



Performance Analysis of Multi-user Diversity Schemes on Interference Limited D-GG Atmospheric Turbulence Channels

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Abstract. With asymmetric channel fading, this research presents the unified analysis of a multiuser radio frequency/free space optics (RF/FSO) amplify-and-forward (AF) relay network. It has been assumed that relay and destination nodes operate in the presence of co-channel interferers (CCIs). The first hop of the AF strategy is in RF domain, which are used for information transmission between users and relay nodes, and an FSO link is utilized for sending information from the relay to destination node. The RF channels have been assumed to undergo the Nakagami- m fading model, whereas the turbulence on the wireless optical channel is expected to follow the Double Generalized Gamma (D-GG) fading model, which takes pointing errors into account. Analytical expression for the outage probability has been presented for the aforementioned system model. Monte Carlo simulations are used to verify all of the theoretical results.

Keywords: Mixed RF/FSO · D-GG channel · Co-channel interferers

1 Introduction

Researchers have paid close attention to the huge unlicensed bandwidth available in free space optical (FSO) communication systems. The atmospheric turbulence and pointing error limit the capabilities of FSO systems, such as high security, quick deployment, and flexibility [1, 2]. In addition, mixed RF/FSO relay systems have arisen as a result of the use of FSO systems as an complementary form of radio frequency (RF) equivalents [3–6]. Because of its simplicity, the amplify-and-forward (AF) relaying approach has gained a lot of traction.

Multiuser diversity (MUD) techniques have been developed in [7] to boost the data rate supplied by wireless networks across RF fading channels. RF systems can employ the diversity gain provided by MUD systems from several users transmitting at the same time to boost throughput. MUD can take advantage

of fluctuating RF channels by assigning common resources to the user with the best channel. The user with the highest signal-to-noise ratio (SNR) is chosen for transmission in this approach. Furthermore, interference from other modes of transmission in the immediate neighborhood limits the performance of any realistic wireless communication system. Importantly, relay networks with various co-channel interferences (CCIs) at the receiving nodes impair the desired signal quality.

In the literature, there has been a lot of work on mixed RF/FSO relaying systems [3, 5, 7–9]. The dual hop mixed RF/FSO asymmetric relaying was proposed in [3], where the authors developed an estimate of the entire system's outage probability (OP). The Rayleigh distribution was used to represent the RF link, whereas Gamma-Gamma atmospheric turbulence was used to model the FSO link. Using pointing faults on the FSO link subject to intensity modulation with direct detection (IM/DD) and heterodyne detection, the authors of [5] completed a unified study of mixed RF/FSO relaying system. Research works [7–9] examined the performance of a dual hop RF/FSO connection using recently proposed generalized \mathcal{M} -distribution and Double Generalized Gamma (D-GG) fading models.

While the majority of the published results for FSO relay-assisted communications have depended on the absence of CCI, it becomes crucial to emphasize that because RF links are involved, asymmetric RF/FSO relay systems can be intrinsically subject to the impact of CCI. In RF systems, frequency re-use also creates a source of interference, according to [10]. The authors of [11] were inspired by the interference-limited performance of mixed RF/FSO systems to construct the expression of OP and bit error rate (BER) for an AF relaying system. To illustrate accurate and asymptotic performance of the whole system in [11], the channel of interfering signals was modeled as the Nakagami- m distribution fading, while the FSO systems was studied using the D-GG turbulence model. The authors in [12] have derived the closed form formulation of OP by observing the effect of interference at the relay node. In addition, [13] describes an examination of a dual-hop FSO/RF cooperative system in which the FSO and RF links undergo Malaga and generalized-K distributed fadings, respectively.

The majority of past research has focused on mixed RF/FSO relay systems with the premise of interference-free transmission. Furthermore, works that address the issue of interference in FSO systems have not addressed the benefits that the MUD scheme can provide. We intend to study asymmetric relaying with generalized fading models in the presence of MUD, which is missing in the literature. To increase the performance of the RF link, we consider an asymmetric mixed RF/FSO fixed gain relay method with sub-carrier intensity modulation (SIM) and MUD at the source in this study. In addition, we evaluate the influence of several CCIs on the communication to make the system model more feasible. Therefore, the system model presented in this paper is different from that given in [3, 5, 7–9, 11–16]. In addition to this, we model the RF link with Nakagami- m fading model [17]. With interference at relay and destination nodes, we study the system subjected highly adverse conditions. Unequal interferers with non-identical statistical characteristics have been considered on both

the links to observe the effect of simultaneous transmission from different kinds communication systems. D-GG fading distribution is used to model atmospheric turbulence on the FSO link.

