Nonbinary LDPC Coded OTFS System Over Mobile Multipath Channels

Yiming Lu, Lin Zhou, Lin Wang, Sanya Liu, and Chen Chen

Abstract As communication technology advances, it becomes more difficult to keep up, low-density parity-check (LDPC) code has become one of the key technologies in channel coding with its superior error correction performance and efficient decoding algorithm. Among them, nonbinary LDPC codes perform better in high-order modulation and burst error channels, especially in short to medium code length conditions, and the tanner pattern of nonbinary LDPC codes is more sparse than that of binary LDPC codes and has a larger envelope length, which is more favorable to the optimization and design of LDPC codes in this form. In this paper, the basic principle and performance of the new waveform technology, for future 6G mobile communication, orthogonal time–frequency space (OTFS) modulation technology is addressed and examined in depth. In the high-band and high-speed mobile scenario based on the choice of nonbinary LDPC as the channel code. The results show that the OTFS modulation technique has better robustness, the lower peak-to-average ratio, and the potential to obtain full diversity gain on nonbinary LDPC channel coding compared with the conventional OFDM modulation system.

1 Introduction

In 1962, Dr. Gallager proposed the low-density parity-check (LDPC) codes [1], but its implementation was not possible due to the backward conditions at that time. Mackay and Neal discovered that before 1995, the capacity of the system with LDPC decoding algorithm in the case of longer code length is nearer to the Shannon limit than that of the Turbo code system. At present, the design, construction, performance analysis and application of the majority of LDPC codes are binary codes, while related studies

Y. Lu · L. Zhou (✉) · S. Liu · C. Chen
Xiamen Key Laboratory of Mobile Multimedia Communications, College of Information Science and Engineering, National Huaqiao University, Xiamen 361021, Fujian, China
e-mail: linzhou@hqu.edu.cn

L. Wang
China Mobile Communications Group Xiamen Co., Ltd. Xiamen, Fujian 361000, China

prove that nonbinary codes outperform binary codes in terms of performance under multi-channel conditions, nonbinary codes have much stronger burst error tolerance than binary codes. The nonbinary LDPC also has the following advantages: (1) it has the ability to eliminate small loops; (2) it can synthesize multiple burst errors into fewer multivariate symbol errors, thus improving the resistance to burst errors, which has great potential in 5G and even 6G systems [2]. In addition, combining the coding technology of nonbinary LDPC codes with multi-antenna systems can significantly increase the user capacity in the field of communication.

With regard to modulation waveform, 4G and 5G use the orthogonal frequency division multiplexing (OFDM) technology. In scenarios with a lot of movement, supported by B5G/6G, such as high-speed trains, vehicle to vehicle (V2V), unmanned aerial vehicle (UAV), satellite communications, etc., the high speed movement will generate large Doppler shifts, and the orthogonality between subcarriers of OFDM will be severely damaged, leading to a dramatic deterioration of performance. Recently, Hadani et al. proposed a brand-new waveform method, OTFS modulation, for high mobility scenarios [3]. Compared with the conventional OFDM modulation technique, OTFS employs a delay-Doppler domain signal representation that takes full advantage of the invariance, separability, as well as the orthogonality of the delay-Doppler domain channel's coupling to information symbols, and has the potential for full diversity gain and good robustness [4]. In this paper, we discuss the peak to average power ratio (PAPR), diversity gain, and coding gain of OTFS in conjunction with nonbinary LDPC channel coding for 6G high mobility scenarios and high frequency band communication scenarios, and discuss its advantageous potential and the issues that need further research and solution.

2 Nonbinary LDPC Codes

2.1 Basic Concept

GF(q) means a Galois domain with q members, with q deriving from prime numbers' powers. The check matrix $H = [h_{i,j}]_{0 \leq i \leq m, 0 \leq j \leq n}$, defined on the finite field GF(q) constitutes a regular LDPC code consisting of the following structural characteristics: (1) fixed column and row weights; (2) no identical non-zero values exist at the corresponding positions of every two rows (or two columns)[5]. If the row weight column weight is not unique, a non-regular LDPC code is described as the code word. The binary LDPC code's check matrix is made up of "0" and "1," but the nonbinary LDPC code's check matrix is made up of domain elements in a finite domain, for example, under the Galois domain GF(2^3), the domain elements include $(\alpha^{-\infty} = 0, \alpha^0, \alpha^1, \ldots \alpha^6)$, where α denotes the native element under the domain element, and all domain elements can be represented by multiple powers of α. There are two main representations of nonbinary LDPC codes, check matrix and Tanner diagram. The check matrix's non-zero components are represented as label values

