Rayleigh Streaming Past a Wavy Wall with No Slip Suction Under a Transverse Magnetic Field

Fathimunnisa and Neetu Srivastava

Abstract This paper investigates the provoked flow pattern due to an impulsive motion of porous wavy wall with no slip suction velocity under the influence of magnetic field. It is assumed that the amplitude of the wall is much smaller than the developed boundary-layer thickness. This engendered flow pattern has two significant flow regimes: Regime-I near to the wall boundary where flow is affected due to waviness and viscosity and Regime-II is adjacent to the boundary-layer region, i.e., in the core region. Flows in these regimes are governed by boundary-layer equations, and their solution is determined by considering the expansion in terms of smallness of the amplitude oscillations. Results obtained are depicted graphically. It is shown that the magnetic field and suction parameter tend to pull down the effect of Reynolds stress.

Keywords Boundary layer · Rayleigh streaming · Wavy wall · Magnetic field

Nomenclature

v_x, v_y	Velocity Component
a^*	Dimensionless amplitude of the wall
B_0	Applied magnetic field

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Greek symbols

ν	Dynamic viscosity
Ψ	Stream function
ρ	Fluid density
σ	Electrical conductivity
ε	Small amplitude of oscillation
λ	Wave length

Non-dimensional Number

Ω_{ω}	Magnetic interaction parameter
S	Suction parameter

1 Introduction

The Rayleigh problem is the subject of determining the flow caused by a sudden movement of an infinitely long plate from rest, which leads to the study of viscous boundary layer. Rayleigh [1] initiated the theoretical explanation on acoustic streaming. The properties of acoustic streaming are typically seen when the size of the flow region is very small compared with the wavelength (λ), but much greater than the boundary-layer thickness δ. Cuevas and Ramos [2] investigated the effects of a uniform, transverse magnetic field in the steady streaming associated with the oscillatory boundary-layer flow of an electrically conducting fluid. Fathimunnisa et al. [3] have analyzed the flow pattern generated due to an interaction of standing wave in the presence of transverse magnetic field with a fluid, which is slowly discharged from the porous wall. In a viscous incompressible fluid, Shankar and Sinha [4] investigated the fluid motion caused by the impulsive motion of a wavy wall. The steady streaming caused by an oscillating viscous flow over a wavy wall is investigated by Lyne [5] using the conformal transformation approach. Schlichting [6] originally treated the steady streaming due to an oscillating incompressible flow over a curved boundary. Assuming the amplitude of oscillation ($U_{\infty}/\omega * \lambda$) and amplitude of the wall (A/δ) to be small, Kaneko and Honji [7] investigated the double structures of steady streaming in an oscillatory viscous flow over a wavy wall. Vittori and Verzicco [8] analyzed the viscous oscillatory flow over a wavy wall of small amplitude, taking into account nonlinear effects and considering amplitude of fluid displacement (a^{*}) to be small, equal, and greater than the wavelength (L^{*}) of the wall perturbation.

The problem analyzed in this paper focuses on the flow about an infinite porous wavy wall executing vibrations about the know solution of the flat wall [Rayleigh]. These drag reduction models depict the effect of transverse magnetic field on the eddies generated due to Reynolds stresses in an oscillatory electrically conducting fluid is commonly used in practical situation such as in the field of biomechanics, civil engineering, in the study of the interaction between gravity water waves and sea bottom in the near shore region, transpiration cooling of re-entry vehicles, and rocket boosters. In this paper, we studied the effect of transverse magnetic field on the fluid flow generated by the uniform impulsive motion of the porous wavy wall with no slip suction velocity. The equation is then solved by the perturbation method with the main objective to investigate the flow pattern. The influence of magnetic field, suction, and waviness of the boundaries on the flow has been shown analytically through the analysis of the velocity and skin friction. To achieve this goal, the paper is organized as follows: In Sect. 2, we have come up with the mathematical formulation of the problem. In Sect. 3, perturbation method is applied to obtain results. In Sect. 4, the results are discussed and conclusions are drawn.

