

# Chapter 70

## Numerical Study of a Horizontal Axis Washing Machine for Linear and Nonlinear Vibration



Mohit Dhanda, Arpan Gupta, and Saurabh Yadav

**Abstract** In this work, linear and nonlinear vibration analyzes of a drum type washing machine having two springs and two dampers have been performed numerically. The effects of parameters such as linear spring stiffness, nonlinear spring stiffness, and damping coefficient have been investigated. A mathematical model of horizontal axis type washing machine by considering the whole system as a single degree of freedom system is formed. Since vibration level is large during spinning cycle, hence nonlinear stiffness of spring is considered, and a comparison of the linear and nonlinear mathematical model is carried out. The numerical solution of this mathematical model is found by using Runge Kutta 4th order method using MATLAB. From this study, it has been found that the damper with a high-damping coefficient and linear spring with low stiffness effectively reduce the vibration level. Also, steady-state vibration level remains almost unaffected by varying the stiffness of spring with cubic nonlinearity.

**Keywords** Vibration · Spring · Stiffness · Damper

### Introduction

The advent of technology and innovation has continuously transformed the life of a human being, and we are surrounded by a plethora of such examples. The washing machine is one of them. However, with time, reducing the manufacturing cost, manpower, weight, etc., and increasing the service life has become the need of the hour [1]. The washing machines are categorized based on washing methods like drum type, agitator type, and pulsator type washing machine. Based upon the rotational axis of the drum, these can also be classified as the horizontal axis and vertical axis washing machines [2]. The traditional washing machine generally consists of two springs and two dampers attached to the washing machine drum. However, the vibration level may get severe due to unbalance mass of wet clothes and not

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selecting the parameters properly such as spring stiffness and damping coefficient. In this study, two springs and two dampers are used, hence consideration of vibration level for cost optimization and service life becomes important. In any fully automatic washing machine, the drying process involves the oscillatory motion of the drum. During the washing operation, the wet clothes clump to the drum inner wall and hence create unbalance mass along with the mass of the drum. This unbalance mass creates severe vibration magnitude. Hence, nonlinearity due to spring stiffness comes into the picture.

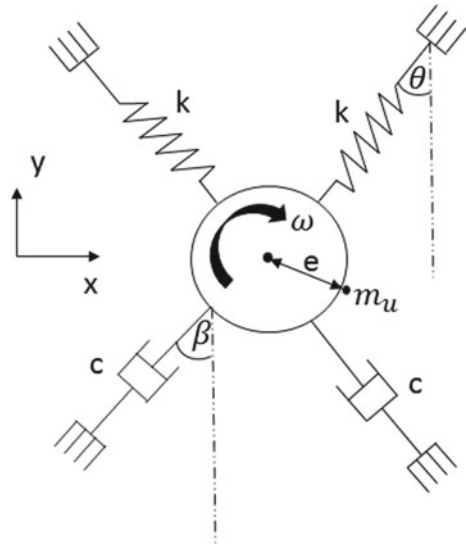
One way of reducing the vibration level in the washing machine is to use a hydraulic balancer that contains saltwater [3]. It rotates along with the drum and acts as a counter mass. It shifts the saltwater to the opposite side of unbalance mass. Another way to reduce the vibration level is the G-fall balancer [4]. The G-fall balancer contained the balance liquid placed at two ends of the spin drum to correct the unbalance and thus reducing vibration level by 70%. Turkay et al. predicted the nonlinear time-variant rigid body dynamic model of the suspension system of horizontal axis washing machine assuming the nonlinear stiffness of bellows [5]. Kang et al. had utilized the Bouc-wen model to describe the nonlinearity of the friction damper. It had assumed the hysteretic behaviour of the damper for nonlinear behaviour [6]. Berbyuk et al. have analyzed the vibration dynamics of a horizontal washing machine using three struts with incorporated spring instead of four strut spring system for cost optimization. Here, each strut consists of a friction damper incorporated. Here, each friction damper is based on Bouc-Wen hysteresis model to represent the nonlinear hysteretic damping force of magnetorheological damper [7].

