Adaptive Observation Noise Variance Algorithm Based on Innovation Repair

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Abstract. When the observation value is abnormal, the traditional robust estimation method only reduces the weight of the abnormal observation value, but does not repair the abnormal residual, and the abnormal residual statistics seriously limit the accuracy of noise variance estimation. Aiming at the above problems, this paper proposes an adaptive observation noise variance algorithm based on innovation repair. First, use the IGG III method to construct an equivalent weight function to reduce or give up weight from abnormal observations. Secondly, considering that abnormal observed values will lead to abnormal innovation, this paper uses the zero-mean constraint of innovation to estimate the sum of abnormal innovation, and then distributes it according to the ratio of abnormal innovation variance, to reduce the influence of abnormal innovation on statistical information. At the same time, considering that the innovation will approach the real situation gradually with the convergence of filtering, the innovation variance is calculated by combining the forgetting factor function to improve the accuracy of its statistical information. Finally, the observation noise variance is estimated in real-time by the function relationship between the innovation variance and the observation value variance. Simulation results show that the proposed method can guarantee the accuracy of noise variance estimation even with gross error and prior observation noise deviation. The accuracy of the filtering result is improved.

Keywords: Observation noise estimation · Innovation repair · Forgetting factor · Robust

1 Introduction

As the core algorithm of GNSS data processing, Kalman filter (KF) results are optimal with minimum mean square error, unbiasedness, and consistency under the assumption that the error is Gaussian distribution, the mathematical model is determined and the noise statistical characteristics are known prior [1, 2]. However, in the actual GNSS measurement, the observation value of gross error is inevitable, which greatly limits the filtering estimation accuracy. Simultaneous observation noise is usually determined by empirical models, such as the height Angle model and the signal-to-noise ratio model [3–5]. The imprecise noise level will affect the determination of the gain matrix and it

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is difficult to guarantee the optimality of filtering results. Therefore, in GNSS precision positioning and data processing, to ensure the accuracy and reliability of data processing results, it is necessary to weaken the influence of gross error as much as possible and obtain a more accurate prior noise variance matrix. Therefore, robust algorithms and adaptive noise variance algorithms are particularly important.

The essence of robust is to construct outlier tests by using residuals and their variances for hypothesis testing, and reduce or give up weight the failed data according to certain rules, to weaken the contribution of abnormal data and ensure the parameters estimation accuracy [6]. The key lies in the construction of equivalent weight functions, commonly used equivalent weight functions include the Huber function [7], the Hampel function [8] and the IGGIII function [9]. All of these functions obtain the weight reduction factor of the residual difference constant observation value through weight selection iteration to obtain the equivalent weight to ensure the estimation accuracy of parameters, but do not repair the abnormal observation value or residual. At the same time, an inaccurate observation noise covariance matrix will also affect the accuracy of the gross error test, resulting in errors and omissions of gross error, thus affecting the robust effect [10].

The essence of noise variance estimation is that the statistical variance based on residuals does not accord with the theoretical variance, and the noise variance is adjusted adaptively. The specific method is to calculate the variance of observation noise in real-time by the functional relationship between the variance of observation and the variance of innovation under the maximum likelihood estimation criterion. However, when the observed values appear gross error, it will cause innovation anomaly, and the abnormal innovation statistics seriously restrict the accuracy of noise variance estimation.

It can be seen that to weaken the influence of gross error as much as possible and obtain a more accurate prior noise variance matrix, it is not the superposition of the robust algorithm and the observation noise variance adaptive algorithm. According to the above problem, this paper proposes an adaptive observation noise variance algorithm based on innovation repair. Finally, the effectiveness of the proposed algorithm is verified by simulation.

2 Adaptive Observation Noise Variance Algorithm Based on Innovation Repair

In this paper, based on the robust algorithm, the constraint of zero-mean and variance ratio of the innovation is considered to repair the abnormal innovation, and the forgetting factor function is combined to reflect the current state of the innovation, so as to ensure the accuracy of noise estimation, and accurate observation noise variance matrix can also improve the robust effect. The specific formula principle will be described below.

