Performance Analysis of Probabilistic Shaped Polar Code in 5G

Congyi Wang, Lin Zhou, Lei Xu, Qiwang Chen, and Yidong Ke

Abstract As a coding scheme in the scenario of enhanced mobile bandwidth in 5G communication, polar codes have been deeply studied due to its communication reliability. Probability shaping is an important method to approach the channel capacity limit. Therefore, it is necessary to study the combination of polar codes and probabilistic shaping. In this paper, we analyze the performance of the shaping systematic polar codes (SPC) and the shaping non-systematic polar codes (NSPC) in SC decoding algorithms. It is proposed to combine systematic polar codes with probabilistic shaping technology, hoping to induce a sub-optimization symbol probability distribution. We found that in the scheme of probabilistic shaping based on the source sequence, the non-systematic polar code encoding method will force the bit probability in the encoded codeword to be equal, which will greatly affect the performance gain brought by the probability shaping. Therefore, in this paper, we use systematic polar encoder to protect the unequal probability characteristics of 0 and 1 in the source sequence, so that the distribution of the channel input symbols tends to the Maxwell Boltzmann distribution and achieve a performance gain of 0.6 dB.

1 Introduction

Arikan first proposed polar codes in 2008 [1]. Polar codes are proven to be a channel coding scheme that can approach the symmetric capacity of binary-input discrete memoryless channel (B-DMC). Afterwards, Arikan proposed the coding method of systematic polar codes and pointed out that the performance of systematic polar codes in bit error rate (BER) is better than NSPC [2]. Because polar codes have lower

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coding and decoding complexity and excellent codeword error correction performance, polar codes have been chosen as the channel coding scheme in 5G enhanced mobile broadband (eMBB) scenarios in recent years.

Many scholars have conducted in-depth research on probabilistic shaping techniques in NSPC. A probabilistic shaping scheme that uses a polar decoder as a shaping code generator is proposed in [3]. This scheme can avoid additional shaping code generators and at the same time avoid the addition of corresponding shaping decoders at the receiving end. The dynamic freeze bit is proposed in [4], which combines the non-systematic polarization code and the many-to-one mapper to generate an ideal channel input symbol distribution. The random number generator uses a recoverable random number seed to generate a sequence of random numbers and put into the frozen bit index. At the receiving end, these pseudo-random numbers are used as prior information to eliminate the ambiguity introduced by the many-to-one mapping. A many-to-one probability shaping technique based on low-density parity-check codes (LDPC) was sent to the fiber channel for experimental simulation in [5]. Experiments show that a nearly Gaussian PAM8 signal obtained from a uniformly distributed PAM16 achieves superior receiver power sensitivity and excellent transmit fiber power optimization in the optical fiber network.

Many scholars have studied the probabilistic shaping with NSPC in depth, but there are not many studies on the probability shaping of systematic polar codes. In this paper, we first describe the systematic and non-systematic coding methods of polar codes, and then introduce probabilistic shaping of the source. The experiment simulates the probability distribution state of the signal points after different coding methods, in order to observe the influence of the systematic polarization codes and the non-systematic polarization codes on the probabilistic shaped source. Finally, we experimentally simulated the performance of the systematic shaped polar codes and the non-systematic shaped polar codes under the SC decoding algorithm, and compared them with the performance of uniformly distributed signal points.

2 Related Work

2.1 Non-Systematic Polar Coding

Polar codes are a kind of (N, K) linear codes, N represent the length of the codeword and K represent the number of information bits. The channel is divided into N virtual sub-channels. Through the Gaussian approximation method and the polarization weight method, K sub-channels with the highest reliability can be selected to transmit information bits, the remaining $N - K$ sub-channels are used to transmit fixed bits known to both the receiver and the transmitter. Fixed bits are called frozen bits.

For a binary source sequence u with code length N, the non-systematic codewords x can be obtained by

$$x = uG_N \tag{1}$$

where x is the codewords, G_N represent a $N \times N$ matrix. The generator matrix G_N can be expressed as

$$G_N = B_N F^{\otimes n}, \quad F = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \tag{2}$$

where B_N represents a column permutation matrix, which separates the odd and even columns in the matrix. $F^{\otimes n}$ represents the Kronecker power.

