Simplified Design of Multilevel Coding Scheme with Polar Codes for High-Order Modulation

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Abstract Polar codes are a type of forward error correction (FEC) codes that can achieve the capacity of a discrete memoryless symmetric channel. Polar codes are accepted by the 5G standard due to the excellent error correction performance. The next-generation mobile communication system requires higher spectrum efficiency, so it is significant to study the easy-to-implement and efficient coded modulation scheme of polar codes for high-order modulation. The commonly used polar coded modulation schemes in communication systems mainly include bit-interleaved coded modulation (BICM) and multilevel coding (MLC). Aiming at the requirements of higher spectrum efficiency and higher reliability of future communication systems, in this work, we propose an improved MLC scheme based on polar codes to reduce the difficulty of MLC scheme design. Since the quadrature amplitude modulation (QAM) can be decomposed into two orthogonal pulse amplitude modulation (PAM) in the MLC system, we performed simulations based on PAM in an additive white Gaussian noise (AWGN) channel. Compared with the conventional BICM scheme, the proposed MLC scheme provides performance gains of 0.4 dB and 1.48 dB under 8-PAM and 16-PAM, respectively. At the same time, while the proposed MLC scheme provides a large amount of performance gain at 16-PAM, the complexity of the soft-decision decoder of the entire system is reduced by 55%.

1 Introduction

Polar codes are a class of error-correcting codes proposed by Arikan in 2009 based on channel polarization theory, and they were theoretically proven to achieve symmetric channel capacity on binary-input discrete memoryless channels (B-DMC) at infinite

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L. C. Jain et al. (eds.), *Advances in Wireless Communications and Applications*, Smart Innovation, Systems and Technologies 299,
https://doi.org/10.1007/978-981-19-2255-8_8

code length [1]. Polar codes, as a new star in the field of channel coding, have received much attention from academia and industry since it was proposed, and became the coding scheme for the control channel in the 5G mobile communication standard enhanced Mobile BroadBand (eMBB) scenario in 2016.

The coded modulation (CM) problem has been a very popular research topic, and the early studies of polar codes were mainly based on binary channels, where all coded bits are transmitted through the same B-DMC and then decoded by the receiver. However, there are some problems in applying polar code directly to higher-order modulation. The construction of polar codes is based on Binary Phase Shift Keying (BPSK) to select a good (very high reliability) sub-channel for transmitting information bits based on the sub-channel reliability. In this case, the direct application of the polar codes constructed for independent and identical distribution will cause a reliability mismatch problem, resulting in performance loss. The combination of polar codes and CM is an important and meaningful research direction [2], and usually, polar coded modulation schemes include multilevel polar coded modulation [3] and bit interleaved polar coded modulation [4].

Among them, the MLC scheme is optimal from the point of view of information theory, because it is completely based on the chain rule of mutual information, which protects bit levels of different reliability in groups, and approaches channel capacity through multi-stage decoding [5]. However, the design of the MLC scheme requires multiple component codes, and the system will be very complicated for high-order modulation. The BICM scheme counteracts interference by introducing an interleaver to discrete burst errors into random errors in the channel [6]. The BICM scheme is endeared because of its simplicity and flexibility. Nevertheless, the performance gain of the BICM scheme under high-order modulation and high spectral efficiency is limited, which is far inferior to the MLC scheme [7].

Recently, another MLC scheme of concatenated codes for high-order modulation was introduced in [8, 9], in which the MLC scheme is designed to be of low-complexity and offers better performance over BICM. Inspired by this, considering that some bit levels in the MLC system are already very reliable at high code rates, we separate these bit levels from the other bit levels and utilize polar codes to separately encode the less reliable bit levels as least significant bits (LSBs), while the other bit levels as the most significant bits (MSBs) are not encoded. We apply the strong error correction capability of the polar codes to ensure the reliability of the bit levels in the LSBs, at the same time, the encoding code length is reduced by using the split layer scheme, which brings the reduction of system power consumption. The simulation results show that the proposed MLC scheme outperforms the BICM scheme.

