Study on the Band Gap Characteristics of Two-Dimensional Local Resonant Phononic Crystals

Yuan Xing, Bo Zhang, Yao Zhang, Jiaxing Song, and Meng Wang

Abstract A local resonant phononic crystal is proposed. The phononic crystal can control vibration and noise effectively. It also can control the frequency range. We can obtain elastic wave dissipation in a wide frequency range. The band structure and transmission characteristics are calculated by the finite element software COMSOL Multiphysics. Through the results, the vibration modes at the initial frequency, cut-off frequency and other typical frequencies are analyzed. So we can elaborate the mechanism of the band gap. The opening of the band gap is the result of the coupling between the long wave traveling wave in the matrix and the resonant characteristics of the oscillator. The band gap frequency range of the phononic crystal structure is 268.7 ~ 1097.9 Hz. The factors affecting the band gap of the phononic crystal were studied. The result show: The density of materials and the geometric parameters of phononic crystals will affect the band gap. The density of scatterers mainly affects the initial frequency. The density of the matrix only affects the cut-off frequency. The geometric size has an effect on the starting frequency and cut-off frequency.

Keywords Phononic crystal · Wave coupling · Band gap

1 Introduction

With the improvement of China's international competitiveness, the manufacturing industry has made great contributions. China takes "Made in China 2025"as the national action program. With the development of manufacturing industry, the problem of vibration and noise has become an unavoidable problem perplexing the development of the industry [1, 2]. Vibration poses a great challenge to the accuracy requirements of manufacturing industry. Noise affects people's life and workers' operating environment. These two problems have become important issues that designers must consider in the process of product development [3–5].

Y. Xing · B. Zhang (✉) · Y. Zhang · J. Song · M. Wang
School of Mechanical Engineering, Ningxia University, Ningxia, China
e-mail: zhangb@nxu.edu.cn

Artificial periodic structure is a new structure found to effectively control this problem in recent years. Phononic crystal is a typical representative. It can be used to control the propagation of elastic waves. In national defense, national economy and daily life, acoustic metamaterials based on phononic crystals have important potential applications in the fields of vibration and noise reduction [6–9]. Phononic crystal is a new physical concept in the field of condensed matter physics. It is a composite material or structure with elastic wave band gap characteristics composed of two or more media. Phononic crystal can be regarded as the extension of the concept of crystal in solid physics in the sense of elastic wave. Its important characteristic is its attenuation domain. The research of phononic crystal can learn from the research of elastic dynamics and solid physics.

Phononic crystals are periodic composites or structures with elastic band gap characteristics. Phononic crystal is a new artificial periodic structure. Although human beings have studied the propagation characteristics of elastic waves in layered periodic media for about 80 years, the concept of phononic crystal and the related theory of phononic crystal have only been studied for more than 20 years [10–12]. In 1993, M.S. Kushwaha and others clearly put forward the concept of phononic crystal for the first time when studying nickel/aluminum two-dimensional solid periodic composite medium. At the same time, they also clearly and firstly put forward that the band gap characteristics of phononic crystals have application prospects in high-precision vibration free environment [13, 14]. In 2000, Liu studied the three-dimensional three-component phononic crystal formed by a simple cubic lattice structure composed of shot put coated with viscoelastic soft material and buried in epoxy resin [15–18]. He found that the wavelength corresponding to the band gap frequency of the phononic crystal was much larger than the lattice size. Therefore, he proposed the local resonance band gap mechanism of phononic crystal, which marked a major breakthrough in the research of phononic crystal. The band gap of local resonant phononic crystal is caused by the resonance of local oscillator [19–21]. In 2001, Vasseur et al. designed two-dimensional solid phononic crystals using steel and epoxy resin. The existence of complete band gaps of elastic wave, shear wave and longitudinal wave is found and formalized. Before that, the band gap frequency of elastic wave is generally in the same order of magnitude as the corresponding wavelength size. The formation mechanism of this band gap is called Bragg scattering mechanism. However, phononic crystals based on Bragg scattering mechanism are sensitive to the size of materials. In order to obtain low-frequency band gap, it is necessary to design large macro size, which is not conducive to practical application. The proposal of local resonance mechanism is a major theoretical breakthrough in the study of phononic crystals [22–25]. On this basis, new concepts such as acoustic black hole, acoustic cloak and acoustic metamaterial are proposed [26].

