An Improved Event-Triggered Control Method Based on Consistency Algorithm in Heterogeneous AUV Swarm Under Communication Delay

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Abstract. The method for continuous time sampling suffers from communication interruptions in AUV (Autonomous Underwater Vehicle) swarm for underwater communication. In this paper, an improved event triggering mechanism is introduced to control the communication among heterogeneous AUV swarm. Each AUV only updates control input at its own event triggering time, without considering the triggering time of neighboring AUVs. In this method, the frequency of control signal updates has been reduced. Meanwhile, two consistency control algorithms for heterogeneous AUV swarm are studied with time delay and without time delay, respectively. The simulation results show that when time delay is reduced, the transient performance of the system will be increased and the steady-state error will be reduced. The heterogeneous AUV swarm under event-triggered control can significantly reduce the number of communications and the Zeno phenomenon will not occur. Consequently, the stability of the system has been improved.

Keywords: Heterogeneous AUV swarm · Event trigger control · Consistency control · Communication delay

1 Introduction

AUV is a cable-free, self-propelled underwater vehicle [1,2]. Due to small size and flexibility of AUV, an increasingly important position in future military activities will be occupied [3]. Because it is difficult for a single AUV to perform complex tasks, multiple AUVs are usually utilized to form a swarm to coordinate tasks [4]. Generally speaking, each AUV usually comes from different manufacturers, so types and platforms of AUVs are different. The formation of a swarm often reflects a heterogeneous type, so there are difficulties in information interaction [5,6]. While the amount of information interaction can be reduced by consistency algorithm [7]. In underwater environment, hydroacoustic communication is widely used among AUVs. Compared with time-sampled communication method, resource consumption can be reduced by event-triggered method

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[8]. Therefore, a formation control method based on event-triggered mechanism for heterogeneous AUV swarm is proposed.

In [9], the event-triggered tracking control problem of fully driven AUVs in the vertical plane was studied. Meanwhile, a reinforcement learning method was introduced to optimize the long-term tracking performance. In [10], a fixed-time leader-following formation control method for a set of AUVs with event-triggered acoustic communications was investigated. In [11], a distributed event-driven adaptive formation control strategy was proposed to achieve three-dimensional formation tracking of AUV swarm. In [12], asynchronous and synchronous communication strategies were proposed. Moreover, the consistency of the algorithm with limited information exchange and distributed communication delays was proved. In [13], the event-triggered consensus problem of multiagent systems with input time delay is investigated, which could be applied to AUV swarm. In [14], the problem of fixed-time event-triggered formation control for multi-AUV systems with external uncertainty was investigated. A distributed control strategy was considered, which can realize the control of arbitrary initial states of multi-AUVs. While the time delay was not taken into account. In [15], an event-triggered integral sliding-mode fixed-time control method was proposed to solve the trajectory tracking problem of AUVs with interference without considering communication delay. In [16], the problem of event-triggered distributed adaptive bipartite consensus control for multi-AUV systems with a fixed topology was studied. Although the competition among AUVs was considered, the heterogeneity of swarm was not taken into account.

In this paper, a distributed event-triggered control algorithm is proposed for heterogeneous AUV swarm under communication delay. In this algorithm, each AUV updates control input at its own trigger moment without considering neighboring AUVs. Consumption of communication resources and the update frequency of control signals is reduced through the algorithm. The consistency stability of the algorithm is proved by Lyapunov theory. Meanwhile, Zeno phenomenon will not occur. Finally, the correctness and the effectiveness of the algorithm are demonstrated by numerical simulations.

2 Problem Description

2.1 Multi-AUV Swarm

The model of AUV is studied in a right-angle coordinate system [16]. Suppose that AUVi can detect its own position and angle from neighboring AUVj. E is defined as the fixed coordinate system while B is defined as the motion coordinate system. Ignoring sea surface interference, mathematical model of AUV can be expressed as Eq. (1).

$$\begin{cases} \dot{\eta}_i = R_i(\psi_i)v_i \\ M_i\dot{v}_i + C_i(v_i)v_i + D_iv_i = \tau_i \end{cases} \tag{1}$$

where $\eta_i = [x_i, y_i, \psi_i]^T$ denotes the position of AUVi, $\upsilon_i = [u_i, \upsilon_i, r_i]^T$ denotes the velocity of AUVi, $\tau_i = [\tau_{1i}, \tau_{2i}, \tau_{3i}]^T$ denotes the motion thrust, M_i denotes inertia matrix, C_i denotes koch force matrix, D_i denotes damping matrix. $R_i(\psi_i)$ denotes coefficient matrix.

$$R_i(\psi_i) = \begin{bmatrix} \cos(\psi_i) & -\sin(\psi_i) & 0 \\ \sin(\psi_i) & \cos(\psi_i) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{2}$$

In this section, a distributed controller is designed under a fixed undirected topology graph, which make all AUVs maintain the same position, attitude, and velocity asymptotically.

$$\begin{array}{l} \lim\limits_{t\to\infty} \eta_i - \eta_j = 0 \\ \lim\limits_{t\to\infty} \upsilon_i - \upsilon_j = 0 \end{array} \tag{3}$$