2 System and Channel Models

In this section, the system and channel models of the considered relaying strategy is discussed.

Table 1. List of symbols.

Symbol	Meaning
$P_{SR,j}$	Power of signal of j^{th} -user on $S - R$ link
$h_{SR,j}$	Fading coefficient of j^{th} -user on $S - R$ link
d_{RF}	Symbol of j^{th} -user from source node
P_{I1}	Power of interferer at relay node
$x_{r,i}$	Symbol from interferer at relay node
ρ	Type of optical demodulation
N_{RF}	AWGN on $S - R$ link
η	Optical to electrical conversion factor
I	Irradiance on optical link
$h_{D,i}$	Power of interferer at destination node
$x_{d,i}$	Symbol from interferer at relay node
N_o	AWGN on $R - D$ link

2.1 System Model

Consider an asymmetric RF/FSO dual hop cooperative relaying system with K RF mobile users intend to send data to the destination node D through the relay node R . The randomness offered by the RF channel increases as the number of users on the RF link increases, which is utilized by the node R for increasing system throughput. Moreover, the K users communicate with the relay node in the vicinity of L CCIs at the relay node. On source-to-relay (SR) link, the signal received $y_{SR,j}$ from j^{th} -user at node R is given by

$$y_{SR,j} = \sqrt{P_{SR,j}}h_{SR,j}d_{RF} + \sqrt{P_{I1}} \sum_{i=1}^L h_{SR,i}x_{r,i} + N_{RF} \quad (1)$$

where suffix j corresponds to the j^{th} -user and all the symbols are defined in Table 1. In the second phase, the relay node amplifies the signal $y_{SR,j}$ and transmits the same towards the destination node D . In the presence of M CCIs at

the destination node, the received signal is expressed as

$$\begin{aligned}
 y_{j,RD} = & (\eta I)^{\frac{\rho}{2}} G \sqrt{P_{SR,j}} h_{SR,j} d_{RF} + (\eta I)^{\frac{\rho}{2}} G \sqrt{P_{I_1}} \sum_{i=1}^L h_{R,i} x_{r,i} \\
 & + (\eta I)^{\frac{\rho}{2}} \sqrt{P_{I_2}} \sum_{i=1}^M h_{D,i} x_{d,i} + N_o
 \end{aligned} \quad (2)$$

The various symbols in (2) have been defined in Table 1. As mentioned in [10], the gain G can be formulated as

$$G^2 = \frac{1}{P_{SR,j} |h_{SR,j}|^2 + P_{I_1} \sum_{i=1}^L |h_{R,i}|^2 + \sigma_n^2} \quad (3)$$

For the cooperative relay strategy, the signal-to-interference-plus-noise (SINR) can be written using (2) as [10]

$$\gamma_{e2e} = \frac{\gamma_{SR,j}^{eff} \gamma_{RD}^{eff}}{\gamma_{SR,j}^{eff} + \gamma_{RD}^{eff} + 1} \quad (4)$$

where the effective instantaneous SINR on the RF link corresponding to the j^{th} user considering both RF fading as well as the interference statistics is denoted by $\gamma_{SR,j}^{eff}$ and is given by the relation $\gamma_{SR,j}^{eff} = \frac{\gamma_k}{1 + \sum_{i=1}^L \gamma_{I_r,i}}$. On the other hand, γ_{RD}^{eff} denotes the effective SINR on the wireless optical link considering atmospheric scintillation, pointing errors, path loss and interference defined as $\gamma_{RD}^{eff} = \frac{\gamma_{RD}}{1 + \sum_{i=1}^M \gamma_{I_d,i}}$. The variables γ_{I_r} and γ_{I_d} denote interfering signal at relay and destination nodes, respectively. For mathematical tractability, consider an upper bound, $\gamma_{e2e} < \gamma_{up}$, such that

$$\gamma_{e2e} = \min \left(\gamma_{SR,j}^{eff}, \gamma_{RD}^{eff} \right) \quad (5)$$