Phononic crystals are further studied in this paper. A traditional local resonant phononic crystal is proposed. The vibration modes are carefully analyzed, including whether each mode produces band gap and the mechanism of band gap. The factors affecting the band gap start frequency and cut-off frequency are deeply studied.

2 Structural Model and Calculation Method

A two-dimensional three component phononic crystal with local resonance is designed. A two-dimensional three component phononic crystal with local resonance is designed. The structure is shown in Fig. 1a.The structure is wrapped by a large mass circular vibrator a with a coating B and embedded into the matrix C. No material is filled between the cladding layers B to form a cavity. The vibrator A is made of lead with high density, the coating B is made of silicone rubber with good elasticity, and the matrix C is made of epoxy resin. The detailed geometric parameters of the phononic crystal structure are shown in Table 1. and the material parameters are shown in Table 2. The control group is also set up, as shown in Fig. 1b. The energy band structure is compared by changing the coating position without changing other parameters.

The research on the mechanism and characteristics of phononic crystal band gap depends on the corresponding calculation methods. In this paper, the band gap is calculated by finite element method, and the finite element software COMSOL Multiphysics is used for analysis and calculation.

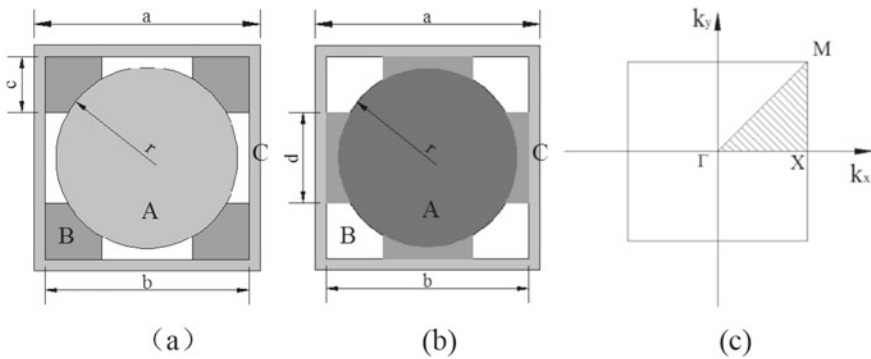The propagation equation of elastic wave in structure is

Fig. 1 Phononic crystal model (**a**) (**b**) and the first Brillouin zone (**c**)

Table 1 Structural parameters

a	b	c	d	r	mm
20	18	5	10	5	–

Table 2 Material parameters

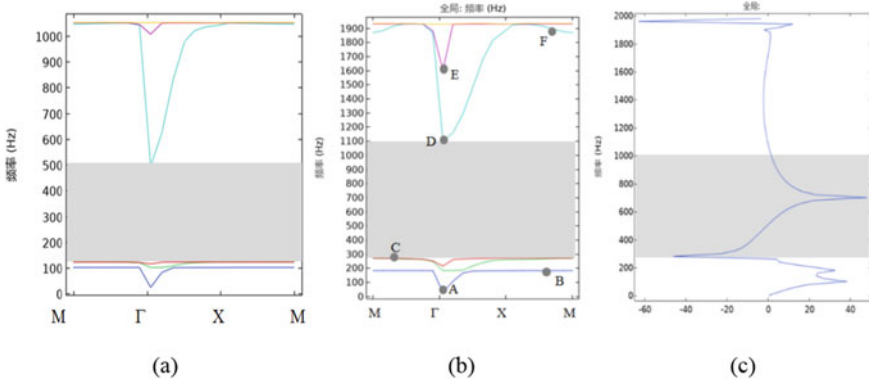| Material	$\rho(\mathrm{kg/m^3})$	$E(10^{10}\mathrm{Pa})$	υ
Lead	11,600	4.08e-10	0.369
Silicon rubber	1300	1.175e-5	0.469
Epoxy resin	1180	0.435	0.368

Fig. 2 The band gap diagram and transmission characteristic curve of Phonon Crystal

$$\sum_{i=1}^{3} \frac{\partial}{\partial x_j} \left(\sum_{l=1}^{3} \sum_{k=1}^{3} C_{ijkl} \frac{\partial u_k}{\partial x_l} \right) = \rho \frac{\partial^2 u_i}{\partial t^2} \tag{1}$$

ρ is density, u_i is the displacement, t is the time, C_{ijkl} is the elastic constant, x_j (j = 1, 2, 3) represents the coordinate variables x, y and z respectively.