Suppose $R_i(\psi_i)\upsilon_i = \gamma_i$. Due to ${R_i}^{-1}(\psi_i) = {R_i}^T(\psi_i)$, $({R_i}^{-1})^T(\psi_i) = {R_i}^{-T}(\psi_i)$, $\upsilon_i = R_i^{-1}(\psi_i)\gamma_i$, Eq. (1) can be written as

$$\begin{cases} \dot{\eta}_i = \gamma_i \\ M_i R_i^{-1}(\psi_i)(\dot{\gamma}_i - \dot{R}_i(\psi_i)R_i^{-1}(\psi_i)\gamma_i) \\ = -C_i(R_i^{-1}(\psi_i)\gamma_i)R_i^{-1}(\psi_i)\gamma_i - D_i R_i^{-1}(\psi_i)\gamma_i + \tau_i \end{cases} \tag{4}$$

Suppose

$$Q_i = {R_i}^{-T}(\psi_i) M_i R_i^{-1}(\psi_i)$$

$$G_i = {R_i}^{-T}(\psi_i)[C_i(R_i^{-1}(\psi_i)\dot{\eta}_i) - M_i R_i^{-1}(\psi_i)\dot{R}_i(\psi_i) - D_i]R_i^{-1}(\psi_i)$$

$$p_{ij} = \eta_i - \eta_j$$

Equation (4) can be written as

$$\begin{cases} \dot{p}_{ij} = \gamma_i - \gamma_j \\ \dot{\gamma}_i = -Q_i^{-1}(G\gamma_i - \tau_{qi}) \end{cases} \tag{5}$$

where $\tau_{qi} = {R_i}^{-T}(\psi_i)\tau_i$ denotes control input.

An event-triggered consistent controller is designed for multi-AUV swarm in Eq. (6).

$$\tau_{qi}(t) = -\sum_{j\in N_i} a_{ij}(\eta_i(t_k^i) - \eta_j(t_k^j)) - \omega\gamma_i(t_k^i), t \in [t_k, t_{k+1}] \tag{6}$$

where $\eta_i(t_k^i)$, $\gamma_i(t_k^i)$ denotes the state information sampled by AUVi at the moment of t_k^i. t_k^i denotes the k trigger moment of AUVi, $k \in \{0, 1, 2, ...\}$. $\eta_j(t_k^j)$ denotes the sampled value of latest trigger moment of AUVj. Position error and velocity error are expressed as Eq. (7).

$$\begin{aligned} e_{\eta i}(t) &= \eta_i(t_k^i) - \eta_i(t) \\ e_{\gamma i}(t) &= \gamma_i(t_k^i) - \gamma_i(t) \end{aligned} \tag{7}$$

Furthermore, Eq. (5) can be written as

$$\begin{cases} \dot{p}_{ij} = \gamma_i - \gamma_j \\ \dot{\gamma}_i = -Q_i^{-1}(G\gamma_i + \sum\limits_{j \in N_i} a_{ij}(\eta_i(t_k^i) - \eta_j(t_k^j)) + \omega\gamma_i(t_k^i) \\ \quad + \sum\limits_{j \in N_i} a_{ij}((e_{\eta i}(t) - e_{\gamma i}(t)) + \omega e_{\gamma i}(t)) \end{cases} \tag{8}$$

Event triggering function is designed as Eq. (9).

$$f_i(t) = \|e_i(t)\| - \zeta_i \|\gamma_i(t)\| - \delta_i(t) \tag{9}$$

where $e_i(t) = [e_{\eta i}^T, e_{\gamma i}^T]^T$ denotes state error, $\zeta_i = \sqrt{\frac{c\alpha_i\beta_i}{2\theta_i}}$, where $0 < \alpha_i < 1$, $\beta_i = \frac{\omega(2-c)}{2} - c|N_i| > 0$, $\theta_i = \max(|N_i|, \frac{\omega}{2}) > 0$. $\delta_i(t) = \kappa_i e^{-\varepsilon(t-t_0)}$ denotes compensation function, where $\kappa_i > 0$, $0 < \varepsilon < 1$. The sequence of trigger moment is shown as Eq. (10).

$$t_{k+1}^i = \inf\{t : t > t_k^i, f_i(t) > 0\} \tag{10}$$

when $f_i(t) > 0$, the control protocol $\tau_{qi}(t)$ of AUVi updates once. Meanwhile, the position state $\eta_i(t_k^i)$ is passed to AUVj. At the same time, the state value updates once: $\eta_i(t_k^i) = \eta_i(t)$, $\gamma_i(t_k^i) = \gamma_i(t)$. The error $e_i(t)$ is set to zero. After AUVj is triggered in $t \in [t_k, t_{k+1}]$, the corresponding sampling information is received by AUVi. The control protocol of AUVi updates. Therefore, the measurement error of AUVi should be known while the state information of AUVj is not necessary. communication burden and bandwidth resource are reduced. by the improved event-trigger method. The system stability is demonstrated below.

