

# **Disturbance Observer-Based Speed Control of Interior Permanent Magnet Synchronous Motors for Electric Vehicles**

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**Abstract.** This paper studies speed control problem for interior permanent magnet synchronous motor (IPMSM) based on a backstepping algorithm. The conventional backstepping control method cannot achieve a robustness controller in the presence of load torque and uncertain parameters. Thus, a high-gain disturbance observer (HGDOB) is proposed to improve the controller and obtain fast transient responses and strong robustness. Consequently, the influence of the disturbances on the motor drive transmission for electric vehicles is effectively reduced. TI C2000 F28377S microcontroller combined with Matlab/Simulink is implemented to demonstrate the effectiveness and feasibility of the proposed control and observer.

**Keywords:** IPMSM · Backstepping · High gain observer · Processor in the loop (PIL)

# **1 Introduction**

Electric cars are making big waves in the automobile industry. Instead of using an internal combustion engine (ICE), electric vehicles (EVs) are driven by electric motors. Electric motors have significant advantages over internal combustion engines in motion control [\[1\]](#page-13-0). These advantages can be summarized as follows: noise-free, pollution-free, highperformance, and the feasibility of applying advanced control methods. From which, the dynamic quality of electric vehicles is improved. In particular, Electric Traction Motors have key characteristics as fast and quick torque response, high torque at low speed, small size, reduced weight. Adopted traction motors in electric vehicles (EVs) and hybrid electric vehicles (HEVs) is Interior Permanent Magnet Synchronous Motor (IPMSM).

To achieve the high performance and high accuracy requirements of PMSM drive systems, various nonlinear control techniques have been widely applied, such as model predictive control  $[2, 3]$  $[2, 3]$  $[2, 3]$ ; backstepping control  $[4, 5]$  $[4, 5]$  $[4, 5]$ , sliding mode control  $[6]$ , etc. to improve motor control efficiency in different aspects. Among the above methods, backstepping control is a recursive design approach for nonlinear systems. By using

variable virtual control law, the initial high-order system is simple so that the final outputs can be systematically computed through suitable Lyapunov functions.

The reality shows that IPMSM drive system faces unavoidable disturbances, such as uncertain parameters and load torque. In order to improve the control quality as well as reduce the influence of the above problems on speed response, disturbance observerbased control systems are widely studied. Origins of disturbance observer was mentioned as Birth of Robust Control (1960s–1980s), and Birth of DOb (1960s–1980s), then the development of DOB-based robust control was proposed by K. Ohnishi in 1983, reviewed in "35th Anniversary Overview" [\[7\]](#page-14-5). Up to now, many disturbance observers have been developed such as linear disturbance and uncertainty estimation (LDUE) techniques, and nonlinear disturbance/uncertainty estimation (NDUE) techniques [\[8\]](#page-14-6).

In [\[9\]](#page-14-7) and [\[10\]](#page-14-8), nonlinear disturbance observers are used for rotor speed control loop and current control loop, respectively. In which rotor speed response is greatly improved. A backstepping control combined with nonlinear disturbance observers for both outer-loop and inner-loop control are presented in [\[11\]](#page-14-9) and obtained good results in the presence of disturbances. In  $[12]$ , a terminal sliding mode single-loop control is used with a nonlinear disturbance observer to estimate the lump disturbance. In order to improve the results further, in this paper, nonlinear high-gain disturbance observers [\[13\]](#page-14-11) combined with the backstepping control system based on Lyapunov stability theory [\[14\]](#page-14-12) is proposed and achieved better rotor speed response, in which the deviation is significantly reduced in the presence of load torque disturbance.

The contribution of the paper is: (i) A high-gain disturbance observer is proposed and achieve high-precision observation results. (ii) The proposed controller is the nonlinear disturbance observer combined backstepping controller that eliminating the disturbance of IPMSM included uncertainty parameters and load torque to obtain fast transient responses and strong robustness.

This paper is structured as follows: the second part is the introduction of mathematical model of PMSM. In the third part, the nonlinear disturbance observer is presented. The fourth part, the backstepping controller is built in combination with the estimation results of the nonlinear disturbances obtained in the previous third part. Simulation methods with the combination of Matlab/Simulink on computer and TI C2000 microcontroller are presented in the fifth part and finally concluded in the sixth part.

