Mathematical Modeling and Parametric FEM Study on the Thermal Management of a Rectangular Microchannel Heat Sink

Ria Ann Zachariah, Neelima Kuttappa Mukkatira, M. Sachin Bharadwaj, V. Krishna, and Babu Rao Ponangi

Abstract Due to the fast-paced depletion of natural energy resources such as petroleum, more sustainable alternatives, like Electric Vehicles (EVs) are destined to become the new norm. This new demand has increased the amount of research happening in this field, especially with regard to the controllers of the EV, which act as the brain of the operation. Of all the factors that affect the efficiency and functioning of any power electronic system, temperature is the most important. Thermal management should therefore be considered early in the design process. In this discourse, microchannels are considered to be the cooling mechanism applied on a microchannel heat sink system. A mathematical model is proposed based on Finite Element Methods which simplifies the discretization and assembly of parallel and tapered microchannel systems and validates their rates of cooling against similar models constructed using CFD methods. The goal of the study is to obtain a robust mathematical model that can be used to generate data for all attempts at optimizing the cooling rates using combinations of genetic algorithms.

Keywords Mathematical modeling · Microchannel · Finite element · Thermal management

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Nomenclature

μ_f Dynamic viscosity of the fluid (Ns/m^2)
C_p Specific heat capacity at constant pressure (J/kgK)
D Hydraulic Diameter of channel (μm, mm)
f Friction factor
H_c Height of the channel containing the coolant (μm, mm)
h_c Overall heat transfer coefficient (W/m^2K)
k Channel thermal conductivity (W/mK)
k_f Coolant thermal conductivity (W/mK)
L Length of the channel containing the coolant (cm)
m Mass flow rate (Kg/s)
Nu Nusselt number
Pr Prandtl number
Re Reynolds number
T Average wall temperature (K)
t Fin half thickness (μm)
T_f Average fluid temperature (K)
V Inlet velocity of the coolant (m/s)
w_c Width of the channel containing the coolant (μm, mm)

1 Introduction

In a sustainable world, innovation revolving around natural resources like petroleum and natural oils cannot survive indefinitely as the core of our industries especially the mobility market. Cleaner alternatives such as Electric Vehicles (EVs) are emerging at the present, to combat the fast depleting petroleum issue. To achieve the best solution for this issue, researchers around the world are working toward optimizing the efficiency and life of EVs. The controller of an EV helps monitoring the overall system and ensure proper flow of energy and balance within the EV and hence its optimal functioning is extremely critical for best vehicle performance. The functioning of the controller is entirely due to power electronics, which are economical, lightweight, compact, integrated, and can operate efficiently at high switching frequencies and operational temperatures [1]. Despite these exceptional qualities, its performance can be sensitive to temperature which can affect the performance of the controller as a whole. For high performance electronics, the heat must be conducted away from the small surface area of the chip to ensure maximum efficiency. The thermal distribution within the substrate can also pose a threat to the overall electronic system [2]. Hence, thermal management of the controller must be considered at an early stage in design. Various techniques have been studied and employed to support the advancing computational power. Passive and active cooling methods were analyzed [3, 4] in 1981, presented a new technique of scaling liquid-cooled heat exchangers down to the microscopic dimensions. This technique, now referred to as microchannel heat sinks, increased the limit of power density from 20W/sqcm to 1000 W/sqcm. The usage of microchannels resulted in compact configurations enabling it to be integrated with the miniaturized silicon chips. This makes microchannels the most sought-after cooling technique and hence was considered to be the system for our study. The analysis of microchannels can be done using different methods, both numerically and with the help of computer aided simulations [1].