2 Polar Codes

2.1 Channel Polarization

Channel polarization is the core theory of polar codes, which consists of channel combining and channel splitting. The phenomenon of channel polarization is that when the code length N tends to infinity, a set of independent binary input channels will be transformed into two types of extreme channels with symmetric capacity approaching 0 or 1, namely, a noiseless channel and a full noise channel. We transmit the information bits on the noiseless channel, and transmit the information agreed by the sender and receiver in advance on the full noise channel, that is, the frozen bits.

2.2 Polar Encoding

Polar Codes apply a generator matrix $\mathbf{G}_N = \mathbf{B}_N\ \mathbf{F}^{\otimes \mathrm{n}}$ for coding, where $N = 2^n$ is the code length, "⊗" is the Kronecker product, **F** represent the kernel matrix:

$$\mathbf{F} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \tag{1}$$

and $\mathbf{B}_N$ is the permutation operation:

$$\mathbf{B}_N = \mathbf{R}_N\left(\mathbf{I}_2 \otimes \mathbf{B}_{N/2}\right) \tag{2}$$

where $\mathbf{I}_2$ represent an identity matrix, $\mathbf{R}_N$ is the permutation matrix.

The encoding operation of polar codes (N, K) is divided into two steps. Firstly, a good set of sub-channels are selected to carry K information bits through the method of coding construction, and the remaining sub-channels carry frozen bits to obtain the message sequence $\boldsymbol{u}_1^K$. In this work, we choose the polarization weight method proposed by HUAWEI [10]. Then, the message sequence $\boldsymbol{u}_1^K$ and the generator matrix $\mathbf{G}_N$ are multiplied to obtain the coded sequence $varvecx_1^N$:

$$\boldsymbol{x}_1^N = \boldsymbol{u}_1^N \mathbf{G}_N \tag{3}$$

2.3 Polar Decoding

Corresponding to the channel splitting, Arikan proved that when the code length of polar codes is large enough, a better asymptotic performance can be obtained by applying the successive cancellation (SC) decoding algorithm. SC decoding is a single-pass algorithm where the decoding process of the ith bit requires the use of the log-likelihood ratio (LLR) of the channel output and the decoding result of the previous $(i-1)$th bits, and upon receiving them, computes the LLR

$$L_N^{(i)}(y_1^N, \hat{u}_1^{i-1}) = \ln \frac{W_N^{(i)}(y_1^N, \hat{u}_1^{i-1}|u_i = 0)}{W_N^{(i)}(y_1^N, \hat{u}_1^{i-1}|u_i = 1)} \quad (4)$$

and makes its decision as

$$\hat{u}_i = \begin{cases} 0, \ L_N^{(i)}(y_1^N, \hat{u}_1^{i-1}) \geq 0 \\ 1, \ L_N^{(i)}(y_1^N, \hat{u}_1^{i-1}) < 0 \end{cases} \quad (5)$$

the complexity of SC decoding mainly comes from the calculation of LLR, which can be calculated by parity grouping recursively

$$L_{2N}^{(2i-1)}\left(y_1^{2N}, \hat{u}_1^{2i-2}\right) = \ln \frac{e^{L_1+L_2}+1}{e^{L_1}+e^{L_2}} \quad (6)$$

$$L_{2N}^{(2i)}\left(y_1^{2N}, \hat{u}_1^{2i-1}\right) = \left(1 - 2\hat{u}_{2i-1}\right)L_1 + L_2 \quad (7)$$

L_1 and L_2 were defined as:

$$\begin{cases} L_1 = L_N^{(i)}\left(y_1^N, \widehat{u}_{1,o}^{2i-2} \oplus \widehat{u}_{1,e}^{2i-2}\right) \\ L_2 = L_N^{(i)}\left(y_{N+1}^{2N}, \widehat{u}_{1,e}^{(2i-2)}\right) \end{cases} \quad (8)$$

$\hat{u}_{1,\mathrm{o}}^{2i-2}$ denotes the odd grouping $\left(\hat{u}_1, \hat{u}_3, \ldots, \hat{u}_{2i-3}\right)$ of $\hat{u}_i$ and $\hat{u}_{1,\mathrm{e}}^{2i-2}$ denotes the even grouping $\left(\hat{u}_2, \hat{u}_4, \ldots, \hat{u}_{2i-2}\right)$ of $\hat{u}_i$. The recursive process starts with the reception of the channel LLR $L_1^{(1)}(y_i)$ and keeps recursion to finally get $L_N^{(i)}(y_1^N, u_1^{i-1})$.