According to Bloch's theorem, the displacement of the outer boundary of a single cell satisfies

$$u(r) = e^{i(kr)} u_k(r) \tag{2}$$

$\mathbf{r = (x, y, z)}$ is position vector, $\mathbf{k}$ is the wave vector of the first irreducible Brillouin region, $\mathbf{u_k(r)}$ is the node displacement field function with the same periodicity as the cell structure.

In the finite element software COMSOL Multiphysics, the boundary of the non Brillouin region M-Γ-X-M of the single cell structure of phononic crystals is scanned by using the solid mechanics module, as shown in Fig. 1c.It is solved to obtain the characteristic frequency, energy band structure and vibration mode diagram, as shown in Fig. 2.

3 Equivalent Simplified Model

The phononic crystal is a typical two-dimensional three component phononic crystal. In principle, it can be simplified into a relatively simple one-dimensional spring oscillator structure. The matrix is simplified as a thin straight beam with uniform mass. The scatterer is simplified as a mass block with fixed mass. The cladding is simplified as a spring with a certain elastic coefficient, as shown in Fig. 3.However, in
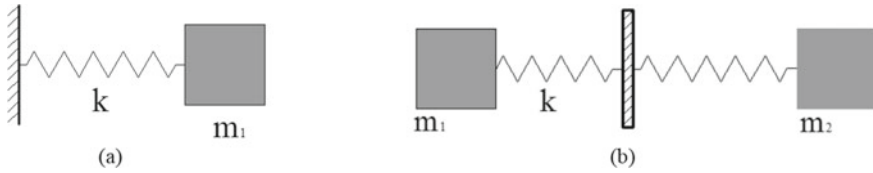
Fig. 3 Equivalent simplified model of local resonance band gap in phononic crystals

different modes, they represent different equivalent masses and elements, which also explains the reason why the band gap and modes of phononic crystals in the control group are different from those in the original group. For example, in the mode (d) of Fig. 4, the vibrator moves up and down. The upper and lower cladding layers vibrate up and down with the vibrator. At this time, it can be considered that the tension and compression of the upper and lower cladding layers play a major role. The cladding of these two parts can only be equivalent to the equivalent stiffness of the spring, and the mass of the cladding can not be ignored. According to the modal diagram, the mass of the cladding of the left and right parts can be distributed to the matrix, and the mass of the cladding translating up and down with the vibrator can be distributed to the vibrator. In Fig. 4c, the vibrator makes a translational movement to the right, and the left and right cladding layers are obviously stretched and compressed. These two parts are equivalent to the stiffness of the spring. In the vibration, according to
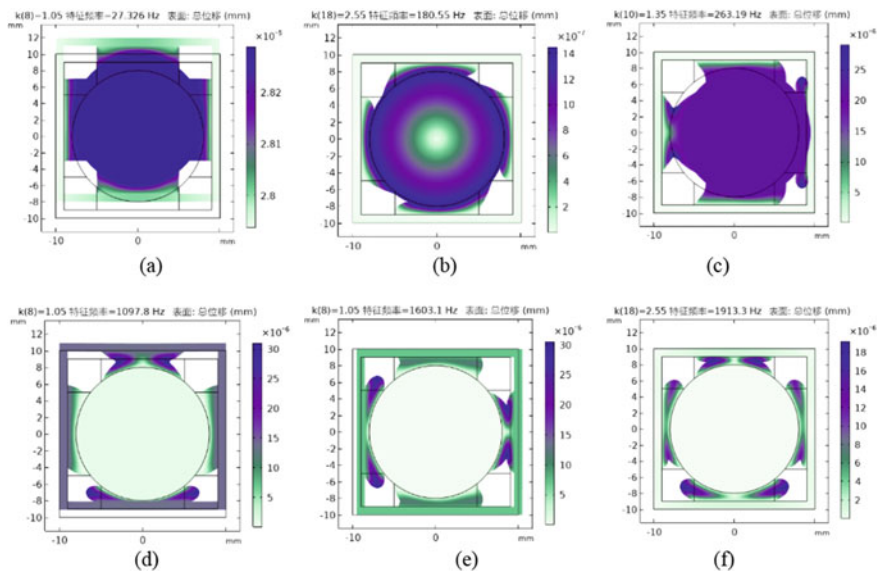

Fig. 4 Mode shapes of the key points at the upper and lower boundary of the band gap of the phononic crystal **a** A-point mode; **b** B-point mode; **c** C-point mode; **d** D-point mode; **e** E-point mode; **f** F-point mode

the values of the modal diagram, the four parts of the cladding are translating with the translation of the vibrator. Therefore, the mass of the four parts of the cladding can be distributed to the mass of the vibrator.