2.2 Channel Model

2.2.1 RF Link

The k^{th} user on the RF link follows the probability density function (PDF) of Nakagami- m distribution as given below [17]

$$f_{\gamma_k}(\gamma) = \frac{\beta^{m_{SR}}}{\Gamma(m_{SR})} \gamma^{m_{SR}-1} \exp(-\beta\gamma) \quad (6)$$

where m_{SR} is the shape parameter and $\beta = \frac{m_{SR}}{\gamma_k}$. Furthermore, using the notion of ordered statistics as follows, the PDF with K users may be produced as

$$f_{\gamma_{SR,j}}(\gamma) = K [F_{\gamma_k}(\gamma)]^{K-1} f_{\gamma_k}(\gamma) \quad (7)$$

Hence, considering (6) and (7), for integer values of fading parameter m_{SR} , the closed-form expression for the PDF of RF link experiencing Nakagami- m fading can be obtained as [6]

$$f_{\gamma_{SR,j}}(\gamma) = \frac{K\beta^{m_{SR}}}{\Gamma(m_{SR})} \sum_{n_1=0}^{K-1} (-1)^{n_1} \binom{K-1}{n_1} \times \sum_{n_2=0}^{n_1(m_{SR}-1)} \zeta_{n_1 n_2}(m_{SR}) \gamma^{m_{SR}+n_2-1} \beta^{n_2} \exp(-\beta(n_1+1)\gamma) \tag{8}$$

where $\zeta_{n_1 n_2}(m_{SR})$ is the multinomial expansion's coefficient involved in the derivation of (8) which can be recursively obtained using the relation $\zeta_{n_1 n_2} = \sum_{b=n_1-x+1}^{n_1} \frac{\lambda_{bn_2-1}}{n_1-b} I_{[0,(n_2-1)(x-1)]}$ where $I_{[a,b]}$ is defined in [17, Eq. (9.120)]. Moreover, the PDF of the total interference-to-noise ratio (INR) $\sum_{i=1}^L \gamma_{I_r,i}$ can be expressed as [6] $f_{I_r}(\gamma) = \left[\frac{m_1}{\Omega_{I1}} \right]^{m_1 L} \frac{\gamma^{m_1 L-1}}{\Gamma(m_1 L)} \exp\left(-\frac{m_1 L}{\Omega_{I1}} \gamma\right)$ where m_1 is Nakagami- m fading parameter and Ω_{I1} is the interference to noise ratio (INR) on the RF link. The effective PDF can be simplified as [6]

$$f_{\gamma_{SR,j}}^{eff}(\gamma) = \sum_{n_1=0}^{K-1} \sum_{n_2=0}^{n_1(m_{SR}-1)} A_2 \gamma^{m_{SR}+n_2-1} e^{[-\beta(n_1+1)\gamma]} \times U(m_1 L, m_1 L + 1 + m_{SR} + n_2, \mathcal{B}_0(\gamma)) \tag{9}$$

where $U(a, b, z)$ is the confluent hypergeometric function of second kind as defined in [18, Eq. (9.211.4)], $\mathcal{B}_0(\gamma) = \beta(n_1 + 1)\gamma + \frac{m_1}{\Omega_{I1}}$, the constant $A_1 = \frac{K\beta^{m_{SR}}}{\Gamma(m_1 L)\Gamma(m_{SR})} \left(\frac{m_1}{\Omega_{I1}}\right)^{m_1 L}$, $A_2 = A_1(-1)^{n_1} \binom{K-1}{n_1} \Gamma(m_1 L) \zeta_{n_1 n_2}(m_{SR}) \beta^{n_2}$. The cumulative distribution function (CDF) on the RF link can be derived as $F_{\gamma_{SR}}^{eff}(\gamma) = \int_0^\infty f_{\gamma_{SR}}^{eff}(\gamma) d\gamma$. After some mathematical manipulations, the CDF of $\gamma_{SR,j}^{eff}$ can be expressed as follows [6]

$$F_{\gamma_{SR,j}}^{eff}(\gamma) = \frac{K}{\Gamma m_{SR}} \sum_{n_1=0}^{K-1} \sum_{n_2=0}^{n_1(m_{SR}-1)} A_3 \left[1 - \sum_{m=0}^{n_2+m_{SR}-1} e^{(-\beta(n_1+1)\gamma)} \times \frac{\{(n_1+1)\beta\gamma\}^m}{m!} U\left(m_1 L, m_1 L + 1 + m, \mathcal{B}_0(\gamma)\right) \right] \tag{10}$$

where A_3 is given by

$$A_3 = (-1)^{n_1} \binom{K-1}{n_1} (m_{SR} + n_2 - 1)! \zeta_{n_1 n_2}(m_{SR}) \times (n_1 + 1)^{-n_2 - m_{SR}} \tag{11}$$