# **2 Mathematical Model**

Assume that the motor's current loss, hysteresis, and eddy are ignored, and the magnetic circuit is unsaturated. The three-phase stator windings of AC machines are sinusoidal in space, then vector control strategy is adopted to obtain decoupled control of motor torque and flux. By using the Park transforms mathematical model, a three-phase system in a reference frame *abc* is converted to a rotating reference frame *dq0* included direct, quadrature, and zero components*.* The stator flux equations of IPMSM in the *d* and *q* axis is given in  $[15]$ :

<span id="page-1-0"></span>
$$
\begin{cases} \phi_d = L_d i_d + \phi \\ \phi_q = L_q i_q \end{cases} \tag{1}
$$

where  $L_d$ ,  $L_q$  denote the  $d-q$  axis stator inductances ( $mH$ ),  $\phi$  is rotor permanent magnet flux (*Wb*);  $i_d$ ,  $i_q$  are the *d-q* axis currents. According to field-oriented theory, the *d-q* axis voltages are expressed as [\[16\]](#page-14-14):

$$
\begin{cases}\n u_d = R_s i_d + L_d \frac{di_d}{dt} - n_p \omega L_q i_q \\
 u_q = R_s i_q + L_q \frac{di_q}{dt} - n_p \omega L_d i_d + n_p \omega \phi\n\end{cases}
$$
\n(2)

where  $R_s$  is stator resistance  $(\Omega)$ ;  $n_p$  is the pole-pair number;  $\omega$  is the rotor speed. Torque equations of IPMSM is given as follows:

<span id="page-2-2"></span>
$$
T_e = \frac{3}{2} n_p \left( \phi_d i_q - \phi_q i_d \right) \tag{3}
$$

<span id="page-2-1"></span><span id="page-2-0"></span>
$$
T_e = J_m \frac{d\omega}{dt} + B\omega + \tau_L \tag{4}
$$

where  $J_m$  is the rotor moment of inertia ( $kgm^2$ ); *B* is the viscous friction coefficient (*Nms*);τ*<sup>L</sup>* is load torque (*Nm*). From the Eqs. [\(1\)](#page-1-0)–[\(4\)](#page-2-0), the mathematical model of IPMSM in the rotor rotation reference system  $(dq)$  can be expressed as follows [\[11\]](#page-14-9):

$$
\begin{cases}\n\frac{d\omega}{dt} = \frac{1}{J_m} \left( n_p \left( (L_d - L_q) i_d i_q + \phi i_q \right) - B\omega \right) \\
+ d_1 \\
\frac{di_q}{dt} = \frac{1}{L_q} \left( -R_s i_q - n_p \omega \phi - n_p \omega L_d i_d \right) \\
+ \frac{1}{L_q} u_q + d_2 \\
\frac{di_d}{dt} = \frac{-R_s i_d + n_p \omega L_q i_q}{L_d} + \frac{1}{L_d} u_d + d_3\n\end{cases} (5)
$$

where  $d_1$ ,  $d_2$ ,  $d_3$  represent disturbances caused by uncertainty parameters and load torque that are defined as follows [\[11\]](#page-14-9):

<span id="page-2-3"></span>
$$
\begin{cases}\nd_1 = -\frac{1}{J_m} \left( \Delta J_m \frac{d\omega}{dt} + \Delta B \omega + \tau_L \right. \\
-n_p \left( \Delta L_d - \Delta L_q \right) i_d i_q - n_p \Delta \phi i_q \right) \\
d_2 = -\frac{1}{L_q} \left( \frac{\Delta R_s i_q + \Delta L_d n_p \omega i_d + \Delta \phi n_p \omega}{\Delta L_q \frac{di_q}{dt}} \right) \\
d_3 = -\frac{1}{L_d} \left( \Delta R_s i_d - \Delta L_q n_p \omega i_q + \Delta L_d \frac{di_d}{dt} \right)\n\end{cases} \tag{6}
$$