The initial frequency of the band gap is due to the overall translational motion of the oscillator. The vibration phases of adjacent oscillators are opposite. The cladding layer is stretched and compressed with the translation of the vibrator. The whole phononic crystal system reaches a dynamic equilibrium state. At the cut-off frequency of the band gap, under the elastic connection of the coating, the oscillator and the substrate form anti phase resonance. The vibration phase of all cells in phononic crystal is the same and reaches dynamic equilibrium. The starting frequency and cut-off frequency of the simplified model are

$$f_1 = \frac{1}{2\pi}\sqrt{\frac{k}{m_1}} \qquad f_2 = \frac{1}{2\pi}\sqrt{\frac{k(m_1 + m_2)}{m_1 m_2}} \qquad (3)$$

4 Analysis of Band Gap Characteristics and Band Gap Formation Mechanism of Phononic Crystals

Figure 3 shows the energy band structure. It can be seen from the figure that the band gap of the phononic crystal shows strong asymmetry. There is a sharp attenuation peak at the resonant frequency of the oscillator. The phononic crystal belongs to local resonance type. There is a wide full band gap. The frequency range is 268.7 ~ 1097.9 Hz. The center frequency is 683.3 Hz. The relative bandgap width is 121.4% (relative bandgap width = actual bandgap width/bandgap center frequency) [27, 28].The control group also produced a complete band gap. The frequency range is 125 ~ 498.5 Hz. The center frequency is 311.75 Hz. The relative band gap width is 119.8%.Compared with the control group, the upper and lower boundaries of the band gap in the original group decreased. Although the band gap is relatively low frequency, the actual band gap width decreases more. This is because the coating area of the coating layer and the vibrator decreases, and the change of the position leads to the change of the vibration mode. The parameters of the corresponding equivalent simplified model will also change at the characteristic frequency, resulting in the change of band gap. The original group has a wider range of applications because of its low frequency and large band gap. This paper focuses on the characteristics and impression factors of primitive phononic crystals.

In this paper, the vibration modal diagrams of six points of A, B, C, D, E and F in the first non Brillouin region are selected for analysis, as shown in Fig. 4. C is the starting frequency of the full band gap. Analyze the modal diagram of point C: the outer boundary matrix basically remains unchanged. The vibrator moves in translation, and the coating produces tensile and compressive deformation. At this point, the vibration resultant force generated by the vibrator as a scatterer is coupled with the long wave traveling wave in the matrix, resulting in a band gap. Point D

is the cut-off point of the band gap. The corresponding frequency is 1097.9 Hz. Analyze the vibration mode diagram of point D: the vibrator of point D basically has no displacement. The outer boundary box of the matrix moves in translation. The cladding layer then undergoes tensile and compressive elastic deformation. The long wave traveling wave of the matrix causes the vibration displacement of the matrix. The scatterer has no motion. It is not coupled with the long wave traveling wave in the matrix. So the band gap is cut off. Point F is the lower boundary of the directional band gap of the flat band part. Analyze the vibration mode diagram of point F: it can be seen that the cladding moves in translation. There is little change in the vibrator and matrix frame. There is no mutual coupling of waves. Therefore, the band gap is cut off. At point A, the matrix frame, cladding and vibrator move in translation under the action of elastic wave, without wave coupling, so there is no band gap. At point B, the vibrator has torsional motion. In this mode, the vibrator only produces torque on the base frame. There is no effect of plane internal force. The long wave traveling wave in the matrix is difficult to couple with the oscillator, so there is no band gap. The outer frame of the matrix at point E has a small translational motion. The vibrator hardly moves. The coating layer has a large translational movement and extrusion deformation with the vibrator and the matrix frame. Therefore, it is difficult to couple the long wave traveling wave in the oscillator and the matrix frame, and there is no band gap. From the above modal diagram analysis, it can be seen that the main reason for opening the band gap of the local resonant phononic crystal is the result of the coupling between the translational motion of the oscillator and the long wave traveling wave in the matrix frame. The existence of the coupling is the key factor to determine whether the band gap can be generated.