The source sequence u can be divided into u_A, $(u_A = u_k)$ and u_{A^c}, $(u_{A^c} = 0)$, where A contains the index of K sub-channels with the highest reliability, and A^c represents the index set of the remaining $N - K$ sub-channels. Therefore, (1) can be written as

$$x = u_k G_A \tag{3}$$

where G_A is the sub-matrix contains the row of index A in G_N.

2.2 Systematic Polar Coding and SC Decoding

The systematic polar encoding divides generator matrix G_N into multiple sub-matrix and codeword x is divided into x_A and x_{A^c}. The systematic polar codewords x_A and x_{A^c} are obtained by formula (4).

$$\begin{aligned} x_A &= u_A G_{AA} + u_{A^c} G_{A^c A} \\ x_{A^c} &= u_A G_{AA^c} + u_{A^c} G_{A^c A^c} \end{aligned} \tag{4}$$

where G_{AA^c} represents a submatrix composed of rows contained in set A and columns contained in set A^c in G_N.

When G_{AA} is an invertible matrix, there is a systematic encoder, u_A can be calculated by the following formula (5)

$$u_A = (x_A - u_{A^c} G_{A^c A})(G_{AA})^{-1} \tag{5}$$

then x_{A^c} can be obtained through (4) and (5). A new systematic polar coding method with lower hardware occupancy resources is proposed in [6]. Compared with the method in [1], this method requires less calculation.

SC decoding is an algorithm proposed in [1], which divides the received bit likelihood ratio (LR) sequence into two parts, A and A^c. For the received ith bit LR, the frozen bit $\hat{u}_i$ can be directly hard-decided to 0 when $i \in A^c$. When $i \in A$, the SC decoder calculates the LR of the ith codeword $\hat{u}_i$ according to the decision result of all $i-1$ bits before $\hat{u}_i$.

$$L_N^i(y_1^N, \hat{u}_1^{i-1}) = \frac{P_N^i(y_1^N, \hat{u}_1^{i-1}|u_i = 0)}{P_N^i(y_1^N, \hat{u}_1^{i-1}|u_i = 1)} \tag{6}$$

where y_1^N represents the channel output vector, $\hat{u}_1^{i-1}$ represents the decision result of $i-1$ bits, and P_N is the channel transition probability. Then, make a hard decision on the LR value to obtain a bit estimate $\hat{u}_i$.

$$\hat{u}_i = \begin{cases} 0, & \text{if } L_N^i(y_1^N, \hat{u}_1^{i-1}) \geq 1 \\ 1, & \text{if } L_N^i(y_1^N, \hat{u}_1^{i-1}) < 1 \end{cases} \tag{7}$$

The decoder keeps the judgment results obtained and uses it to help calculate the LR of all bits after the ith bit. Due to the problem of error propagation in SC decoding, a decoding method that preserves multiple sdecoding paths is proposed in [7].

2.3 *Probabilistic Shaping*

For traditional communications, the channel input symbol distribution is uniformly distributed. However, this is not optimal for most channels. For example, the optimal symbols probability distribution of the additive white Gaussian noise (AWGN) channel is the Maxwell Boltzmann distribution, that is, the MB distribution. Therefore, the uniform distribution of channel input symbols will bring performance loss to the system, which we call shaping loss. To solve this problem, probabilistic shaping technology has been introduced into channel coding. Probabilistic shaping increases the frequency of low-energy signal points, reduces the frequency of high-energy points, and reduces the average power. This paper introduces the probabilistic shaping technique proposed in [8].

3 Probabilistically Shaped Systematic Polar Code

A structure combining probability shaping and channel coding in a single-layer BICM is proposed in [9]. We use the structure in this paper to combine the systematic polar code with probabilistic shaping, and its structure is shown in Fig. 1. The difference from the literature is that in the structure adopted in this paper, the probability shaping does not depend on the mapper, but converts the binary equal-probability source sequence to the non-equal-probability source before encoding.