2 Methodology

2.1 Problem Formulation

Microchannel cooling is commonly achieved with the aid of a heat sink that contains a large number of small channels. A suitable coolant is passed through these channels and hence heat is transferred through convection by the fluid, thereby cooling the device. Analysis for these microchannels with their own specific environments can be done using different methods, both numerical and with the help of computer aided simulation. The primary intent in this paper is to create a mathematical FEA model for a microchannel system which can be generalized for any geometric configuration. This model would be further used to determine the maximum wall temperatures and the pressure drop experienced across the channel for a given flow. This model was further used to analyze a parametric sweep study for varying height and width of the channel.

3 Mathematical Model

3.1 Creating a Mathematical Model for Wall Surface Temperature

Quadir et al. [5] were considered as the base study upon which a mathematical FEA model of a microchannel system was built. This would be applicable on any future geometrical configuration.

In Fig. 1, a uniform heat flux from a chip is imposed on the externally exposed face of the base. The other side of the base contains a series of rectangular channels which help in directing the flow of the coolant to enable heat transfer. The first step was to discretize the domain under study, which is described in Fig. 2. Each elemental microchannel wall was further defined using four noded bilinear elements. Conduction heat transfer was considered in two directions in both the vertical walls. The base wall had conduction from the heat sources and convective cooling from

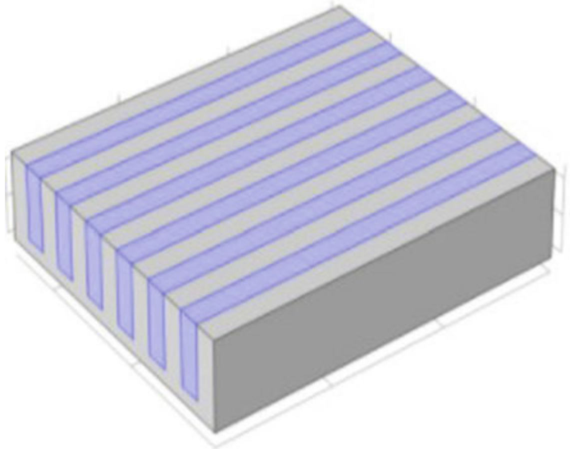

Fig. 1 General structure of the rectangular microchannel arrangement

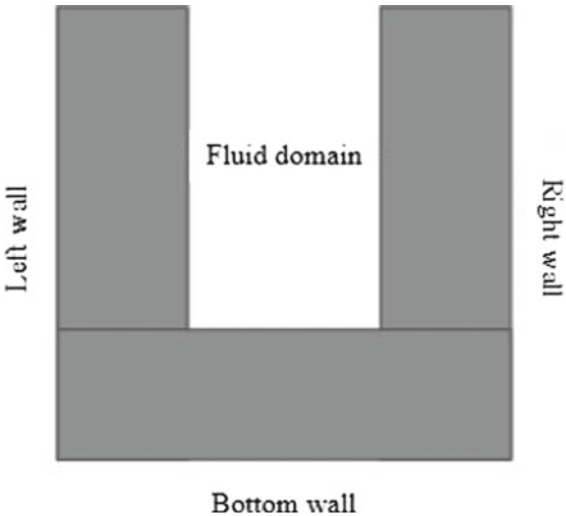

Fig. 2 Discretized domain under study

the coolant flowing above. Once the control volume was defined, energy balance equations for two walls and the coolant were stated.

Left:

$$k\frac{d^2T}{dx^2} + k\frac{d^2T}{dz^2} - \frac{h_c}{t}\left(T - T_f\right)_{Right} = 0 \quad (1)$$

Right:

$$k\frac{d^2T}{dx^2} + k\frac{d^2T}{dz^2} - \frac{h_c}{t}\left(T - T_f\right)_{Right} = 0 \quad (2)$$

Table 1 Coolant properties

Properties	Value
Fluid velocity	1.973 m/s
Coolant inlet temperature	10
Coolant thermal conductivity(k_f)	0.609 W/mK
Coolant absolute viscosity(μ_f)	$9.8 \times 10^{-4} Ns/m^2$
Coolant specific heat capacity(C_{pf})	4180 J/kgK
Coolant density(ρ_f)	1000 kg/m^3

