Non-Darcian Gravitactic Bioconvection with a Porous Saturated Vertical Vibration

K. Srikanth and Virendra Kumar

Abstract A linear stability analysis is applied to study the onset of bioconvection in a suspension of negatively geotactic (gravitactic) swimmers saturated with a non-Darcy porous fluid layer under the effect of high-frequency and small-amplitude vertical vibrations. The time-averaged formulation is used to write the closed system of equations for average quantities and amplitudes of pulsation quantities in the fluid, porous layer. The eigenvalue problem is solved using the Galerkin method. An analytical expression for the modified critical bioconvection Rayleigh–Darcy number dependence on parameters like vibrational Rayleigh–Darcy number, wave number, modified Darcy number, and Péclet number has been obtained for both rigid–rigid and rigid-free cases. The presence of a non-Darcy porous medium lessens the magnitude of critical bioconvection Rayleigh–Darcy number compared to its absence. Numerical results and discussions, along with their graphical comparisons, are explored.

Keywords Gravitactic swimmers · Rigid-free boundary · Non-Darcy porous medium · The time-averaged method · Vertical vibration

1 Introduction

In cultures of various protozoa, many of the microorganisms such as Paramecium, flagellate Euglena gracilis, Tetrahymena pyriformis, and ciliated protozoan freely swim preferentially upwards and gather near the top of the culture medium [1]. These kinds of microorganisms are named negative geotaxis (gravitaxis). Due to the

K. Srikanth (✉) · V. Kumar
Department of Mathematics, School of Mathematics and Computer Sciences, Central University of Tamil Nadu, Thiruvarur 610005, India
e-mail: srikanthehow@gmail.com

V. Kumar
e-mail: virendrakumar@cutn.ac.in

S. S. Ray et al. (eds.), *Applied Analysis, Computation and Mathematical Modelling in Engineering*, Lecture Notes in Electrical Engineering 897,
https://doi.org/10.1007/978-981-19-1824-7_4

density difference between fluid and cells, a layer with a higher density than the fluid below occurs, which can cause a Rayleigh Bénard type instability [1, 2].

Theoretical models on bioconvection and stability theory were developed by Pedly and coworkers [2, 3]. The effect of vibration can be utilized to control the stability of fluid systems and may be necessary for specific pharmaceutical and bioengineering processes [4]. Kuznetsov and Jiang first studied a model of bioconvection of negatively geotactic particles in a porous medium, which accounts for cell deposition and declogging [5], and observed the permeability, the rate of cell deposition are the important factors that affect the development of bioconvection. Nield and coworkers investigated the onset of instability of suspension of gyrotaxis in a horizontal porous layer [6]. The thermal behavior of the system is analyzed for the onset of convection in a vertically vibrated porous saturated fluid layer [7]. Nguyen and coworkers studied the suspension of gravitactic swimmers in a layer of finite depth in the absence of porous media [8]. The effect of high-frequency, low-amplitude vertical vibration in a suspension of different motile microorganisms confined in a shallow horizontal fluid layer was studied [9, 10]. They reported that the strength of vibration stabilizes the suspension. Bilgen and coworkers investigated the suspension of gravitactic swimmers in a horizontal thermally stratified fluid layer [11]. Gravitactic bioconvection with double diffusion in a thermally stratified porous layer was investigated and revealed that over-stability may take place when the diffusivity of the stabilizing quantity is weaker than that of the destabilizing quantity [12]. Many thin films have been designed by the 2D system of bio-fluid mechanics through the Hele-Shaw apparatus. Nguyen-Quang and coworkers investigated the 2D gravitactic bioconvection in a system of Hele-Shaw cells [13].

The other study, such as nanofluid bioconvection in a porous saturated layer, was addressed in [14]. Vertically vibrated suspension of active swimmers in a fluid layer and porous saturated fluid layer was analyzed by [15–17]. Saini and Sharma investigated the effect of vertical flow on the onset of nanofluid thermo-gravitactic bioconvection in porous media. They disclosed that vertical through-flow disturbs the formation of bioconvection patterns necessary for the growth of bioconvection [18]. Further, they studied linear and nonlinear stability analysis of thermal convection in a suspension of gravitactic swimmers in a fluid layer by energy method [19].

In recent times, researchers have published their work on the study of bioconvection formation due to suspension of motile microorganisms in porous media and utilized the Darcy–Brinkman model [14, 20]. The onset of instability of vertically vibrated suspension of gyrotactic swimmers in a thermally stratified fluid layer was analyzed by Kumar and Srikanth [21]. In this paper, the study by Kuznetsov [9] is continued. The mathematical model for this study has been based on the deterministic formulation of a suspension of gravitactic swimmers. Here the non-Darcy model is utilized to investigate the onset of bioconvection in a high-porosity porous layer subjected to vertical vibration. The porosity of the porous medium is presumed to be big enough so that the microorganism can freely swim. Weakly nonlinear analysis of the present study and the other studies, such as the phenomena of oscillatory instability of the bio-thermally vertically vibrated suspension of negatively geotactic microorganisms, would also be of great interest.

