Impact of Two Temperatures on a Generalized Thermoelastic Plate with Thermal Loading

Ankit Bajpai and P. K. Sharma

Abstract This study investigates the effects of two temperatures on a generalized thermoelastic plate in light of generalized thermoelasticity. The plate is infinite in *x*- and *y*-direction and has a finite thickness in the *z*-direction. The origin of the coordinate system is taken on the middle plane of the plate. Various field quantities are taken as functions of *x*, *z*, and t only. The boundary of the plate is rigidly fixed and subjected to thermal loading. The governing equations are non-dimensionalized. To solve the governing equations, potential functions are introduced, and harmonic solutions are obtained. With the help of obtained solutions, the stress and displacement components, and conductive and thermodynamic temperatures are determined analytically in the closed-form. Using boundary conditions, the constants in the solutions are obtained. To show the results graphically, numerical results are computed for copper material. The variation of stress components and conductive and thermodynamic temperatures, is presented graphically for different values of two-temperature parameters and compared with one temperature thermoelasticity.

Keywords Generalized thermoelasticity · Two temperature · Harmonic solution

1 Introduction

The theory of thermoelasticity is the coupling of thermal and mechanical fields. In the classical theory of thermoelasticity, there are two shortcomings. The first one is that the heat conduction equation has no elastic term. So, elastic changes do not affect the temperature. The second one is that it has a parabolic type heat conduction equation providing infinite speed for heatwaves' deliverance. Biot [1] introduced the coupled theory of thermoelasticity, eliminating the first drawback. To overcome the second shortcoming, generalized theories of thermoelasticity have been developed by many researchers predicting the finite speed of heatwaves. Lord and Shulman [2] proposed

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the first generalized theory of thermoelasticity by introducing one relaxation time. Green and Lindsay [3] have submitted another approach of thermoelasticity involving two relaxation times.

In problems of ultra-short laser heating, where very high heat flux generates in the body for a brief time interval (about 10^{-7}), the two-temperature model's role becomes more significant. It explains more realistic results compared to one temperature theory. For heat conduction in deformable bodies, Chen and Gurtin [4] and Chen et al. [5] suggested two different temperatures, viz. conductive and thermodynamic temperatures that arise due to thermal and mechanical processes, respectively, separated by heat supply for time-independent situations, and hence when heat supply vanishes, both temperatures will be equal. But for time-dependent problems, two temperatures are in general distinct even though heat supply is zero. Warren and Chen [6] explained that, in two-temperature theory, the propagation speed increases and discontinuities in strain and conductive and thermodynamic temperatures become smooth. Thereafter, for many years, this theory was underestimated and ignored. But, in recent time, two-temperature theory (2TT) has been noticed by many researchers. They further obtained advancement in two-temperature theory and explained their applications, primarily describing the continuity of stress function as it is discontinuous for one temperature thermoelasticity (1TT) [7]. Various authors have presented a good number of the problems of two-temperature thermoelasticity by considering various models and boundary conditions [8–17].

The present study is motivated by the broad applications of two-temperature theory in pulsed laser technologies in material processing and nondestructive detecting, and explaining the continuity of the stress function [7]. Also, the plate structures are broadly utilized in many engineering fields, for example, aerospace, mechanical, and automotive engineering disciplines. The plate theory is a significant piece of transport engineering, whereby the utilization of plate and shell structures is common, particularly in aerospace engineering.

In this study, authors formulated a thermoelastic plate problem to analyze the impact of two temperatures on symmetric and skew-symmetric modes of various field quantities due to thermal loading. The Youssef model [18] of the generalized thermoelasticity with two temperatures has been used to investigate the deformation in thermoelastic plate. For numerical computations, copper material is taken, and numerical results are obtained using MATLAB programming. The numerical results obtained are presented graphically for symmetric as well as skew-symmetric cases in light of Lord and Shulman's theory with and without two temperatures. Stress and displacement components, and conductive and thermodynamic temperatures are displayed graphically to show the two-temperature effect. The results are compared graphically with the one temperature thermoelasticity (1TT).

2 Mathematical Modeling and Solution of the Problem

In this section, basic equations, formulation of the problem, and its solution will be presented.