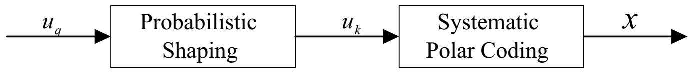


Fig. 1 Probabilistic shaping structure of systematic polar codes

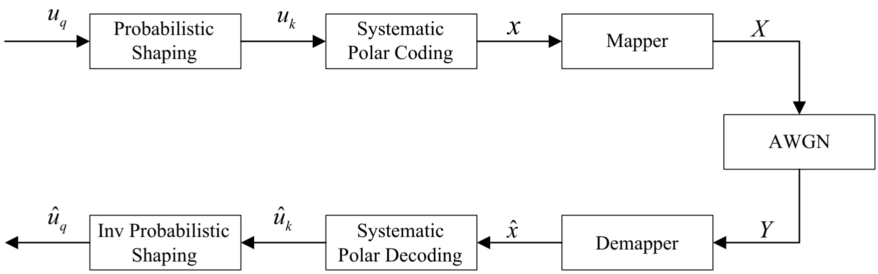


Fig. 2 Single-layer shaped polar codes scheme

Due to the limitation of the generation matrix of traditional systematic polar coding, we are adopting a kind of systematic polar coding scheme consisting of two-step non-systematic coding [10]. The system frame diagram is shown in Fig. 2.

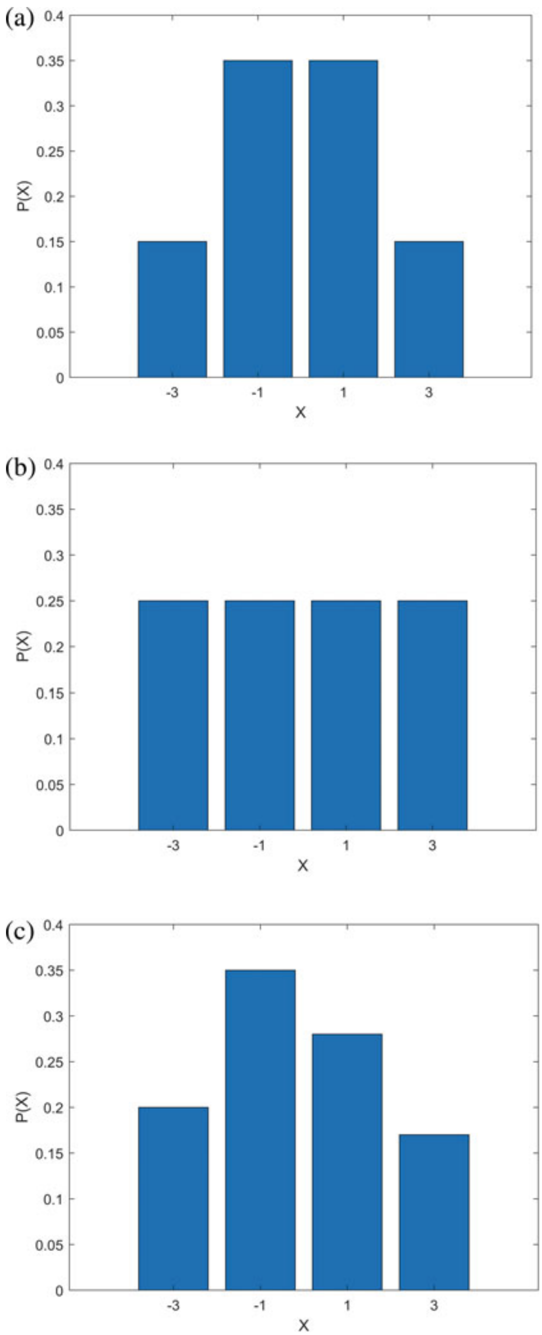
At the transmitting end, a binary uniformly distributed bit source u_q of length q is converted into a non-equal probability binary vector u_k of length k by the shaping code encoder. The vector u_k is sent to a systematic polar encoder with a code rate of k/N to generate a code vector x, which is then mapped to 2^m-PAM signal points by a mapper. The PAM symbol sequence X is sent into the AWGN channel.

