An Approach to Experimental Data-Based Mathematical Modelling for a Green Roof

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Abstract Planted roof or green roof is an environmentally friendly approach to overcome climatic challenges and environmental issues created in the urban area. Literature reviews highlight that for many years planted roof building has been beneficial to the society as it reduces the roof temperature. So it is very important to study various geometrical parameters of the plant roof to optimize the dimensional parameters by means of independent and dependent variables to generate an exact mathematical model between different geometrics, and to develop the mathematical model based on the experimental data recorded during the experimentation. The experimental outputs, such as heat flux through planted roof, are obtained by varying the inputs such as solar radiation, velocity, humidity and various temperatures recorded at different layers of planted roof. This correlation is formulated using the approach suggested by H. Schenck Jr. in his book 'Theories of Engineering Experimentation'. Thus, the factors influencing the performance of the planted roof activity have been identified, so as to optimize the performance of the heat flow through planted roof.

Keywords Planted roof · Mathematical model · Solar radiation · Optimization · Dimensional parameters · Green roof · Global warning

1 Introduction

Due to global warming, the average indoor and outdoor temperature has increased. Mechanical air-conditioning systems are used for comfort in India. Energy consumption in conventional domestic housing in India is high, and the equipment is highly

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energy intensive. Planted roofs are now an established technology for improvement of thermal efficiency or heat reduction process of buildings.

A detailed literature review about the thermal performance of planted roofs, planted roof materials and construction, feasibility of planted roofs in arid regions, the effect of planted roof in energy efficiency of the buildings, etc., has been done. Paper demonstrates the formulation of experimental data-based model and the influence of the individual pie terms based on its indices to optimize the performance of planted roof so as to control the heat flow through planted roof. The mathematical model investigates the effectiveness of green roof in terms of energy efficiency on small building depending on the heat flux. The formulated mathematical equation of the green roof highlights the most effective pie terms so that accordingly the care of that pie term will be taken to obtain best performance of planted roof [1].

2 Working Principal of Planted Roof

Green roof consists of different layers such as

Planted Roof Layer is as follows:

(1) Plants (chosen depending on unique qualities).
(2) Mixture of soil, minerals and nutrients.
(3) Retention medium to hold the water.
(4) Drainage layer that sometimes has a built in reservoir.
(5) Water proofing membrane with a root barrier.
(6) Original roof structural material (Concrete slab)

Figure 1 shows the different layers of the planted roof under the extensive roofing with thin medium layer for plantation of different plants and vegetation [2–4].

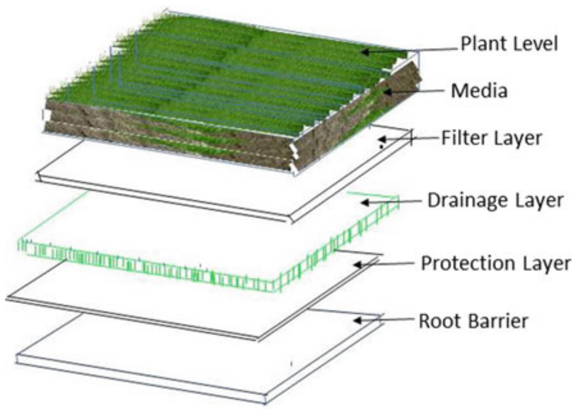


Fig. 1 Different layers of green roofing

3 Model Formulation

Model formulation of any activities related to field or experimental investigation plays an important role in the analysis. The recorded observations were studied by following the analysis steps of formulation as given below.

- Recognize various inputs and corresponding outputs related to the system.
- As it is difficult to handle many inputs in the equation, dimensionless analysis is used to minimize the number of inputs under the group of suitable pie.
- Once the pie terms are selected, it is necessary to fix the range.
- The data which is not within the range is rejected.
- Final stage is to correlate the data by formulating the mathematical model.

The independent variable or parameters involved in the experimental or field data are identified as the Theories of Experimentation. The inputs and output involved in the green roof is identify to effects of these variables.

4 Variables of Planted Roof

The input variables of the green roof and corresponding output are given in Table 1.

5 Dimensionless Pie Terms

5.1 Formation of Pie Terms

All the input terms involved in the experimental set-up are grouped to the individual pie term so that the effect of each pie term, i.e. the group of variable, can be studied effectively. The group of these variables is given in Table 2.