2 Model and Governing Equations

We consider a shallow horizontal sparsely packed high-porosity porous layer saturated with the suspension of gravitactic swimmers of depth l is confined between two parallel plates ($z = 0$ and $z = l$) with vertically downward gravity g acting on it as shown in the Fig. 1. It is assumed that the porous matrix does not absorb the microorganisms, imposed vertical vibration does not affect the behavior of gravitactic swimmers, and the fluid is incompressible, then mass conservation equation:

$$\nabla.\mathbf{v} = 0 \tag{1}$$

Here $\mathbf{v}$ is the fluid filtration velocity vector.

By volume averaging the equation and adding Brinkman term which account for the inertia effects [22], then the momentum equation:

$$\rho_0 c_a\left[\partial\mathbf{v}/\partial t\right] = -\nabla p + \tilde{\mu}\nabla^2\mathbf{v} - \left(\mu/K\right)\mathbf{v} + n\theta\Delta\rho\left(\mathbf{g} + \hat{b}\omega^2\cos\omega t\mathbf{k}\right) \tag{2}$$

Here ρ_0 and c_a are the density of fluid and the acceleration coefficient; p and $\tilde{\mu}$ are excess pressure and effective viscosity; μ and K are the dynamic viscosity of suspension and the permeability of the non-Darcy porous medium; n and $\mathbf{g}$ are number density of microorganisms and gravity vector; θ and $\Delta\rho$ are average volume of microorganism, density difference($\rho_c - \rho_0$); $\hat{b}$ and ω are vibration amplitude and vibration angular frequency; t and $\mathbf{k}$ are time and vertically upward unit vector.

And cell conservation equation:

$$\phi\left[\partial n/\partial t\right] = -\nabla.\left(n\mathbf{v} + nq_c\mathbf{k} - D_c\nabla n\right) \tag{3}$$

where q_c and $\mathbf{k}$ are the average upswimming velocity of microorganisms in the porous medium and upward unit vector in the z-direction; D_c and ϕ are the effective diffusivity of microorganisms in the porous medium and porosity.

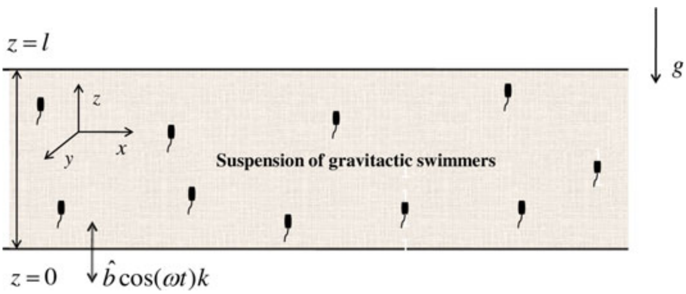

Fig. 1 Schematic diagram of the problem

To formulate the governing equation for the present study, the time-averaged method has been adopted [7]. This method provides the time-averaged system of equations which is valid if the following constraints are utilized [17]. Firstly, frequency of vibration is sufficiently high which makes the vibration period low compared to all the characteristic times scales, i.e., $\tau_{\text{vibrational}} = \min(\tau_{\text{diffusive}}, \tau_{\text{buoyancy}}, \tau_{\text{viscous}})$. Secondly, vibrational amplitude is small enough so that the components corresponding to the rapid variation in velocity can be omitted, i.e., $l/[\theta(\overline{n}_1 - \overline{n}_2)(\Delta\rho/\rho_0)] \gg \hat{b}$. Here $(\overline{n}_1 - \overline{n}_2)$ is the density difference (of the cell concentration). Utilizing these assumptions, the variable quantities are decomposed as the sum of mean (slowly varying) and rapidly oscillating (with time period $\tau = 2\pi/\omega$) components to obtain the suitable equations for vibrational suspension. By using this time-averaging technique, we obtain the following systems of equations:

$$\nabla.\overline{\mathbf{v}} = 0 \tag{4}$$

$$\rho_0 c_a\Big[\partial\overline{\mathbf{v}}/\partial t\Big] = -\nabla\overline{p} + \tilde{\mu}\nabla^2\overline{\mathbf{v}} - \Big(\mu/K\Big)\overline{\mathbf{v}} + \overline{n}\theta\Delta\rho\mathbf{g} + \Big(\rho_0/2\Big)\Big[\Big(\Delta\rho/\rho_0\Big)\hat{b}\omega\Big]^2\Big[\theta^2\Big(\overline{\mathbf{w}}.\nabla\Big)\Big(\overline{n}\mathbf{k} - \overline{\mathbf{w}}\Big)\Big] \tag{5}$$

$$\phi\Big[\partial\overline{n}/\partial t\Big] = -\nabla.\Big(\overline{n}\,\overline{\mathbf{v}} + \overline{n}\mathbf{k}q_c - D_c\nabla\overline{n}\Big) \tag{6}$$