Fig. 1 Tanner graph of a nonbinary LDPC
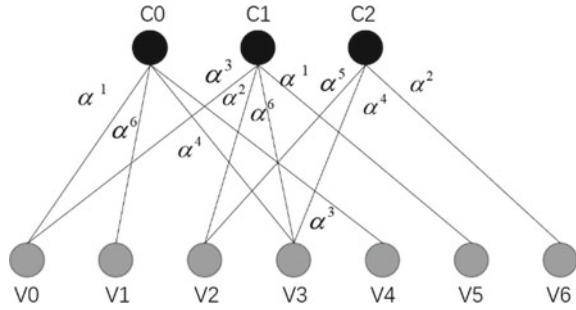
on the connected edges in the Tanner diagram. The Hq of the nonbinary LDPC code defined in $GF(2^3)$ is given in the following equation. The corresponding Tanner diagram is shown in Fig. 1.

$$H_q = \begin{bmatrix} \alpha^1 & \alpha^6 & 0 & \alpha^4 & \alpha^3 & 0 & 0 \\ \alpha^3 & 0 & \alpha^2 & \alpha^6 & 0 & \alpha^1 & 0 \\ 0 & 0 & \alpha^5 & \alpha^4 & 0 & 0 & \alpha^2 \end{bmatrix} \tag{1}$$

2.2 Construction

Nonbinary LDPC codes may be classified into two types of building methods, namely, randomized construction methods and structured construction methods. The progressive edge growth (PEG) algorithm is one of the commonly used randomized construction methods, which can gradually establish the link between check and variable nodes under known degree distribution, and is used to establish the Tanner graph of nonbinary LDPC codes with larger ring length. The Approximate Cycle EMD (ACE) algorithm is also one of the well-known computer-based random construction algorithms, which increases the influence of overlapping rings on the decoding code based on the consideration of rings, as a way to improve the flow of extra messages during decoding iterations. The above random construction method requires a large number of computer search operations, and constructed code words are irregular. Although the randomly constructed LDPC codes have good performance, the coding complexity is high [6].

3 New Waveform Technology—OTFS Modulation

For high mobility scenarios in 6G and high frequency band communication scenarios such as the millimeter wave, OTFS modulation technology can extend each information symbol in the time-delayed-Doppler domain to the entire time–frequency domain using the inverse symplectic finite fourier transform (ISFFT) compared to OFDM modulation technology. As a result, each transmitted symbol has a nearly constant channel gain with considerable resilience.

Figure 1 depicts the basic block diagram of the OTFS modulation approach. Assume a data burst packet has a total duration of NT seconds and the total bandwidth of MΔf Hz, with N being the number of OTFS symbols, M being the number of OTFS subcarriers, T being the symbol time interval, and Δf being the subcarrier frequency interval. The modulation symbols of length MN obtained from the message sequence u by constellation mapping are arranged into a two-dimensional matrix $x \in \mathbb{C}^{N \times M}$, which is the vector of information symbols on the DD plane, effective symbols at the k-th Doppler and l-th time-delayed grid point on the time-delayed-Doppler grid: $x[k, l] \in x, 0 \leq k \leq N - 1, 0 \leq l \leq M - 1$. First, the transmitter uses ISFFT to map the message symbols $X[n, m], 0 \leq n \leq N - 1, 0 \leq k \leq M - 1$:

$$X[n, m] = SFFT^{-1}(x[k, l]) = \frac{1}{\sqrt{NM}} \sum_{k=0}^{N-1} \sum_{l=0}^{M-1} x[k, l] e^{j2\pi(\frac{nk}{N} - \frac{ml}{M})} \qquad (2)$$

The Heisenberg transformation may then be used to retrieve the time domain signal as follows:

$$s(t) = \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} X[n, m] g_{tx}(t - nT_t) e^{j2\pi m \Delta f(t - nT_t)} \qquad (3)$$

$g_{tx}(t)$ is the filter responsible for pulse shaping.

The linear time varying (LTV) channel [7] is represented for the DD plane class as:

$$h(\tau, v) = \sum_{i=1}^{P} h_i \delta(\tau - \tau_i) \delta(v - v_i) \qquad (4)$$

where P denotes the number of pathways, and the ith path's path gain, time delay, and Doppler shift are indicated by h_i, τ_i and v_i. The ith path's time delay and Doppler shift may be represented as:

$$\tau_i = \frac{l_{\tau_i}}{M\Delta f}$$

$$v_i = \frac{k_{v_i} + \kappa_{v_i}}{NT} \tag{5}$$

where l_{τ_i} and k_{v_i} denote the tapping parameters with relation to the time delay expansion and Doppler shift, respectively.