2.1 Basic Equations

In the framework of generalized two-temperature thermoelasticity following Youssef [18], governing equations and constitutive relations in the absence of body forces and heat sources are

$$(\lambda + \mu)u_{j,\mathrm{ij}} + \mu u_{i,\mathrm{jj}} - \gamma\left(\theta + \nu\frac{\partial\theta}{\partial t}\right)_{,i} = \rho\frac{\partial^2 u_i}{\partial t^2} \tag{1}$$

$$K\phi_{,ii} = \rho C_E\left(\frac{\partial\theta}{\partial t} + \tau_0\frac{\partial^2\theta}{\partial t^2}\right) + \gamma T_0\left(\frac{\partial u_{i,i}}{\partial t} + n_0\tau_0\frac{\partial^2 u_{i,i}}{\partial t^2}\right) \tag{2}$$

$$\sigma_{\mathrm{ij}} = 2\mu e_{\mathrm{ij}} + \lambda e_{\mathrm{kk}}\delta_{\mathrm{ij}} - \gamma\left(\theta + \nu\frac{\partial\theta}{\partial t}\right)\delta_{\mathrm{ij}} \tag{3}$$

$$e_{\mathrm{ij}} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right) \tag{4}$$

The relation between two temperatures:

$$\phi - \theta = a^*\phi_{,\mathrm{ii}}, \theta = |T - T_0| \text{ with } \frac{\theta}{T_0} \ll 1 \tag{5}$$

where $u_i (i = 1, 2, 3)$—displacement components, σ_{ij}—stress components, e_{ij}—strain components, θ—thermodynamic temperature, ϕ—conductive temperature, T_0—reference temperature, λ, μ—Lame's parameters, K—thermal conductivity, ρ—mass density, C_E—specific heat at constant strain, $\gamma = (3\lambda + 2\mu)\alpha_t$, α_t-linear thermal expansion, "a^*"—two-temperature parameter (2TP), τ_0, ν—relaxation times, and n_0—parameter.

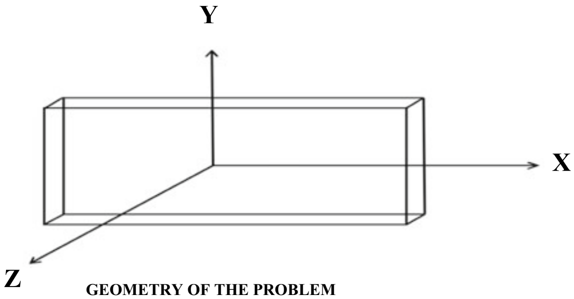
By putting $\tau_0 = \nu = 0$, the governing equations correspond to coupled thermoelasticity; $n_0 = 1, \nu = 0$ correspond to LS theory and $n_0 = 0$ correspond to GL theory of thermoelasticity. On taking $a^* = 0$, the respective thermoelastic models in 1TT can be obtained.

2.2 Formulation of the Problem

Consider an infinite elastic plate of finite width "2d," which is homogeneous, isotropic, and thermally conducting with initial uniform temperature T_0. Initially, the plate is assumed to be unstrained and unstressed. The middle plane of the plate coincides with the x–y plane such that $-d \le z \le d$ and $-\infty < x, y < \infty$ as shown in figure below. The origin of the coordinate system is taken at any point of the middle plane. The boundary surfaces $z = \pm d$ are considered to be rigidly fixed with thermal loading.

We consider the x–z plane as the plane of incidence and restrict our analysis to this plane so that various quantities are functions of only x, z, and t. Hence, the displacement components and temperatures are given by

$$u = u(x,\ z,\ t), v = 0, w = w(x,\ z,\ t), \phi = \phi(x, z, t)\ \&\ \theta = \theta(x, z, t) \tag{6}$$



GEOMETRY OF THE PROBLEM

Equations (1)–(5) along with (6) can be written in nondimensional form (after hiding primes) as

$$\frac{\partial^2 u}{\partial x^2} + \left(1 - \delta^2\right)\frac{\partial^2 w}{\partial x \partial z} + \delta^2 \frac{\partial^2 u}{\partial z^2} - \frac{\partial}{\partial x}\left(\theta + \nu \frac{\partial \theta}{\partial t}\right) = \frac{\partial^2 u}{\partial t^2} \tag{7}$$

$$\frac{\partial^2 w}{\partial z^2} + \left(1 - \delta^2\right)\frac{\partial^2 u}{\partial x \partial z} + \delta^2 \frac{\partial^2 w}{\partial x^2} - \frac{\partial}{\partial z}\left(\theta + \nu \frac{\partial \theta}{\partial t}\right) = \frac{\partial^2 w}{\partial t^2} \tag{8}$$