Coolant:

$$mC_p\frac{dT_f}{dx} - h_cH_c\left(T - T_f\right)_{Left} - h_cH_c\left(T - T_f\right)_{Right} - h_cw_c\left(T - T_f\right)_{Bottom} = 0 \tag{3}$$

The finite element formulation of these equations was performed using the Galerkin method [6]. The temperature driving potential for the convective heat transfer was assumed to be the weighted residual integral. These equations were solved on the Maple software simultaneously for each node in the control volume. The coefficients obtained were arranged to attain a global matrix for an elemental microchannel. Subsequently, an algorithm for an assembly of such elemental microchannels was created, which enabled us to get further converging results with [5]. The dimensions used to build the geometry in Fig. 1 are as follows:

- Channel height—200 μm.
- Channel width—56 μm.
- Fin half thickness—12 μm.
- Channel length—0.5 cm.

The channel is made of silicon and its thermal conductivity is 125 W/mK. The coolant considered is water and its general properties are given in Table 1. The overall heat transfer coefficient for this study is 54,290 W/m^2K.

3.2 Parametric Study

In parametric studies, the mathematical model formed was further tested by varying channel height and width while keeping the channel length constant which helped analyze the cooling effect of the heat sink using the specific coolant.

Two variations were analyzed as follows:

Varying height with width constant.
Varying width with height constant.

The parameter values used are as follows:

Height: 11 mm, 12 mm, 13 mm, 14 mm.
Width: 3.80mm, 3.85mm, 3.90mm, 3.95mm.

The other dimensions and properties of the channel are as follows:

Half fin thickness—2 mm.
Channel length—60 mm.
Channel Thermal Conductivity (Aluminum)—237 W/mK.

The coolant considered in this scenario was water and its properties are given in Table 1.

3.3 Maximum Wall Surface Temperature

To calculate the maximum wall surface temperature, heat transfer coefficient for this was calculated using two different formulas for Nusselt number.

Equation 1:

$$Nu = 0.023 Re^{0.8} Pr^{n} \tag{4}$$

where n = 0.4 for heating of fluids Eq. 2:

$$Nu = \frac{\left(\frac{f}{8}\right) Re Pr}{1.07 + 12.7\left(\frac{f}{8}\right)^{0.5}\left[Pr^{0.67} - 1\right]} \left(\frac{\mu_m}{\mu_w}\right)^{n} \tag{5}$$

where

$$\frac{\mu_m}{\mu_w} \tag{6}$$

was assumed to be 1. Here, $f = 0.184 Re^{-0.2}$.

3.4 Pressure Drop Across Microchannel

The friction factor was calculated using a generalized formula for smooth tubes

$$f = 0.184 Re^{-0.2};\ Re > 10^{4} \tag{7}$$

From this the pressure drop was calculated using

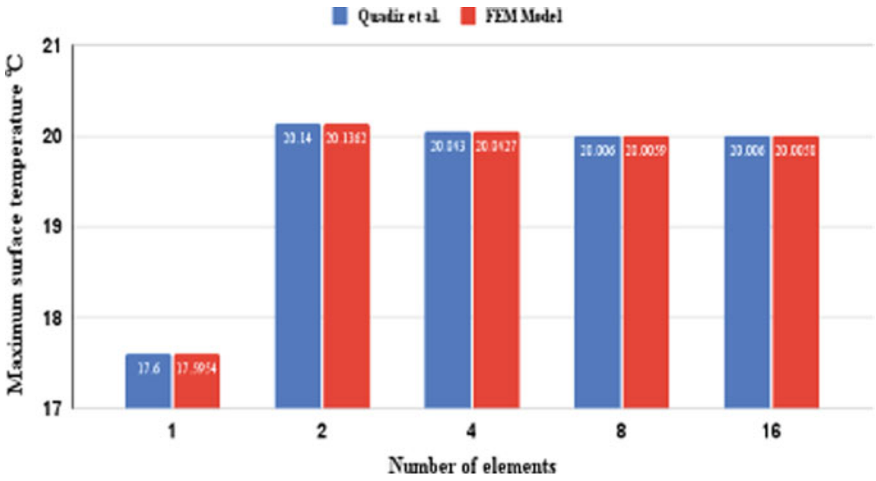

Fig. 3 Maximum surface temperature obtained using the developed mathematical model against the values given in Quadir et al