where  $\Delta R_s = R_{st} - R_s$ ,  $\Delta L_d = L_{dt} - L_d$ ,  $\Delta L_q = L_{qt} - L_q$ ,  $\Delta \phi = \phi_t - \phi$ ,  $\Delta J_m =$  $J_{mt} - J_m$ ,  $\Delta B = B_t - B$  with  $R_s$ ,  $L_d$ ,  $L_q$ ,  $\phi$ ,  $J_m$ ,  $B$  are the nominal parameter values;  $R_{st}$ ,  $L_{dt}$ ,  $L_{qt}$ ,  $\phi_t$ ,  $J_{mt}$  and  $B_t$  are the actual parameter values. Let  $\omega = x_1$ ,  $i_q = x_2$ ,  $i_d = x_3$ , Eq.  $(5)$  is rewritten as follows:

$$
\begin{cases}\n\dot{x}_1 = -\frac{Bx_1}{J_m} + \frac{n_p((L_d - L_q)x_3 + \phi)}{J_m} x_2 + d_1 \\
\dot{x}_2 = \frac{-R_s x_2 - n_p x_1 \phi - n_p x_1 L_d x_3}{L_q} + \frac{1}{L_q} u_q + d_2 \\
\dot{x}_3 = \frac{-R_s x_3 + n_p x_1 L_q x_2}{L_d} + \frac{1}{L_d} u_d + d_3\n\end{cases} (7)
$$

<span id="page-3-3"></span>**Assumption 1.** The state variables  $\omega$ ,  $i_q$ ,  $i_d$  are physically bounded, exist that:

$$
|\omega| < \omega_{\text{max}}, \ |i_q| < i_{q,\text{max}}, \ |i_d| < i_{d,\text{max}} \tag{8}
$$

where  $\omega_{\text{max}}$ ,  $i_{q,\text{max}}$ ,  $i_{d,\text{max}}$  are constants. Besides, the unknown disturbances  $d_1$ ,  $d_2$ , and  $d_3$  vary slowly and are bounded. Thus, exist such that the constraint constants  $\delta_1$ ,  $\delta_2$ ,  $\delta_3$ satisfy [\[10\]](#page-14-8):

<span id="page-3-2"></span>
$$
|\dot{d}_1| \le \delta_1, |\dot{d}_2| \le \delta_2, |\dot{d}_3| \le \delta_3 \tag{9}
$$

## **3 Robust Backstepping Controller Design**

### **3.1 Speed Controller Design**

In order to maintain constant flux operations of the motor, the *d*-axis current  $i_d$  is usually set to be zero. From which, the motor model given in Eq. [\(5\)](#page-2-1) can be modified as follow:

<span id="page-3-1"></span>
$$
\dot{x}_1 = \frac{n_p \phi x_2 - B x_1}{J_m} + d_1 \tag{10}
$$

Define speed error as  $e_1 = x_1^* - x_1$ , where  $x_1^*$  is the reference speed. The derivative of speed error is:

$$
\dot{e}_1 = \dot{x}_1^* - \dot{x}_1 = \dot{x}_1^* - \frac{n_p \phi x_2}{J_m} + \frac{B x_1}{J_m} - d_1 \tag{11}
$$

Choose the Lyapunov candidate function as  $V_1 = (1/2)e_1^2$ . The time derivative of  $V_1$  is:

$$
\dot{V}_1 = -\frac{e_1}{J_m} \left( -J_m \dot{x}_1^* + n_p \phi x_2 - B x_1 + J_m d_1 \right) \tag{12}
$$

To force the derivative of  $V_1$  to become negative definite, the virtual control  $x_2^*$  is designed as follows:

<span id="page-3-0"></span>
$$
x_2^* = \frac{1}{n_p \phi} \left( Bx_1 - J_m d_1 + J_m \dot{x}_1^* + k_1 J_m e_1 \right) \tag{13}
$$

Substituting Eq. [\(13\)](#page-3-0) into Eq. [\(12\)](#page-3-1):  $\dot{V}_1 = -k_1 e_1^2 \le 0$ , where  $k_1 > 0$ . When the derivative of  $V_1$  is negative definite, the error of the speed controller will increasingly approach zero.