2 Formulation of the Problem

Consider an incompressible electrically conducting fluid in a semi-infinite region bounded by a wavy wall $y = A\cos\left(\frac{2\pi x}{\lambda}\right)$ in the presence of a uniform transverse magnetic field with strength B_0. In our analysis, the assumptions made are as follows (Fig. 1):

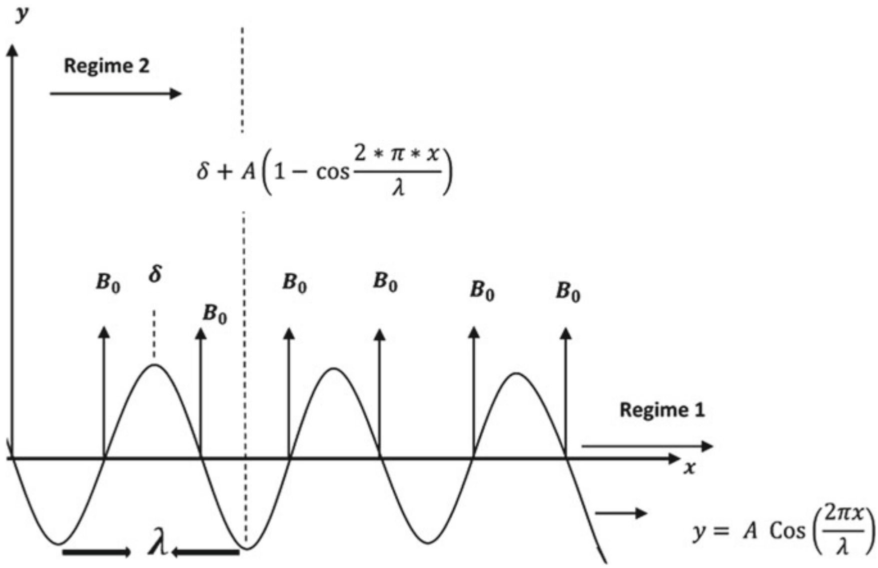

Fig. 1 Schematic of the problem

(i) We assumed that the amplitude of the wall is much smaller than the boundary-layer thickness.
(ii) We assumed that the quantity of fluid removed from the stream is so small that only fluid particles in the immediate neighborhood of wall are sucked away. This ensures the presence of no slip with suction.
(iii) We assumed that the induced magnetic field is much smaller than the applied magnetic field. This will be true, if the magnetic Reynolds number $R_m = \mu_e \sigma \lambda U_0$ is much less than unity.

For this two-dimensional situation, the equation and boundary conditions describing the motion of the incompressible fluid flow are as follows:

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0, \tag{1}$$

$$\frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \nu\left(\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2}\right) - \frac{\sigma B_0}{\rho}(E_z + v_x B_0), \tag{2}$$

$$\frac{\partial v_y}{\partial t} + v_x \frac{\partial v_y}{\partial x} + v_y \frac{\partial v_y}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial y} + \nu\left(\frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2}\right), \tag{3}$$

$$\left.\begin{array}{ll} v_x = 0 \quad v_y = V_0 & @y = A\cos\left(\frac{2\pi x}{\lambda}\right) \\ v_x = e^{i\omega t} & \text{as} y \to \infty \end{array}\right\}, \tag{4}$$

where v_x, v_y is the velocity component in the x, y direction and a is the dimensionless amplitude of the wall. Since the boundary layer is thin, it is clear that the flow in it takes place parallel to the surface (i.e., y-component flow velocity v is very small as compared to x-component flow velocity). Since the velocity varies slowly along the x-axis, $\frac{\partial^2 v_x}{\partial x^2}$ may be neglected in comparison with $\frac{\partial^2 v_x}{\partial y^2}$ and comparing (2) with (3) we see that the derivative $\frac{\partial P}{\partial y}$ is small in comparison with $\frac{\partial P}{\partial x}$, i.e., $\frac{\partial P}{\partial y} \approx 0$.