The last term in the right-hand side of Eq. (5) is the average body force of vibrational nature and the vector $\overline{\mathbf{w}}$ is the solenoidal part of $\overline{n}\mathbf{k}$. Here this vector $\overline{\mathbf{w}}$ satisfies the Helmholtz decomposition [7, 23]:

$$\mathbf{curl}\overline{\mathbf{w}} = \nabla\overline{n} \times \mathbf{k}, \quad \text{div}\overline{\mathbf{w}} = 0 \tag{7}$$

The rigid boundary conditions at lower and upper layer are taken as:

$$\left.\text{at } z = 0, 1: \quad \overline{\mathbf{v}} = 0, \quad \Big(\overline{n}\,\overline{\mathbf{v}} + \overline{n}q_c\mathbf{k} - D_c\nabla\overline{n}\Big).\mathbf{k} = 0, \quad \overline{\mathbf{w}}.\mathbf{k} = 0\right\} \tag{8}$$

2.1 Basic State Solution

In this state, we assume that the parameters' velocity, pressure, and number density vary in z-direction only. The time-independent quiescent solution of Equations [4–7] are:

$$\left.\begin{aligned} \overline{\mathbf{v}}^b = \mathbf{0}, \quad \overline{n}^b\big(z\big) = \nu\exp\big(q_c z/D_c\big), \quad Pe = q_c l/D_c \quad \overline{\mathbf{w}}^b = \mathbf{0}, \\ \overline{p}^b(z) = p_0 + \Big[\exp(\text{Pe}) - \exp\big(q_c z/D_c\big)\Big]g\nu\theta\Delta\rho\Big(D_c/q_c\Big) \end{aligned}\right\} \tag{9}$$

where the superscript 'b' denotes the basic state, $\overline{n}_{av}$ is the average concentration of cells, integration constant $\nu = \overline{n}_{av}Pe/\Big(\exp\big(Pe\big) - 1\Big)$ represents the basic number density at the bottom of the layer, and Pe is the bioconvection Péclet number which represents the ratio of the mean cell swimming speed to the speed of bulk fluid motions.

2.2 Linear Stability Analysis

For the linear stability analysis, applying the small perturbation $\overline{\mathbf{v}} = \overline{\mathbf{v}}^*$, $\overline{p} = \overline{p}^b + \overline{p}^*$, $\overline{n} = \overline{n}^b + \overline{n}^*$, $\overline{\mathbf{w}} = \overline{\mathbf{w}}^*$ to the basic state , we have:

$$\nabla.\overline{\mathbf{v}}^* = 0 \tag{10}$$

$$c_a\rho_0\left(\partial\overline{\mathbf{v}}^*/\partial t\right) = -\nabla\overline{p}^* + \tilde{\mu}\nabla^2\overline{\mathbf{v}}^* - \left(\mu/K\right)\overline{\mathbf{v}}^* + \mathbf{g}\overline{n}^*\theta\Delta\rho + \left(\rho_0/2\right)\left[\left(\Delta\rho/\rho_0\right)\hat{b}\omega\right]^2\left[\theta^2\left(\overline{\mathbf{w}}^*.\nabla\right)\left(\overline{n}^*\mathbf{k} - \overline{\mathbf{w}}^*\right)\right] \tag{11}$$

$$\phi\left[\partial\overline{\mathbf{n}}^*/\partial t\right] = -v_z\left(\partial\overline{n}^b/\partial z\right) - q_c\left(\partial\overline{n}^*/\partial z\right) + D_c\nabla^2\overline{n}^* \tag{12}$$

$$\mathbf{curl}\overline{\mathbf{w}}^* = \nabla\overline{n}^* \times \mathbf{k} \tag{13}$$

Here $\overline{\mathbf{w}}^* = (w_x, w_y, w_z)$, $\overline{\mathbf{v}}^* = (v_x, v_y, v_z)$, and $\overline{n}^*$, which represents vibrational body force, perturbations to velocity and number density of micro-organisms, respectively.
Operating **k**.**curlcurl** on Eq. (11) and **curl** on Eq. (13) we get:

$$c_a\rho_0\left(\partial/\partial t\right)\left(\nabla^2 v_z\right) = -\theta\Delta\rho\nabla_1\overline{n}^* g + \left(\rho_0/2\right)\left[\left(\Delta\rho/\rho_0\right)\hat{b}\omega\right]^2 \times\left[\theta^2\left(\partial n_b/\partial z\right)\nabla_1\left(w_z\right)\right] + \tilde{\mu}\left[\partial^4 v_z/\partial x^4 + \partial^4 v_z/\partial y^4 + \partial^4 v_z/\partial z^4 + 2\left(\partial^4 v_z/\partial x^2\partial y^2 + \partial^4 v_z/\partial y^2\partial z^2 + \partial^4 v_z/\partial z^2\partial x^2\right)\right] - \left(\mu/K\right)\nabla^2 v_z \tag{14}$$

$$\nabla^2 w_z = \nabla_1\overline{n}^* \tag{15}$$

Here $\nabla_1 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ is the two-dimensional Laplacian operator. To analyze the disturbances into normal modes, the perturbation quantities are taken as follows:

$$\left(v_z, \quad \overline{n}^*, \quad w_z\right) = \left(V_z, \quad N_z, \quad W_z\right) f(x, y) \exp\left(\sigma t\right) \tag{16}$$