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2} = \left(\frac{\partial \theta}{\partial t} + \tau_0 \frac{\partial^2 \theta}{\partial t^2}\right) + \in \left(\frac{\partial}{\partial t} + n_0 \tau_0 \frac{\partial^2}{\partial t^2}\right)\left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}\right) \tag{9}$$

$$\sigma_{zz} = \frac{\partial w}{\partial z} + \left(1 - 2\delta^2\right)\frac{\partial u}{\partial x} - \left(\theta + \nu \frac{\partial \theta}{\partial t}\right) \tag{10}$$

$$\sigma_{xz} = \delta^2\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) \tag{11}$$

where nondimensional quantities are introduced as

$$\left(x',\ z'\right) = \frac{\omega^*}{c_1}(x,\ z), \quad \left(u',\ w'\right) = \frac{\omega^* c_1 \rho}{\gamma T_0}(u,\ w), \quad t' = \omega^* t,$$
$$\nu' = \omega^* \nu\,,\ \tau_0' = \omega^* \tau_0,\ a^{*\prime} = \frac{a^* \omega^{*2}}{c_1^2},$$
$$\theta' = \frac{\theta}{T_0},\ \phi' = \frac{\phi}{T_0},\ \sigma_{ij}' = \frac{\sigma_{ij}}{\gamma T_0},\ c_1^2 = \frac{(\lambda + 2\mu)}{\rho}\,,$$
$$\omega^* = \frac{C_E(\lambda + 2\mu)}{K},\ \delta^2 = \frac{\mu}{(\lambda + 2\mu)}\,,\ \in = \frac{T_0 \gamma^2}{\rho C_E(\lambda + 2\mu)}. \tag{12}$$

2.3 Solution of the Problem

To simplify Eqs. (7)–(9), potential functions Φ and Ψ are introduced by Helmholtz decomposition theorem as [19]

$$u = \frac{\partial \Phi}{\partial x} + \frac{\partial \Psi}{\partial z},\quad w = \frac{\partial \Phi}{\partial z} - \frac{\partial \Psi}{\partial x} \tag{13}$$

Equations (7)–(9) with (13) can be obtained as

$$\nabla^2 \phi = (\dot{\phi} - a^* \nabla^2 \dot{\phi}) + \tau_0 (\ddot{\phi} - a^* \nabla^2 \ddot{\phi}) + \in [\nabla^2 \dot{\Phi} + n_0 \tau_0 (\nabla^2 \ddot{\Phi})] \tag{14}$$

$$\nabla^2 \Phi - \frac{\partial^2 \Phi}{\partial t^2} = \theta + \nu \frac{\partial \theta}{\partial t} \tag{15}$$

$$\nabla^2 \Psi - \frac{1}{\delta^2} \frac{\partial^2 \Psi}{\partial t^2} = 0 \tag{16}$$

We take the solutions following [19] as

$$(\Phi, \Psi, \phi) = [f(z), g(z), h(z)] \exp[i\xi(x - ct)] \tag{17}$$

where $c = \frac{\omega}{\xi}$—phase velocity, ω—frequency, and ξ—wave number.

Equations (14)–(16) along with (13) and (17), after some simplifications, yield

$$\begin{aligned} u(x, z, t) =& [i\xi(C_3 \cos m_1 z + C_4 \sin m_1 z + C_5 \cos m_2 z + C_6 \sin m_2 z) \\ & - \beta C_7 \sin \beta z + \beta C_8 \cos \beta z] \exp[i\xi(x - ct)] \end{aligned} \tag{18}$$

$$\begin{aligned} w(x, z, t) =& [-m_1 C_3 \sin m_1 z + m_1 C_4 \cos m_1 z - m_2 C_5 \sin m_2 z + m_2 C_6 \cos m_2 z \\ & - i\xi(C_7 \cos \beta z + C_8 \sin \beta z)] \exp[i\xi(x - ct)] \end{aligned} \tag{19}$$