Hence, the governing equations are given by

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0, \tag{5}$$

$$\frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} = U\frac{\partial U}{\partial x} + \frac{\partial U}{\partial t} + \nu\left(\frac{\partial^2 v_x}{\partial y^2}\right) - \frac{\sigma B_0^2}{\rho}(v_x - U), \tag{6}$$

Because of the ponderomotive forces $\vec{J} \times \vec{B}$ per unit volume along x-axis which is equal to $\sigma B_0(E_z + v_x B_0)$, the condition for the pressure gradient along x-axis is as follows:

$$-\frac{1}{\rho}\frac{\partial p}{\partial x} - \frac{\sigma B_0}{\rho}(E_z + v_x B_0) = U\frac{\partial U}{\partial x} + \frac{\partial U}{\partial t}, \tag{7}$$

Equation (7) confirms the assumption that the velocity field generated in boundary layer will not be affected by the E_z, but it will modify the pressure gradient.

Let us define the dimensionless variables;

$$\tau = \omega t, \quad X = \frac{x}{\lambda}, \quad Y = \frac{y}{\delta}, \quad \delta = \sqrt{\frac{2\nu}{\omega}}, \quad v_x^* = \frac{v_x}{U_\infty},$$
$$v_y^* = \frac{v_y}{U_\infty * \delta/\lambda}, \quad U^* = \frac{U}{U_\infty}, \quad V_0^* = \frac{V_0}{U_\infty * \delta/\lambda}, \tag{7a}$$

After non-dimensionalization of Eqs. (5) and (6) using Eq. (7a) and on neglecting the asterisk, the equation of motion and the corresponding boundary conditions can be rewritten as follows:

$$\frac{\partial v_x}{\partial X} + \frac{\partial v_y}{\partial Y} = 0, \tag{8}$$

$$2\frac{\partial v_x}{\partial \tau} + \varepsilon\left(v_x * \frac{\partial v_x}{\partial X} + v_y * \frac{\partial v_x}{\partial Y}\right) = 2 * \frac{\partial U}{\partial \tau} + \varepsilon * U\frac{\partial U}{\partial X} + \left(\frac{\partial^2 v_x}{\partial y^2}\right) - \Omega_\omega (v_x - U), \tag{9}$$

$$\left.\begin{array}{ll} v_x = 0 \quad v_y = S \; as \; Y = Y_w = a^* \cos(2 * \pi * X) \\ v_x = e^{i\tau} \qquad\qquad as \; Y \to \infty \end{array}\right\}, \tag{10}$$

where $\varepsilon = \frac{2*U_\infty}{\omega*\lambda}$, $\Omega_\omega = \frac{2*\sigma B_0^2}{\rho*\omega}$ and $S = \frac{V_0}{U_\infty*\delta/\lambda}$ are dimensionless parameters, respectively, and Ω_ω is the magnetic interaction parameter, which is defined as the ratio of the amplitude of the oscillation to the wave length λ and the ratio of the magnetic to the inertial forces, respectively. S is the suction parameter. In the present problem, we assume that there is small amplitude of oscillation, i.e., $\varepsilon \ll 1$ is considered; this assure that boundary-layer separation will not arise.

Let us define a new co-ordinate system in terms of old co-ordinates as follows:

$$X = \chi, \eta = Y - Y_w(X), \tag{11}$$

and therefore
$$\frac{\partial}{\partial X} = \frac{\partial}{\partial \chi} + 2 * \pi * a^* \sin(2 * \pi * \chi)\frac{\partial}{\partial \eta}, \quad \frac{\partial}{\partial Y} = \frac{\partial}{\partial \eta}, \tag{12}$$

Equation (12) represents the change to the reference frame from the flat to wavy wall. Since waviness of the wall affects the volumetric flow conservation, it necessitates defining the flow defect equation.$U_\infty \delta$ is the volume flow defect at the crest of the wavy wall due to the presence of the boundary layer. The incompressibility condition signifies that this quantity must be preserved in any transversal section.