Here, $\left(\nabla_1 f + \alpha^2\right) f(x, y) = 0$, and '$\alpha$' is the horizontal wave number. Introducing the following dimensionless quantities:

$$\left.z' = z/l, \quad \alpha' = \alpha l, \quad V_z{}' = \nu\theta q_c l^2 V_z/D_c^2, \quad Pe = q_c l/D_c, \quad W_z' = W_z\theta, \quad N_z' = N\theta\right\} \tag{17}$$

We get the non-dimensionalized system of equations as:

$$\left(\tilde{\mu}/\mu\right)V_z^{\prime IV} - \left[2\alpha'^2\left(\tilde{\mu}/\mu\right) + \left(1/Da\right) + \sigma\left(C_a\rho_0 l^2/\mu\right)\right]V_z^{\prime\prime} \\ +\left[\alpha'^4\left(\tilde{\mu}/\mu\right) + \left(\alpha'^2/Da\right) + \sigma\left(\alpha'^2 C_a\rho_0 l^2/\mu\right)\right]V_z' \\ +\alpha'^2\left(RbPe\right)N_z' - \alpha'^2\left(R\upsilon\right)\exp\left(z'Pe\right)W_z' = 0 \tag{18}$$

$$N_z^{\prime\prime} - PeN_z^{\prime\prime\prime} - \left[\alpha'^2 + \sigma\left(l^2\phi/D_c\right)\right]N_z' - V_z'\exp\left(z'Pe\right) = 0 \tag{19}$$

$$W_z^{\prime\prime} + \alpha'^2\left(N_z' - W_z'\right) = 0 \tag{20}$$

Here, $Da = K/l^2$ is the Darcy number, $\left(RbPe\right) = q_c g\nu\theta\Delta\rho l^4/\mu D_c^2$ is the modified bioconvection Rayleigh number, Pe is the Péclet number, $R\upsilon = \rho_0\left[\theta\hat{b}\omega\nu q_c l^2\left(\Delta\rho/\rho_0\right)\right]^2/2\mu {D_c}^3$ is the vibrational Rayleigh number, σ is the growth rate. The principal of exchange of stabilities [24] is valid for this problem; therefore, in Eqs. (18)–(20) σ is set to zero for the onset of stationary convection.

$$Da_*\left[V_z^{\prime IV} - 2\alpha'^2 V_z^{\prime\prime} + \alpha'^4 V_z'\right] + \left[\alpha'^2 V_z' - V_z^{\prime\prime}\right] \\ +\alpha'^2 R_b N_z' - \alpha'^2 R_\upsilon \exp\left(z'Pe\right)W_z' = 0 \tag{21}$$

$$N_z^{\prime\prime} - PeN_z^{\prime\prime\prime} - \alpha'^2 N_z' - V_z'\exp\left(z'Pe\right) = 0 \tag{22}$$

$$W_z^{\prime\prime} + \alpha'^2\left(N_z' - W_z'\right) = 0 \tag{23}$$

$Da_* = K\mu/l^2\tilde{\mu}$ is the modified Darcy number, $R_b = \left(DaRbPe\right)$ is the bioconvection Rayleigh–Darcy number, $R_\upsilon = \left(R\upsilon Da\right)$ is the vibrational Rayleigh–Darcy number. To obtain an approximate solution to the system of Eqs. (21)–(23), we apply a Galerkin-type weighted residuals method [25]. And select a trial solution (which satisfy the boundary conditions) and write

$$\left.V_z' = \sum_{j=1}^{M} A_j\left(V_z'\right)_j, \quad N_z' = \sum_{j=1}^{M} B_j\left(N_z'\right)_j, \quad W_z' = \sum_{j=1}^{M} C_j\left(W_z'\right)_j\right\} \tag{24}$$

Substituting Eq. (24) into system of Eqs. (21)–(23) and applying the standard Galerkin procedure, we get a system of 3M algebraic equations in 3M variables $A_j, \quad B_j, \quad C_j, \quad (j = 1, 2, 3 \ldots M)$. A non-trivial solution of this system (vanishing of the determinant of coefficients) leads to an interesting eigenvalue equation.

