusion

GNSS technology provides a convenient, efficient and objective mean for precise orbit determination, relative orbit determination and time synchronization solution of LEO constellation. Based on GNSS system, this paper calculated the orbit and clock difference of a small constellation composed of six LEO satellites. The simulation results showed that LEO satellites could achieve centimeter level absolute orbit determination and relative orbit determination accuracy and sub nanosecond time synchronization accuracy.

Under the multi-layer architecture of GNSS system and LEO satellite, LEO constellation relies on GNSS system to maintain its relatively reliable space-time benchmark. This provides a technical basis for LEO satellites to provide navigation and positioning backup services for ground users. When the number of LEO satellites available is small, the combination of pseudo range and pseudo range rate observation data can better improve the positioning results of pseudo range only or pseudo range rate only, especially the geometric singularity with the worst positioning results.

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