## **3.2 Current Controller Design**

The reference q-axis current is the virtual control variable from Eq. [\(13\)](#page-3-0), and the reference d-axis current  $x_3^*$  is normally set to zero to maintain constant flux. The current error on the d-q axis is determined as follows:

$$
e_2 = x_2^* - x_2
$$
  
\n
$$
e_3 = x_3^* - x_3
$$
\n(14)

The time derivative of  $e_2$ ,  $e_3$  are given as:

$$
\dot{e}_2 = \frac{1}{n_p \phi} \left( \frac{B \left( \frac{n_p \phi x_2 - B x_1}{J_m} + d_1 \right) + J_m \ddot{x}_1^*}{+ k_1 (J_m \dot{x}_1^* - n_p \phi x_2 + B x_1 - J_m d_1)} \right) + \frac{R_s x_2 + n_p x_1 L_d x_3 + n_p x_1 \phi - u_q}{L_q} - d_2
$$
\n
$$
\dot{e}_3 = \frac{R_s x_3 - n_p x_1 L_q x_2}{L_d} - \frac{1}{L_d} u_d - d_3
$$
\n(15)

Define Lyapunov candidate function:

$$
V_2 = V_1 + \frac{1}{2}e_2^2 + \frac{1}{2}e_3^2\tag{16}
$$

Then,

$$
\dot{V}_2 = -k_1 e_1^2 + e_3 \left( \frac{R_s x_3 - n_p x_1 L_q x_2}{L_d} - \frac{1}{L_d} u_d - d_3 \right)
$$
\n
$$
+ e_2 \left( \frac{1}{n_p \phi} \left( \frac{B \left( \frac{n_p \phi x_2 - B x_1}{J_m} + d_1 \right) + J_m \ddot{x}_1^*}{+k_1 (J_m \dot{x}_1^* - n_p \phi x_2 + B x_1 - J_m d_1)} \right) + \frac{R_s x_2 + n_p x_1 L_d x_3 + n_p x_1 \phi - u_q}{L_q} - d_2 \right)
$$
\n(17)

The current errors can be stabilized if the control laws are designed as follows:

$$
u_q = \frac{L_q}{n_p \phi} \left( B \left( \frac{n_p \phi i_q - B\omega}{J_m} + d_1 \right) + J_m \ddot{\omega}^* \right)
$$
  
+ 
$$
k_1 (J_m \dot{\omega}^* - n_p \phi i_q + B\omega - J_m d_1) \right)
$$
  
+ 
$$
R_s i_q + n_p \omega L_d i_d + n_p \omega \phi - L_q d_2 + k_2 e_2
$$
  

$$
u_d = L_d \left( \frac{R_s i_d - n_p \omega L_q i_q}{L_d} - d_3 + k_3 e_3 \right)
$$
 (18)

where  $k_2, k_3 > 0$ . Substituting Eq. [\(18\)](#page-4-0) into the derivative of Lyapunov function  $\dot{V}_2$ , we get:

<span id="page-4-1"></span><span id="page-4-0"></span>
$$
\dot{V}_2 = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 \le 0 \tag{19}
$$

<span id="page-5-4"></span>**Remark 1.** In order to achieve this convergence as Eq. [\(19\)](#page-4-1), then the disturbances *d*1, *d*2, *d*<sup>3</sup> must be explicitly known or zero. However, in practice, the disturbances of IPMSM are always existed and are unknown. Thus, the derivative of Lyapunov function  $\dot{V}_2$  results in:

$$
\dot{V}_2 = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 + e_1 d_1 + e_2 d_2 + e_3 d_3 \tag{20}
$$

Let,

<span id="page-5-0"></span>
$$
\begin{cases}\n \zeta = e_1 d_1 + e_2 d_2 + e_3 d_3 \\
 \chi = \min\{k_1, k_2, k_3\}\n \end{cases}
$$
\n(21)

Then, Eq. [\(20\)](#page-5-0) is rewritten as:

<span id="page-5-1"></span>
$$
\dot{V} \le -2\chi V + \varsigma \tag{22}
$$

From the Eq. [\(22\)](#page-5-1) can realize that the deviation of the control system is strongly influenced by the disturbances  $d_1$ ,  $d_2$ ,  $d_3$ . Thus, to improve the control system, a highgain disturbance observer is proposed that provides disturbance information such as uncertainty parameters and load torque to the controller.