Hence, the volume defect balance equation is given as follows:

$$U_\infty * \delta = u_0\left[\delta + A\left(1 - \cos\frac{2*\pi*x}{\lambda}\right)\right],$$

where u_0 is the modified velocity due to the wavy wall. In dimensionless form, the above expression can be written as follows:

$$U_0(\chi) = \frac{u_0}{U_\infty} = \left[1 + a^*(1 - \cos 2*\pi*\chi)\right]^{-1}, \tag{13}$$

Thus, the dimensionless outer flow is as follows:

$$U(\chi, \tau) = U_0(\chi) * e^{i\tau}.$$

Using the transformation (11), Eqs. (8)–(10) can be rewritten as follows:

$$\frac{\partial v_x}{\partial \chi} + 2*\pi*a^* \sin(2*\pi*\chi) * \frac{\partial v_x}{\partial \chi} + \frac{\partial v_y}{\partial \eta} = 0, \tag{14}$$

$$2\frac{\partial v_x}{\partial \tau} + \varepsilon\left(v_x * \left[\frac{\partial v_x}{\partial \chi} + 2*\pi*a^* \sin(2*\pi*\chi) * \frac{\partial v_x}{\partial \eta}\right] + v_y * \frac{\partial v_x}{\partial \eta}\right)$$
$$= 2*\frac{\partial U}{\partial \tau} + \varepsilon * U\frac{\partial U}{\partial \chi} + \left(\frac{\partial^2 v_x}{\partial \eta^2}\right) - \Omega_\omega(v_x - U), \tag{15}$$

$$\left.\begin{array}{lll} v_x = 0 & v_y = S\ as & \eta = 0 \\ v_x = U_0(\chi) * e^{i\tau} & as & \eta \to \infty \end{array}\right\}, \tag{16}$$

To solve the above system of equations by the method of perturbation [Schlichting[6]], let us express the velocity in the form

$$v_x(\chi, \eta, \tau, \varepsilon, \Omega_\omega) = v_x^{(1)}(\chi, \eta, \tau, \varepsilon, \Omega_\omega) + \varepsilon * v_x^{(2)}(\chi, \eta, \tau, \varepsilon, \Omega_\omega) + O\left(\varepsilon^2\right), \tag{17}$$

where the superscripts 1 and 2 represent the first approximation and second approximation, respectively.

3 Solution of the Problem

3.1 First Approximation ($\boldsymbol{\varepsilon}^{(0)}$)

Substituting Eq. (17) in Eq. (15) and equating the terms independent of ε, we get at $O\left(\varepsilon^0\right)$

$$2\frac{\partial v_x^{(1)}}{\partial \tau} - \frac{\partial^2 v_x^{(1)}}{\partial \eta^2} + \Omega_\omega\left(v_x^{(1)} - U\right) = 2 * \frac{\partial U}{\partial \tau}, \tag{18}$$

Let us substitute,
$v_x^{(1)} = F(\eta) * e^{i\tau}$ in Eq. (18), we obtain the solution as follows:

$$v_x^{(1)} = \mathrm{Re}\left\{\left[1 + a^*(1 - \cos 2 * \pi * \chi)\right]^{-1} * \left(1 - e^{J\eta}\right) * e^{i\tau}\right\}, \tag{19}$$

where $J = \sqrt{\Omega_\omega + 2 * I}$.
The corresponding stream function can be written as follows:

$$\Psi^{(1)} = -S\chi + \mathrm{Re}\left\{\left[1 + a^*(1 - \cos(2 * \pi * \chi))\right]^{-1} * \xi^{(1)}(\eta) * e^{i\tau}\right\}. \tag{20}$$

where $\xi^{(1)}(\eta) = \eta + \frac{e^{-J*\eta}}{J} - \frac{1}{J}$,

3.2 Second Approximation ($\boldsymbol{\varepsilon}^{(1)}$)

The second approximation in terms of stream function can be written as follows:

$$2\frac{\partial^2 \Psi}{\partial \tau * \partial \eta} - \frac{\partial^3 \Psi^{(2)}}{\partial \eta^3} + \Omega_\omega \frac{\partial \Psi^{(2)}}{\partial \eta} = \varepsilon\left(U\frac{\partial U}{\partial \chi} - \frac{\partial \Psi^{(1)}}{\partial \eta} * \frac{\partial^2 \Psi^{(1)}}{\partial \chi * \partial \eta} + \frac{\partial \Psi^{(1)}}{\partial \chi} * \frac{\partial^3 \Psi^{(1)}}{\partial \eta^2}\right), \tag{21}$$

