

Sub-nanometer Displacement Measurement Using Heterodyne Interferometer and Down-Beat Frequency Technique

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Abstract. The paper presents mechanical displacement measurements with subnanometer order for a double-path heterodyne interferometer using a laboratorymade phase meter based on a single phase-locked loop. We use a Zeemanstabilized He-Ne laser source in the interferometer, whose beat frequency is in the megahertz regime. It requires a high-cost, high-sampling rate analog-to-digital converter (ADC) for the megahertz-regime interference signals. A down-beat frequency technique is implemented to obtain these interference signals for a lowcost, low-sampling rate ADC. The result measured by the laboratory-made phase meter is compared with that of a reference one. The results of both phase meters show that a mechanical displacement of ~2.6 nm is confirmed. The noise reduction of the laboratory-made phase meter under 1 nm is more efficient than that of the reference one. It demonstrates that the interferometer combined with the downbeat technique is capable of achieving sub-nanometer mechanical displacement measurements.

Keywords: Displacement measurement · Heterodyne interferometer · Down-beat technique · Single phase-locked loop

1 Introduction

Over the last decade, nano and picometer technology has utilized high-precision highresolution positioning sensors and systems to measure or manipulate objects at the subnanometer or even picometer level $[1-3]$ $[1-3]$. Interferometry is one of the most candidates for acquiring such measurements and manipulations because of its traceability to the length standard of the meter. Many types of interferometry such as homodyne [\[4\]](#page-6-1) and heterodyne [\[5,](#page-6-2) [6\]](#page-6-3) interferometers, homodyne interferometers with frequency modulation [\[7\]](#page-6-4) or phase modulation [\[8\]](#page-6-5) have been developed. The motivation of the paper is to execute step-wise displacement measurements within the sub-nanometer scale or at the smallest available resolution (i.e., steps of 20 pm or even down to 10 pm).

Displacement interferometry is usually based on the Michelson configuration or some variant of that basic design. Interferometers of this type measure the displacement

proportional to the phase change interpolated from a sinusoidal interference signal with frequency *f*. Therefore, processing the measured signal from the avalanche photodiode (APD) is critical for obtaining the measurement results. A conventional heterodyne interferometer often has a beat frequency of the interference signal in the megahertz regime due to traditional Zeeman-stabilized He-Ne heads (i.e., Zygo and Agilent interferometers [\[9–](#page-6-6)[11\]](#page-6-7)). Such interference signals require a high-cost high-sampling frequency analogto-digital converter (ADC) equipped with a digital signal processor/field-programmable gate array (FPGA) $[12-14]$ $[12-14]$. It will limit the broad applications of the conventional heterodyne interferometer in industry.

This paper presents mechanical displacement measurements with sub-nanometer order using a conventional double-path heterodyne interferometer and a laboratory-made phase meter based on the single phase-locked loop (PLL) [\[6\]](#page-6-3). We show a traditional Michelson setup and a down-beat frequency technique, in which the beat frequency is downed from 3.6 MHz into 20 kHz. The laboratory-made phase meter equipped with an ADC, whose sampling frequency is 250 kHz, is used for these displacement measurements. The measurement results are compared with that of a reference phase meter $[15]$. The results give a mechanical displacement of ~ 2.6 nm. A comparison of the noise reduction between the phase meters is estimated under the same system and measurement conditions. The PLL phase meter gives more efficient noise reduction than the reference one. It shows that the interferometer can achieve sub-nanometer mechanical displacement measurements.

The paper is organized as follows. Section [2](#page-1-0) describes the principles of the heterodyne interferometer and the down-beat technique. Section [3](#page-3-0) provides experiments and results. Section [4](#page-5-1) shows the conclusions.

2 Methodology

2.1 Heterodyne Interferometer

Figure [1](#page-3-1) shows the schematic of a heterodyne interferometer. A target mirror is driven by a piezoelectric stage (PZT), and the heterodyne interferometer measures its displacement. The interferometer consists of a frequency-stabilized He-Ne laser, which has two slightly different and orthogonally polarizing frequencies f_1 and f_2 , a beam splitter (BS), a double-pass configuration (it includes a polarization beam splitter (PBS), a retroreflector, two quarter-wave plates (QWP), two mirrors (a fixed mirror (FM) and a target mirror (TM)), two polarization plates (PPs) and two photodetectors (PD). After BS, the laser beam is separated into two beams, reflected and transmitted beams. The reflected beam passes through PP towards PD1, and an interferometer signal *Iref* is generated. The transmitted beam comes to interferometer in which PBS splits the incident light into reflected S-polarized (vertical polarization, f_1) and transmitted P-polarized (horizontal polarization, f_2) beams. In the reference arm, the S-polarized beam passes through QWP1, reflected by FM and retroreflector respectively, becomes P-polarized beam, toward FM second time, passes through QWP1, becomes S-polarized beam and return PBS. In the measurement arm, the P-polarized beam passes through QWP2, reflected by TM and retroreflector respectively, becomes S-polarized beam, toward TM second time, passes through QWP2, becomes P-polarized beam and return PBS. By adjusting