The above-mentioned channel receives the time plane signal as:

$$r(t) = \iint h(\tau, v)s(t - \tau)e^{j2\pi v(t-\tau)} d\tau dv + \overline{w}(t) \tag{6}$$

Next, the Winger transform (Heisenberg inversion) yields the time–frequency domain signal as:

$$Y[n, m] = \int r(t)g_{rx}^*(t - nT_t)e^{-j2\pi m\Delta f(t-nT_t)} dt \tag{7}$$

The equivalent symplectic finite fourier transform (SFFT) may be used to get the received signal in the DD plane.:

$$y[k, l] = SFFT(Y[n, m]) = \frac{1}{\sqrt{NM}} \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} Y[n, m]e^{-j2\pi(\frac{nk}{N} - \frac{ml}{M})} + w[k, l] \tag{8}$$

where w is the sampling point of the corresponding DD domain noise.

As seen in the graph, OTFS can also obtain full partial set gain in time and frequency while maintaining a lower PAPR, which is clearly better compared to the PAPR of OFDM [8], and the figure depicts a simple example of the PAPR comparison of OTFS with OFDM. Where the horizontal coordinate is the threshold value of the PAPR in dB form and the vertical coordinate Pr is the probability that either OFDM or OTFS is greater than the threshold value. The PAPR of OTFS is much lower than the PAPR of OFDM for the same.

4 The Simulation Result of Nonbinary LDPC Coded OTFS

The message vector received by the receiver is $y_n = (y_{n,0}, y_{n,1}, \ldots, y_{n,b-1})$, the vector length is b, and each vector corresponds to a transmitted code element as x_n, then:

$$L(x_n = a) = \log \frac{p(y_n/x_n = a)}{p(y_n/x_n = 0)} \tag{9}$$

Fig. 2 Communication simulation system nonbinary LDPC decoding

The logarithmic domain Q sum product algorithm (QSPA) algorithm is as follows [9]: initialization first,

$$L(x_n) = \ln \frac{p(y_n/x_n = a)}{p(y_n/x_n = 0)} \tag{10}$$

$$L(q_{mn}^a) = L(v_n), \ L(r_{mn}^a) = 0 \tag{11}$$

Then the message update of the check node and the message update of the variable node are performed:

$$L(r_{mn}^a) = L(h_{m,n_m}^{-1} \sigma_{m,n_{m,l}} + h_{m,n_m}^{-1} \rho_{m,n_{m,l}}) \tag{12}$$

$$L(q_{mn}^a) = L(x_n) + \sum_{m' \in M(n) \backslash m} L(r_{mn}^a) \tag{13}$$

The final judgement, similar to the standard SPA decoding, is that after the sign judgement, the decoding of the all-zero vector of the accompanying equation is completed, otherwise it returns to the next iteration until the accompanying equation is satisfied or the preset iteration maximum is reached, and the decoding is completed, reaching the desired result [10] (Fig. 2).

We use nonbinary LDPC code in GF(16) domain for simulation in channel coding, which has a 512-byte code length and a 1/2 coding rate, where the number of symbols of OTFS N = 8, the number of subcarriers M = 16, and the moving speed is 250 km/h. The graph compares the performance of OTFS modulation with OFDM modulation under 16QAM and 256QAM, and the OTFS condition can also be found in the diagram. The simulation results show that with 16QAM modulation, the OTFS system achieves a gain of 1.8 dB over the OFDM system under multipath fading channel, and similar simulation results are obtained when modulated with 256QAM (Figs. 3 and 4).