$$\phi(x,z,t) = \left[\frac{1}{\alpha_1}(C_3\cos m_1 z + C_4 \sin m_1 z) + \frac{1}{\alpha_2}(C_5 \cos m_2 + C_6 \sin m_2 z)\right] \exp[i\xi(x-ct)] \quad (20)$$

$$\theta(x,z,t) = \left[\begin{array}{l} \frac{\{1+a^* g_1\}}{\alpha_1}(C_3\cos m_1 z + C_4 \sin m_1 z) + \\ \frac{\{1+a^* g_2\}}{\alpha_2}(C_5 \cos m_2 + C_6 \sin m_2 z) \end{array}\right] \exp[i\xi(x-ct)] \quad (21)$$

where

$$\alpha_j = \frac{-i\omega t_2\{1+a^* g_j\}}{\alpha^2 - m_j^2}, g_j = \left(m_j^2 + \xi^2\right), (j=1,2),$$

$$\alpha^2 = \xi^2\left(c^2-1\right), \beta^2 = \xi^2\left(\frac{c^2}{\delta^2}-1\right),$$

$$m_1^2 = \frac{1}{2}\left(A + \sqrt{A^2-4B}\right), m_2^2 = \frac{1}{2}\left(A - \sqrt{A^2-4B}\right),$$

$$A = \frac{2\xi^2 - \omega^2\left(1 + t_0 - i \in \omega t_1 t_2 + a^*\left(\omega^2 t_0 + 2i\xi^2\omega t_1 t_2 - 2\xi^2 t_0\right)\right)}{-1 + a^*\omega^2(t_0 - i \in \omega t_1 t_2)},$$

$$B = \frac{-\xi^4 - \omega^4 t_0 + \omega^2\xi^2\left(1 + t_0 - i \in \omega t_1 t_2 + a^*\left(-\omega^2 t_0 - i \in \xi^2\omega t_1 t_2 + \xi^2 t_0\right)\right)}{-1 + a^*\omega^2(t_0 - i \in \omega t_1 t_2)},$$

$$t_0 = \tau_0 + i\omega^{-1}, t_1 = \nu + i\omega^{-1}, t_2 = n_0\tau_0 + i\omega^{-1}.$$

Terms corresponding to two-temperature parameter “a^*” stand for 2TT. If the two-temperature parameter “$a^* = 0$,” one will get the results for 1TT.

2.4 Boundary Conditions

The boundary conditions are taken as under.

Boundary conditions in nondimensional form at $z = \pm$ d are given as.

I. Mechanical boundary conditions: The surfaces of plate are considered rigidly fixed, hence

$$u = w = 0, \quad (22)$$

II. Thermal boundary conditions: Thermal load is applied on surfaces as

$$\phi = G_1 e^{i\xi(x-ct)}, \quad (23)$$

G_1 is the constant temperature applied on the boundary.

3 Amplitudes of Stress and Displacement Components, and Conductive and Thermodynamic Temperatures

Invoking boundary conditions (22)–(23) with the help of solutions given in (18)–(21) and relations (10)–(11), the analytical expressions of stress and displacement components, and conductive and thermodynamic temperatures for symmetric as well as skew-symmetric cases are obtained as

$$\left.\begin{array}{l}(\sigma_{zz})_{sym}(x,z,t) = [q_1 C_3 \cos m_1 z + q_2 C_5 \cos m_2 z + p_1 C_8 \beta \cos \beta z]\exp[i\xi(x-\mathrm{ct})] \\ (\sigma_{zz})_{sksym}(x,z,t) = [q_1 C_4 \sin m_1 z + q_2 C_6 \sin m_2 z + p_1 C_7 \beta \sin \beta z]\exp[i\xi(x-\mathrm{ct})]\end{array}\right\} \tag{24}$$

$$\left.\begin{array}{l}(\sigma_{xz})_{sym}(x,z,t) = [q_3 C_3 \cos m_1 z + q_4 C_5 \cos m_2 z + p_2 C_8 \beta \cos \beta z]\exp[i\xi(x-\mathrm{ct})] \\ (\sigma_{xz})_{sksym}(x,z,t) = [q_3 C_4 \sin m_1 z + q_4 C_6 \sin m_2 z + p_2 C_7 \beta \sin \beta z]\exp[i\xi(x-\mathrm{ct})]\end{array}\right\} \tag{25}$$