PP to the same polarization plane of the beams f_1 and f_2 , the interference signal I_{meas} is generated at PD2. The interferometer signals are described as

$$
I_{ref} = I_1 \cos(2\pi \Delta ft), \tag{1}
$$

$$
I_{meas} = I_2 \cos(2\pi \Delta ft + \delta),\tag{2}
$$

where I_1 , I_2 , Δf , and δ are the amplitude of the two signals, signal frequency, and the phase difference, which is related to the displacement of the TM, respectively. For discrete signal processing, the phase δ is shifted over an interval time of 0 to t_N , given by

$$
\delta = \sum_{t=0}^{t=t_N} d\delta \tag{3}
$$

where dδ is the instantaneous phase difference obtained over d*t*. Here assume that the TM velocity is not changed over dt and $dt = 1/f_s$ is the sampling time interval of the ADC. For a double-path heterodyne interferometer, $d\delta$ can be calculated as

$$
d\delta = \frac{8\pi n}{\lambda} dL, \tag{4}
$$

where dL , *n*, and λ are displacements of TM over dt when the mirror is in uniform motion, the refractive index of air, and the laser source wavelength in vacuum, respectively. Substituting Eq. [\(4\)](#page-2-0) to Eq. [\(3\)](#page-2-1), we have the relationship between δ and the value of displacement measurement mirror *L*

$$
\delta = \sum_{t=0}^{t=t_N = Ndt} \frac{8\pi n}{\lambda} dL = \frac{8\pi n}{\lambda} L. \tag{5}
$$

If dδ is larger than 2π rad over d*t*, the ambiguity should be mixed in Eq. [\(5\)](#page-2-2).

2.2 Down-Beat Technique

In the setup, a down-beat frequency technique is implemented to obtain these interference signals using a low-cost, low-sampling rate ADC. *Iref* and *Imeas* are mixed with a pure electronic signal *I** generated from a function generator (FG), which is given by

$$
I^* = A \cdot \cos(2\pi f^* \cdot t), \tag{6}
$$

where A and f^* are the signal amplitude and frequency, respectively. The products are then passed through by low-pass filters (LPFs) to remove higher-frequency terms $(\Delta f + f^*)$ and maintain lower-frequency terms ($\delta f = \Delta f - f^*$). The reference I_{ref}^* and measurement *I*[∗]_{*meas*} signals after the LPFs are expressed as

$$
I_{ref}^* = I^* \cdot I_{ref} = 0.5AI_1 \cos[2\pi (\delta f)t],
$$
 (7)

$$
I_{meas}^* = I^* \cdot I_{meas} = 0.5AI_2 \cos[2\pi (\delta f)t + \delta]. \tag{8}
$$

Finally, the interference signals are fed into peripheral component interconnect (PCI) equipped with the ADC at f_s . A laboratory-made phase meter based on the PLL is employed to determine δ [\[6\]](#page-6-3).

Fig. 1. Schematic of the heterodyne interferometer. BS, beam splitter; PBS, polarization beam splitter; QWPs, quarter-wave plates; PZT, piezoelectric stage; PPs, polarization plates; PD1 and PD2, photodetectors; LPF, low-pass filter; PCI, peripheral component interconnect; PC, personal computer.

3 Experiments and Results

Figure [2](#page-3-2) shows the photograph of the experimental setup. In the setup, we use a He-Ne head (Zygo ZMI 7705, beat frequency $= 3.6$ MHz) [\[10\]](#page-6-11) as a light source for the interferometer. To minimize the environmental effects, the measurement mirror is set close to PBS as much as possible. In addition, we use a commercial double-path plane mirror interferometer (Agilent 10706B) to limit the effects, reduce the length of the dead-path and feedback of beam light. A travel translation stage (Thorlabs NFL5D20P/M, sensitivity = 267 nm/V, range = 20 μ m) [\[16\]](#page-6-12) is driven by a voltage driver (Thorlabs KPZ101) that is used to generate mechanical displacements of the target mirror. The interference signals are collected by two PDs (Thorlabs PDA36A2) for signal processing. The whole setup is located on an antivibration table with an enclosure, and the experimental conditions are shown in Table [1.](#page-4-0)

Fig. 2. Photograph of the double-path heterodyne interferometer. BS, beam splitter; TM, target mirror; PZT, piezoelectric; PD1 and PD2, photodetectors.