At the receiving end, the channel output symbol Y is sent to the de-mapper to calculate the log-likelihood ratio (LLR) of the encoded codeword x, and the source bit estimate $\hat{u}_k$ is calculated through a systematic polar decoder, and then it is sent to the shaping code decoder.

4 Experiment Result and Data Analysis

In this part, we simulate and analyze the influence of the two encoding methods of the polar code on the probabilistically shaped source sequence. At the same time, the performance of probabilistic shaped systematic polar codes and probabilistic shaped NSPC under same code rates is compared. The influence of systematic polar coding and non-systematic polar coding on probability shaping is shown in Fig. 3. It can be seen that the NSPC eliminates the unequal probability characteristics of 0 and 1 in the formed source sequence, resulting in a uniform distribution of the mapped PAM4 signal points. Compared with NSPC, systematic polar codes greatly remain the “0” and “1” bit unequal probability characteristics brought about by probabilistic shaping. Therefore, the combination of system polar coding and source probability shaping

Fig. 3 **a** is the shaped PAM4 symbol probability distribution without polar coding. **b, c** are the symbol probability distributions of probability-shaped non-systematic polarization codes and probability-shaped systematic polarization codes after PAM4 mapping



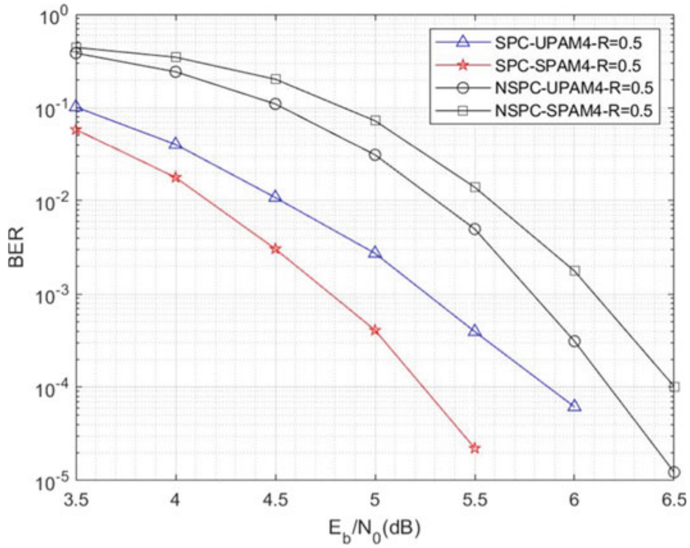


Fig. 4 Performance of PS-SPC and PS-NSPC under SC decoding algorithm

can form a constellation point probability distribution with progressive standard MB distribution.

Therefore, this paper adopts a scheme that combines probabilistic shaping with systematic polar codes, and retains the shaping gain brought by probability shaping. The corresponding performance of this scheme is shown in Fig. 4.

Compared with the uniformly distributed systematic polar codes with code length $N = 1024$ and code rate $R = 0.5$, the systematic polar codes after probabilistic shaping have better BER performance. The shaping gain reaches about 0.6 dB when $\text{BER} = 10^{-4}$. The NSPC eliminate the shaping gain brought by the probabilistic shaping. At the same time, because probabilistic shaping introduces $k - q$ redundant bits before encoding, resulting in a loss of code rate, the performance is worse than that of uniformly NSPC.

5 Conclusions

Systematic polar codes are superior to NSPC in terms of bits error rate. The combination of probability shaping and systematic polar codes is an important means to approximate the channel capacity limit. Therefore, it is very meaningful to study their combinations. In this paper, we introduced a combination of systematic polar codes and probabilistic shaping, and proposed that NSPC will eliminate the influence of probabilistic shaping. At the same time, the shaping method will introduce redundancy in the source part, so that the performance of NSPC for shaping is poor, while systematic polar codes can remain the shaping gain brought by probabilistic shaping. The application of NSPC and shaped systematic polar codes to multi-level coded modulation systems can be further study.

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