#### **3.3 Disturbance Observer Design**

Define the estimated disturbances as  $d_1$ ,  $d_2$ ,  $d_3$  the estimated errors are defined as follows:

$$
\tilde{d}_1 = d_1 - \hat{d}_1, \ \tilde{d}_2 = d_2 - \hat{d}_2, \ \tilde{d}_3 = d_3 - \hat{d}_3 \tag{23}
$$

With  $\varepsilon_1$ ,  $\varepsilon_2$ ,  $\varepsilon_3$  are the observation coefficients, dynamic equations of the estimated disturbances are designed as follows:

$$
\dot{\hat{d}}_1 = \frac{1}{\varepsilon_1} \left( \dot{x}_1 + \frac{Bx_1}{J_m} - \frac{n_p((L_d - L_q)x_3 + \phi)}{J_m} x_2 \right)
$$
\n
$$
\dot{\hat{d}}_2 = \frac{1}{\varepsilon_2} \left( \dot{x}_1 - \frac{-R_s x_2 - n_p x_1 \phi - n_p x_1 L_d x_3}{L_q} - \frac{1}{L_q} u_q - \hat{d}_2 \right)
$$
\n
$$
\dot{\hat{d}}_3 = \frac{1}{\varepsilon_3} \left( \dot{x}_3 - \frac{-R_s x_3 + n_p x_1 L_q x_2}{L_d} - \frac{1}{L_d} u_d \right)
$$
\n
$$
-\hat{d}_3
$$
\n(24)

Define the auxiliary state variables:

<span id="page-5-3"></span><span id="page-5-2"></span>
$$
\xi_i = \hat{d}_i - \frac{x_i}{\varepsilon_i}, \ i = [1, 3] \tag{25}
$$

The dynamic equations of the auxiliary state variables are:

<span id="page-6-0"></span>
$$
\dot{\xi}_1 = -\frac{1}{\varepsilon_1} \left( -\frac{Bx_1}{J_m} + \frac{n_p((L_d - L_q)x_3 + \phi)}{J_m} x_2 \right)
$$

$$
- \frac{1}{\varepsilon_1} \left( \xi_1 + \frac{x_1}{\varepsilon_1} \right)
$$

$$
\dot{\xi}_2 = -\frac{1}{\varepsilon_2} \left( \frac{1}{L_q} u_q + \frac{-R_s x_2 - n_p x_1 \phi - n_p x_1 L_d x_3}{L_q} \right)
$$

$$
- \frac{1}{\varepsilon_2} \left( \xi_2 + \frac{x_2}{\varepsilon_2} \right)
$$

$$
\dot{\xi}_3 = -\frac{1}{\varepsilon_3} \left( \frac{1}{L_d} u_d + \frac{-R_s x_3 + n_p x_1 L_q x_2}{L_d} \right)
$$

$$
- \frac{1}{\varepsilon_3} \left( \xi_3 + \frac{x_3}{\varepsilon_3} \right)
$$
(26)

*Theorem 1. For the nonlinear system of the interior permanent magnet synchronous motor in Eqs.*  $(1)$ – $(7)$ *, the unknown disturbances are bounded and shown in Eqs.*  $(6)$ *, [\(9\)](#page-3-2) can be observed by using the high-gain disturbance observers as Eq. [\(26\)](#page-6-0) combined the auxiliary state variables in Eq. [\(25\)](#page-5-2).*

**Proof of Theorem 1.** The following disturbance estimation error dynamics is obtained as [\[13\]](#page-14-11):

<span id="page-6-2"></span><span id="page-6-1"></span>
$$
\dot{\xi}_i = \dot{\tilde{d}}_i - \frac{\dot{x}_i}{\varepsilon_i} \tag{27}
$$

From Eqs. [\(26\)](#page-6-0) and [\(27\)](#page-6-1), estimation error dynamics of the observers is obtained as:

$$
\dot{\tilde{d}}_i = \dot{d}_i - \left(\frac{1}{\varepsilon_i}\right)\tilde{d}_i
$$
\n(28)

Consequently,

$$
\left|\tilde{d}_i\right| \leq e^{-(1/\varepsilon_i)t} \left|\tilde{d}_i(0)\right| + \varepsilon_i \rho_i(t) \tag{29}
$$

Then, the upper bound of  $|\tilde{d}_i(\infty)|$  becomes smaller by  $\varepsilon_i$  gets smaller.