$$\Delta P = f\frac{L}{D}(\frac{V^2}{2}) \tag{8}$$

4 Results and Discussions

4.1 Creating a Mathematical Model for Wall Surface Temperature

The maximum surface temperature obtained using the mathematical model mentioned above is plotted in Fig. 3 against the values given in [5]. This also portrays that post eight elements, the maximum surface temperature is independent of the number of elements undertaken for study. With this Quadir et al. were validated and the developed mathematical model is ready to be used for any given configuration.

4.2 Parametric Study: Maximum Wall Surface Temperature

The first variation was done by keeping the width constant and varying the height. The maximum surface temperature obtained for this, with both the heat transfer values have been plotted in Fig. 4. From this it can be concluded that by keeping

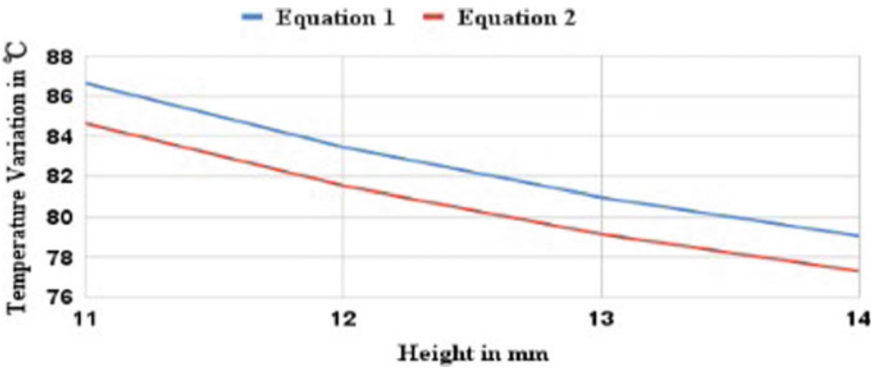

Fig. 4 Maximum surface temperature obtained at constant 3.8 mm width with varying height

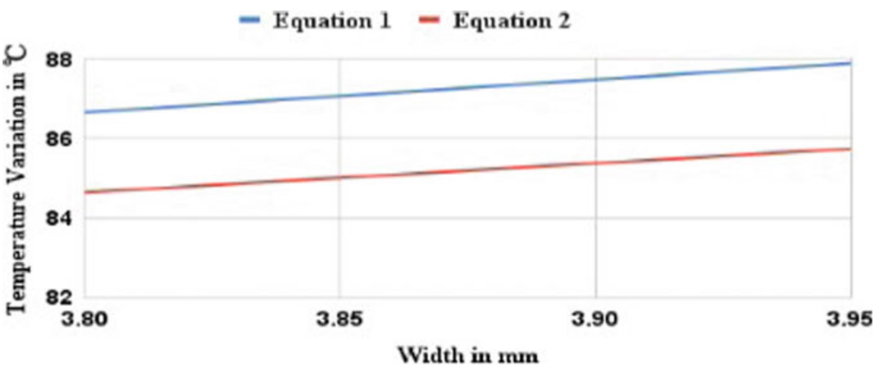

Fig. 5 Maximum surface temperature obtained at constant 11 mm height with varying width

the width constant and increasing the height, maximum surface temperature tends to decrease.