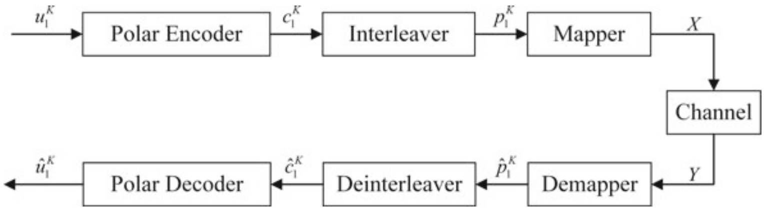

Fig. 1 The BICM scheme model

3 BICM Scheme and the Proposed MLC Scheme

3.1 BICM Scheme

BICM is a practical coded modulation scheme, which is widely used in engineering practice. The purpose is to use an interleaver to disrupt the correlation of the coded bit sequence and to disperse the bits with continuous errors into different constellation symbols, so as to obtain a larger codeword dispersion. The BICM scheme of the Polar codes is a simple and practical high-order coded modulation scheme. The design of the interleaver is the focus, we use different interleaving rules to disrupt the coded bit sequence and discretize burst errors, thereby improving system performance. Figure 1 is a bit-interleaved polar coded modulation scheme model. Compared with the high-order modulation system in which coding and modulation are independent of each other, an interleaver is added between the encoder and the mapper, and a de-interleaver is added correspondingly at the receiver. This reflects the advantages of the BICM. The advantage of the BICM scheme is its low complexity, but it is not optimal in information theory.

3.2 The Proposed MLC Scheme

Let A denote the set of signal constellation points, and its cardinality given by $|A| = 2^m$. Let $X \in A$ denote the transmit symbol composed of codewords and Y is the receive signal. Then, the channel can be equivalent to m parallel sub-channels according to the mutual information chain rule:

$$\begin{aligned} I(Y;X) &= I\left(Y;x^{(1)},\ldots,x^{(m)}\right) \\ &= I\left(Y;x^{(1)}\right) + I\left(Y;x^{(2)}|x^{(1)}\right) + \cdots + I\left(Y;x^{(m)}|x^{(1)},\ldots,x^{(m-1)}\right) \end{aligned} \tag{9}$$

the capacity C^i of the equivalent sub-channel is given by the respective mutual information:

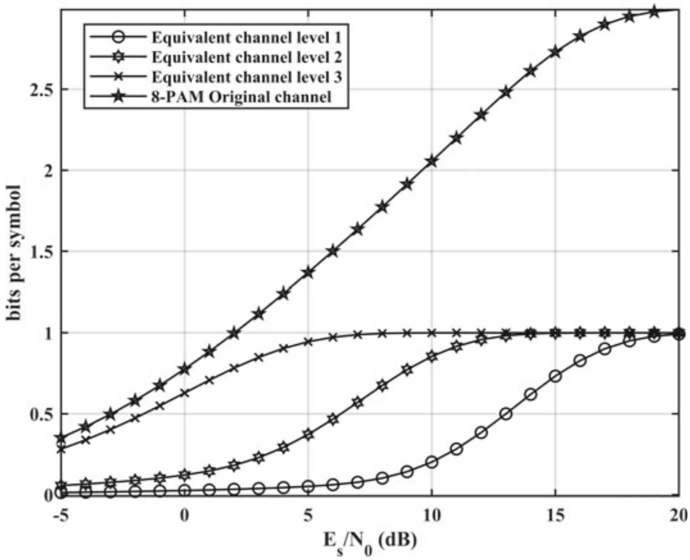

Fig. 2 The capacity of equivalent channels with 8-PAM

$$\begin{aligned} C^i &= I\left(Y; x^{(m)} | x^{(1)}, \ldots, x^{(m-1)}\right) \\ &= \mathrm{E}_{x^0 x^1 \ldots x^{i-1}}\left\{C\left(A\left(x^0 x^1 \ldots x^{i-1}\right)\right)\right\} - \mathrm{E}_{x^0 x^1 \ldots x^i}\left\{C\left(A\left(x^0 x^1 \ldots x^i\right)\right)\right\} \end{aligned} \quad (10)$$