   The above band gap diagram and analysis of phononic crystals are based on the ideal case. The calculation results are obtained under the assumption of infinite periodic structure. Some of the smaller attenuation will increase cumulatively due to the increase of periodic structure. But in reality, there is no infinite period. The band gap can not be directly used as the physical properties of materials in applications. In practical application, the finite periodic structure and the transmission loss of elastic wave have more practical significance. Figure 5 is a periodic arrangement of 10 phononic crystals along the X direction. The finite element simulation calculation is carried out by COMSOL Multiphysics software. The transmission loss diagram is obtained. According to the band gap characteristics of phononic crystals, the transmission loss should be positive. The magnitude of the absolute value indicates the degree of attenuation. It is basically consistent with the energy band structure. The negative peak analysis in the middle produces resonance because the natural frequency of the material is consistent with the transmission frequency. Other frequencies are not in the band gap range, so the transmission loss is very small.

Fig. 5 Calculating the finite periodic structure of transmission spectrum

5   Influencing Factors of Band Gap

The density of materials and the geometric size of structure are the key factors affecting the band gap of phononic crystals. In this paper, the effects of material density and phononic crystal geometry on phononic crystal band gap will be investigated.

Several simulations are carried out without changing the geometry and size of phononic crystals. A density variation diagram as shown in Fig. 6 is obtained. From Fig. 6a, we can see that due to the increase of scatterer density, the upper boundary of the band gap decreases slightly at the beginning, and then tends to be flat and slightly changed. The lower boundary of the band gap decreases gradually with the increase of scatterer density, and the band gap width widens gradually and moves to low frequency. This shows that the density of the scatterer mainly affects the initial frequency of the band gap and has little effect on the cut-off frequency. This is consistent with the equivalent simplified model of the phononic crystal. The particle at the initial frequency of the band gap resonates under the action of the spring. The resonant frequency is determined by the resonant frequency of the translational motion of the local oscillator. The greater the density and mass of the vibrator, the less the stiffness of the spring. And the rate at which the vibrator mass m increases is greater than the rate at which the spring stiffness K decreases, so the resonant frequency decreases. Therefore, by increasing the density of the oscillator, the starting frequency of the band gap moves to the low frequency.

The density of the substrate mainly affects the cut-off frequency of phononic crystals. Figure 6b shows that the upper boundary of the band gap moves rapidly downward as the substrate density increases. The lower boundary does not change, and the band gap width decreases gradually. This shows that the density of the matrix directly affects the band gap cut-off frequency of phononic crystals, and has little effect on the initial frequency.

When considering the influence of geometric size on the band gap of phononic crystals, the lattice size a remains unchanged, that is, the size of the matrix frame
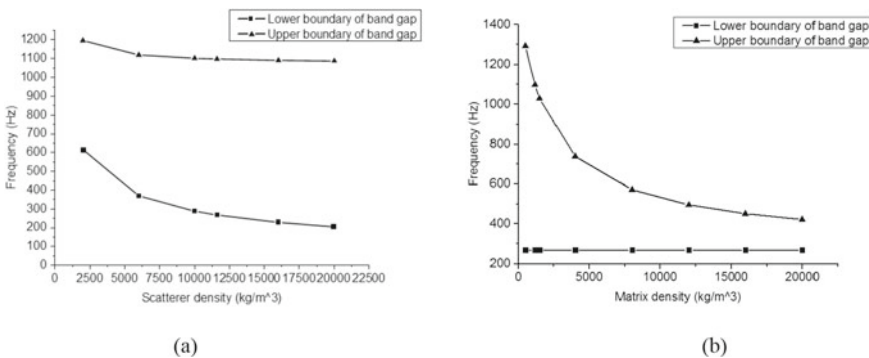

(a)   (b)

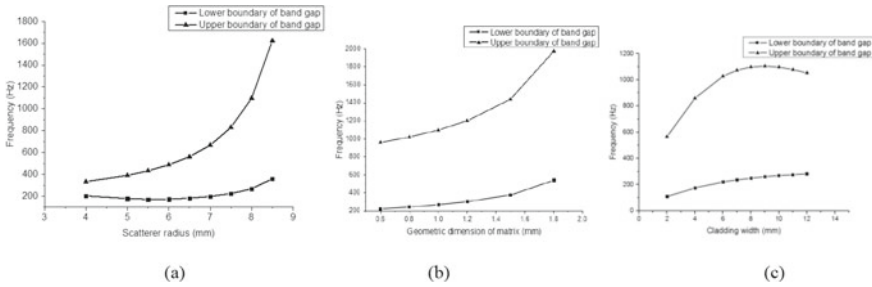**Fig. 6**   Effect of material parameters on band gap