On the other hand, it is assumed that $R \rightarrow D$ link undergoes the turbulence-induced fading I_a and the pointing errors I_p such that $I = I_a I_p$. The pointing

error parameters are obtained from the reference [1]. The atmospheric turbulence fading I_a on the FSO link is modeled as double generalized gamma (GG) fading model [9] and the PDF of I_a can be expressed as [9]

$$f(I_a) = \frac{y\sigma^{\beta_1 - \frac{1}{2}}\lambda^{\beta_2 - \frac{1}{2}}(2\pi)^{1 - \frac{\sigma + \lambda}{2}}}{\Gamma(\beta_1)\Gamma(\beta_2)I_a} G_{\lambda + \sigma, 0}^{0, \lambda + \sigma} \left[\left(\frac{\Omega_2}{I_a^{\alpha_2}} \right)^\lambda \frac{\lambda^\lambda \sigma^\sigma \Omega_1}{\beta_1^\sigma \beta_2^\lambda} \middle| 1 - \tau_0 \right] \quad (12)$$

where $\tau_0 = [\Delta(\sigma : \beta_1), \Delta(\lambda : \beta_2)]$ with $\Delta(z : x)$ representing $\left[\frac{x}{z}, \frac{x+1}{z}, \dots, \frac{x+z-1}{z} \right]$, where $G_{p,q}^{m,n} \left[\cdot \right]$ is the Meijer-G function defined in [18, Eq. (9.301)] and $y = \alpha_2 \lambda$. Moreover, the SNR PDF on FSO link experiencing a double GG fading model with the effect of pointing errors is given by [9]

$$f_{RD}(\gamma) = \frac{\mathcal{D}_1}{r\gamma} G_{1, \lambda + \sigma + 1}^{\lambda + \sigma + 1, 0} \left[\mathcal{D}_2 z^y \left(\frac{\gamma}{\mu_r} \right)^{\frac{y}{r}} \middle| \tau_1 \right] \quad (13)$$

with

$$\mathcal{D}_1 = \frac{\xi^2 \sigma^{\beta_1 - \frac{1}{2}} \sigma^{\beta_2 - \frac{1}{2}} (2\pi)^{1 - \frac{\sigma + \lambda}{2}}}{\Gamma(\beta_1)\Gamma(\beta_2)} \quad (14)$$

$$\mathcal{D}_2 = \frac{\beta_1^\sigma \beta_2^\lambda}{\lambda^\lambda \sigma^\sigma \Omega_1^\sigma \Omega_2^\lambda} \quad (15)$$

where $\tau_1 = \left[\frac{\xi^2}{y}, \Delta(\sigma : \beta_1), \Delta(\lambda : \beta_2) \right]$, $\tau_2 = \left[1 + \frac{\xi^2}{y} \right]$, $\mu_r = \frac{(\eta \mathbb{E}(I))^r}{N_0}$ and r denotes the kind of demodulation scheme [9]. Considering $\mathcal{D}_3 = \prod_{g=1}^{\sigma + \lambda} \Gamma \left(\frac{1}{y} + \tau_{0,g,3.5pt} \right)$, z can be given as $z = \frac{\mathcal{D}_1 (\mathcal{D}_2)^{1/y} \mathcal{D}_3}{(1 + \xi^2)}$.

Furthermore, the following is the unified CDF of the FSO channel over double GG air turbulence with pointing errors [9]

$$F_{RD}(\gamma) = \mathcal{D}_4 G_{r+1, n+1}^{n, 1} \left[\mathcal{D}_5 \left(\frac{\gamma}{\mu_r} \right)^y \middle| \begin{matrix} 1, \tau_3 \\ \tau_4, 0 \end{matrix} \right] \quad (16)$$

where $n = r(\lambda + \sigma + 1)$, $\mathcal{D}_4 = \frac{\xi^2 \sigma^{\beta_1 - \frac{1}{2}} \sigma^{\beta_2 - \frac{1}{2}} (2\pi)^{1 - \frac{r(\sigma + \lambda)}{2}} r^{(\beta_1 + \beta_2 - 2)}}{y \Gamma(\beta_1) \Gamma(\beta_2)}$, $\mathcal{D}_5 = \left(\frac{\mathcal{D}_2 z^y}{r \sigma^{\lambda}} \right)^r$, $\tau_3 = [\Delta(r : \tau_2)]$ and $\tau_4 = [\Delta(r : \tau_1)]$ comprising of $r(\lambda + \sigma + 1)$ terms. Moreover, the interference on the destination is assumed to follow the PDF [19]