where $C\left(A\left(x^0 x^1 \ldots x^i\right)\right)$ denotes the coded modulation capacity, and E represents an expectation operation. For example, Fig. 2 shows the channel capacity of each bit level using Ungerboeck partitioning with 8-PAM modulation over the AWGN channel.

It is important to assign code rates to each level according to specific rules, and the capacity rule is one of the effective design rules in traditional MLC schemes. However, the capacity C^i of the higher bit levels of the MLC scheme tends to 1 at high spectral efficiency, when still coding these bit levels for protection brings less benefit and generates a lot of unnecessary system overhead.

Therefore, we simplify the MLC scheme to the two-level scheme shown in Fig. 3, bind the bit level with capacity C^i tending to 1 as the MSBs and the other bit levels as the LSBs. Then, the LSBs are encoded with a polar encoder, while the MSBs are not encoded. At the receiver, the input Y is divided into MSBs and LSBs, the LSBs are demodulated to obtain LLR, which is sent to a polar decoder to obtain the decoding result $\hat{\boldsymbol{u}}^{(2)}$ of LSBs. After that, $\hat{\boldsymbol{u}}^{(2)}$ will be applied to assist the hard-decided demodulator of MSBs to obtain the decision result $\hat{\boldsymbol{u}}^{(1)}$.

We describe the subset partitioning and constellation labeling rules as follows. We divide the constellation signal point label into two subsets: MSBs and LSBs, and they are divided according to Ungerboeck partitioning. Meanwhile, Gray code is used for labeling inside the subset. For example, for an 8-PAM constellation consisting of

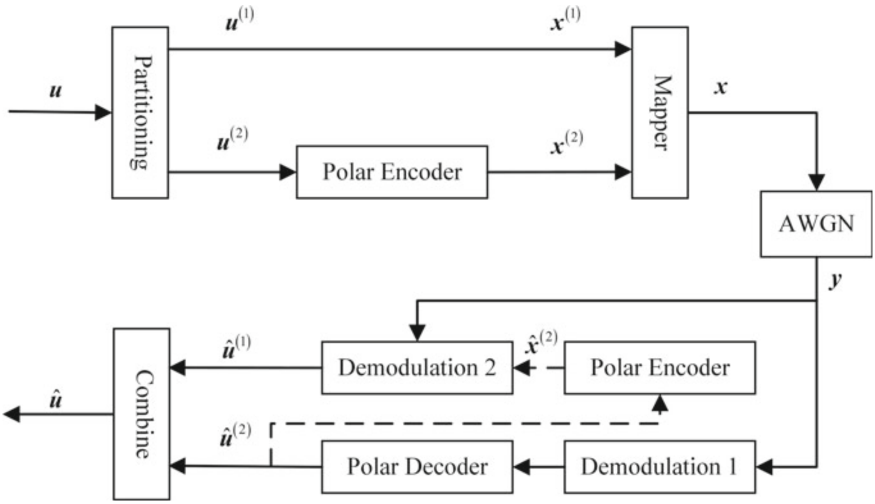

Fig. 3 The proposed MLC scheme model

three bits per constellation point, we use the least reliable bit level as the LSB and the remaining two bit levels as the MSBs. We apply the following labeling for the 8-PAM constellation: $A_{8-\mathrm{PAM}} = (000, 010, 110, 100, 001, 011, 111, 101)$. Through this labeling, it is possible to clearly distinguish the reliability between the LSB and the MSBs, thus making the MSBs channel sufficiently reliable.