Fig. 7 The influence of the change of geometry on the band gap

remains unchanged. First, change the radius of the scatterer. The frequency variation diagram of the upper and lower boundaries of the band gap is obtained, as shown in Fig. 7a. It can be seen that in the process of increasing the radius of the scatterer, the lower boundary of the band gap first decreases slightly and then increases steadily. And the change is not great relative to the upper boundary. The upper boundary of the band gap increases with the increase of the radius of the scatterer, and the change rate is large. The band gap width increases and moves to high frequency. The mechanism is that the filling ratio of the scatterer in the phononic crystal increases with the increase of the radius of the scatterer. It has a great influence on the upper boundary of the band gap. The larger the filling rate is, the greater the frequency of the upper boundary of the band gap is, and the smaller the influence on the lower boundary is.

The effect of substrate thickness on band gap is considered. Ensure that the lattice size a remains unchanged and increase the thickness of the matrix inward to obtain Fig. 7b. It can be seen from the figure that with the increase of substrate thickness, the upper and lower boundaries of band gap move to high frequency. The upper boundary rises rapidly. The width of the band gap increases gradually. The mechanism is that with the increase of matrix thickness, the mass increases, the relative mass of scatterers decreases, and the spring stiffness pair decreases. Therefore, the band gap moves to high frequency.

Consider the influence of cladding width. Change the width of the coating and keep it in the middle of the substrate. Figure 7c is obtained. As shown in the figure, the lower boundary of the band gap rises gradually, and the lower boundary rises first and then decreases. The peak value is reached when the cladding width is 9 mm. The band gap width first increases and then decreases. The mechanism can be explained by equivalent simplified model. According to formula (3), when the width of the cladding layer increases, the equivalent stiffness K of the spring increases and the initial frequency increases. In the cut-off frequency formula, the spring stiffness plays a major role at the beginning. The principle is the same as the starting frequency. Combined with the equivalent model and modal diagram, with the increasing width of the cladding layer, the mass distribution of the cladding layer in the equivalent model at point (c) of Fig. 4 is as mentioned above. At this time, the mass plays a

major role in the cut-off frequency formula of formula (3), so the cut-off frequency is reduced.

6   Conclusion

A typical local resonant phononic crystal structure is proposed in this paper. The equivalent simplified model is abstracted. The band gap characteristics and transmission loss of the phononic crystal with finite period structure are analyzed by finite element method. The band gap changes of phononic crystals under different material densities and different geometric structures are analyzed, and the following conclusions are drawn: The local resonant phononic crystal produces a wide complete band gap at 268.7 ~ 1097.9 Hz. The band gap width is 832.2 Hz. The main mechanism is the translational motion of the local resonance of the lead scatterer, which is coupled with the long wave traveling wave in the matrix. The transmission characteristics of phononic crystals with finite periodic structure also verify the ability to suppress waves in the band gap frequency range. By changing the material density and geometric parameters of phononic crystals, the band gap range of phononic crystals can be changed. The upper and lower boundaries of the band gap can be controlled purposefully.

References

1. Xu J, Cui H, Hong M (2021) Study on sound insulation performance of phononic crystal sandwich plate structure. Vibrat Impact 40(09):285–291
2. Zhang S, Xia CG, Fang N (2011) Broadband acoustic cloak for ultrasound waves. J Phys Rev Lett 106(2):024301
3. Cheng Y, Zhou C, Yuan BG (2015) Ultra-sparse metasurface for high reflection of low-frequency sound based on artificial Mie resonances. Nature Mater 14(10):1013–1019
4. Zhang H, Xiao Y, Wen JH (2016) Ultra-thin smart acoustic metasurface for low-frequency sound insulation. Appl Phys Lett 108(14):141902
5. Lu X,Qian S, Wei C, Li X, Zhu X (2021) Analysis of noise and vibration control of distribution transformer. Transformer 58(01):34–38
6. Qi P, Du J, Jiang J, Dong Y, Zhang J (2016) Sound insulation mechanism and characteristics of two-dimensional phononic crystals. J Silicate 44(10):1458–1464
7. Peng Z, Li C, Gao Y (2020) Study on low frequency band gap of two-dimensional open-hole locally resonant phononic crystals. Mech Design 037(04):73–77
8. Sun X, Yan Q, Guo X (2021) Analysis of low frequency band gap characteristics and structural improvement of single-sided cylindrical local resonance phononic crystals. J Intraocular Lenses 50(07):1378–1385