The dimensionless boundary conditions are taken as follows:

$$\text{at} \quad z' = 0, \quad 1; \quad V_z' = 0, \quad PeN_z' = DN_z', \quad W_z' = 0 \tag{25}$$

and the trial solutions satisfying the boundaries Eq. (25) are chosen as:

$$\left.\begin{aligned}\left(V_z'\right)_1 = z'\left(1 - z'\right), \quad \left(N_z'\right)_1 = 2 - Pe\left(1 - 2z'\right) - \left(Pe\right)^2\left(z' - z'^2\right), \\ \text{and} \quad \left(W_z'\right)_1 = \left(z' - z'^2\right)\end{aligned}\right\} \tag{26}$$

Substituting Eq. (26) into Eqs. (21)–(23) and applying the Galerkin method [25], the eigenvalue problem takes the form:

$$\left(R_b\right)_{cr} = \underset{\alpha'\geq 0}{\text{Min}}\Bigg\{\Bigg(\Bigg\{\Big[10\left(Pe\right)^4 + \alpha'^2\left(120 - 10\left(Pe\right)^2 + \left(Pe\right)^4\right)\Big]\left(10 + \alpha'^2\right)$$
$$\times\Big[Da_*\left(\alpha'^4 + 20\alpha'^2\right) + \left(10 + \alpha'^2\right)\Big]\Bigg\} + \Bigg\{900\xi_1\alpha'^4\left(10 - \left(Pe\right)^2\right)R_v$$
$$\times\Big[\left(24/\left(Pe\right)^5 + 2/\left(Pe\right)^3\right)\left(\exp\left(Pe\right) - 1\right) - \left(12/\left(Pe\right)^4\right)$$
$$\times\left(\exp\left(Pe\right) + 1\right)\Big]\Bigg\}\Bigg)\Big/\Big[30\xi_1\alpha'^2\left(10 + \alpha'^2\right)\left(10 - \left(Pe\right)^2\right)\Big]\Bigg\} \tag{27}$$

where

$$\xi_1 = \left(8/\left(Pe\right)^2\right)\left(\exp\left(Pe\right) + 1\right) - \left(16/\left(Pe\right)^3 + 1/Pe\right)\left(\exp\left(Pe\right) - 1\right) \tag{28}$$

In the absence of vertical vibration, Eq. (27) collapses to:

$$\left(R_b\right)_{cr} = \underset{\alpha'\geq 0}{\text{Min}}\Bigg(\Bigg\{\Big[Da_*\left(\alpha'^4 + 20\alpha'^2\right) + \left(10 + \alpha'^2\right)\Big]$$
$$\times\Big[10\left(Pe\right)^4 + \alpha'^2\left(120 - 10\left(Pe\right)^2 + \left(Pe\right)^4\right)\Big]\Bigg\}\Big/\Big[30\xi_1\alpha'^2\left(10 - \left(Pe\right)^2\right)\Big]\Bigg) \tag{29}$$

When Pe tends to zero, above expression reduces to:

$$\left(R_b\right)_{cr} = \underset{\alpha'\geq 0}{\text{Min}}\Bigg(\Big\{2Da_*\left(\alpha'^4 + 20\alpha'^2\right) + \left(20 + 2\alpha'^2\right)\Big\}\Big/\left(5\xi_1\right)_{Pe\to 0}\Bigg) \tag{30}$$

Here, Eq. (30) represents the expression for the critical bioconvection Rayleigh–Darcy number for high-porosity porous layer case. And for this case $\left(R_b\right)_{cr}$ is obtained as 12 for the corresponding critical value of wave number α'_{cr} which is zero when the value of Da_* tending to 0.

As a special case, the permeability K approaches to zero, this Eq. (30) set off as:

$$\left(R_b\right)_{cr} = \underset{\alpha'\geq 0}{\text{Min}}\Big\{2\left(10 + \alpha'^2\right)\Big/\left[5\xi_1\right]_{Pe\to 0}\Big\} = \underset{\alpha'\geq 0}{\text{Min}}\Big\{6\left(10 + \alpha'^2\right)\Big/5\Big\} \tag{31}$$

The above expression Eq. (31) holds for low porosity(Darcy model) and matches completely with the results obtained by Nield and coworkers for the case of fluid layer confined between two rigid boundaries [6]. Also this result coincides exactly with the known results for two impermeable boundaries obtained by Virendra kumar [15, 17].

For the case of stress-free upper and rigid lower surfaces, the analysis is same as that for rigid–rigid boundary case with the exception of boundary condition [in Eq. (25)] as: at $z' = 0$; $V'_z = 0$, $DV'_z = 0$ and $z' = 1$; $V'_z = 0$, $D^2V'_z = 0$ and trial solution (in Eq. (26)) for $\left(V'_z\right)_1$ as $z'^2\left(1 - z'\right)\left(3 - 2z'\right)$. An analytical relation of $\left(R_b\right)_{cr}$ is obtained [see in Appendix A.1]. And for limiting case, modified Darcy number $Da_* \to 0$, when Pe approaches to 0, the critical bioconvection Rayleigh–Darcy number is $\left(R_b\right)_{cr} = 7.619$ for the corresponding critical value of $\alpha'_{cr} = 0$. This $\left(R_b\right)_{cr}$ value is less in comparison with rigid–rigid case, means the suspension is less stable in the system with rigid-free boundaries.