$$f_{I_d}(\gamma) = \frac{\gamma^{M-1}}{\Omega_{I_2} (M-1)!} \exp \left(-\frac{\gamma}{\Omega_{I_2}} \right) \quad (17)$$

where Ω_{I_2} is the INR on the FSO link. Given that I_d and γ_{RD} are independent, the CDF of γ_{RD}^{eff} can be derived using

$$F_{\gamma_{RD}^{eff}}(\gamma) = \int_0^\infty F_{RD}(\gamma(1+x)) f_{I_d}(x) dx \quad (18)$$

Substituting (16) and (17) in (18), and assuming $1 + y = z$, above integral may be formulated as

$$F_{\gamma_{RD}}^{eff}(\gamma) = \mathcal{D}_6 (\mathcal{J}_1 - \mathcal{J}_2) \tag{19}$$

where $\mathcal{D}_6 = \frac{\mathcal{D}_4}{\Omega_{I2}^M (M-1)!} \exp(1/\Omega_{I2})$ and

$$\mathcal{J}_1 = \int_0^\infty (z - 1)^{M-1} e^{-\frac{z}{\Omega_{I2}}} G_{r+1,n+1}^{n,1} \left[\mathcal{D}_5 \left(\frac{z\gamma}{\mu_r} \right)^y \middle| \begin{matrix} 1, \tau_3 \\ \tau_4, 0 \end{matrix} \right] \tag{20}$$

Resorting to binomial expansion $(z - 1)^{M-1} = \sum_{i=0}^{M-1} \binom{M-1}{i} (-1)^{M-1-i} z^i$ as per [18, Eq. (1.11)] for integer values of M and with the aid of [20, Eq. (07.34.21.0013.01)], integral \mathcal{J}_1 can be given as

$$\mathcal{J}_1 = \sum_{i=0}^{M-1} x_i \mathcal{D}_{10} G_{r+y+1,n+1}^{n,y+1} \left[\mathcal{D}_7 (\gamma)^y \middle| \begin{matrix} 1, \tau_3, \tau_5 \\ \tau_4, 0 \end{matrix} \right] \tag{21}$$

where $\mathcal{D}_7 = \mathcal{D}_5 (y\Omega_{I2}/\mu_r)^y$, $\mathcal{D}_{10} = \frac{y^{i+\frac{1}{2}} (\Omega_{I2})^{i+1}}{(2\pi)^{\frac{y-1}{2}}}$ and $\tau_5 = \left[\frac{(-i)}{y}, \dots, \frac{(v-1-i)}{y} \right]$.

On the other hand, assuming $x_i = \binom{M-1}{i} (-1)^{M-1-i}$, the integral \mathcal{J}_2 can be given as

$$\mathcal{J}_2 = \sum_{i=0}^{M-1} x_i \int_0^1 (z)^i e^{-\frac{z}{\Omega_{I2}}} G_{r+1,n+1}^{n,1} \left[\mathcal{D}_5 \left(\frac{z\gamma}{\mu_r} \right)^y \middle| \begin{matrix} 1, \tau_3 \\ \tau_4, 0 \end{matrix} \right] \tag{22}$$

The integral \mathcal{J}_2 can be expressed in complex integral form on using [20, Eq. (07.34.02.0001.01)]. By virtue of [18, Eq. (8.380)] and [21, Eq. (28-30)] and with some mathematical manipulations, the integral \mathcal{J}_2 shown in (23) where

$$\begin{aligned} \mathcal{J}_2 &= \sum_{i=0}^{M-1} x_i H_{1,1:0,1:r+1,n+1}^{0,1:1,0:n,1} \left[\begin{matrix} (-i : 1, v) \\ (-i - 1 : 1, v) \end{matrix} \middle| \begin{matrix} \text{---} \\ (0, 1) \end{matrix} \middle| \begin{matrix} (1, 1), (\tau_3, [1]_{\tau_3}) \\ (\tau_4, [1]_{\tau_4}), (o, 1) \end{matrix} \middle| \frac{1}{\Omega_{I2}}, \mathcal{D}_5 \left(\frac{z\gamma}{\mu_r} \right)^y \right] \\ &= \sum_{i=0}^{M-1} x_i \mathcal{H} \left[\Omega_{I2}, \left(\frac{\gamma}{\mu_r} \right)^y \right] \end{aligned} \tag{23}$$