The right-hand side of Eq. (21) contains terms proportional to $\cos^2(\tau)$. This generates time-independent terms, which leads to steady streaming flow. Considering only this part of the velocity, the stream function $\Psi^{(2)}$ is written as follows:

$$\Psi^{(2)} = -S\chi - 2\pi a^* \sin(2 * \pi * \chi)\left[\xi^{(1)}(\eta) - \xi^{(2)}(\eta)\right]. \tag{22}$$

Once Eqs. (20) and (22) is introduced in Eq. (21), we arrive the equation which is as follows:

$$\xi^{(2)'''}(\eta) - \Omega_\omega \xi^{(2)'}(\eta) = -2 * e^{-a\eta}\sin(b\eta) - \frac{2 * \varepsilon}{[1 + a^*(1 - \cos(2 * \pi * \chi))]^3} - \frac{\varepsilon * e^{-2a\eta}}{[1 + a^*(1 - \cos(2 * \pi * \chi))]^3}\{1 + \cos(b\eta)\} + \frac{3 * \varepsilon * e^{-a\eta} * \cos(b\eta)}{[1 + a^*(1 - \cos(2 * \pi * \chi))]^3} - \frac{\varepsilon * e^{-a\eta} * \eta * a * \cos(b\eta)}{[1 + a^*(1 - \cos(2 * \pi * \chi))]^3}$$

$$-\frac{\varepsilon * e^{-a\eta} * \eta * b * \sin(b\eta)}{[1 + a^*(1 - \cos(2 * \pi * \chi))]^3}, \tag{23}$$

On solving Eq. (23) and eliminating the arbitrary constant with the boundary conditions, the equation obtained is as follows:

$$\begin{aligned}\xi^{(2)}(\eta) =& \frac{-[\gamma_{1p} + \gamma_{3p} + \gamma_{4p} + \gamma_{6p}]}{\sqrt{\Omega_\omega}} - D_{1p} + D_{4p} + D_{6p} \\ &+ \frac{(\gamma_{1p} + \gamma_{3p} + \gamma_{4p} + \gamma_{6p})}{\sqrt{\Omega_\omega}} * e^{-\sqrt{\Omega_\omega}\eta} \\ &+ e^{-a\eta}[D_{1p} * \cos(b\eta) + D_{2p} * \sin(b\eta)] + D_{3p} * \eta - D_{4p} * e^{-2a\eta} \\ &- e^{-2a\eta}[D_{5p} * \sin(b\eta) + D_{6p} * \cos(b\eta)] \\ &+ e^{-a\eta} * \eta[D_{7p}\cos(b\eta) + D_{8p} * \sin(b\eta)],\end{aligned} \tag{24}$$

On differentiating Eq. (24), we get the equation as follows:

$$\begin{aligned}\xi^{(2)\prime}(\eta) =& -(\gamma_{1p} + \gamma_{3p} + \gamma_{4p} + \gamma_{6p}) * e^{-\sqrt{\Omega_\omega}\eta} \\ &+ e^{-a\eta}[\gamma_{1p} * \cos(b\eta) + \gamma_{2p} * \sin(b\eta)] \\ &+ \gamma_{3p} + \gamma_{4p} * e^{-2a\eta} \\ &+ e^{-2a\eta}[\gamma_{5p} * \sin(b\eta) + \gamma_{6p} * \cos(b\eta)] \\ &+ e^{-a\eta} * \eta[\gamma_{7p}\cos(b\eta) + \gamma_{8p} * \sin(b\eta)].\end{aligned} \tag{25}$$

The above constants are defined in Appendix.