Suppose a Lyapunov function:

$$V = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}p_{ij}^T p_{ij} + \frac{1}{2}\sum_{i=1}^{n} \gamma_i^T Q_i \gamma_i \tag{11}$$

where Q_i is a symmetric and positive definite matrix. $V > 0$ when p_{ij} and γ_i are not simultaneously 0. The derivative of V is

$$\begin{aligned}
\dot{V} &= \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij} p_{ij}^T(\gamma_i - \gamma_j) + \sum_{i=1}^{n} {\gamma_i}^T Q_i \dot{\gamma}_i + \frac{1}{2}{\gamma_i}^T \dot{Q}_i \gamma_i \\
&= \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij} p_{ij}^T(\gamma_i - \gamma_j) - \sum_{i=1}^{n} {\gamma_i}^T \left(G_i\gamma_i + \sum_{j\in N_i} a_{ij}\left(\eta_i(t) - \eta_j(t)\right)\right) \\
&\quad - \sum_{i=1}^{n} {\gamma_i}^T \left(\omega\gamma_i(t) + \sum_{j\in N_i} a_{ij}\left((e_{\eta i}(t) - e_{\gamma i}(t)) + \omega e_{\gamma i}(t)\right)\right) \\
&\quad + \frac{1}{2}\sum_{i=1}^{n} {\gamma_i}^T \dot{Q}_i \gamma_i \\
&\le \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij} p_{ij}^T(\gamma_i - \gamma_j) - \sum_{i=1}^{n} {\gamma_i}^T \left(G_i\gamma_i + \sum_{j\in N_i} a_{ij}(\eta_i(t) - \eta_j(t))\right) \\
&\quad - \sum_{i=1}^{n} {\gamma_i}^T \left(\omega\gamma_i(t) + \sum_{j\in N_i} a_{ij}((e_{\eta i}(t) - e_{\gamma i}(t)) + \omega e_{\gamma i}(t))\right) \\
&\le \sum_{i=1}^{n} \gamma_i \sum_{j=1}^{n} a_{ij} p_{ij}^T - \sum_{i=1}^{n} {\gamma_i}^T \sum_{j=1}^{n} a_{ij}(\eta_i(t) - \eta_j(t)) \\
&\quad - \sum_{i=1}^{n} \omega {\gamma_i}^T \gamma_i - \sum_{i=1}^{n} {\gamma_i}^T \sum_{j=1}^{n} a_{ij}((e_{\eta i}(t) - e_{\gamma i}(t)) - \sum_{i=1}^{n} \omega {\gamma_i}^T e_{\gamma i}(t) \\
&\le -\sum_{i=1}^{n} \omega {\gamma_i}^T \gamma_i - \sum_{i=1}^{n} {\gamma_i}^T \sum_{j=1}^{n} a_{ij} e_{\eta i}(t) \\
&\quad + \sum_{i=1}^{n} {\gamma_i}^T \sum_{j=1}^{n} a_{ij} e_{\eta j}(t) - \sum_{i=1}^{n} \omega {\gamma_i}^T e_{\gamma i}(t)
\end{aligned} \tag{12}$$

Due to

$$\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}\left\| e_{\eta i}(t)\right\| = \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}\left\| e_{\eta j}(t)\right\| \tag{13}$$

where $a_{ij} = 1$ when AUVi maintains communication with AUVj , $a_{ij} = 0$ when AUVi disconnect from AUVj. According to $xy \le \frac{c}{2}x^2 + \frac{1}{2c}y^2$ and $0 < c <$

$\frac{2\omega}{(2|N_i|+\omega)}$, the upper bound of $\dot{V}$ is obtained as Eq. (14).

$$\begin{aligned}\dot{V} \le & -\sum_{i=1}^{n}\omega{\gamma_i}^T\gamma_i + 2\sum_{i=1}^{n}\left\|{\gamma_i}^T\right\|\sum_{j=1}^{n}a_{ij}\left\|e_{\eta i}(t)\right\| - \sum_{i=1}^{n}\omega{\gamma_i}^T e_{\gamma i}(t)\\ \le & -\sum_{i=1}^{n}\omega{\gamma_i}^T\gamma_i + \sum_{i=1}^{n}|N_i|\left(c{\gamma_i}^T\gamma + \frac{1}{c}{e_{\eta i}}^T(t)e_{\eta i}(t)\right)\\ & + \sum_{i=1}^{n}\omega\left(\frac{c}{2}{\gamma_i}^T\gamma_i + \frac{1}{2c}{e_{\gamma i}}^T(t)e_{\gamma i}(t)\right)\\ \le & \sum_{i=1}^{n}\left(-\omega + c\left(|N_i| + \frac{\omega}{2}\right)\right){\gamma_i}^T\gamma_i\\ & + \sum_{i=1}^{n}\frac{|N_i|}{c}{e_{\eta i}}^T(t)e_{\eta i}(t) + \sum_{i=1}^{n}\frac{\omega}{2c}{e_{\gamma i}}^T(t)e_{\gamma i}(t)\end{aligned} \tag{14}$$

When $f_i(t) \le 0$,

$$\left\|e_i(t)\right\|^2 \le 2{\zeta_i}^2\left\|\gamma_i(t)\right\|^2 + 2{\delta_i}^2(t) \tag{15}$$

Consequently, Eq. (13) can be written as

$$\begin{aligned}\dot{V} \le & -\sum_{i=1}^{n}\beta_i\left\|\gamma_i\right\|^2 + \sum_{i=1}^{n}\frac{|N_i|}{c}\left\|e_{\eta i}\right\|^2 + \sum_{i=1}^{n}\frac{\omega}{2c}\left\|e_{\gamma i}\right\|^2\\ \le & -\sum_{i=1}^{n}\beta_i\left\|\gamma_i\right\|^2 + \sum_{i=1}^{n}\frac{\theta_i}{c}\left\|e\right\|^2\\ \le & -\sum_{i=1}^{n}(1-\alpha_i)\beta_i\left\|\gamma_i\right\|^2 + \sum_{i=1}^{n}\frac{2\theta_i}{c}{\kappa_i}^2 e^{-2\varepsilon(t-t_0)}\end{aligned} \tag{16}$$