$$\left.\begin{array}{l}(u)_{sym}(x,z,t) = [i\xi(C_3 \cos m_1 z + C_5 \cos m_2 z + \beta C_8 \cos \beta z]\exp[i\xi(x-\mathrm{ct})] \\ (u)_{sksym}(x,z,t) = [i\xi(C_4 \sin m_1 z + C_6 \sin m_2 z) - \beta C_7 \sin \beta z]\exp[i\xi(x-\mathrm{ct})]\end{array}\right\} \tag{26}$$

$$\left.\begin{array}{l}(w)_{sym}(x,z,t) = [-m_1 C_3 \sin m_1 z - m_2 C_5 \sin m_2 z - i\xi C_8 \sin \beta z]\exp[i\xi(x-\mathrm{ct})] \\ (w)_{sksym}(x,z,t) = [m_1 C_4 \cos m_1 z + m_2 C_6 \cos m_2 z - i\xi C_7 \cos \beta z]\exp[i\xi(x-ct)]\end{array}\right\} \tag{27}$$

$$\left.\begin{array}{l}(\phi)_{sym}(x,z,t) = \left[\dfrac{1}{\alpha_1}(C_3 \cos m_1 z) + \dfrac{1}{\alpha_2}(C_5 \cos m_2 z)\right]\exp[i\xi(x-\mathrm{ct})] \\ (\phi)_{sksym}(x,z,t) = \left[\dfrac{1}{\alpha_1}(C_4 \sin m_1 z) + \dfrac{1}{\alpha_2}(C_6 \sin m_2 z)\right]\exp[i\xi(x-\mathrm{ct})]\end{array}\right\} \tag{28}$$

$$\left.\begin{array}{l}(\theta)_{sym}(x,z,t) = \left[\dfrac{\{1+a^* g_1\}}{\alpha_1}(C_3 \cos m_1 z) + \dfrac{\{1+a^* g_2\}}{\alpha_2}(C_5 \cos m_2 z)\right]\exp[i\xi(x-\mathrm{ct})] \\ (\theta)_{sksym}(x,z,t) = \left[\dfrac{\{1+a^* g_1\}}{\alpha_1}(C_4 \sin m_1 z) + \dfrac{\{1+a^* g_2\}}{\alpha_2}(C_6 \sin m_2 z)\right]\exp[i\xi(x-\mathrm{ct})]\end{array}\right\} \tag{29}$$

where constants $C_i (i = 3, 4, 5, 6, 7, 8)$ are obtained by using boundary conditions (22) and (23) such that

$$C_i = \frac{\Delta_i}{\Delta}, (i = 3, 4, 5, 6, 7, 8)$$

where,

$$\Delta = \det(A),\ A = \begin{pmatrix} i\xi cs_1 & i\xi sn_1 & i\xi cs_2 & i\xi sn_2 & -\beta sn_3 & \beta cs_3 \\ i\xi cs_1 & -i\xi sn_1 & i\xi cs_2 & -i\xi sn_2 & \beta sn_3 & \beta cs_3 \\ -m_1 sn_1 & m_1 cs_1 & -m_2 sn_2 & m_2 cs_2 & -i\xi cs_3 & -i\xi sn_3 \\ m_1 sn_1 & m_1 cs_1 & m_2 sn_2 & m_2 cs_2 & -i\xi cs_3 & i\xi sn_3 \\ \frac{1}{\alpha_1} cs_1 & \frac{1}{\alpha_1} sn_1 & \frac{1}{\alpha_2} cs_2 & \frac{1}{\alpha_2} sn_2 & 0 & 0 \\ \frac{1}{\alpha_1} cs_1 & -\frac{1}{\alpha_1} sn_1 & \frac{1}{\alpha_2} cs_2 & -\frac{1}{\alpha_2} sn_2 & 0 & 0 \end{pmatrix},$$