3.3 Skin Friction

The skin friction is defined as the shearing stress exerted by the fluid on the surface over which it flows, and is given by

$$\tau_{xy} = \mu * \frac{\partial v_x^{(1)}}{\partial \eta}, \tag{26}$$

On differentiating Eq. (19) and substituting in Eq. (26), we get the equation as follows:

$$\tau_{xy} = \frac{\mu}{[1 + a^*(1 - \cos(2 * \pi * \chi))]} * \{a * e^{-a\eta}\cos(\tau - b\eta) + b * e^{-a\eta}\sin(b\eta - \tau)\}. \tag{27}$$

The skin friction coefficient at the wall is as follows:$c_f = \frac{2*\tau}{\rho*U^2}$, Substituting Eq. (27) in above expression, the equation we get is as follows:

$$c_f = 2 * \nu * \left[1 + a^*(1 - \cos(2 * \pi * \chi))\right] \left\{a * e^{-a\eta} \cos(\tau - b\eta) + b * e^{-a\eta} \sin(b\eta - \tau)\right\}. \tag{28}$$

The above constants are defined in Appendix.

4 Result and Discussion

Intricacy of the considered problem is analyzed by the figures plotted using **MATHEMATICA 11.1**. These figures show the significant variation in the flow pattern due to the presence of the magnetic field and waviness. Fathimunnisa et.al [3] have presented the model of porous flat plate with the velocity distribution expression as follows:

For Flat Porous Plate

$$\begin{aligned}\zeta^{(2)} =& \delta\Bigg\{-\frac{(\beta_{2p} + \beta_{3p} + \beta_{5p})}{\sqrt{2}\Gamma_B} - \alpha_{2p} + \alpha_{3p} - \alpha_{5p} + \frac{(\beta_{2p} + \beta_{3p} + \beta_{5p})}{\sqrt{2}\Omega_\omega} \\ & * e^{-\sqrt{2}\Gamma_B\eta} + e^{-a\eta}\left[\alpha_{1p}\sin(b\eta) + \alpha_{2p}\cos(b\eta)\right] \\ & + \eta e^{-a\eta}\left[\alpha_{6p}\sin(b\eta) + \alpha_{7p}\cos(b\eta)\right] - \alpha_{3p}e^{-2a\eta} \\ & + e^{-2a\eta}\left[\alpha_{4p}\sin(2b\eta) + \alpha_{5p}\cos(2b\eta)\right]\Bigg\}.\end{aligned}$$

In this manuscript, the velocity expression will take a form.

For Porous Wavy Plate

$$\begin{aligned}\xi^{(2)}(\eta) =& \frac{-\left[\gamma_{1p} + \gamma_{3p} + \gamma_{4p} + \gamma_{6p}\right]}{\sqrt{\Omega_\omega}} - \beta_{1p} + \beta_{4p} + \beta_{6p} \\ & + \frac{(\gamma_{1p} + \gamma_{3p} + \gamma_{4p} + \gamma_{6p})}{\sqrt{\Omega_\omega}} * e^{-\sqrt{\Omega_\omega}\eta} \\ & + e^{-a\eta}\left[D_{1p} * \cos(b\eta) + D_{2p} * \sin(b\eta)\right] \\ & + D_{3p} * \eta - D_{4p} * e^{-2a\eta} - e^{-2a\eta}\left[D_{5p} * \sin(b\eta) + D_{6p} * \cos(b\eta)\right] \\ & + e^{-a\eta} * \eta\left[D_{7p}\cos(b\eta) + D_{8p} * \sin(b\eta)\right].\end{aligned}$$

where all the notations in the above expressions are presented in [3] appendix as well as in current appendix. The velocity distribution of the flow pattern developed in the current manuscript is compared with the velocity distribution established in [3].