2.1 Robust Kalman Filtering

Kalman filter is a linear model, which can estimate system state by inputting observation data, function model and prior noise covariance matrix. Its basic formula is as follows:

$$\begin{cases} \boldsymbol{X}_k = \boldsymbol{A}\boldsymbol{X}_{k-1} + \boldsymbol{W}_{k-1} \quad \boldsymbol{Q} \\ \boldsymbol{Z}_k = \boldsymbol{C}\boldsymbol{X}_k + \boldsymbol{V}_k \quad \boldsymbol{R} \end{cases} \tag{1}$$

where, $\boldsymbol{X}_{k-1}$ is the state vector of $k-1$ epoch, $\boldsymbol{Z}_k$ is the observed value of k epoch, $\boldsymbol{A}$ is the state transition matrix, $\boldsymbol{C}$ is the observation coefficient matrix, $\boldsymbol{W}_{k-1}$ is the process noise, $\boldsymbol{V}_k$ is the observed noise, suppose $\boldsymbol{W}_{k-1}$ and $\boldsymbol{V}_k$ is white gaussian noise with variances $\boldsymbol{Q}$ and $\boldsymbol{R}$ respectively. Robust Kalman filter introduces equivalent weight observation noise variance based on Kalman filter, in the following form:

$$\overline{\boldsymbol{R}}(i,i) = \frac{1}{\alpha_i}\boldsymbol{R}(i,i) \tag{2}$$

where, $\boldsymbol{R}(i,i)$ is the i-th row and i-th column element of the $\boldsymbol{R}$ matrix, α_i a is the adaptive downgrading factor and takes a value in the range of (0, 1]. Through adaptive factors, the weight of observation value can be adjusted adaptively, to reduce the contribution of gross error observation value and suppress the influence of gross error treatment on estimation parameters.

Because the abnormal observation value will lead to the abnormal innovation, and normally, the innovation follows the zero-mean white noise distribution, the detection and recognition of gross error can be realized through the innovation sequence. The innovation $\boldsymbol{Y}_k$ and its variance-covariance matrix is calculated:

$$\boldsymbol{Y}_k = \boldsymbol{Z}_k - \boldsymbol{C}\boldsymbol{X}_k^- \tag{3}$$

$$\boldsymbol{D}_Y = \boldsymbol{C}\boldsymbol{P}_k^-\boldsymbol{C}^T + \boldsymbol{R} \tag{4}$$

where, $\boldsymbol{X}_k^-$ and $\boldsymbol{P}_k^-$ are the state predicted value and their variance of k epoch respectively. In actual GNSS observation, it is usually unequal precision observation, so it is necessary to unify the accuracy of innovation and calculate standardized innovation:

$$\overline{\boldsymbol{Y}}_k(i) = \boldsymbol{Y}_k(i)/\sqrt{\boldsymbol{D}_v(i,i)} \tag{5}$$

After obtaining the standardized innovation vector of k epoch, the hypothesis testing can be constructed by the mean square error. However, the gross error will affect the calculation of the mean square error, so median is used to calculate the mean square error [11]:

$$\sigma = median(\overline{\boldsymbol{Y}}_k)/0.6745 \tag{6}$$

The test quantity is:

$$\tilde{\boldsymbol{Y}}_k = \overline{\boldsymbol{Y}}_k/\sigma \tag{7}$$

This paper falling weight factor calculated with the IGG III method:

$$\alpha_i = \begin{cases} 1 & \left|\tilde{Y}_i\right| \le k_0 \\ \frac{k_0}{\left|\tilde{Y}_i\right|}\left(\frac{k_1-\left|\tilde{Y}_i\right|}{k_1-k_0}\right)^2 & k_0 < \left|\tilde{Y}_i\right| < k_1 \\ 1\times 10^{-10} & \left|\tilde{Y}_i\right| > k_1 \end{cases} \tag{8}$$

where, $\tilde{Y}_i$ is the i-th member of the $\tilde{Y}_k$ vector, k_0 is constant, usually 1.5–3.0, k_1 is constant, usually 3.5–8.0. By iteration to convergence, the best adaptive weight reduction factor can be obtained to resist gross error.

2.2 Abnormal Innovation Repair Program

The above robust method only reduces the weight of abnormal innovation, but does not repair it, which will affect its statistical information and then affect the noise estimation result. Therefore, this paper proposes an innovation restoration method considering the innovation variance ratio and the innovation zero-mean distribution constraints.

First, the innovation was regarded as abnormal innovation and marked as a group $\boldsymbol{Y'}$, and the rest as a normal innovation group $\boldsymbol{Y''}$. Under normal circumstances, the innovation follows the zero-mean distribution, so the zero-mean constraint can be expressed as:

$$0 = \sum \hat{\boldsymbol{Y}}' + \sum \boldsymbol{Y''} \tag{9}$$

where, $\hat{\boldsymbol{Y}}'$ is the optimal valuation of abnormal innovation, that is repaired innovation.