Δ_i = determinant of matrix A when ith column of A is replaced by column vector,$B = \begin{pmatrix} 0\ 0\ 0\ 0\ G_1\ G_1 \end{pmatrix}^T$

$$\begin{aligned} \mathrm{cs}_i &= \cos m_i d,\ \mathrm{sn}_i = \sin m_i d, \quad i = 1, 2 \\ \mathrm{cs}_3 &= \cos \beta d,\ \mathrm{sn}_3 = \sin \beta d, \\ q_j &= -\, m_j^2 \alpha_j - \xi^2 (1 - 2\delta^2) \alpha_j - \left(1 + a(m_j^2 + \xi^2)\right)(1 - i\xi \mathrm{c}\nu), \quad (j = 1, 2) \\ q_k &= 2i\xi m_{k-2} \alpha_{k-2}, \quad (k = 3, 4) \\ p_1 &= 2i\xi\beta\delta^2, \\ p_2 &= (\xi^2 - \beta^2). \end{aligned}$$

sksym = Skew symmetric, sym = Symmetric.

4 Numerical Results and Discussion

In order to portray theoretical results presented in preceding sections, we have chosen copper material (following [12]) for evaluation of numerical results and physical data for which is as given below

$$\begin{aligned} &\lambda = 7.76 \times 10^{10} \mathrm{Kg\,m^{-1}} s^{-2},\ \mu = 3.86 \times 10^{10} \mathrm{Kg\,m^{-1} s^{-2}}, \\ &\varepsilon = 0.0168,\ \rho = 8954 \mathrm{Kg\,m^{-3}}, \\ &T_0 = 293\,\mathrm{K},\ C_E = 383.1 \mathrm{J\,Kg^{-1} K^{-1}}, \\ &K = 386 \mathrm{W\,m^{-1} K^{-1}},\ \alpha_t = 1.78 \times 10^{-5} \mathrm{K^{-1}},\ \omega = -0.3 \mathrm{s^{-1}}, \\ &\tau_0 = 0.003\,\mathrm{s},\ \nu = 0\,\mathrm{s},\ t = 0.1\,\mathrm{s},\ x = 1\,\mathrm{m},\ \xi = 1 \mathrm{m^{-1}},\ G_1 = 1. \end{aligned}$$

The variation of amplitudes of stress component σ_{zz}, displacement component w, thermodynamic temperature (T), and conductive temperature (ϕ) for skew-symmetric and symmetric modes of vibration with the thickness of the plate z in Figs. 1, 2, 3, 4, 5, and 6 for $x = 1$ and with the length of the plate x in Figs. 7 and 8 for $z = 1$ are demonstrated for three different values of two-temperature parameter (2TP), viz.$a^* = 0$, 0.5, 0.9 in the context of two-temperature LS theory. The solid