**Remark 2.** In Eq. [\(26\)](#page-6-0), observers with auxiliary state variables do not need to use derivatives of  $\omega$ ,  $i_d$ ,  $i_q$  as in Eq. [\(24\)](#page-5-3). Then the measurement noise amplification by using the high gain  $1/\varepsilon$ <sub>i</sub> will be reduced, thus observers will be more feasible when applied in practice.

In order to improve the efficiency of the backstepping controller, the observed values  $d_1, d_2, d_3$  from the Eq. [\(26\)](#page-6-0) are substituted into Eqs. [\(13\)](#page-3-0) and [\(18\)](#page-4-0) as follows:

<span id="page-7-0"></span>
$$
x_{2}^{*} = \frac{1}{n_{p}\phi} \left( Bx_{1} - J_{m}\hat{d}_{1} + J_{m}\dot{x}_{1}^{*} + k_{1}J_{m}e_{1} \right)
$$
  
\n
$$
u_{q} = \frac{L_{q}}{n_{p}\phi} \left( \frac{B\left(\frac{n_{p}\phi i_{q} - B\omega}{J_{m}} + \hat{d}_{1}\right) + J_{m}\ddot{\omega}^{*}}{k_{1}\left(J_{m}\dot{\omega}^{*} - n_{p}\phi i_{q} + B\omega - J_{m}\hat{d}_{1}\right)} \right)
$$
  
\n
$$
+ R_{s}i_{q} + n_{p}\omega L_{d}i_{d} + n_{p}\omega\phi - L_{q}\hat{d}_{2} + k_{2}e_{2}
$$
  
\n
$$
u_{d} = L_{d} \left( \frac{R_{s}i_{d} - n_{p}\omega L_{q}i_{q}}{L_{d}} - \hat{d}_{3} + k_{3}e_{3} \right)
$$
  
\n(30)

Compared with the conventional backstepping control law in Eqs. [\(13\)](#page-3-0) and [\(18\)](#page-4-0), the novel control law Eq. [\(30\)](#page-7-0) is more convenient to be adjusted with the observed values  $d_1, d_2, d_3$  from Eqs. [\(25\)](#page-5-2) and [\(26\)](#page-6-0) that are continually updated for the control system.

## **4 Stability Analysis**

*Theorem 2. Consider the interior permanent magnet synchronous motor form as Eq. [\(5\)](#page-2-1) under the bounded disturbance as Assumption [1.](#page-3-3) By using the control law in Eq. [\(30\)](#page-7-0) with the positive constants.*

 $k_1, k_2, k_3$  *and the observer gains*  $1/\varepsilon_1$ ,  $1/\varepsilon_2$ ,  $1/\varepsilon_3$  *of the high-gain disturbance observer Eqs. [\(25\)](#page-5-2) and [\(26\)](#page-6-0) guarantee the Input-to-State Stability* [\[17\]](#page-14-15) *of the closedcontrol system.*

**Proof of Theorem 2.** Define the Lyapunov candidate function as follows:

<span id="page-7-1"></span>
$$
V = \frac{1}{2}e_1^2 + \frac{1}{2}\tilde{d}_1^2 + \frac{1}{2}e_2^2 + \frac{1}{2}\tilde{d}_2^2 + \frac{1}{2}e_3^2 + \frac{1}{2}\tilde{d}_3^2
$$
 (31)

Taking derivative of Eq. [\(31\)](#page-7-1) with respect to time yields:

<span id="page-7-2"></span>
$$
\dot{V} = e_1 \dot{e}_1 + \tilde{d}_1 \dot{\tilde{d}}_1 + e_2 \dot{e}_2 + \tilde{d}_2 \dot{\tilde{d}}_2 + e_3 \dot{e}_3 + \tilde{d}_3 \dot{\tilde{d}}_3 \tag{32}
$$