5.2 Formulation of Experimental Data-Based Model

All the variables described in Table 1 are grouped as given in Table 2. Further model formulation is done to identify the curve fitting constant and the indices of each pie term. The total nine pies are P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8 and P_9 with dependent pie term $\pi_{D.}$ The approximate mathematical model is presented as follow [5],

$$
\begin{aligned}
(z_1) = k * [(P_1)^{a_1} * (P_2)^{b_1} * (P_3)^{c_1} * (P_4)^{d_1} * (P_5)^{e_1} * (P_6)^{f_1} \\
* (P_7)^{g_1} * (P_8)^{h_1} * (P_9)^{i_1}]
\end{aligned}
\quad (1)
$$

Table 1 Planted roof–variable

S. No	Description of variables	Type of variable	Symbol	Dimension
1	Heat flux	Dependent	Q_f	$M^1L^0T^{-3}$
2	Heat transferred into the room	Independent	Q_{tr}	M^1L^0T
3	Evaporation rate	Independent	R_{ep}	$M^1L^{-2}T^{-1}$
4	Transpiration rate	Independent	R_{tp}	$M^1L^{-2}T^{-1}$
5	Heat generation by respiration	Independent	Q_{rp}	M^1L^0T
6	Temperature	Independent	T_{sr}	$M^0L^0T^0$
7	Wind velocity at surrounding	Independent	W_{vo}	$M^0L^1T^{-1}$
8	Wind velocity inside room	Independent	W_{vi}	$M^0L^1T^{-1}$
9	Humidity	Independent	H_u	$M^0L^0T^0$
10	Ambient temperature	Independent	T_a	$M^0L^0T^0$
11	Nutrient concentration in plant	Independent	N_c	$M^0L^0T^0$
12	Water content of plant tissue	Independent	W_c	$M^0L^0T^0$
13	Plant type	Independent	P_t	$M^0L^0T^0$
14	Humidity of plant	Independent	H	$M^0L^0T^0$
15	Leaf index of plant	Independent	L_i	$M^0L^0T^0$
16	Leaf area	Independent	L_a	$M^0L^2T^0$
17	Leaf area ratio	Independent	L_{ar}	$M^{-1}L^2T^0$
18	Areal density of plant	Independent	A_d	$M^1L^{-3}T^0$
19	Plant height	Independent	P_h	$M^0L^1T^0$
20	Heat storage of plant	Independent	H_s	$M^1L^2T^{-2}$
21	Solar storage converted by photosynthesis	Independent	S_e	$M^1L^2T^{-2}$
22	Plant temperature	Independent	P_{tc}	$M^0L^0T^0$
23	Thickness layer of soil	Independent	T_s	$M^0L^1T^0$
24	Soil temperature	Independent	S_t	$M^0L^{-2}T^{-2}$
25	Thermal conductivity of soil	Independent	T_{cs}	$M^1L^1T^{-2}$
26	Areal density of soil	Independent	D_e	$M^1L^{-3}T^0$
27	Sp. heat of soil	Independent	S_{hs}	$M^0L^{-2}T^{-2}$
28	Vol. heat capacity	Independent	V_{hc}	$M^1L^{-1}T^{-2}$
29	Latent heat of soil	Independent	L_s	$M^0L^{-2}T^{-2}$
30	Surface area of soil	Independent	S_a	$M^0L^2T^0$
31	Evaporation rate	Independent	E_r	$M^1L^{-2}T^{-1}$
32	Velocity	Independent	V	$M^0L^1T^{-1}$
33	Density	Independent	D_e	$M^1L^{-3}T^0$
34	Temperature	Independent	T	$M^0L^0T^0$
35	Evaporation rate	Independent	E_r	$M^1L^{-2}T^{-1}$
36	Area of flow of water	Independent	A_w	$M^0L^2T^0$

(continued)

Table 1 (continued)

S. No	Description of variables	Type of variable	Symbol	Dimension
37	Rate of flow of water	Independent	F_w	$M^1L^0T^{-1}$
38	Heat storage	Independent	H_s	$M^1L^2T^{-2}$
39	Sp. Heat of concrete	Independent	S_p	$M^0L^{-2}T^{-2}$
40	Bulk density	Independent	B_d	$M^1L^{-3}T^0$
41	Thickness of concrete	Independent	T	$M^0L^1T^0$
42	Area of concrete	Independent	A_r	$M^0L^2T^0$