$[1]_x$ represents sequence of 1^s of length x and $H_{p_1, q_1: p_2, q_2: p_3, q_3}^{m_1, n_1: m_2, n_2: m_3, n_3} [\mathcal{A}_1, \mathcal{A}_2] = \mathcal{H}[\mathcal{A}_1, \mathcal{A}_2]$ is the bivariate Fox-H function given in [22, Eq. (28)]. Substituting (21) and (23) into (19), we obtain the closed form equation of CDF on FSO link in the presence of multiple interferers as given below

$$\begin{aligned} F_{\gamma_{RD}}^{eff}(\gamma) &= \mathcal{D}_6 \sum_{i=0}^{M-1} x_i \mathcal{D}_{10} G_{r+y+1,n+1}^{n,y+1} \left[\mathcal{D}_7 (\gamma)^y \middle| \begin{matrix} 1, \tau_3, \tau_5 \\ \tau_4, 0 \end{matrix} \right] \\ &\quad - \mathcal{H} \left[\Omega_{I2}, \left(\frac{\gamma}{\mu_r} \right)^y \right] \end{aligned} \tag{24}$$

Moreover, the PDF of end-to-end SNR can be derived by utilizing the relationship (18). Using the identity [20, Eq. (07.34.21.0013.01)] and [22], the closed form expression for the overall PDF can be written as shown below

$$f_{\gamma_{RD}}^{eff}(\gamma) = \mathcal{D}_{11} \sum_{i=0}^{M-1} x_i \left[\mathcal{D}_{12} G_{r+y, r(\lambda+\sigma+1)}^{r(\lambda+\sigma+1), y} \left[\mathcal{D}_8 \gamma^y \left| \begin{matrix} \tau_6 \\ \tau_7 \end{matrix} \right. \right] - \mathcal{H} \left[\Omega_{I2}, \mathcal{D}_2 z^y \left(\frac{\gamma}{\mu_r} \right)^{y/r} \right] \right] \quad (25)$$

In (25), the parameters involved in Meijer-G function has been derived as $\tau_6 = \left[\frac{\tau_2}{r}, \dots, \frac{\tau_2+r-1}{r}, \frac{1-(i+2)}{y}, \dots, \frac{y-(i+2)}{y} \right]$ and $\tau_7 = \left[\frac{\tau_1}{r}, \dots, \frac{\tau_1+r-1}{r}, \dots, \frac{\tau_1(\lambda+\sigma+1)+r-1}{r} \right]$. The argument of the Meijer-G equals $\mathcal{D}_8 = \mathcal{D}_2 z^y \{r\}^{-r(\lambda+\sigma)} \left(\frac{\Omega_{I2}}{y\mu_r} \right)^y$, $\mathcal{D}_{11} = \frac{\mathcal{D}_1 \exp(\frac{1}{\Omega_{I2}})}{r\lambda\Omega_{I2}^M (M-1)!}$ and $\mathcal{D}_{12} = \frac{y^{i+\frac{3}{2}} (\Omega_{I2})^{i+2} (r)^{c*}}{(2\pi)^{\{y-1+\frac{(r-1)(\lambda+\sigma)}{2}\}}}$. The bivariate Fox's H-function is defined in (26).

$$\begin{aligned} & H_{1,1:0,\lambda+\sigma+1,1}^{0,1:1,0:\lambda+\sigma+1,1} \left[\begin{matrix} (-i-1 : 1, y/r) \\ (-i-2 : 1, y/r) \end{matrix} \middle| \begin{matrix} - \\ (0, 1) \end{matrix} \middle| \begin{matrix} (\tau_2, [1]_{\tau_2}) \\ (\tau_1, [1]_{\tau_1}) \end{matrix} \middle| \frac{1}{\Omega_{I2}}, \mathcal{D}_2 z^y \left(\frac{\gamma}{\mu_r} \right)^{y/r} \right] \\ &= \mathcal{H} \left[\Omega_{I2}, \mathcal{D}_2 z^y \left(\frac{\gamma}{\mu_r} \right)^{y/r} \right] \end{aligned} \quad (26)$$

3 Performance Analysis

In this section, we investigate various performance metrics of relay system based on the aforementioned SNR bound.