Substituting the disturbance estimation error dynamics Eq. [\(28\)](#page-6-2) into Eq. [\(32\)](#page-7-2) results in:

$$
\dot{V} = e_1 \left( \dot{x}_1^* - \frac{n_p \phi x_2}{J_m} + \frac{Bx_1}{J_m} - d_1 \right) + \tilde{d}_1 \left( \dot{d}_1 - \left( \frac{1}{\varepsilon_1} \right) \tilde{d}_1 \right)
$$
\n
$$
+ e_2 \left( \frac{1}{n_p \phi} \left( \frac{B \left( \frac{n_p \phi x_2 - Bx_1}{J_m} + d_1 \right) + J_m \ddot{x}_1^*}{\mu_k (J_m \dot{x}_1^* - n_p \phi x_2 + Bx_1 - J_m d_1)} \right) + \frac{R_s x_2 + n_p x_1 L_d x_3 + n_p x_1 \phi - u_q}{L_q} - d_2 \right)
$$
\n
$$
+ e_3 \left( \frac{R_s x_3 - n_p x_1 L_q x_2}{L_d} - \frac{1}{L_d} u_d - d_3 \right)
$$
\n
$$
+ \tilde{d}_2 \left( \dot{d}_2 - \left( \frac{1}{\varepsilon_2} \right) \tilde{d}_2 \right) + \tilde{d}_3 \left( \dot{d}_3 - \left( \frac{1}{\varepsilon_3} \right) \tilde{d}_3 \right)
$$
\n(33)

With the control laws in Eq. [\(30\)](#page-7-0), the time derivative of Lyapunov candidate function *V* is obtained as follows:

<span id="page-8-0"></span>
$$
\dot{V} = e_1 \left( -k_1 e_1 - \tilde{d}_1 \right) + \tilde{d}_1 \left( \dot{d}_1 - \left( \frac{1}{\varepsilon_1} \right) \tilde{d}_1 \right)
$$
\n
$$
+ e_2 \left( -k_2 e_2 - \tilde{d}_2 \right) + \tilde{d}_2 \left( \dot{d}_2 - \left( \frac{1}{\varepsilon_2} \right) \tilde{d}_2 \right)
$$
\n
$$
+ e_3 \left( -k_3 e_3 - \tilde{d}_3 \right) + \tilde{d}_3 \left( \dot{d}_3 - \left( \frac{1}{\varepsilon_3} \right) \tilde{d}_3 \right)
$$
\n
$$
= \sum_{i=1}^3 \left( -k_i e_i^2 - e_1 \tilde{d}_i - \frac{1}{\varepsilon_i} \tilde{d}_i + \tilde{d}_i \dot{d}_i \right)
$$
\n(34)

Using the inequality  $|a||b| \ge ab$ , Eq. [\(34\)](#page-8-0) is obtained as follows:

$$
\dot{V} \leq -\sum_{i=1}^{3} \left( k_i \left( \frac{e_i}{\alpha_i} + \frac{\alpha_i}{2k_i} \tilde{d}_i \right)^2 + \tau_i \left( \left| \tilde{d}_i \right| - \frac{1}{2\tau_i} \left| \dot{d}_i \right| \right)^2 \right) \n- \sum_{i=1}^{3} \left( \left( 1 - \frac{1}{\alpha_i^2} \right) k_i e_i^2 + \frac{\beta_i^2}{4k_i} d_i^2 \right) + \sum_{i=1}^{3} \left( \frac{1}{4\tau_i} \left| \dot{d}_i \right|^2 \right)
$$
\n(35)

where  $\tau_i = \left(\frac{1}{\varepsilon_i} - \frac{\alpha_i^2 + \beta_i^2}{4k_i}\right)$  $\bigg); \alpha_i > 0; \beta_i \neq 0.$  Exist that:

<span id="page-8-1"></span>
$$
\gamma = \sum_{i=1}^{3} \left( \frac{1}{4\tau_i} \delta_i^2 \right)
$$
  
\n
$$
\kappa = \min \left\{ \left( 1 - \frac{1}{\alpha_i^2} \right) k_i, \frac{\beta_i^2}{4k_i} \right\}
$$
\n(36)

Then, Eq. [\(35\)](#page-8-1) can be rewritten as:

<span id="page-9-0"></span>
$$
\dot{V} \le -2\kappa V + \gamma \tag{37}
$$

Consequently,

$$
V(t) \le V(0)e^{-2\kappa t} + \frac{\gamma}{2\kappa} \left(1 - e^{-2\kappa t}\right)
$$
 (38)

**Remark 3.** The Eq. [\(38\)](#page-9-0) shows that the tracking errors *e*1, *e*2, *e*3, and estimation errors *d*<sub>1</sub>, *d*<sub>2</sub>, *d*<sub>3</sub> exponentially converge to an arbitrarily small ball  $\gamma/2\kappa$  that can be shrunk by γ via the high observer gains  $1/\varepsilon_1$ ,  $1/\varepsilon_2$ ,  $1/\varepsilon_3$  [\[18\]](#page-15-0). This adjustment is different from when using only the conventional backstepping as in Remark [1,](#page-5-4) which is greatly influenced by the disturbances.