The Eq. (9) can be used to calculate the sum of the abnormal innovation after repair, and then allocate it according to the variance ratio of the abnormal innovation, so that the result of each abnormal innovation after repair can be obtained:

$$\hat{Y}'_j = \frac{D_{Y'_j}}{tr(\boldsymbol{D}_{Y'})} \sum \hat{\boldsymbol{Y}}' \tag{10}$$

The restored innovation series regains the characteristics of the zero-mean distribution, while the innovation value coupons is strictly assigned according to their respective precision. The innovation with large variances also have larger values after restoration than the rest of the innovation, in line with the distribution of the actual innovation and thus with the correct statistical significance.

2.3 Observation Noise Estimation Method Based on Abnormal Innovation Repair

The observation noise level changes slowly in the actual GNSS navigation and positioning, and the earliest historical information cannot well describe the current observation level. Therefore, to adapt to the contribution of old and new innovation, the forgetting factor function is added to calculate the innovation variance. The forgetting factor function is expressed as follows:

$$\beta_i = \frac{i^2}{k(k+1)(2k+1)}, i = 1, 2, \ldots k \tag{11}$$

Use $\hat{\boldsymbol{Y}}$ to represent the innovation vector after repair, the optimal estimation of the innovation actual variance of the $k+1$ epoch over a window with length k is:

$$\hat{\boldsymbol{D}}_{Y_{k+1}} = \sum_{j=1}^{k} \beta_j \hat{\boldsymbol{Y}}_j \hat{\boldsymbol{Y}}_j^T \tag{12}$$

Compared with the traditional statistical variance method, after adding the forgetting factor function the new innovation has a larger weight ratio and the old innovation has a smaller contribution, which can weaken the influence of the initial period jitter of filtering on the innovation, and its statistical information can better reflect the current innovation state.

After obtaining the optimal valuation of the innovation variance, let $\boldsymbol{D}_Y = \hat{\boldsymbol{D}}_Y$, through the functional relationship between the innovation variance $\boldsymbol{D}_Y$ and the observed noise variance $\boldsymbol{R}$, obtain the optimal valuation of the observed noise $\hat{\boldsymbol{R}}$:

$$\hat{\boldsymbol{R}} = \hat{\boldsymbol{D}}_Y - \boldsymbol{C}\boldsymbol{P}_k^- \boldsymbol{C}^T \tag{13}$$

Since the noise of carrier phase/pseudo-range observation values between different satellites is not correlated, to avoid estimation results exceeding the actual situation, the following constraints should be applied in actual GNSS measurement [10]:

$$\hat{\boldsymbol{R}}'(i,j) = \begin{cases} \hat{\boldsymbol{R}}(i,j)\ m_1 \le \frac{\hat{\boldsymbol{R}}(i,j)}{\boldsymbol{R}(i,j)} \le m_2,\, i=j \\ m_1\boldsymbol{R}(i,j)\ \frac{\hat{\boldsymbol{R}}(i,j)}{\boldsymbol{R}(i,j)} \le m_1,\, i=j \\ m_2\boldsymbol{R}(i,j)\ \frac{\hat{\boldsymbol{R}}(i,j)}{\boldsymbol{R}(i,j)} \ge m_2,\, i=j \\ 0 \quad i \ne j \end{cases} \tag{14}$$

where, i and j represent the rows and columns of the matrix respectively; $\boldsymbol{R}$ is the observed noise variance under the empirical model; m_1 is $0.1 \sim 0.5$; m_2 is 2–5. Users can select the value based on actual observation.

The proposed algorithm firstly uses the variance expansion method to weaken the contribution of abnormal observations, and does not directly eliminate abnormal observations is to improve the utilization rate of data and ensure the reliability of the algorithm (for example, when only four satellites can be observed, deleted data will not be able to position). Secondly, abnormal innovation is repaired, and combine the forgetting factors to calculate the innovation variance that matches the current state. Finally, the covariance matching method is used to estimate the observed noise variance, and the estimation is limited within a reasonable range.