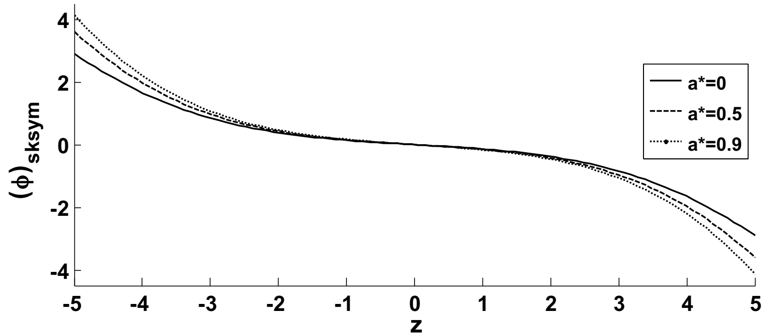


Fig. 1 Variation of skew symmetric conductive temperature with plate thickness

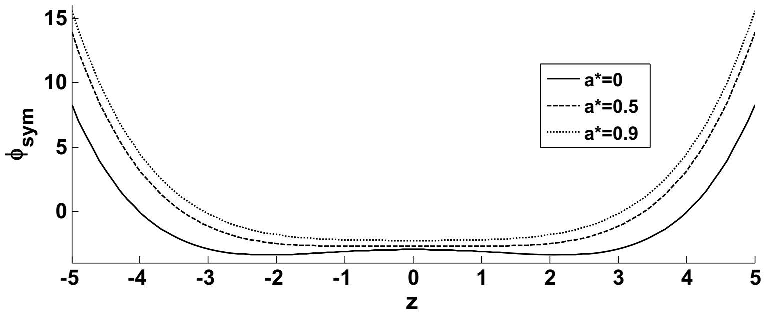


Fig. 2 Variation of symmetric conductive temperature with plate thickness

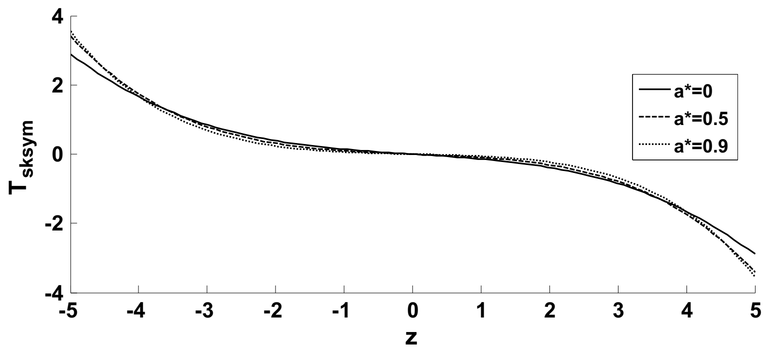


Fig. 3 Variation of skew symmetric thermodynamic temperature with plate thickness