Figure 2b represents the variation of stream function with magnetic interaction parameter. This graph depicts that the flow is influenced by the magnetic interaction

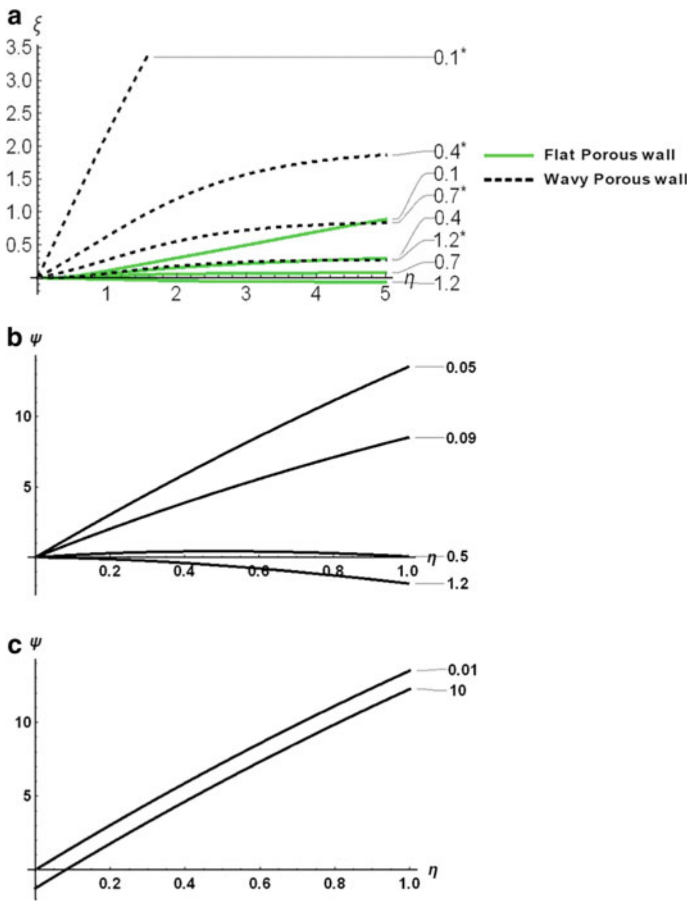

Fig. 2 **a** Velocity distribution perpendicular to porous wavy and flat wall for different Ω_ω. **b** Stream function plot for $S = 0.01$ and $\Omega_\omega = 0.05, 0.09, 0.5, 1.2$. **c** Stream function plot for $S = 0.01$, 10 and $\Omega_\omega = 0.05$

parameter, whereas Fig. 2c shows that for the fixed magnetic interaction parameter, the streamlines are damped as the suction parameter is increased.

Figure 3 represents the skin friction plot for different magnetic interaction parameters. It describes the flow Regime-I. It shows that skin friction is increasing with increase in magnetic field whereas it is decreasing from (0.4652, 0.0811) onwards. This figure depicts that the magnetic field will tend to retard the flow after $\eta = 0.4652$. From $\eta = 0$ to 0.46, flow will behave in a reverse way.

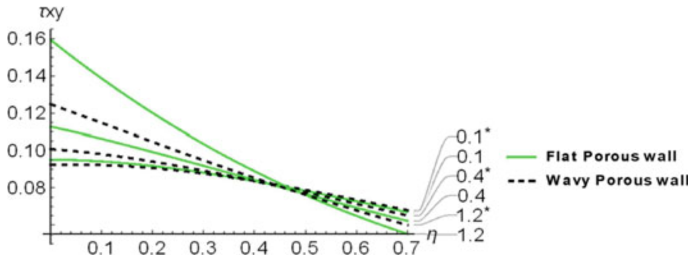

Fig. 3 Skin friction for Flat and Porous wavy wall for $\Omega_{\omega} = 0.1, 0.4, 1.2$

5 Conclusions

1. Damping phenomenon confirms that the increase in magnetic field tends to pull down the Reynold stress in both cases, i.e., wavy wall and flat plate. Hence, due to this phenomenon, a retarded zone is produced in the flow pattern.
2. Noteworthy increase in velocity for flow phenomena in case of wavy wall is seen due to the roughness of the wall.
3. In the considered flow field, the electromagnetic force is more dominant over the oscillatory motion present in the fluid. This helps in dampening the oscillatory flow. Also, this depicts that the flow changes due to the suction parameter.
4. Increase in magnetic parameter results the flow to have a retard zone due to the large shearing stress.
5. This figure also depicts that flat porous wall has large shearing stress near the boundary as compared to the wavy porous wall. This conveys that rough surface will help to reduce the large shearing stress as compared to the flat surface.