3 Experiments

To verify the correctness and effectiveness of the proposed method in this paper, a constant velocity target tracking experiment is designed. The observed value is assumed to be 10 dimensions, and the target's motion state includes position and velocity. The filtering model is as follows:

$$\boldsymbol{X}_{k+1} = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} \boldsymbol{X}_k + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \boldsymbol{W}_k \tag{15}$$

$$\boldsymbol{Z}_k = \begin{bmatrix} 1 & 0 \\ \vdots & \vdots \\ 1 & 0 \end{bmatrix} \boldsymbol{X}_k + \boldsymbol{V}_k \tag{16}$$

where, T is the sampling interval, set to 1s here, the length of the design data is 5000 epoch, and the initial truth value of the state is: $\boldsymbol{X}_1 = \begin{bmatrix} 1 \\ 0.1 \end{bmatrix}$, the initial displacement is 1m and the initial velocity is 0.1 m/s, maintaining uniform motion. GNSS actual measurement with satellite and observation environment changes, the observation noise exists slow change process, to more intuitive reflect the algorithm of this paper can track the change of the environment, the observation noise is set to the form of order variation, therefore, the a priori noise covariance matrix is designed as:

$$\boldsymbol{Q} = q\begin{bmatrix} T^3/3 & T^2/2 \\ T^2/2 & T \end{bmatrix} \tag{17}$$

$$\boldsymbol{R} = \begin{bmatrix} 1+r & 0 & \cdots & 0 \\ 0 & 1+r & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1+r \end{bmatrix}_{10\times 10} \tag{18}$$

where, state parameter: $q = 1 \times 10^{-8}$, r is the amount of observed noise variation, which is 0 initially and increases by 0.25 every 1000 epoch. The innovation variance was counted from the first epoch, and the observation noise was estimated online from the 500th epoch. The RMSE of the state valuation quality is used as an evaluation indicator for the experimental results:

$$\hat{X}^k_{RMSE} = \sqrt{\frac{1}{k}\sum_{i=1}^{k}\left(\hat{\boldsymbol{X}}_k - \overline{\boldsymbol{X}}_k\right)^2} \tag{19}$$

where k denotes the kth epoch; $\hat{\boldsymbol{X}}_k$ denotes the state valuation of the kth epoch; $\overline{\boldsymbol{X}}_k$ denotes the state truth value of the kth epoch.

3.1 Analysis of Variance Estimation Results of Observation Noise

To verify the correctness of the estimation of the variance of the observation noise proposed in this paper, the gross error is not added temporarily, and the diagonal matrix with the variance of the initial observation noise is set as 10, and two experimental schemes are set:

- Scheme 1: Processing with the Kalman filter;
- Scheme 2: Processing with the method in this paper.

Figure 1 shows the noise estimation results of the second observation component of Scheme 2, and the other observation noise estimation results are similar. Table 1 shows the RMSE of filtering results under the two schemes.

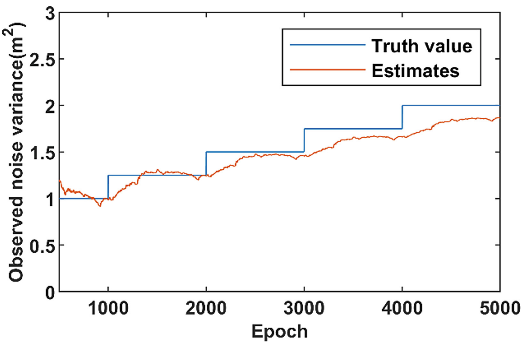

Fig. 1. Observation noise $\boldsymbol{R}(2,2)$ estimation results

Table 1. Comparison of RMSE between the two schemes

	Displacement (m)	Speed (m/s)
Scheme 1	0.1028	0.0025
Scheme 2	0.0800	0.0024

As can be seen from the above results:

(1) When the prior observed noise is biased, the noise variance estimation result of the method in this paper is close to the true value. When the real observed noise variance changes, the estimation result of this method also begins to change and gradually approaches the true value, but with a certain lag, requiring a convergence space of about 300 epochs. It can be concluded that the observation noise variance estimation algorithm proposed in this paper is correct, and can track the changes of the observation environment and adapt to the noise variance matrix.

(2) After the proposed algorithm adaptively observation noise matrix, RMSE of state component in filtering result is improved by 22.18% compared with Scheme 1, and velocity component is slightly better than that of Scheme 1. It can be concluded that the adaptive observation noise variance can significantly improve the filtering accuracy, and the wrong prior observation noise variance will seriously affect the filtering result. Therefore, the adaptive observation noise is of great significance in the application of precision data processing.

3.2 Analysis of the Effect of Resisting Abnormal Innovation

To verify the effectiveness of the method of repairing abnormal innovation in this paper, the gross error was added to the observation value, a diagonal matrix with the variance of initial observation noise of 10 was set, and two experimental schemes were set:

- Scheme 1: Adding gross error and prior observation noise deviation, do not repair abnormal innovation, robust and noise estimation processing;

- Scheme 2: Adding gross error and prior observation noise deviation, using the method presented in this paper.