2.3 Results and Discussion

For the fixed values of the dimensionless parameters $\alpha^{\iota} = 1.9$, $(R_b) = 100$, and Pe (0–3) [9, 17], desired control parameter values are computed and the results are depicted with graphs. Figures 2 and 3 illustrate the effect of the bioconvection Péclet number, Pe, on the modified critical bioconvection Rayleigh–Darcy number, $(R_b)_{cr}$, for different values of vibrational Ralyeigh–Darcy number and modified Darcy number respectively. For different values of R_v (0 to 5000) and modified Darcy number, Da_* (0.0001 to 1), the bioconvection Rayleigh–Darcy number $(R_b)_{cr}$ increases exponentially as Pe increases. Similar trends were attained in earlier reported study of Kuznetsov in the absence of porous media [9]. The vibration Rayleigh–Darcy number characterizes the effect of high-frequency and low-amplitude vertical vibration across the non-Darcy porous fluid layer, as R_v increases, magnitude of $(R_b)_{cr}$ increases, means vibrations have stabilizing effect on bioconvection. As modified Darcy number enlarges, the magnitude of $(R_b)_{cr}$ enlarges, which means this modified Darcy number makes the suspension stable.

In Figs. 4 and 5, the variation of α^{ι}_{cr} against bioconvection Péclet number, Pe, is examined graphically for distinct values of vibrational Rayleigh–Darcy number, modified Darcy number, respectively. From Fig. 4, it is observed that $\alpha^{\iota}_{c}r$ first increases, after certain range it takes on a maximum value, then decreases and also $\alpha^{\iota}_{c}r$ enlarges as R_v enlarges, shows the stronger vibrations correlate to larger critical wave number. For different values of modified Darcy number, the significant changes in the critical wave number have shown in Fig. 5. It is noticed that, with an increase in Péclet number (in a certain range, i.e., 0–2) critical wave number increases. Hence, the parameter Pe reduce the size of cells in this range. The magnitude of $\alpha^{\iota}_{c}r$ enlarges for decreased modified Darcy values.

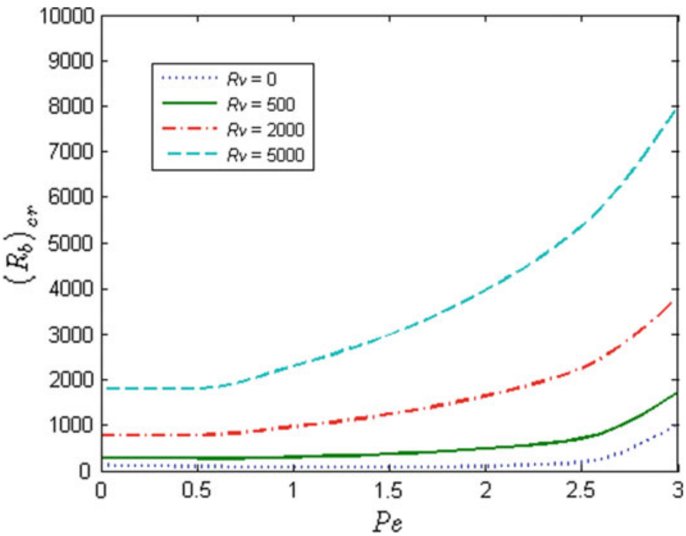

Fig. 2 Dependence of critical bioconvection Rayleigh–Darcy number $(R_b)_{cr}$ on Péclet number (Pe) for different values of vibrational Rayleigh–Darcy number (R_v)

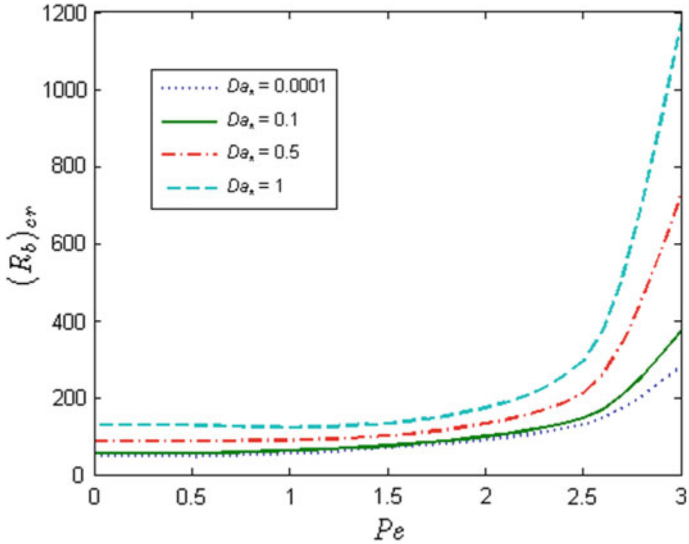

Fig. 3 Dependence of critical bioconvection Rayleigh–Darcy number $(R_b)_{cr}$ on Péclet number (Pe) for different values of modified Darcy number Da_*