So far, negligible work has been reported considering the nonlinear stiffness of the spring. So present work focuses on estimating the vibration level during the spinning cycle by using two springs and two damper of a drum type horizontal axis washing machine with cubic spring nonlinearity. Here, effects of parameters such as damping coefficient, spring linear stiffness, and nonlinear spring stiffness with cubic nonlinearity on vibration magnitude have been investigated.

## Mathematical Model

The mathematical model of the washing machine is formulated assuming it as a single degree of freedom system as shown in Fig. 70.1. Small deflection of spring and damper is assumed for linear vibration, and the equation of motion is derived by using Newton second law of motion. Separate analysis for vibration magnitude is carried out in  $x$ - and  $y$ -direction. For nonlinear vibration analysis, a large amplitude of vibration is considered, and spring stiffness is assumed to be cubic. Equations (70.1) and (70.2) represent the mathematical model of horizontal washing machine in  $x$ - and  $y$ -direction, respectively, considering linear stiffness of spring. Equations (70.3) and (70.4) represent the mathematical model of horizontal washing in  $x$ - and  $y$ -direction, respectively, considering the nonlinear stiffness of spring. Since the superposition principle is not satisfied for Eqs. (70.3) and (70.4), hence equation of motion found to

**Fig. 70.1** Free body diagram of a washing machine with two spring and two dampers



resemble the famous nonlinear Duffing equation whose numerical solution is found using the fourth-order Runge Kutta method.

$$(m_u + m)\ddot{x} + 2(c_e)_x\dot{x} + 2(k_e)_x x = m_u e \omega^2 \sin \omega t \tag{70.1}$$

$$(m_u + m)\ddot{y} + 2(c_e)_y\dot{y} + 2(k_e)_y y = m_u e \omega^2 \cos \omega t \tag{70.2}$$

$$(m_u + m)\ddot{x} + 2(c_e)_x\dot{x} + 2(k_e)_x x + k_3 x^3 = m_u e \omega^2 \sin \omega t \tag{70.3}$$

$$(m_u + m)\ddot{y} + 2(c_e)_y\dot{y} + 2(k_e)_y y + k_3 y^3 = m_u e \omega^2 \cos \omega t \tag{70.4}$$

where  $m$  is mass of drum,  $k$  is spring stiffness,  $k_3$  is spring stiffness due to cubic nonlinearity,  $c$  is damping coefficient,  $\omega$  is the speed of rotation of the drum,  $e$  is the eccentricity of unbalance mass,  $m_u$  is unbalanced mass due to wet clothes,  $\theta$  is the inclination angle of spring with vertical, and  $\beta$  is the inclination angle of the damper with vertical.  $(k_e)_x$  and  $(k_e)_y$  are the effective stiffness in  $x$ - and  $y$ -direction, respectively. The values of  $(k_e)_x$  and  $(k_e)_y$  are  $k \sin^2 \theta$  and  $k \cos^2 \theta$ , respectively. Similarly,  $(c_e)_x$  and  $(c_e)_y$  are the effective damping coefficient in  $x$ - and  $y$ -direction, respectively. The values of  $(c_e)_x$  and  $(c_e)_y$  are  $c \sin^2 \beta$  and  $c \cos^2 \beta$ , respectively.

The values of different parameters used for the analysis are given in Table 70.1.