3.1 Exact Outage Probability

The outage probability (OP) of a wireless communication system is an important performance measure. The likelihood that the instantaneous SNR goes below a pre-specified threshold γ_{th} is defined as OP. The end-to-end CDF $F_{e_2e}(\gamma)$ of the overall system can be written as

$$F_{e_2e}(\gamma) = F_{\gamma_{SR,j}}^{eff}(\gamma) + F_{\gamma_{RD}}^{eff}(\gamma) - F_{\gamma_{SR,j}}^{eff}(\gamma)F_{\gamma_{RD}}^{eff}(\gamma) \quad (27)$$

Hence, substituting (10) and (24) into (27), and by putting $\gamma = \gamma_{th}$ we can obtain the overall CDF of the considered system.

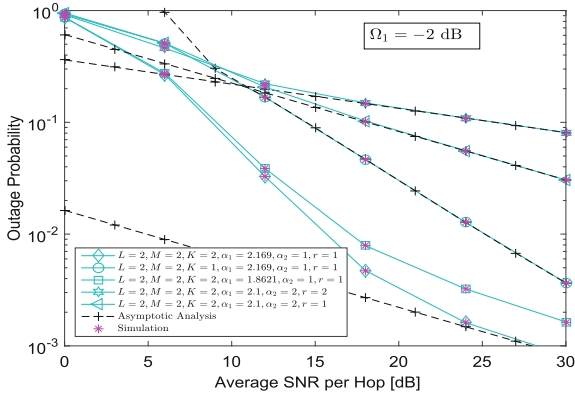


Fig. 1. Outage Probability of dual hop mixed RF/FSO relaying system with varying number of CCIs.

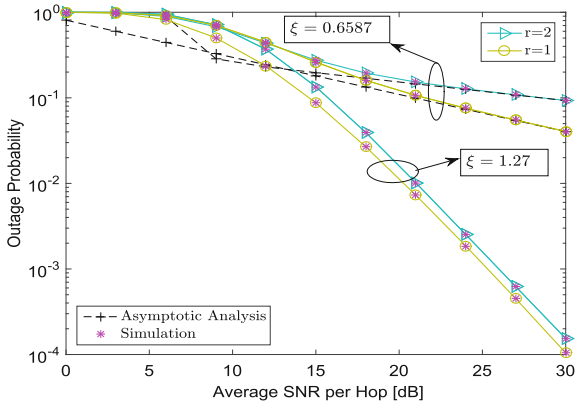


Fig. 2. Outage Probability of dual hop mixed RF/FSO relaying system for varying FSO link parameters.

4 Results

The results of the research study have been discussed in this section. The outage performance of the mixed RF/FSO relaying system is shown in Fig. 1. The outage probability has been plotted against the average SNR per hop in this graph. The comparison was made for a variety of RF users, as well as the number of interferers at the relay and destination nodes. The graphic has actually showed the fluctuating strength of air turbulence. It can be shown that as the number of CCIs grows, so does the outage. The asymptotic representation of the outage probability is also shown in the plot. Monte-Carlo simulations were used to verify the results. The effect of pointing inaccuracy and optical demodulation type is displayed in Fig. 2. It can be seen that the outage probability rises as the pointing inaccuracy rises (lower value of ξ). The chance of an outage is also influenced by

optical demodulation. The plot shows that when coherent detection is used $r = 1$, the system's outage probability is much higher than when coherent detection is used $r = 2$. Also shown are the asymptotic and Monte-Carlo results.

5 Conclusion

The analysis of a mixed RF/FSO relaying system in the presence of interferers on the relay and destination nodes is presented in this paper. The fading of the RF link was modeled using the Nakagami- m distribution, whereas the turbulence of the FSO link was studied using the D-GG distribution. We offer the analysis for the OP of the overall system for the considered system model.