Integrating over Eq. (16):

$$\sum_{i=1}^{n}(1-\alpha_i)\beta_i\int_0^t\left\|\gamma_i(\partial)\right\|^2 d\partial \le V(0) + \sum_{i=1}^{n}\frac{\theta_i\kappa_i^2}{c\varepsilon_i} \tag{17}$$

In Eq. (17), V and $\int_0^t\left\|\gamma_i(\partial)\right\|^2 d\partial$ is bounded. Equation (11) shows that both position error p_{ij} and velocity γ_i are bounded, then $\|p_{ij}\| \le p_{\max}$ and $\|\gamma_i(t)\| \le \gamma_{\max}$. According to $\dot{\gamma}_i = -Q_i^{-1}(G\gamma_i + \sum\limits_{j\in N_i}a_{ij}(\eta_i(t_k^i) - \eta_j(t_k^j)) + \omega\gamma_i(t_k^i))$, $\|\dot{\gamma}_i\| \le \left\|Q_i^{-1}\right\|((\|G_i\|+1)\gamma_{\max} + \sum\limits_{j\in N_i}a_{ij}p_{\max}$. Due to Q_i and G_i are bounded, $\dot{\gamma}_i$ is bounded. Therefore, $\lim\limits_{t\to\infty}\gamma_i = 0$, $\dot{\gamma}_i \to 0$, $\eta_i(t_k^i) - \eta_j(t_k^j) \to 0$ when $t \to \infty$. Consequently, multi-AUV swarm can remain consistent and stable.

Lemma 1 is given to show that Zeno phenomenon does not exist in multi-AUV swarm based on event-triggered mechanisms.

Lemma 1. *In a directionless communication topology, when trigger function satisfies* $f_i(t) \leq 0$ *and* $0 < c < \frac{2\omega}{(2|N_i|+\omega)}$, *the neighboring trigger moment time interval satisfies* $\Delta t_k^i > 0$ *[2].*

2.2 Heterogeneous AUV Swarm

A formation control problem for a heterogeneous AUV swarm is studied in this section. When The swarm consists of N AUVs, the dynamic equation of AUVi is given by [17], which is shown as Eq. (18).

$$\dot{x}_i(t) = Ax_i(t) + Bu_{x,i}(t) + \theta_i^*, i = 1, 2, ..., N \tag{18}$$

where $x_i(t) \in R^n$ denotes state variables, $u_{x,i}(t) \in R^n$ denotes control inputs, $\theta_i^* \in R^n$ denotes bounded vectors of AUVi. $A \in R^{n\times n}$ and $B \in R^{n\times n}$ are system parameters. Each disparate $\theta_i^* \in R^n$ is characteristic of the heterogeneous AUV swarm.

$$rank([\theta_i^*, B]) = rank(B), i = 1, 2, \ldots, N \tag{19}$$

$$B\theta_i = \theta_i^*, i = 1, 2, \ldots, N \tag{20}$$

Then Eq. (18) can be written as

$$\dot{x}_i(t) = Ax_i(t) + B(u_{x,i}(t) + \theta_i), i = 1, 2, \ldots, N \tag{21}$$

Event trigger policy is activated when the system state function or measurement data exceeds a certain threshold value. Event trigger time series is composed of a strictly increasing sequence $\{t_k^i, k \geq 0\}$. The control input $u_{x,i}(t)$ keeps constant at $t \in [t_k^i, t_{k+1}^i)$.

$$u_{x,i}(t) = z_i(t) + K \sum_{j\in N_i} a_{ij}(\hat{x}_j(t) - \hat{x}_i(t)), t \in [t_k^i, t_{k+1}^i) \tag{22}$$

where $K \in R^{n\times n}$ is control gain matrix, $\hat{x}_i(t) = e^{A(t-t_k^i)}x_i(t_k^i)$ is estimated value at t. $E_{x,i}(t) = \hat{x}_i(t) - x_i(t)$ denotes corresponding estimation error. a_{ij} is the i,j element of the adjacency matrix A in communication network topology. $z_i(t)$ is subsequent compensation variable. In order to compensate for the heterogeneity in AUV swarm, variable $z_i(t)$ is introduced. $z_i(t)$ updates when event trigger condition is activated.

$$\dot{z}_i(t) = u_{z,i}(t) = H \sum_{j\in N_i} a_{ij}(\hat{z}_j(t) - \hat{z}_i(t)) \tag{23}$$

where $z_i(t) \in R^n$, $\hat{z}_i(t) = \theta_i + z_i(t_k^i)$. $H \in R^{n\times n}$ is the corresponding control gain matrix. $E_{z,i}(t) = z_i(t_k^i) - z_i(t)$ is the measurement error.

3 Controller Design

3.1 Heterogeneous AUV Swarm Consistency Control Algorithm Based on Event-Triggered

In this section, consistency problem of heterogeneous AUV swarm under fixed topology is studied. A distributed consistency control algorithm based on event-triggered is given as [18].