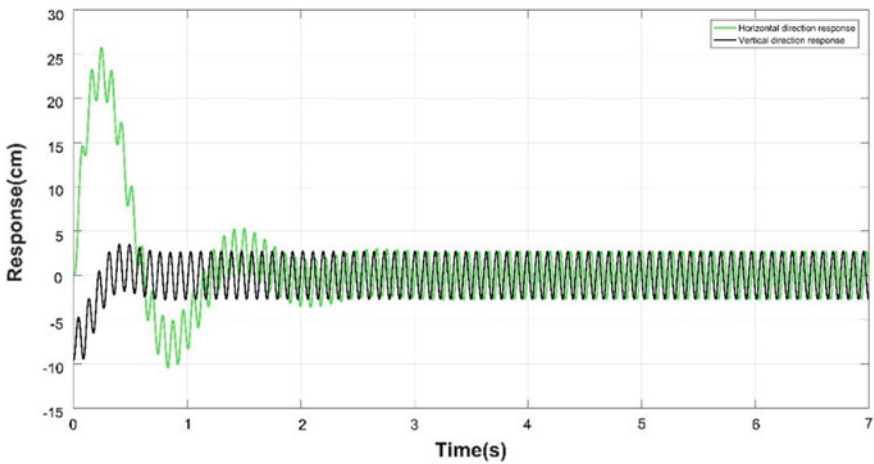
**Table 70.1** Values of the parameters used for analysis

$m_u$ (kg)	$m$ (kg)	$\omega$ (rad/s)	$e$ (m)	$\theta$ ( $^\circ$ )	$\beta$ ( $^\circ$ )
7	10	70	0.066	30	30

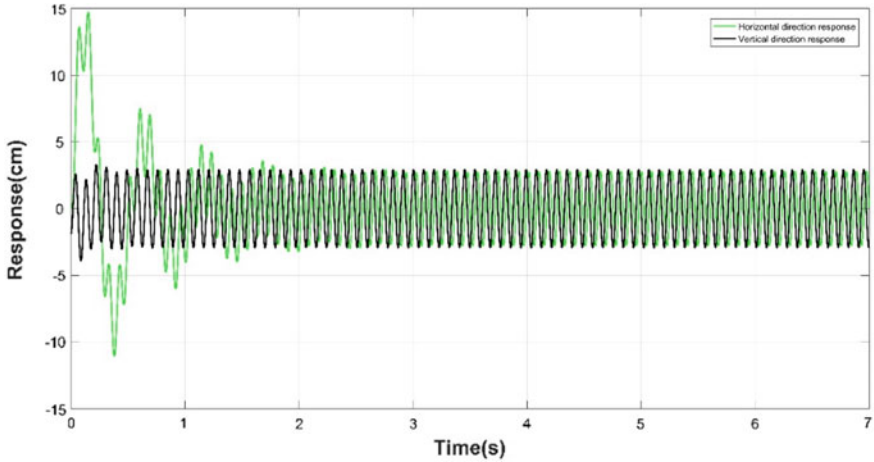
## Results and Discussion

The mathematical model formulated above is solved by the fourth-order Runge Kutta method using MATLAB. The second-order differential equation is converted into two first-order differential equations. Firstly, the system is solved without considering spring the nonlinearity in the spring. The system response is calculated for linear spring stiffness of 1000, 5000, and 15,000 N/m. The value of the damping coefficient is kept constant at 120 Ns/m. The results are shown in Figs. 70.2, 70.3, and 70.4, respectively. It is found that with the increase in the value of linear spring stiffness, vibration magnitude in the steady-state condition in both horizontal and vertical direction increases.