References

1. Farid, A.A., Hranilovic, S.: Outage capacity optimization for free-space optical links with pointing errors. *J. Lightwave Technol.* **25**(7), 1702–1710 (2007)
2. Goel, A., Bhatia, R.: Hybrid RF/MIMO-FSO relaying systems over gamma-gamma fading channels. In: Gupta, D., Khanna, A., Bhattacharyya, S., Hassaniien, A.E., Anand, S., Jaiswal, A. (eds.) *International Conference on Innovative Computing and Communications. AISC*, vol. 1165, pp. 607–615. Springer, Singapore (2021). <https://doi.org/10.1007/978-981-15-5113-0-49>
3. Lee, E., Park, J., Han, D., Yoon, G.: Performance analysis of the asymmetric dual-hop relay transmission with mixed RF/FSO links. *IEEE Photonics Technol. Lett.* **23**(21), 1642–1644 (2011)
4. Goel, A., Bhatia, R.: Joint impact of interference and hardware impairments on the performance of mixed RF/FSO cooperative relay networks. *Opt. Quant. Electron.* **53**(9), 1–15 (2021). <https://doi.org/10.1007/s11082-021-03064-x>
5. Zedini, E., Ansari, I.S., Alouini, M.S.: Performance analysis of mixed Nakagami- m and Gamma Gamma dual-hop FSO transmission systems. *IEEE Photonics J.* **7**(1), 1–20 (2015)
6. Goel, A., Bhatia, R.: On the performance of mixed user diversity-RF/spatial diversity FSO cooperative relaying AF systems. *Opt. Commun.* **8** (2020)
7. Yang, L., Hasna, M.O., Ansari, I.S.: Unified performance analysis for multiuser mixed $\eta - \mu$ and \mathcal{M} - distribution dual-hop RF/FSO systems. *IEEE Trans. Commun.* **65**(8), 3601–3613 (2017)
8. Wang, P., Wang, R., Guo, L., Cao, T., Yang, Y.: On the performances of relay-aided FSO system over \mathcal{M} distribution with pointing errors in presence of various weather conditions. *Opt. Commun.* **367**, 59–67 (2016)
9. AlQuwaiee, H., Ansari, I.S., Alouini, M.: On the performance of free-space optical communication systems over Double Generalized Gamma channel. *IEEE J. Sel. Areas Commun.* **33**(9), 1829–1840 (2015)
10. Ikki, S.S., Aissa, S.: Performance evaluation and optimization of dual-hop communication over nakagami- m fading channels in the presence of co-channel interferences. *IEEE Commun. Lett.* **16**(8), 1149–1152 (2012)
11. Soleimani-Nasab, E., Uysal, M.: Generalized performance analysis of mixed RF/FSO cooperative systems. *IEEE Trans. Wireless Commun.* **15**(1), 714–727 (2016)

12. Petkovic, M.I., Cvetkovic, A.M., Djordjevic, G.T., Karagiannidis, G.K.: Outage performance of the mixed RF/FSO relaying channel in the presence of interference. *Wireless Pers. Commun.* **96**, 2999–3014 (2017). <https://doi.org/10.1007/s11277-017-4336-7>
13. Trigui, I., Cherif, N., Affes, S., Wang, X., Leung, V., Stephenne, A.: Interference-limited mixed Málaga-m and generalized-k dual-hop FSO/RF systems. In: 2017 IEEE 28th Annual International Symposium on Personal, Indoor, and Mobile Radio Communications (PIMRC), pp. 1–6, October 2017
14. Balti, E., Guizani, M.: Mixed RF/FSO cooperative relaying systems with co-channel interference. *IEEE Trans. Commun.* **66**(9), 4014–4027 (2018)
15. Abouei, J., Plataniotis, K.N.: Multiuser diversity scheduling in free-space optical communications. *J. Lightwave Technol.* **30**(9), 1351–1358 (2012)
16. Chen, L., Wang, W., Zhang, C.: Multiuser diversity over parallel and hybrid FSO/RF links and its performance analysis. *IEEE Photonics J.* **8**(3), 1–9 (2016)
17. Simon, M.K., Alouini, M.: *Digital Communication over Fading Channels*. Wiley, Hoboken (2000)
18. Gradshteyn, I.S., Ryzhik, I.M.: *Table of Integrals, Series and Products*. Academic, New York (2000)
19. Hasna, M.O., Alouini, M.S., Bastami, A., Ebbini, E.S.: Performance analysis of cellular mobile systems with successive co-channel interference cancellation. *IEEE Trans. Wireless Commun.* **2**(1), 29–40 (2004)
20. Wolfram, I.: *Wolfram, research, mathematica edition: Version 10.0*. Champaign, Wolfram Research Inc. (2010)
21. Mittal, P., Gupta, K.: An integral involving generalized function of two variables. *Proc. Indian Acad. Sci. Section A* **75**(3), 117–123 (1972). <https://doi.org/10.1007/BF03049732>
22. Alhennawi, H.R., Ayadi, M.M.H.E., Ismail, M.H., Mourad, H.A.M.: Closed-form exact and asymptotic expressions for the symbol error rate and capacity of the H-function fading channel. *IEEE Trans. Veh. Technol.* **65**(4), 1957–1974 (2016)