Suppose event trigger time sequence $\{t_k^i\} = \inf_{k\in^+}\{t > t_{k-1}^i : f_i(t) > 0\}$. The corresponding event trigger function is shown as Eq. (24).

$$f_i(t) = \|e_i(t)\| - c_1 \left\| \sum_{j\in N_i} a_{ij}(\hat{x}_j(t) - \hat{x}_i(t)) + \sum_{j\in N_i} a_{ij}(\hat{z}_j(t) - \hat{z}_i(t)) \right\| - c_2 e^{-\alpha t} \tag{24}$$

where $e_i(t) = \left[E_{x,i}^T(t), E_{z,i}^T(t)\right]^T$.

When a known constant scalar $\lambda > 0$ and control gains $K = M_1\bar{P}_1^{-1}$, $H = M_2\bar{P}_2^{-1}$, there exist positive definite matrices $\bar{P} = diag\{\bar{P}_1, \bar{P}_2\}$ and $\bar{M} = diag\{M_1, M_2\}$. The stability of the system can be achieved when Eq. (25) is satisfied.

$$\begin{cases} 0 < \alpha < \frac{\lambda}{2} \\ 0 < c_1 < \min\left\{ \sqrt{\frac{1}{\lambda_{\max}(\bar{P}^{-1})}}, \frac{1}{N\|L\otimes I_{2n}\|} \right\} \\ 0 < c_2 < \sqrt{\frac{1}{\lambda_{\max}(\bar{P}^{-1})}} \end{cases} \tag{25}$$

Proof. A Lyapunov function is chosen for a fixed topology [19].

$$V(\delta(t)) = \delta(t)^T \mathrm{P} \delta(t) \tag{26}$$

where $\mathrm{P} = I_N \otimes P$, $P = diag\{P_1, P_2\}$. Taking the derivative of $V(\delta(t))$:

$$\begin{aligned} \dot{V}(\delta(t)) &= 2\delta(t)^T \mathrm{P}[\hat{A}\delta(t) + \hat{B}\delta(t) + \hat{B}e(t)] \\ &\le \tilde{\eta}(t)^T \prod_1 \tilde{\eta}(t) - \lambda\delta(t)^T \mathrm{P}\delta(t) + e(t)^T \mathrm{P} e(t) \\ &\le \tilde{\eta}(t)^T \prod_1 \tilde{\eta}(t) - \lambda\delta(t)^T \mathrm{P}\delta(t) + \lambda_{\max}(P) e(t)^T e(t) \end{aligned} \tag{27}$$

According to Eq. (28)

$$\begin{aligned} \|e_i(t)\| &\le c_1 \left\| \sum_{j\in N_i} a_{ij}(\hat{x}_j(t) - \hat{x}_i(t)) + \sum_{j\in N_i} a_{ij}(\hat{z}_j(t) - \hat{z}_i(t)) \right\| + c_2 e^{-\alpha t} \\ &\le c_1 \|(L\otimes I_{2n})\delta(t) + (L\otimes I_{2n})e(t)\| + c_2 e^{-\alpha t} \end{aligned} \tag{28}$$

After deflating and accumulating, Eq. 28 can be written as

$$\begin{aligned}\sum_{i=1}^{N}\|e_i(t)\|^2 \le\ & 4N{c_1}^2\|(L\otimes I_{2n})\delta(t)\|^2 \\ & + 4N{c_1}^2\|(L\otimes I_{2n})e(t)\|^2 \\ & + 2N{c_2}^2e^{-2\alpha t}\end{aligned} \tag{29}$$

Due to $c_1^2c_2^2 \in (0, \lambda_{\max}(P))$, Eq. (27) can be written as

$$\begin{aligned}\dot{V}(\delta(t)) &\le \tilde{\eta}(t)^T\prod_2\tilde{\eta}(t) - \lambda\delta(t)^T\mathrm{P}\delta(t) + 2Ne^{-2\alpha t} \\ &\le -\lambda V(\delta(t)) + 2Ne^{-2\alpha t}\end{aligned} \tag{30}$$

Integrating over Eq. (30):

$$\begin{aligned}V(\delta(t)) &\le V(\delta(t_0))e^{-\lambda(t-t_0)} + 2N\int_{t_0}^{t}e^{-2\alpha t}ds \\ &= V(\delta(t_0))e^{-\lambda(t-t_0)} + \frac{2N}{\lambda-2\alpha}e^{-2\alpha t} - \frac{2Ne^{(\lambda-2\alpha)t_0}}{\lambda-2\alpha}e^{-\lambda t} \\ &\le V(\delta(t_0))e^{-\lambda(t-t_0)} + \frac{2N}{\lambda-2\alpha}e^{-2\alpha t}\end{aligned} \tag{31}$$

Suppose $q_1 = \lambda_{\min}(P)$, $q_2 = \lambda_{\max}(P)$

$$q_1\|\delta(t)\|^2 \le V(\delta(t)) \le q_2\|\delta(t)\|^2 \tag{32}$$

$$\|\delta(t)\|^2 < \frac{q_2\|\delta(t)\|^2}{q_1}e^{\lambda(t-t_0)} + \frac{2N}{q_1(\lambda-2\alpha)}e^{-2\alpha t} \tag{33}$$

In summary, the heterogeneous AUV swarm can reach agreement with the control gain of $K = M_1\bar{P}_1^{-1}$, $H = M_2\bar{P}_2^{-1}$. According to Lemma 1, Zeno phenomenon will not occur.