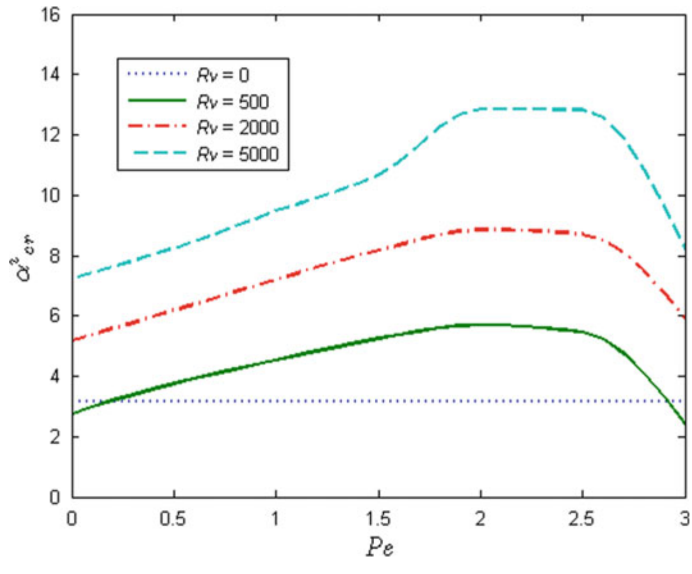

Fig. 4 Dependence of critical wave number(α_{cr}^{ι}) on Péclet number (Pe) for different values of vibrational Rayleigh–Darcy number (R_v)

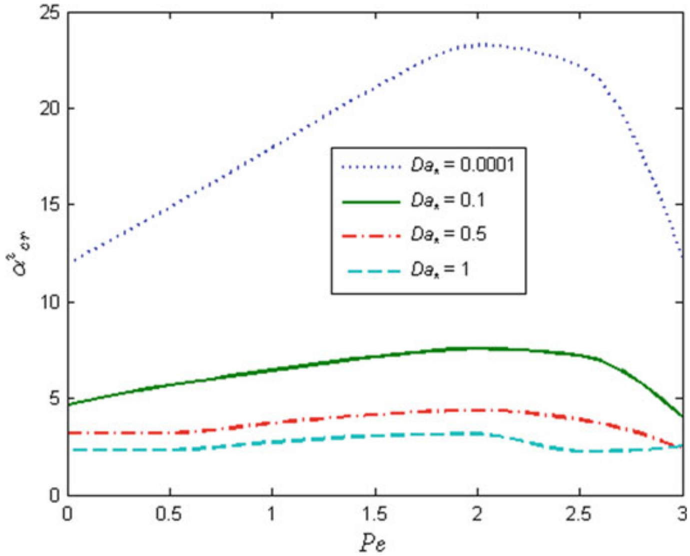

Fig. 5 Dependence of critical wave number(α^{ι}_{cr}) on Péclet number (Pe) for different values of modified Darcy number Da_*

3 Conclusions

This paper analyzes the effect of high-frequency and low-amplitude vertical vibration of the onset of bioconvection in a suspension of gravitactic swimmers in fluid saturated with a horizontal non-Darcy porous layer of finite depth. The system is solved analytically using the Galerkin technique, and an expression for the critical bioconvection Rayleigh number for the non-oscillatory convection is obtained. The numerical results are in a good match-up with formerly published results. The main conclusions are drawn. The influence of the vibration effect stabilizes the suspension, and high-frequency vibrations can be applied to control (suppress) bioconvection. The impact of the modified Darcy number is to stabilize the system. Due to the presence of a non-Darcy porous medium, the magnitude of the parameters, bioconvection Rayleigh–Darcy number, wave number is less in comparison with its absence. The suspension with rigid-rigid boundaries is more stable than the system with rigid-free boundaries.

Appendix

An analytic expression for the dependence of critical bioconvection Rayleigh–Darcy number for rigid-free boundaries case is given below:

$$\left(R_b\right)_{cr} = \underset{\alpha' \geq 0}{\mathrm{Min}} \Bigg\{ \Bigg(\Bigg\{ \Big[Da_* \Big(19\alpha'^4 + 432\alpha'^2 + 4536 \Big) + \Big(19\alpha'^2 + 216 \Big) \Big] \Big(10 + \alpha'^2 \Big)$$
$$\times \Big[10(Pe)^4 + \alpha'^2 \Big(120 - 10(Pe)^2 + (Pe)^4 \Big) \Big] \Bigg\} + \Bigg\{ 37800\psi_1 \alpha'^4 \Big(10 - (Pe)^2 \Big) R_v$$
$$\times \Big[2\exp(Pe)/(Pe)^3 - \Big(6\exp(Pe) - 18 \Big)/(Pe)^4 - \Big(72\exp(Pe) - 192 \Big)/(Pe)^5$$
$$+ \Big(600\exp(Pe) + 840 \Big)/(Pe)^6 - 1440 \Big(\exp(Pe) - 1 \Big)/(Pe)^7 \Big] \Bigg\} \Bigg)$$
$$\Bigg/ \Big[90\psi_1 \alpha'^2 \Big(10 + \alpha'^2 \Big) \Big(126 + 7Pe - 13(Pe)^2 \Big) \Big] \Bigg\} \quad (32)$$

where $\psi_1 = -\exp(Pe)/Pe + \Big(4\exp(Pe) - 12\Big)/(Pe)^2 + \Big(54\exp(pe) - 138\Big)/(Pe)^3 - \Big(444\exp(Pe) + 612\Big)/(Pe)^4 + 1056\Big(\exp(Pe) - 1\Big)/(Pe)^5$ and $\left(\psi_1\right)_{Pe \to 0} = 3/10$