Table 2 Grouped independent pie terms

S. No	Independent dimensionless ratios	Nature of basic physical quantities
01	$\pi_1 = [(Q\text{rp}/\text{Qtr})(R_{\text{ep}}/R_{\text{tp}})$	Solar radiation
02	$\pi_2 = [W_{\text{vo}}/W_{\text{vi}}]$	Wind velocity
03	$\pi_3 = [H_{\text{u}}]$	Relative humidity
04	$\pi 4 = [T_a]$	Ambient temperature
05	$\pi_5 = [(N_c/Wc_)(P_t\text{H}/L_i)(L_a L_{\text{ar}} A_d/P_h)(H_s/S_e)(P_{\text{tc}})]$	Temperature gradient across plant layer
06	$\pi_6 = [(S_{\text{hs}}A_t/V_{\text{hc}})(T_s S_t T_{\text{cs}} D_s/L_s S_a E_r)$	Temperature Gradient across Soil Layer
07	$\pi_7 = [E_r T/V D_e)$	Temperature gradient across retention layer
08	$\pi_8 = [(TErAw/F_w)$	Temperature gradient across drainage layer
09	$\pi_9 = [H_s/S_p B_d T A_r]$	Temperature gradient across concrete layer
10	π_D	Heat flux through planted roof

Equation 1 shows the exponential form of model formed between the pie terms. In the above equation, dependent variable z_1 represents heat flux through planted roof. Independent variable π_1 represents solar radiation data, π_2 represents wind velocity data, π_3 represents relative humidity, π_4 represents ambient temperature, π_5 represents roof plant layer data, π_6 represents roof soil layer data, π_7 represents roof retention layer data, π_8 represents roof drainage layer data and π_9 represents specification concrete layer data. To determine the values of k, a_1, b_1, c_1, $d_{1,}$ $e_{1,}$ $f_{1,}$ $g_{1,}$ h_1 and i_1 to arrive at the regression hyper plane, the above equations are presented as follows. Equation 2 is formed by taking the log on both sides of Eq. 1.

$$\begin{aligned}\text{Log}\, z_1 = \log k + a_1 \log P_1 + b_1 \log P_2 + c_1 \log P_3 + d_1 \log P_4 \\ + e_1 \log P_5 + f_1 \log P_6 + g_1 \log P_7 + h_1 \log P_8 + i_1 \log P_9 \end{aligned} \quad (2)$$

Let, $Z_1 = \log z_1$, $K = \log k_1$, $A = \log P_1$, $B = \log P_2$, $C = \log P_3$, $D = \log P_4$, $E = \log P_5$, $F = \log P_{6,}$, $G = \log P_7$, $H = \log P_8$, and $I = \log P_9$.

Substituting the above terms in Eq. 2 to simplify the equation to find out the unknown.

$$Z_1 = K + a_1 A + b_1 B + c_1 C + d_1 + e_1 E + f_1 F + g_1 G + h_1 H + i_1 I \quad (3)$$

As in the experimental set-up, the total nine pie terms with curve fitting constant K can be evaluated by formulating the total ten equations. This is done by taking the summation of each pie term as shown in Eq. 4.

$$\sum Z_1 = nK + a_1 \sum A + b_1 \sum B + c_1 \sum C + d_1 \sum D + e_1 \sum E + f_1 \sum F + g_1 \sum G + h_1 \sum H + i_1 \sum I \quad (4)$$

In the above equations, n represents the number of reading and A, B, C, D, E, F, G, H and I represent the independent p_i terms P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8 and P_9 while Z represents dependent p_i term. All the equations are represented in the matrix form as shown below.

$$[Z] = [W] * [X]$$

All the equations are arranged in the matrix form as presented above. The matrix W is a 10×10 matrix with the multipliers of k, a_1, b_1, c_1, d_1, e_1, f_1, g_1, h_1 and i_1.

Matrix

$$Z_1\ x \begin{bmatrix} n \\ A \\ B \\ C \\ D \\ E \\ F \\ G \\ H \\ I \end{bmatrix} = \begin{bmatrix} n & A & B & C & D & E & F & G & H & I \\ A & A^2 & BA & CA & DA & EA & FA & GA & HA & IA \\ B & AB & B^2 & CB & DB & EB & FB & GB & HB & IB \\ C & AC & BC & C^2 & DC & EC & FC & GC & HC & IC \\ D & AD & BD & CD & D^2 & ED & FD & GD & HD & ID \\ E & AE & BE & CE & DE & E^2 & FE & GE & HE & IE \\ F & AF & BF & CF & DF & EF & F^2 & GF & HF & IF \\ G & AG & BG & CG & DG & EG & FG & G^2 & HG & IG \\ H & AH & BH & CH & DH & EH & FH & GH & H^2 & IH \\ I & AI & BI & CI & DI & EI & FI & GI & HI & I^2 \end{bmatrix} x \begin{bmatrix} k \\ a_1 \\ b_1 \\ c_1 \\ d_1 \\ e_1 \\ f_1 \\ g_1 \\ h_1 \\ i_1 \end{bmatrix}$$

The unknown of the above matrix is evaluated with the help of MATLAB software and substituted in the exponential mathematical model to form the final mathematical model of green roof.