4 Simulation Results and Analysis

In this section, we show the simulation results to compare the proposed MLC scheme and BICM scheme under AWGN channel. We performed simulations with 8-PAM and 16-PAM modulation. In the 8-PAM scheme, the spectral efficiency β is 2.5 bits per symbol, and the code length N is 1536 for the proposed MLC scheme and 2046 for BICM (obtained using punching). For 16-PAM scheme, β is 3 bits per symbol and N is 1024 are used for simulation. In addition, the last two bit-levels are selected to the MSBs, because their reliability is high enough.

As shown in Fig. 4, for the proposed MLC scheme, the performance gains are 0.4 dB and 1.48 dB under 8-PAM and 16-PAM modulation compared to BICM at BER = 10–5. From the simulation results, it can be seen that the proposed scheme has higher performance gain at higher modulation orders. At the same time, in the proposed MLC scheme, since only LSBs are encoded and decoded, the number of encoders and decoders is partially decreased, which reduces the complexity of the system. In the polar coded modulation scheme, the system complexity mainly comes from the soft-decision decoder of polar codes, which involves a large number of floating-point operations. Therefore, we can define the relative complexity reduction

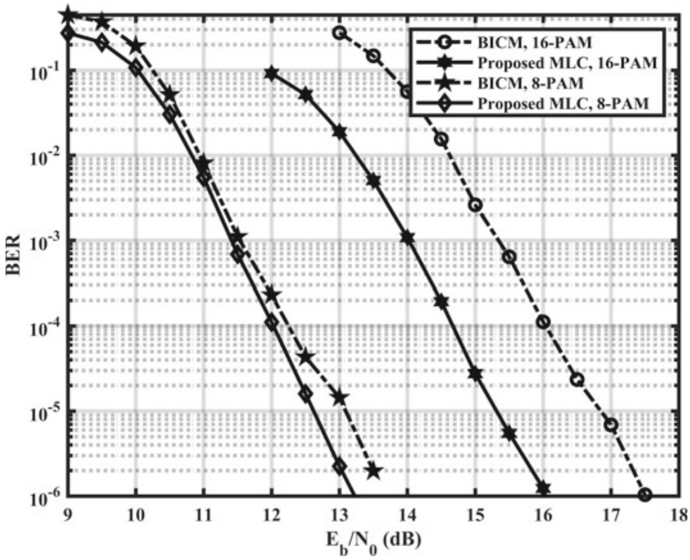

Fig. 4 BER performance of the proposed MLC scheme and BICM

of the entire system as the complexity reduction of the decoder in the system. Since the decoding complexity of the SC decoder is $O(N \log_2 N)$, the relative complexity reduction of the proposed MLC scheme to BICM scheme can be calculated as:

$$T_{\text{reduce}} = 1 - \frac{T_{\text{MLC}}}{T_{\text{BICM}}} \tag{11}$$

T_{MLC} denotes the decoding complexity of the proposed MLC scheme and T_{BICM} denoted BICM decoding complexity. For 16-PAM, the proposed MLC scheme offers 55% reduction of SC decoder. Compared with the difficult implementation of the traditional MLC scheme and the poor performance of the BICM system, the proposed scheme achieves a compromise between performance and complexity.

5 Conclusion

Facing the requirements of high spectral efficiency and high-order modulation in future communication systems, in this article, we propose an improved two-level MLC scheme that can be applied when the code rate is high enough. To reduce the difficulty of implementing the MLC scheme, the structure is redesigned to have only two levels, in which LSBs and MSBs are divided according to Ungerboeck partitioning, and gray labeling is used inside the set to maximize the reliability difference between them. In this case, the bits in the LSBs are protected by polar coding and the remaining bits are transmitted through the MSBs channel and remain uncoded state. The simulation analysis was performed in two scenarios of 8-PAM and 16-PAM over AWGN channel. The results show that the proposed MLC scheme is better than the BICM scheme in the 5G standard, and the higher the modulation order, the greater the performance gain. In addition, since only the LSBs are coded, the overall code length of the system is decreased, resulting in a reduction in the complexity of the system. The proposed scheme is very effective with high baud rate, and how to use the scheme to obtain the ideal performance with low rate is a problem that we need to consider in our future work. In addition, we only consider one-dimensional modulation at present, and we will optimize the proposed scheme under higher-dimensional modulation in the future.

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