## **5 PIL Simulation and Results**

#### **5.1 PIL Simulation**

This paper uses PIL test technique [\[19\]](#page-15-1), i.e., Texas Instrument's TI C2000 F28377S microcontroller to validate the controller, while the plant will be built on Matlab/Simulink environment. The specifics of the 0.72 kW 3-pole IPMSM are given as [\[20\]](#page-15-2) rated speed is 3000 r/min; rated torque is 2.3 Nm;  $R_s = 4.8 \Omega$ ;  $L_d = 19.5 \text{ mH}$ ;  $L_q = 27.5 \text{ mH}$ ;  $\phi = 0.15$  Wb;  $J_m = 0.001$  kgm<sup>2</sup>. The general PIL simulation diagram of the control system for IPMSM is shown in Fig. [1.](#page-9-1) Choose constants as:  $k_1 = 5000$ ,  $k_2 = 9000$ ,  $k_3$  = 3000, observation coefficients:  $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 0.0001$ . Furthermore, actual parameter values are set with an error of about 10% compared to nominal values.



<span id="page-9-1"></span>**Fig. 1.** Overview simulation diagram

### **5.2 Simulation Results**

To demonstrate the efficiency of the proposed backstepping algorithm and the high-gain observer, the speed value and load torque are set to the controller as in Fig. [2](#page-10-0) (a), (b).

Figure [3](#page-10-1) shows the rotor speed response using the controllers as PID, the conventional backstepping control, and the high-gain disturbance observer-based backstepping control. When the load torque changes suddenly at 2 s, the rotor speed response using the PID controller is significantly overshot, and the conventional backstepping is the appearance of steady state error, as shown in Figs.  $3$  and  $4(a)$  $4(a)$ . The steady state error of the traditional backstepping is caused by the disturbances as given in Remark [1.](#page-5-4) In order to deal with these problems, the control system using the high-gain disturbance observerbased backstepping control is proposed and obtains fast transient responses and strong robustness. The deviation of the proposed controller is minimal, and the control quality is clearly improved compared to the PID controller and the conventional backstepping, as in Figs. [3](#page-10-1) and [4.](#page-11-0)



**Fig. 2.** Reference load torque.

<span id="page-10-0"></span>

<span id="page-10-1"></span>**Fig. 3.** Rotor speed response of IPMSM.

Figures [5,](#page-12-0) [6](#page-12-1) and [7](#page-13-1) show the estimation performance of the disturbances  $d_1$ ,  $d_2$ , and *d*3, respectively. The disturbance included uncertainty parameters, and the load torque of IPMSM is accurately estimated using the high-gain disturbance observer. The estimation errors  $d_1$ ,  $d_2$  and  $d_3$  are minimal, as shown in Figs. [5\(](#page-12-0)b), [6\(](#page-12-1)b), and [7\(](#page-13-1)b), from which the controller has the information of the disturbance to improve the accuracy of the closed-control system.



<span id="page-11-0"></span>**Fig. 4.** (a) Speed error without disturbance observer. (b) Speed error using disturbance observer.



**Fig. 5.** Disturbance estimation  $d_1$ 

<span id="page-12-0"></span>

<span id="page-12-1"></span>**Fig. 6.** Disturbance estimation  $d_2$ 



**Fig. 7.** Disturbance estimation *d*3

# <span id="page-13-1"></span>**6 Conclusion**

This paper presents the robust speed control for IPMSM using a backstepping controller combined with a high-gain disturbance observer. Disturbances include uncertainty parameters and external load torque. In order to improve the accuracy of the controller, the nonlinear observer is applied to calculate the disturbance components in the system. The controller combines the advantages of a backstepping controller and the high-gain observer. The effectiveness and feasibility of the proposed control and observer are demonstrated by using the PIL test technique.

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