References

1. Childress S, Levandowsky M, Spiegel EA (1975) Pattern formation in a suspension of swimming microorganisms: equations and stability theory. J Fluid Mech 63:591–613
2. Hill NA, Pedley TJ, Kessler JO (1989) Growth of bioconvection patterns in a suspension of gyrotactic micro-organisms in a layer of finite depth. J Fluid Mech 208:509–543
3. Pedley TJ (1992) Hydrodynamic phenomena in a suspension of swimming microorganisms. Annu Rev Fluid Mech 24:313–358
4. Stewart TL, Fogler HS (2001) Biomass plug development and propagation in porous media. Biotech Bioeng 72:353–63
5. Kuznetsov AV, Jiang N (2003) Bioconvection of negatively geotactic microorganisms in a porous medium: the effect of cell deposition and declogging. Int J Numer Methods Heat Fluid Flow 13:341–364
6. Nield DA, Kuznetsov AV, Avramenko AA (2004) The onset of bioconvection in a horizontal porous-medium layer. Trans Porous Med 54:335–344
7. Cisse I, Bardan G, Mojtabi A (2004) Rayleigh benard instability of a fluid under high frequency vibration. Int J Heat Mass Trans 47:4101–4112
8. Bahloul A, Nguyen-Quang T, Nguyen TH (2005) Bioconvection of gravitactic microorganisms in a fluid layer. Int Commun Heat Mass Transfer 32:64–71
9. Kuznetsov AV (2005) The onset of bioconvection in a suspension of negatively geotactic microorganisms with high-frequency vertical vibration. Int Commun Heat Mass Trans 32:1119–1127
10. Kuznetsov AV (2006) Investigation of the onset of bioconvection in a suspension of oxytactic microorganisms subjected to high-frequency vertical vibration. Theor Comput Fluid Dyn 20:73–87
11. Alloui Z, Nguyen TH, Bilgen E (2006) Stability analysis of thermo-bioconvection in suspensions of gravitactic microorganisms in a fluid layer. Int Commun Heat Mass Transfer 33:1198–1206
12. Tri Nguyen-Quang, Lepalec Georges (2008) Gravitactic bioconvection in a fluid-saturated porous medium with double diffusion. J Porous Med 11:751–764
13. Nguyen-Quang T, Nguyen H, Guichard F, Nicolau A, Szatmari G, LePalec G, Dusser M, Lafossee J, Bonnet JL, Bohatier J (2009) Two-dimensional gravitactic bioconvection in a protozoan (Tetrahymena pyriformis) culture. Zoolog Sci 26:54–65

14. Kuznetsov AV, Nield DA (2010) Thermal instability in a porous medium layer saturated by a nanofluid: Brinkman model. Transp Porous Med 81:409–422
15. Sharma YD, Kumar V (2012) Effect of high-frequency vertical vibration in a suspension of negatively geotactic microorganisms saturating a porous medium. Int J Appl Math Mech 8:47–58
16. Sharma YD, Kumar V (2012) The effect of high-frequency vertical vibration in a suspension of gyrotactic microorganisms. Mech Res Commun 44:40–46
17. Kumar V, Sharma YD (2014) Instability analysis of gyrotactic microorganisms: a combined effect of high frequency vertical vibration and porous media. Transp Porous Med 102:153–165
18. Saini S, Sharma YD (2018) Numerical study of nanofluid thermo-bioconvection containing gravitactic microorganisms in porous media: effect of vertical throughflow. Adv Powder Technol 29:2725–2732
19. Saini S, Sharma YD (2018) Analysis of onset of bio-thermal convection in a fluid containing gravitactic microorganisms by the energy method. Chinese J Phys 56:2031–2038
20. Zhao M, Wang S, Wang H, Mahabaleshwar US (2019) Darcy-brinkman bio-thermal convection in a suspension of gyrotactic microorganisms in a porous medium. Neural Comput Applic 31:1061–1067
21. Kumar V, Srikanth K (2021) An overstability analysis of vertically vibrated suspension of active swimmers subjected to thermal stratification. SN Appl Sci 3:612
22. Nield DA, Bejan A (2006) Convection in porous media, 3rd edn. Springer, New York
23. Gershuni GZ, Lyubimov DU (1998) Thermal vibrational convection. Wiley, New York
24. Chandrasekhar S (1981) Hydrodynamic and hydromagnetic stability. Dover, New York
25. Finlayson BA (1972) The method of weighted residuals and variational principles. Academic Press, NewYork