The second variation was done by keeping the height constant and varying the width. The maximum surface temperature obtained for this, with both the heat transfer values have been plotted in Fig. 5. From this it can be concluded that by keeping height constant and increasing the width, maximum surface temperature tends to increase.

Both these trends are consistent with those observed in [7].

4.3 Parametric Study: Pressure Drop Across Microchannel

The first variation was done by keeping the width constant and varying the height. The pressure drop values obtained have been plotted in Fig. 6. From this it can be

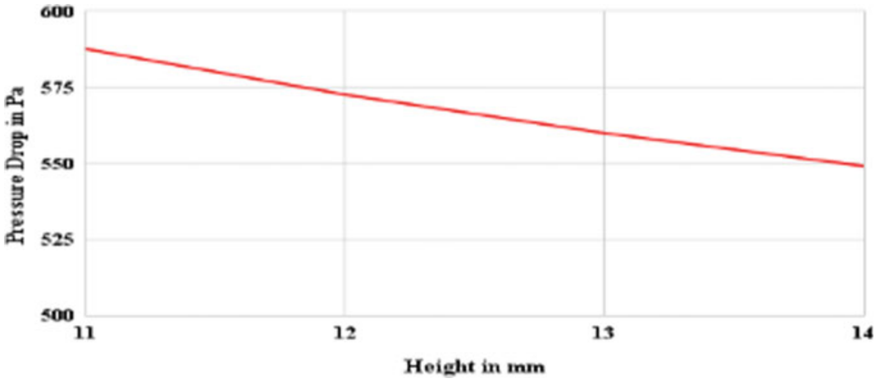

Fig. 6 Pressure drop obtained at constant 3.8 mm width with varying height

concluded that with the increase in height, keeping width constant, the pressure drop value decreases.

The second variation was done by keeping the height constant and varying the width. The pressure drop values obtained have been plotted in Fig. 7. From this it can be concluded that with the increase in width, keeping height constant, the pressure drop value decreases.

These trends are consistent with similar studies that correlate the varying dimensions of the microchannel setup to the pressure drop across the channel. Pressure drop decreases marginally both with the increase in height and width [7].

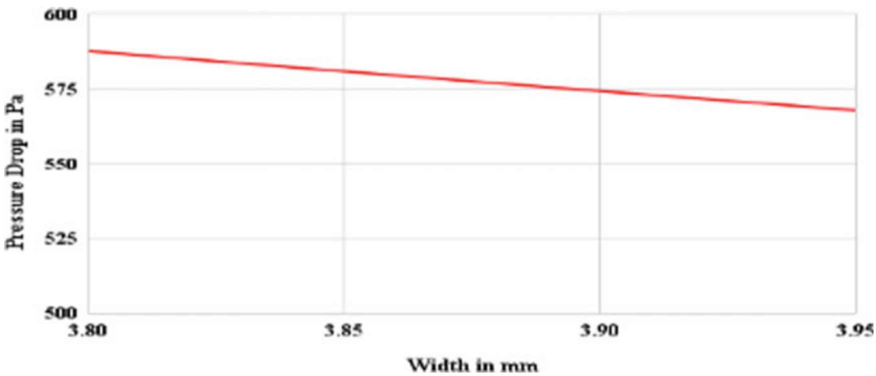

Fig. 7 Pressure drop obtained at constant 11 mm height with varying width

5 Conclusion

Quadir et al. were validated and the maximum wall surface temperature values were attained with no error. The mathematical model generated in this validation was further used in the parametric study of the microchannel for varying values of the height and width to analyze the sensitivity of these parameters and the optimal performance of the heat sink under the specified coolant. Through this it was observed that the maximum surface temperature is directly proportional to the increase in width and inversely proportional to the increase in height. Another conclusion obtained from the parametric study was the variation of pressure drop across the channel with the change in height and width of the microchannel. The pressure drop was found to decreases by 18.28% for 1 mm increase in width compared to 2.56% decrease for 1mm increase in height.

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