5.3 Proposed Form of Model for Dependent Variables of Planted Roof

The proposed form of models for dependent variables of heat flux of planted roof is as under.

$$Q = (Z_1) = 3.25 * \left[(P_1)^{2.5}(P_2)^{-0.4}(P_3)^{0.33}(P_4)^{1.7}(P_5)^{1.32}(P_6)^{-2.5} (P_7)^{-0.5}(P_8)^{0.3}(P_9)^{-3.7}\right]$$

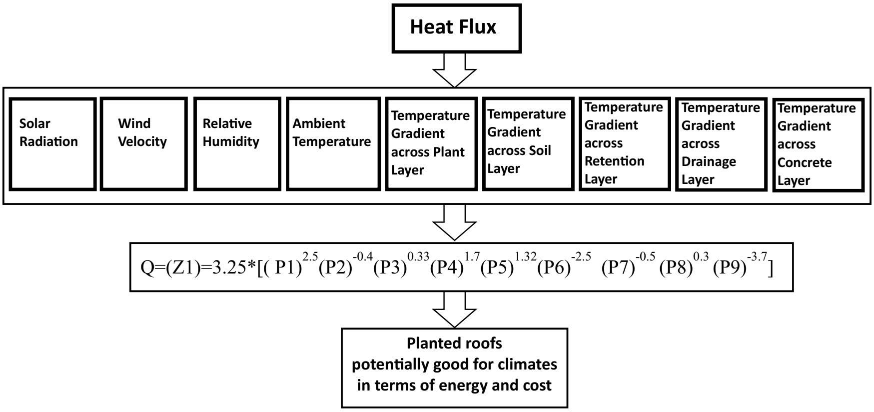
In the above equations, (Z_1) is relating to response variable for heat flux of planted roof. As the soil depth increases, it reduces the temperature in the room, at the same time thicker roof would increase the dead weight upon the concrete roof. Around 20–30 cm thickness of soil gives the best performance. Heat transfer is controlled by leaf cover area. With the increase in leaf area index, heat transfer reduces. The experimental investigation shows that the room air temperature of green roofs is always lower than normal roof throughout the experimentation. The average difference can be predicted by varying the experimental data, i.e. pie terms of independent variables. Thus, the model can be utilized as a tool to find the optimum parameter and shows that green roof is one through which energy consumption impacts the residential building.

6 Results and Conclusions

The indices of the mathematical models of green roof show the effect of the individual pie term so that the index with high value or low value will indicate the dominating nature over the output heat flux of the green roof.

Curve fitting constant k and the indices of each pie term show the influence of the causes on the effects. The curve fitting constant k is the collectively combined effect of the entire extra variable which is not included in the experimentation. If the value of k is 1, it means the model is perfect. If it is too low, the causes are overestimated; if it is too high, the causes are underestimated. This would decide when to repeat the investigation again or to refine the approach in subsequent attempts. The magnitude of exponents of the causes indicates the degree of influences of those causes on the specific response. The results indicate that the planted roof can greatly affect the room temperature profile. Planted roofs are potentially good for climates in terms of energy and cost.

The value of the curve fitting constant in this model for (Z_1) is 3.25. This collectively represents the combined effect of all extraneous variables such as soil properties, concrete materials and plant types. Further, as it is positive, this indicates that these causes have an increasing influence on heat flux through planted roof.



Flow chart of performance of planted roof

1. The absolute index of π_1 is the highest, viz. 2.5. Thus, the term related to the specification of the solar radiation involved the most influencing π term in this model. The value of this index is positive indicating that the heat flux through planted roof (Z_1) is directly proportional to term related to the specification of heat generation by respiration and evaporation rate. Heat flux increases with the increase in heat generation by respiration and evaporation rate.
2. The absolute index of π_3 is the lowest, viz. 0.3. Thus, the term related is the effect influencing π term in this model. The value of this index is positive indicating that the heat flux through planted roof (Z_1) is directly proportional to the term related to temperature gradient across drainage layer [π_8]. The heat flux through planted roof increases as [π_8] increases on the effect of predetermining parameters such as temperature, evaporation rate, area of flow of water and rate of flow of water. Suggestions regarding selection of appropriate area of flow of water will reduce the heat flux.
3. The sequence of influence of another independent π term present in this model is π_4, π_5, π_3, π_8, π_2, π_6 and π_9 having absolute indices as 1.7, 1.32, 0.33, 0.3, −0.4, −2.5 and −3.7 in the order, respectively.

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