5 Conclusion

Under mobile multipath channel, OTFS became a waveform technology with development potential in 6G mobile communication by virtue of its excellent full diversity potential, low PAPR, and good robustness. In this paper, this modulation technique is combined with nonbinary LDPC channel coding. On the one hand, the construction
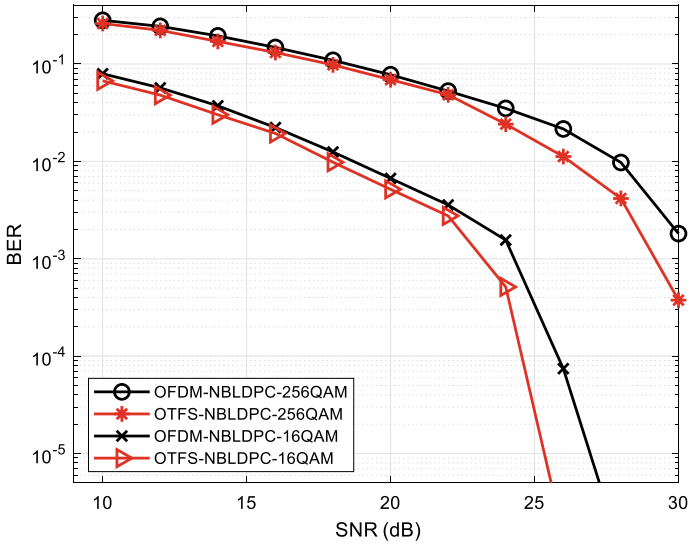
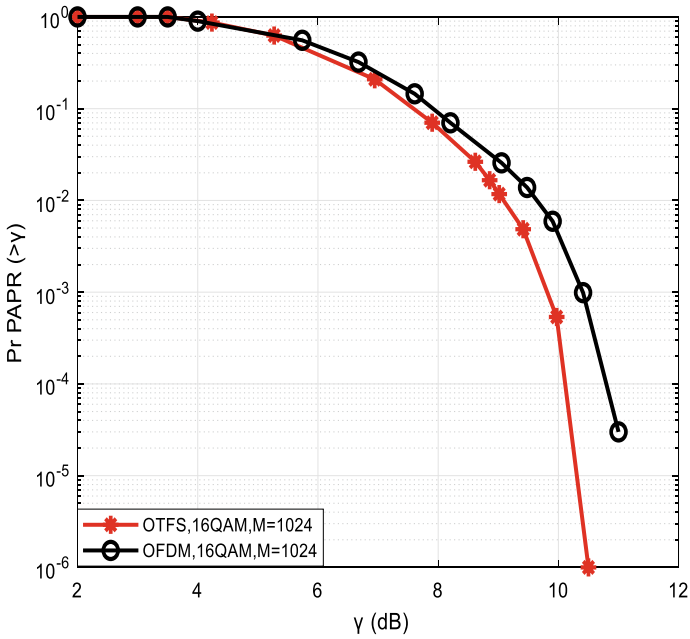**Fig. 3** BER performance under different modulation method

Fig. 4 PAPR comparison of OTFS and OFDM

compilation code process of nonbinary LDPC and OTFS principle are analyzed. On the other hand, the simulation systems of both are established. Nonbinary LDPC codes' performance with various modulation schemes in multipath fading channels is simulated using the QSPA decoding algorithm, and the system BER graph and the PAPR comparison graph of OTFS are obtained. Since the OTFS system has the important feature of being relatively insensitive to time changes, high-motion sceneries and multipath fading scenes are good candidates for the OTFS technology. In the future, the system can be further optimized in terms of channel estimation and the application of different channel coding.

References

1. Gallager, R.: Low-density parity-check codes. IRE Trans. Inf. Theor. **8**(1), 21–28 (1962)
2. You, X., Wang, C X., Huang, J.: Towards 6G wireless communication network. Vision, enabling technologies, and new paradigm shifts. Sci. China Inf. Sci. **64**(1), 5–78 (2021)
3. Hadani, R., Rakib S., Tsatsanis, M.: Orthogonal Time Frequency Space Modulation. arXiv: 1808.00519v1 (2018)
4. Tiwari, S., Das, S.S., Rangamgari, V.: Low complexity LMMSE receiver for OTFS. IEEE Commun. Lett. **23**(12), 2205–2209 (2019)
5. Davey, M.C., Mackay, D.J.C.: Low density parity check codes over GF(q). In: 1998 Information Theory Workshop (Cat.No.98EX131), pp. 70–71 (1998)
6. Rong, B., Jiang, T., Li, X., Soleymani, M.R.: Combine LDPC codes over GF(q) with q-ary modulations for bandwidth efficient transmission. IEEE Trans. Broadcasting **54**(1), 78–84 (2008)
7. Raviteja, P., Phan, K.T., Hong, Y., Viterbo, E.: Interference cancellation and iterative detection for orthogonal time frequency space modulation. IEEE Trans. Wirel. Commun. **17**(10), 6501–6515 (2018)
8. Surabhi, G.D., Augustine, R.M., Chockalingam, A.: Peak-to-average power ratio of OTFS modulation. IEEE Commun. Lett. **23**(6), 999–1002 (2019)
9. Wymeersch, H., Steendam, H., Moeneclaey, M.: Log-domain decoding of LDPC codes over GF(q). In: 2004 IEEE International Conference on Communications (IEEE Cat. No.04CH37577), 2004, vol. 2 pp. 772–776 (2004)
10. Feng, D., He, Q., Bai, B., Zheng, J., Liu, M.: Spatial modulation with multi-dimensional constellations. IEEE Wirel. Commun. Lett. **9**(1), 99–102 (2020)