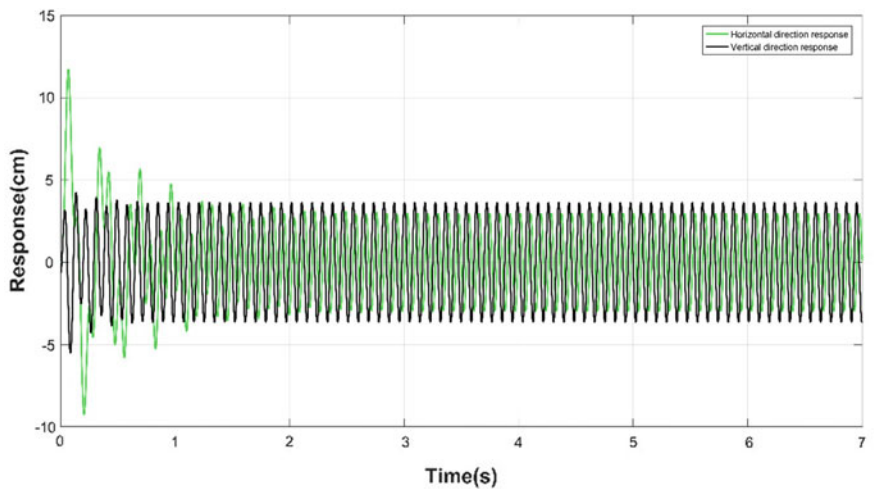
However, considering the same system with cubic spring nonlinearity, the system response reduces slightly at a steady state. But the total response of the system changes due to cubic nonlinearity. The value of linear spring stiffness is taken as 1000, 5000, and 15,000 N/m with nonlinear stiffness as 5000 N/m. The value of the damping coefficient remains unchanged (120 Ns/m). The results are shown in Figs. 70.5, 70.6, and 70.7, respectively.



**Fig. 70.2** Response of the system versus time (at linear spring stiffness of 1000 N/m and damping coefficient of 120 Ns/m)

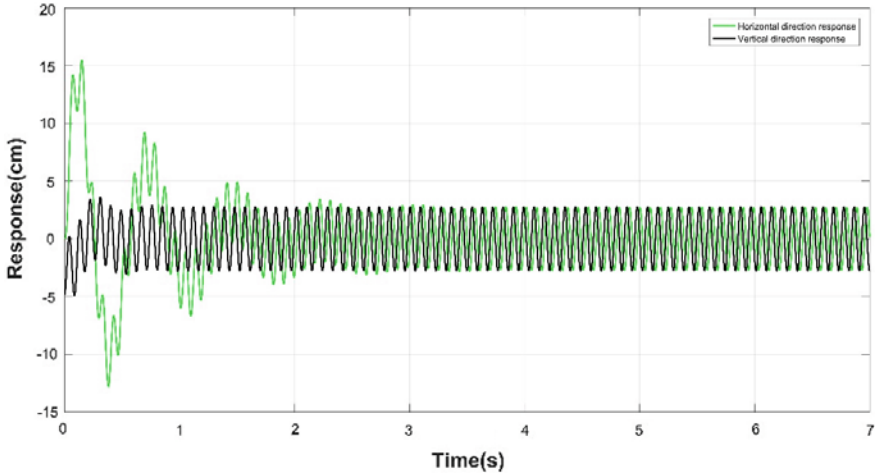


**Fig. 70.3** Response of the system versus time (at linear spring stiffness of 5000 N/m and damping coefficient of 120 Ns/m)

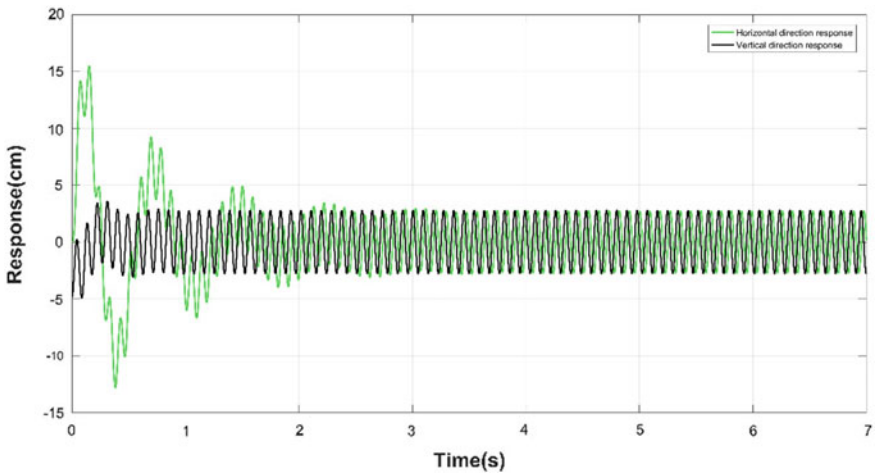


**Fig. 70.4** Response of the system versus time (at linear spring stiffness of 15,000 N/m and damping coefficient of 120 Ns/m)

Next, the damping coefficient is varied by keeping spring stiffness constant. The response of the system is investigated for the damping coefficients of 200, 600, and 800 Ns/m. From this study, it is observed that, with the increase in the value of damping coefficient, vibration magnitude in vertical direction reduces, but in the horizontal direction, the reduction is not significant (Figs. 70.8, 70.9 and 70.10).