In this section, a consistency control algorithm based on event-triggered mechanism is proposed for heterogeneous AUV swarm. Due to the adoption of estimation-based trigger conditions, the designed algorithm can effectively extend the release time and reduce the number of event trigger.

3.2 Heterogeneous AUV Group Consistency Algorithm Based on Event-Triggered Control Under Communication Time Delay

In this section, a distributed event-triggered controller is implemented using a state feedback approach. When communication delay exists, the output of all AUVs is progressively consistent. Meanwhile, Zeno phenomenon will not occur. The system dynamic equation is shown as Eq. (21).

The event trigger controller under communication time delay is shown as Eq. (34).

$$\begin{cases} \dot{\eta}_i(t) = S\eta_i(t) - \sum_{j\in N_i} a_{ij}\left(e^{S(t-t^i_{k_i})}\eta_i(t^i_{k_i}) - e^{S(t-t^j_{k_j})}\eta_j(t^j_{k_j})\right) \\ u_i(t) = K_i\left(x_i(t-\tau_i(t)) - \prod_i \eta_i(t)\right) + \eta_i(t) - \alpha_i \mathrm{sgn}(x_i - \prod_i \eta_i) \end{cases} \tag{34}$$

where $i, j = 1, \ldots, N$, $\tau_i(t) > 0$ denotes the underwater communication time delay, $t^i_{k_i}$ is the most recent trigger moment of AUVi, $k_i = 1, 2, \ldots, \eta_i(t^i_{k_i})$ is the controller state of the last broadcast of AUVi, α_i is the normal number gain, sgn is the sign function. Define a measurement error based on the controller state:

$$e_i(t) = e^{S(t-t^i_{k_i})}\eta_i(t^i_{k_i}) - \eta_i(t) \tag{35}$$

The trigger function of AUVi is given by Eq. (36).

$$f_i(t, e_i(t)) = \|e_i(t)\| - ce^{-\alpha t} \tag{36}$$

where $c > 0$, α is a normal number to be determined. When $f_i(t, e_i(t)) > 0$, AUVi triggers an event. Simultaneously, AUVi updates its controller. At the same time, the measurement error is set to zero. If the error is less than a given threshold, no communication between intelligences is required until the next event is triggered.

Suppose t^* denotes the most recent trigger moment. The tracking error $\delta_i(t) = x_i(t) - \prod_i \eta_i(t)$ is defined. When $c > 0$, $0 < \alpha < -\max_i \mathrm{Re}(\lambda_i)$, $\alpha_i > \max_i \|f_i\|$, system consistency can be achieved. Moreover, Zeno phenomenon will not occur based on Lemma 1.

4 Simulation and Analysis

4.1 Simulation of Heterogeneous AUV Cluster Consistency Algorithm under Event Trigger Control

In this section, a simulation is conducted for the consistency control algorithm of heterogeneous AUV swarm under event-triggered control proposed in Sect. 3.1. Suppose a heterogeneous AUV swarm consists of a navigator and four followers, where $x_i(t) = [s_i(t), v_i(t), a_i(t)]^T$ denotes state variables of AUVi, $s_i(t)$ denotes distance, $v_i(t)$ denotes velocity, and $a_i(t)$ denotes acceleration. Suppose that the initial position of navigator is randomly distributed between [−10, 10]. The initial position of each follower is randomly distributed in the interval [−20, 20]. The initial combined velocity is 5 m/s. The initial values of other state variables are set to 0. The expected formation is designed as a parallel formation with the leader as the center and the four followers evenly distributed around it. The heterogeneity of the swarm is ensured by setting the constant vector of each AUV with $\theta_1^* = [0, -0.0325, 0]^T$, $\theta_2^* = [0, -0.0319, 0]^T$, $\theta_3^* = [0, -0.0428, 0]^T$, $\theta_4^* = [0, -0.0258, 0]^T$, $\theta_5^* = [0, -0.0637, 0]^T$. The control input of leader AUV is

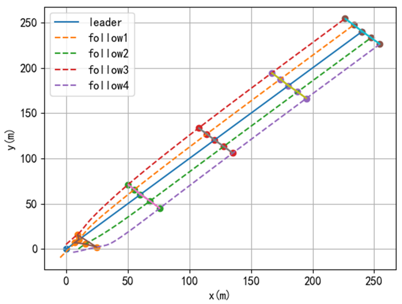

Fig. 1. Heterogeneous AUV group formation (2-D)

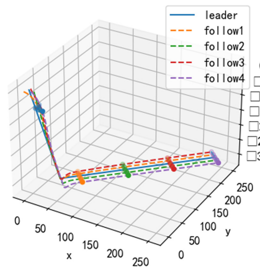

Fig. 2. Heterogeneous AUV group formation (3-D)