9. Guo X, Cui H, Hong M (2021) Study on vibration and noise reduction of local resonant phononic crystal plate. Ship Mech 25(04):509–516

10. Tan P, Sun X, Song T, Wen X, Liu X, Liu Z (2021) Simulation of band gap characteristics of surface wave phononic crystals with spherical composite columns. J Phys 70(14):243–252

11. Tang R, Pan C, Zheng W, He H, Tang J (2021) Study on propagation band gap characteristics of open ring phononic crystals. J Intraocular Lenses 50(03):428–434

12. Lu Y, Cao D, Shen Y, Chen X (2021) Study on defect state band gap and energy capture characteristics of local resonant phononic crystal plate. J Mech 53(04):1114–1123

13. Huang W, Yan S, Li X, Lu M, Li Y, Wang Z, Zhang Y, Wu S, Guo Y, Fan Q, Qian S, Zhang H, Sun Y, Lu C, Chen Y (2021) Discussion on terminology of acoustic metamaterials. Progr Mater China 40(01):1–6 + 20–21

14. Chen Q, Zhang B, Bai Y, Wang L (2021) Band gap characteristics of a new composite local resonance phononic crystal. Acoustic Technol 40(02):157–166

15. Guo X, Sun X, Zhu Y (2020) Study on band gap characteristics of two-dimensional square lattice tungsten silicone rubber phononic crystals. J Intraocular Crystals 49(09):1583–1589

16. Peng Z, Li C, Gao Y (2020) Study on vibration based on band gap characteristics of two-dimensional three component phononic crystals. J Chongqing Jiaotong Univ (Natural Science Edition) 39(08):134–138

17. Kang T, Sun X, Song T, Sun W, Liao T, Tan P (2020) Low frequency band gap characteristics and formation mechanism of two-dimensional hollow scatterer phononic crystal plate . Acta Acoustic A Sinica 45(04):601–608

18. Huang H, Ran J, Chen K, Zhang Y, Liu C (2020) Study on band gap characteristics of local resonant cylindrical shell phononic crystals. J Intraocular Crystals 49(06):1078–1082 + 1106

19. Wei S(2020) Study on the effect of edge bulge of phononic crystal scatterer on band gap . Beijing Univ Technol

20. Chen L, You S, Zhao X (2020) Study on sound insulation characteristics of two-dimensional phononic crystal thin plate. Materials Guide 34(S1):90–93

21. Chen Q (2020) Study on low frequency band gap characteristics and structural design of phononic crystals [D]. Ningxia University

22. Peng Z, LI C, Gao Y (2020) Study on low frequency band gap of two-dimensional perforated local resonant phononic crystals. Mech Design 37(04):73–77

23. Sun W, Wang T, Sun X, Kang T, Tan P, Liu Z (2019) Defect states and vibration energy recovery of a new two-dimensional three component piezoelectric phononic crystal plate. J Phys 68(23):145–153

24. Jia D, Ge Y, Yuan S, Sun H (2019) Dual band acoustic topological insulator based on honeycomb lattice phononic crystal. Acta Phys 68(22):235–241

25. Zhu J, Chen J, Chen J, Xu L, Wang X (2019) Dual channel filtering of cross arranged elliptical hole phononic crystal plates. Sci Bullet 64(26):2703–2709

26. Zhong L, Chen J, Liu J, Ling C, Fu X, Cai M, Huang Z (2019). Acoustic waveguide design based on line defect phononic crystals. Phys Exper 39(08):9–13

27. Shi G, Wang Z, Hunag S, Feng F, Zhang B (2019) Study on band gap of two-dimensional internal mass structure triangular lattice phononic crystals. J Yunnan Univ (Natural Science Edition) 41(03):545–550

28. He Z, Zhao J, Yao H, Jiang J, Zhang S, Chen X (2019) Band gap characteristics and vibration isolation performance of honeycomb phononic crystals. J Silicate 47(07):983–989