As far as we have mentioned in our down-beat technique, the measurement and reference signals are directly mixed with the purely electronic signal (*f** frequency and peak-to-peak amplitude $= 600$ mV) generated from a function generator (HM8030-5) by two mixers (Mini-circuits ZAD-1+). The products are digitized and filtered two digital LPFs with a cutoff frequency of 48.83 kHz (Moku: Lab, sample rate 488.3 kHz, windowing Blackman). The outputs of the digital LPFs are lower frequency components that have a beat frequency $\delta f = 20$ kHz. The intensities of the interference signals are shown in Eqs. [\(7\)](#page-2-3) and [\(8\)](#page-2-4). The outputs are then down-sampled to 244 kHz and fed into the PCI (NI-PCI 6143) equipped with an ADC (resolution $= 16$ bits, sampling frequency $=$ 250 kHz). The single PLL phase meter is used to find δ and a corresponding displacement ΔL . The final sampling frequency of δ and ΔL is 100 Hz.

The displacement of TM is created with a 0.01 V step signal driven by the PZT stage. A theoretical corresponding step displacement of \sim 2.67 nm (= 0.01 V \times 20 μ m/75 V) is determined. The actual displacement of TM is measured by the interferometer equipped with our laboratory-made phase meter. To compare with the results, we use a reference phase meter (Moku:Lab, resolution $= 1$ mdeg, input and output sampling frequency $=$ 500 MHz and 122 Hz, respectively) [\[15\]](#page-6-10). The measurement results determined by the phase meters are then compared with the displacement step of 2.67 nm.

A distance of 24 nm in steps of \sim 2.67 nm (0.01 V per step) is generated using the PZT stage to induce a shift for the TM. Figure [3](#page-5-2) shows the measurement results of both phase meters. The results show that a distance of 24 nm in steps of \sim 2.6 nm is determined by both phase meters. These measurement results are slightly different from the reference data (2.67 nm/0.01 V) derived from $[16]$ since it may be possible that the PZT table with an extra load is actuated. The standard deviations on each step segment displacement are 1.3 nm and 0.14 nm for the reference and laboratory-made phase meters, respectively. Noise reduction of our phase meter is better than that of the reference phase meter. It helps the interferometer be capable of attaining sub-nanometer mechanical displacement measurements. In the near future, it will be a direction of our research to measure sub-nanometer movements or possibly reach 10 pm.

(1) He-Ne heterodyne laser	
3.6 ± 0.3 MHz	
633 nm	
(2) PZT stage	
267 nm/V	
$20 \mu m$	
100 Hz	
(4) Environmental conditions	
25° C	
67%	
1012 hPa	

Table 1. Instruments and conditions for the experiments.

Fig. 3. The measurement results of the phase meter. PM, phase meter.

4 Conclusions

This paper presents mechanical displacement measurements with sub-nanometer order using a double-path heterodyne interferometer and a laboratory-handmade phase meter replied on the single PLL. A down-beat frequency technique is implemented to acquire the interference signals correctly, whose frequency is in the megahertz range, for a low-cost ADC with hundreds of kilohertz sampling rates. A comparison of the results between the laboratory-handmade phase meter supported with the down-beat frequency and a reference phase meter is executed for the interferometer. The results indicate the interferometer can achieve a mechanical displacement of \sim 2.6 nm. The noises over each step are 0.14 nm and 1.3 nm for the PLL and reference phase meters, respectively. The PLL phase meter has more efficient noise reduction than the reference one. It demonstrates that this interferometer and the down-beat technique are capable of attaining sub-nanometer mechanical displacement measurements. However, unexpected effects such as environmental variations, vibrations, etc., affecting our results are not investigated for the interferometer. The investigation is one of future works. We will also develop a modified interferometer with two spatially separated beams to minimize periodic errors and outside effects and reach a picometer-order displacement measurement. Finally, creating a low-cost, high-resolution phase meter is one of the future works.

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