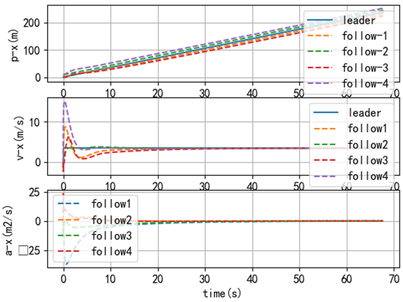

Fig. 3. Position, velocity and acceleration state quantities in x-direction without communication time

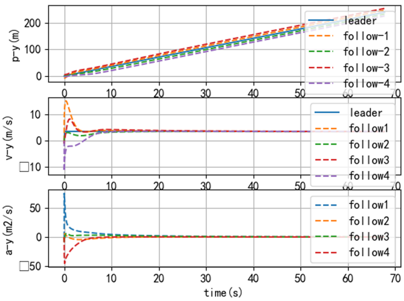

Fig. 4. Position, velocity and acceleration state quantities in y-direction without communication time

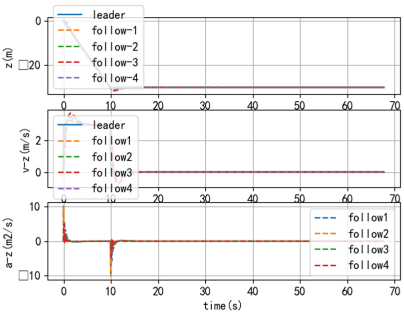

Fig. 5. Position, velocity and acceleration state quantities in z-direction without communication time delay

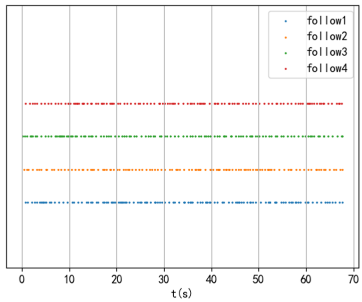

Fig. 6. Event trigger time without communication time delay

set to $u_0(t) = t \sin t$ and the trigger interval is set to $0.1s$. The communication delay is not considered. According to Eq. (22), the simulation results are shown from Fig. 1 to Fig. 6.

The formation process of the heterogeneous AUV swarm can be seen in Fig. 1 and Fig. 2. The position and velocity of the follower can reach the leader, and the acceleration of the follower eventually converges to zero as shown in Fig. 3, Fig. 4, and Fig. 5. The event trigger moments are shown in Fig. 6. It can be seen that the proposed control method can significantly reduce the number of communications. Meanwhile, there is no Zeno behavior.

The simulation results show that the algorithm proposed in Sect. 3.1 updates the control signal of each follower AUVs at its own event-triggered moment, which can effectively reduce a large amount of communication information among AUVs. Moreover, the number of event triggering is reduced and the release time of event triggering is extended by using estimation-based trigger conditions. The speed and position state convergence is better. Consistency can be obtained faster.

4.2 Simulation of Heterogeneous AUV Cluster Consistency Algorithm with Time Delay Under Event Trigger Control

In this section, the simulation is carried out for the algorithm proposed in Sect. 3.2. In order to conduct the comparative analysis, one leader and four followers are also considered to form a heterogeneous AUV group. The rest of the state initial conditions are the same as Sect. 4.1. The following simulation experiments are designed with time delays of 0.1 s and 0.5 s, respectively. The simulation results for a time delay of 0.5 s are shown in Fig. 7, 8, 9, 8, 9, 10, 11 and Fig. 12. The simulation results for a time delay of 0.5 s are shown in Fig. 13, 14, 15, 16, 17 and Fig. 18.

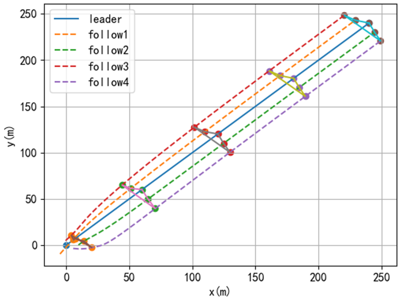

Fig. 7. Heterogeneous AUV group formation (2-D) with communication time delay = 0.5

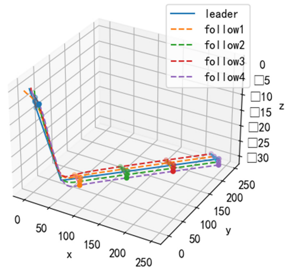

Fig. 8. Heterogeneous AUV group formation (3-D) with communication time delay = 0.5

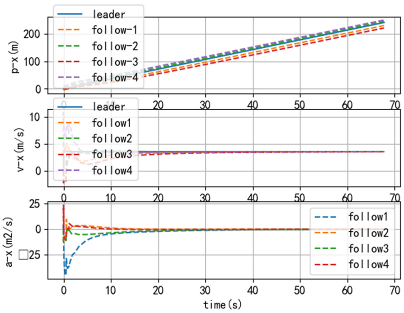

Fig. 9. Position, velocity and acceleration state quantities in x-direction with time delay = 0.5

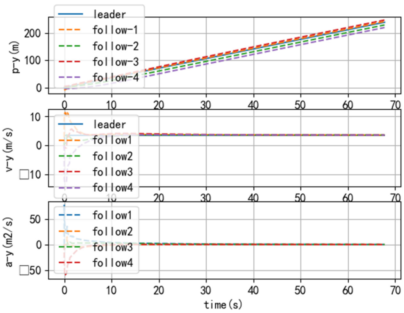

Fig. 10. Position, velocity and acceleration state quantities in y-direction with time delay = 0.5