The method of adding gross errors is as follows: starting from 1000 epoch, normal distribution random gross errors with root mean square error of 100 are added into the second and seventh observation components every 100 epoch. Figure 2 shows the innovation time series of the second observed components with gross errors in the two schemes. Figure 3 shows the estimation results of the observed noise variance of the second observed component with gross errors in the two schemes. The RMSE of the two schemes is given in Table 2.

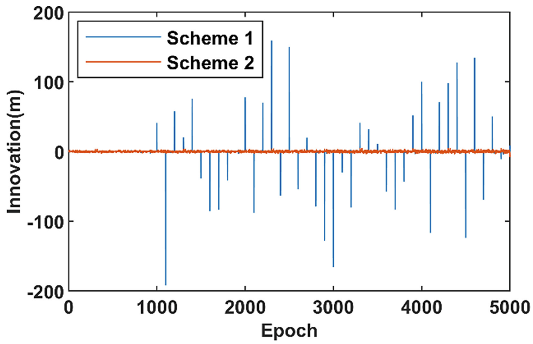

Fig. 2. Innovation $\boldsymbol{Y}(2)$ time series

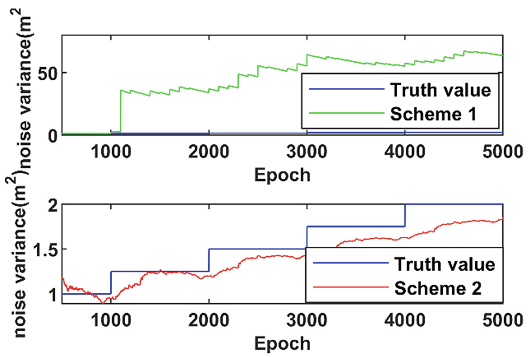

Fig. 3. Observation noise $\boldsymbol{R}(2,2)$ estimation results

As can be seen from the above results:

(1) When the observed value has a gross error, its corresponding innovation will also appear abnormal. Traditional robust methods do not repair the abnormal innovation, and there are still many anomalies in the innovation sequence. In Scheme 2, the innovation repair method proposed in this paper was used, and abnormal innovation was successfully repaired without any abnormal situation similar to Scheme 1, and

Table 2. Comparison of RMSE between the two schemes

	Displacement (m)	Speed (m/s)
Scheme 1	0.0915	0.0025
Scheme 2	0.0800	0.0024

the innovation sequence after the overall repair was relatively stable. It can be concluded that the innovation repair method proposed in this paper is effective and can successfully repair all abnormal innovations.

(2) When innovation is abnormal, the estimation result of observation noise will have serious deviation, as shown in Fig. 3 Without innovation repair, the estimation result of observation noise variance has reached about 60, seriously deviating from the truth value, and the estimation result of noise variance is significantly affected by the gross error, with many order changes. In Scheme 2, abnormal innovation is repaired by the method presented in this paper, and the estimation result of observation noise variance is still close to the real situation, and the change of observation environment can be tracked, and the result of noise estimation is close to that of the observation value without gross error. It can be concluded that abnormal innovation will seriously affect the estimation result of observation noise, and the estimation accuracy of observation noise is guaranteed after the innovation is repaired by the method in this paper.

(3) The proposed algorithm is used to repair abnormal innovation and perform observation noise estimation. The RMSE of the state component of the filtering result is 0.0800, which is equivalent to the filtering accuracy without gross error and 16.39% higher than that of Scheme 1. The RMSE of the velocity component is 0.0024, which is equivalent to the filtering accuracy without gross error and slightly better than the result of Scheme 1. Can be concluded from this, not repairing the abnormal innovation directly estimate the observation noise can seriously affect the filtering result. The method proposed in this paper can effectively weaken the gross error influence on the observation noise variance estimation, in the presence of gross error in the observations and the prior observation noise variance has a deviation, still can ensure the accuracy of filtering, enough to verify the method is correct and effective.

4 Conclusion

In this paper, the effect of gross error observation on noise estimation is studied, and it is concluded that abnormal observation values will cause abnormal innovation, and the statistical information of abnormal will seriously restrict the accuracy of noise estimation. And proposes an adaptive observation noise variance algorithm based on innovation repair, simulation experiments have verified the correctness and effectiveness of the method proposed in this paper, it can effectively weaken the abnormal innovation on the effects of observation noise variance estimation, the observation noise variance estimation results close to the true value, and can be adaptive changes in the observed noise, filtering precision is improved.

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