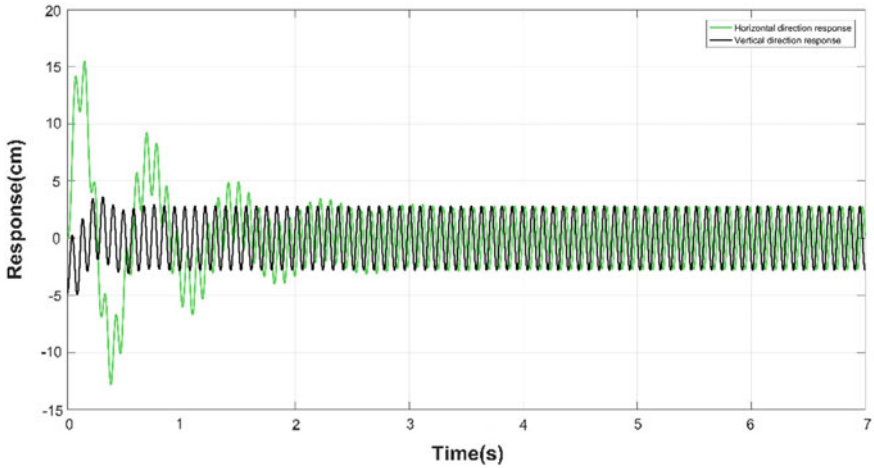
**Fig. 70.5** Response of the system versus time (at linear spring stiffness of 1000 N/m, nonlinear stiffness of 5000 N/m and damping coefficient of 120 Ns/m)



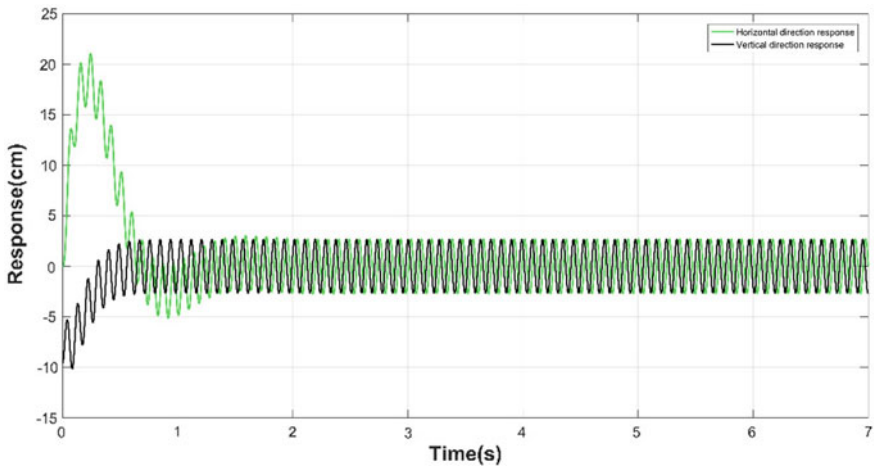
**Fig. 70.6** Response of the system versus time (at linear spring stiffness of 5000 N/m, nonlinear stiffness of 5000 N/m and damping coefficient of 120 Ns/m)