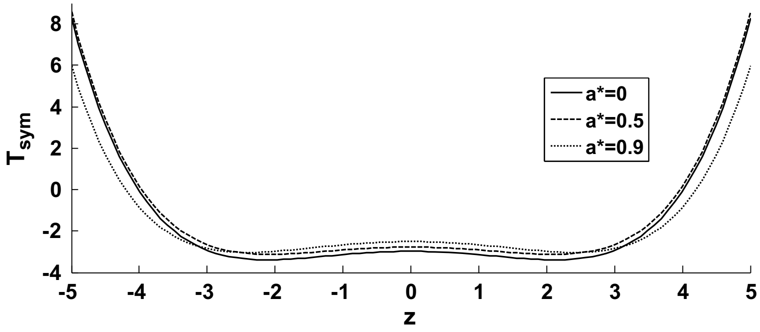


Fig. 4 Variation of symmetric thermodynamic temperature with plate thickness

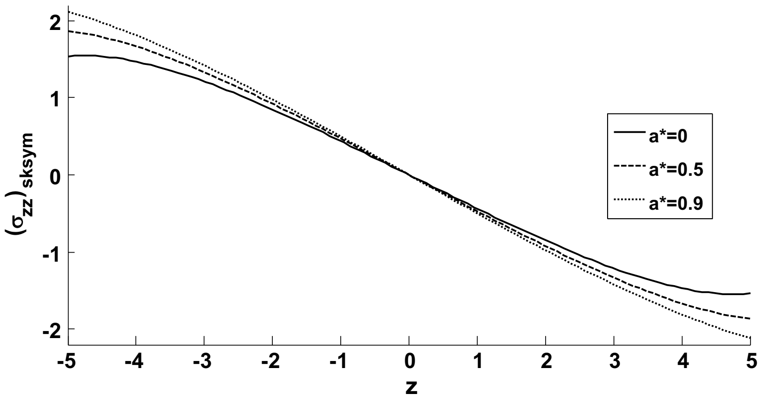


Fig. 5 Variation of skew symmetric normal stress component with plate thickness

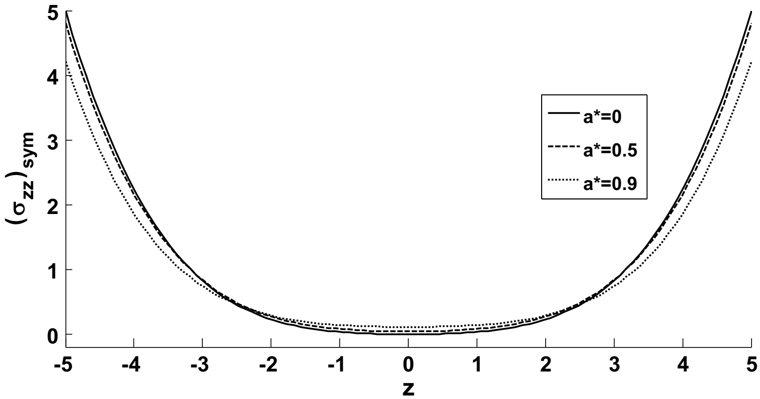


Fig. 6 Variation of symmetric normal stress component with plate thickness

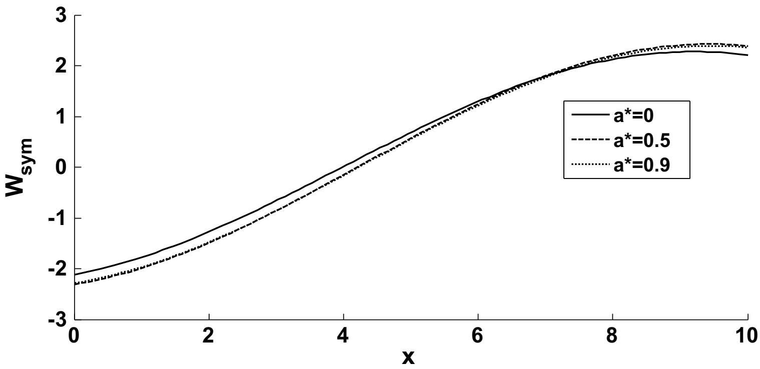


Fig. 7 Variation of symmetric displacement *W* with plate length *x*

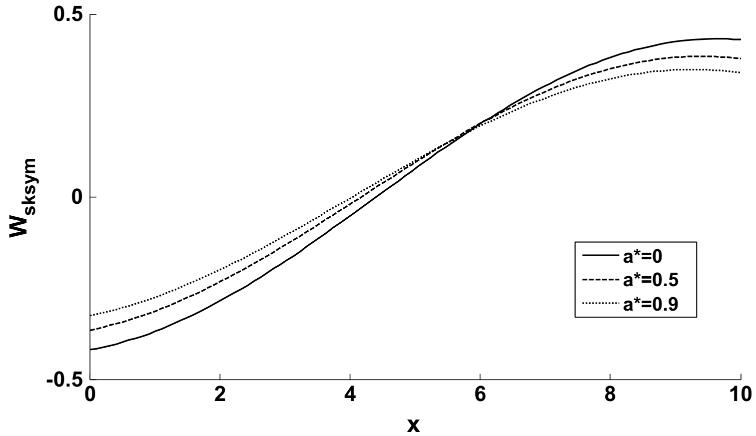


Fig. 8 Variation of skew symmetric displacement *W* with plate length *x*

lines (—), dashed lines (- - -), and dotted lines (· · ·) correspond to 1TT ($a^* = 0$) and 2TT ($a^* = 0.5$, $a^* = 0.9$), respectively.

Figures 1, 2, 3, and 4 represent the skew-symmetric and symmetric conductive and thermodynamic temperature variation along the plate's thickness due to thermal loading, respectively. The symmetric and skew-symmetric components of conductive and thermodynamic temperature have similar behavior with different magnitudes. 2TT has a maximum impact near the boundary surfaces. Its impact reduces on moving away from the boundary to the middle of the plate. Figures 5 and 6 show the symmetric variation and skew-symmetric normal stress components with plate thickness. It is clear from the figures that normal stress components for symmetric and skew-symmetric cases have maximum impact of two-temperature parameter near the load. Figures 7 and 8 depict the skew-symmetric and symmetric displacement component

W variation along the length of the plate x. Displacement W has a similar pattern for both symmetric and skew-symmetric case, but magnitude for both cases is different. Similar to other field quantities, 2TT has its maximum impact near the loading boundaries and reduces on moving away from the boundaries.

5 Conclusion

The effect of two-temperature parameter on various quantities in both modes is maximum on the boundary and decreases on moving toward the plate's middle plane except skew-symmetric normal stress and conductive temperature. A similar pattern of variations in these quantities is observed in 1TT and 2TT. However, various quantities show higher magnitudes in thermal load environment. Skew-symmetric thermodynamic temperature is less affected by two-temperature parameters as compared to symmetric one. Conductive and thermodynamic temperatures and normal stress have their minima in the middle of the plate in symmetric modes. The two-temperature parameter is observed to have more effects in the symmetric case than the skew-symmetric one near the plate's boundary. The two-temperature parameter leads to a change in the magnitude of various computed quantities; however, trends of variation remain unaffected.

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