Our studies make it easier to develop powerful acoustofluidic devices for a variety of chemical and biomedical applications.

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Conflict of Interest Authors have no conflict of interest.

Appendix

$$a = \left(4 + \Omega_{\omega}^{2}\right)^{\frac{1}{4}} * \cos\left(\frac{1}{2}\mathrm{Arg}[2 * i + \Omega_{\omega}]\right), \tag{29}$$

$$b = \left(4 + \Omega_{\omega}^{2}\right)^{\frac{1}{4}} * \sin\left(\frac{1}{2}\mathrm{Arg}[2 * i + \Omega_{\omega}]\right), \tag{30}$$

$$A_1 = \frac{3b^2a - a^3 + \Omega_\omega * a}{3a^2 - \Omega_\omega - b^2}, \tag{31}$$

$$B_1 = \frac{6b^2a - 8a^3 + 2 * \Omega_\omega * a}{12a^2 - \Omega_\omega - b^2}, \tag{32}$$

$$E = \frac{A_1^2 - b^2}{2A_1}, \tag{33}$$

$$M_{1p} = \frac{2}{\left(3 * a^2 - \Omega_\omega - b^2\right) * \left(b^2 + A_1^2\right)}, \tag{34}$$

$$M_{2p} = \frac{2 * \varepsilon}{\Omega_\omega [1 + a^*(1 - \cos(2 * \pi * \chi))]^3}, \tag{35}$$

$$M_{3p} = \frac{\varepsilon}{[1 + a^*(1 - \cos(2 * \pi * \chi))]^3 * \left(2 * a * \Omega_\omega - 8 * a^2\right)}, \tag{36}$$

$$M_{4p} = \frac{\varepsilon}{[1 + a^*(1 - \cos(2 * \pi * \chi))]^3 * \left(12 * a^2 - \Omega_\omega - b^2\right) * \left(b^2 + B_1^2\right)}, \tag{37}$$

$$M_{5p} = \frac{3 * \varepsilon}{[1 + a^*(1 - \cos(2 * \pi * \chi))]^3 * \left(3 * a^2 - \Omega_\omega - b^2\right) * \left(b^2 + A_1^2\right)}, \tag{38}$$

$$M_{6p} = \frac{\varepsilon}{[1 + a^*(1 - \cos(2 * \pi * \chi))]^3 * \left(3 * a^2 - \Omega_\omega - b^2\right) * 2A_1\left(b^2 + E^2\right)}, \tag{39}$$

$$D_{1p} = M_{1p} * b + M_{5p} * A_1 + M_{6p} * a * E - M_{6p} * b^2, \tag{40}$$

$$D_{2p} = -M_{1p} * A_1 + M_{5p} * b + M_{6p} * a * b - M_{6p} * b * E, \tag{41}$$

$$D_{3p} = M_{2p}, \tag{42}$$

$$D_{4p} = M_{3p}, \tag{43}$$

$$D_{5p} = M_{4p} * b, \tag{44}$$

$$D_{6p} = M_{4p} * B_1, \tag{45}$$

$$D_{7p} = -M_{5p} * A_1 * a + M_{5p} * b^2, \tag{46}$$

$$D_{8p} = -M_{5p} * b * a - M_{5p} * A_1 * b, \tag{47}$$

$$\gamma_{1p} = -a * D_{1p} + b * D_{2p} + D_{7p}, \tag{48}$$

$$\gamma_{2p} = -b * D_{1p} - a * D_{2p} + D_{8p}, \tag{49}$$

$$\gamma_{3p} = D_{3p}, \tag{50}$$

$$\gamma_{4p} = 2 * a * D_{4p}, \tag{51}$$

$$\gamma_{5p} = 2 * a * D_{5p} + b * D_{6p} \tag{52}$$

$$\gamma_{6p} = -b * D_{5p} + 2 * a * D_{6p}, \tag{53}$$

$$\gamma_{7p} = -a * D_{7p} + b * D_{8p}, \tag{54}$$

$$\gamma_{8p} = -b * D_{7p} - a * D_{8p}. \tag{55}$$

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