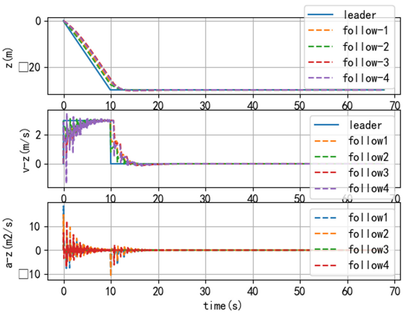

Fig. 11. Position, velocity and acceleration state quantities in z-direction with time delay = 0.5

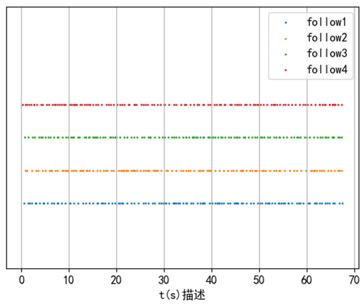

Fig. 12. Event trigger time with time delay = 0.5

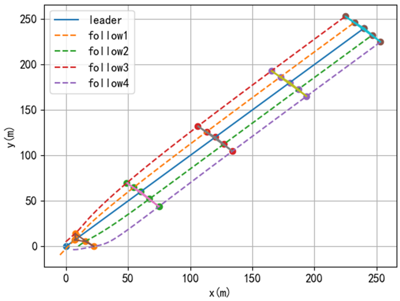

Fig. 13. Heterogeneous AUV group formation (2-D) with communication time delay = 0.1

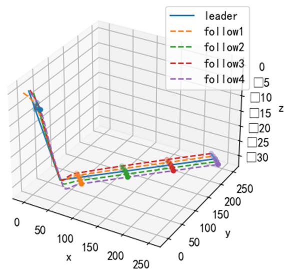

Fig. 14. Heterogeneous AUV group formation (3-D) with communication time delay = 0.1

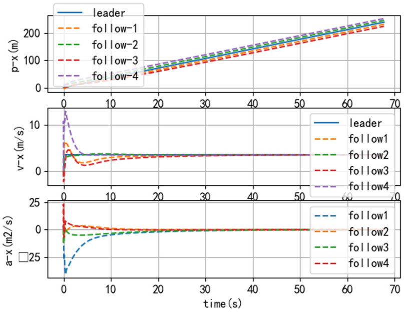

Fig. 15. Position, velocity and acceleration state quantities in x-direction with time delay = 0.1

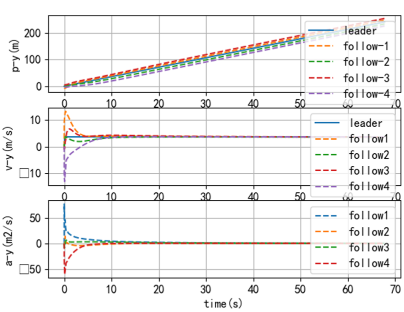

Fig. 16. Position, velocity and acceleration state quantities in y-direction with time delay = 0.1

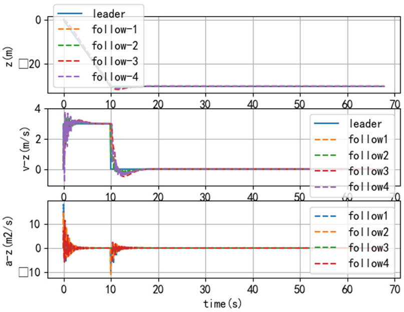

Fig. 17. Position, velocity and acceleration state quantities in z-direction with time delay = 0.1

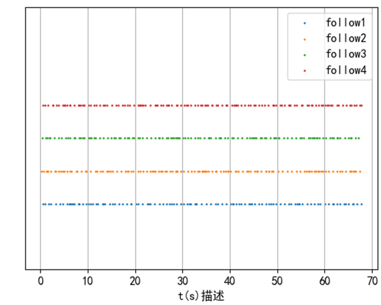

Fig. 18. Event trigger time with time delay = 0.1

The simulation results show that the heterogeneous AUV swarm based on event-triggered control algorithm can still maintain a stable formation under different time delay conditions. The number of communications can be reduced significantly while there is no Zeno behavior. Comparing the simulation results under different time delay conditions, the transient performance will increase and steady-state error will be reduced when time delay is reduced.

5 Conclusion

In this paper, a distributed consistency control algorithm based on event-triggered mechanism for heterogeneous AUV swarm under communicaton delay is proposed. Firstly, an improved method is proposed by using estimation-based trigger conditions. In this method, the release time of event trigger is extended and the number of event triggers is reduced. Secondly, the consistency stability of the algorithm is proved by Lyapunov theory and Zeno phenomenon will not occur. Thirdly, two algorithms with communication delay and without communication delay are proposed respectively. Finally, simulations are conducted to demonstrate the effectiveness of the algorithm. The simulation results show that a fixed formation can be achieved by the two algorithms under different conditions. Meanwhile, the number of communications is reduced significantly and no Zeno phenomenon is found. In order to establish a comparison, simulations under different time delay conditions are conducted. When time delay reduces, the transient performance increases and the steady-state error reduces. In the future, a better compensator can be set in order to adjust the heterogeneity of AUV swarm.

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