### Conclusion

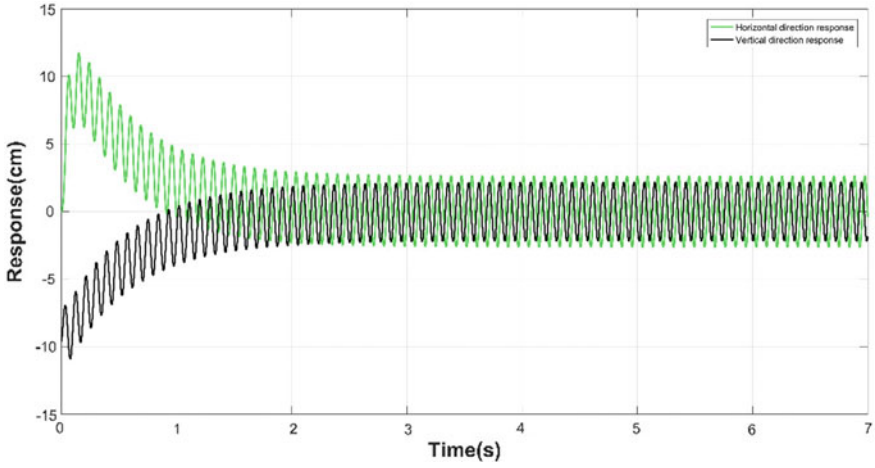
In this study, the vibration response of a washing machine drum with two springs and two dampers is studied. The nonlinearity of the spring is also considered for the study. The effects of the parameters like linear spring stiffness, nonlinear cubic stiffness, and the damping coefficient are studied. It is found that with increase in linear spring stiffness, the steady-state vibration level in both horizontal and vertical direction



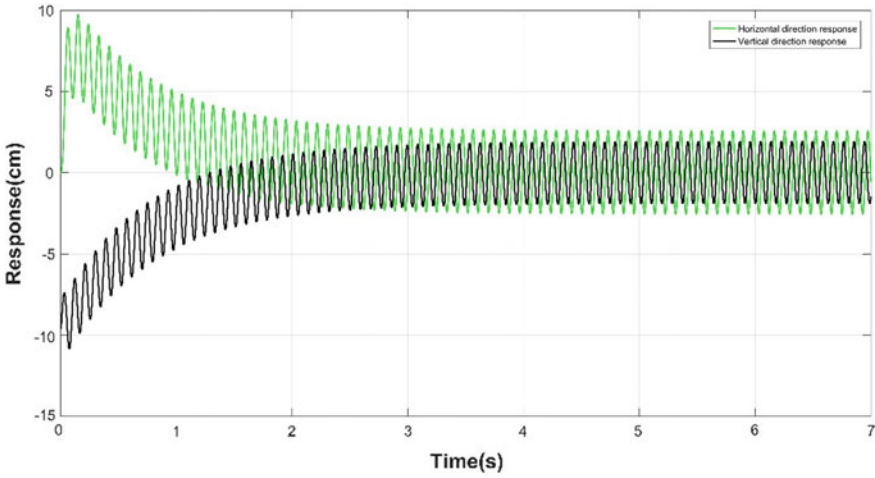
**Fig. 70.7** Response of the system versus time (at linear spring stiffness of 15,000 N/m, nonlinear stiffness of 5000 N/m and damping coefficient of 120 Ns/m)



**Fig. 70.8** Response of the system versus time (at linear spring stiffness of 1000 N/m, damping coefficient of 200 Ns/m and without spring nonlinearity)



**Fig. 70.9** Response of the system versus time (at linear spring stiffness of 1000 N/m, damping coefficient of 600 Ns/m and without spring nonlinearity)



**Fig. 70.10** Response of the system versus time (at linear spring stiffness of 1000 N/m, damping coefficient of 800 Ns/m and without spring nonlinearity)



increases. However, by adding a spring with cubic nonlinearity, the vibration level remains unaltered for steady state but total response of the system changes. With increase in value of damping coefficient, the magnitude of vibration decreases. It is also found that the vibration amplitude of the system (washing machine) can be minimized with the lower spring stiffness and higher damping coefficient. This study can be extended to design a new control technique for the control of different parameters such as spring stiffness, damping coefficient, and spring nonlinearity for future work.

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