An Approach for Optimal Coverage of Heterogeneous AUV Swarm Based on Consistency Theory

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Abstract. Target search mission in given 3-D underwater environments is a challenge in heterogeneous AUV swarms exploration. In this paper, an effective and low consumption strategy is focused for the challenge. With consistency theory, two problems are proposed: optimal partition of regions and cooperative search of targets. First, the original computational geometry of spatial structures is exploited using centroidal Voronoi tessellation. Then, the optimal distribution of the regions under weighted condition of the target probability is obtained by using the mission load dynamic model. Next, a distributed cooperative protocol based on consensus strategy is proposed to solve the cooperative search problem. Finally, theoretical results are validated through simulations on heterogeneous AUV swarms.

Keywords: Consensus theory · Optimal partition of regions · Bioinspired model · Cooperative target search · Optimal coverage control

1 Introduction

Autonomous underwater vehicle (AUV) is a submarine robot that can cruise underwater freely without the restriction of umbilical cable. With a variety of sensors, it is capable of performing various underwater tasks, such as marine resource exploitation, underwater scientific research and hydrological environmental exploration [1–4]. However, with the increasing complexity and dangerous of underwater tasks, a single AUV or a homogeneous AUV swarm is difficult to complete the tasks. In order to improve efficiency of such situation, AUVs with different characteristics and abilities are grouped to perform tasks collaboratively, which is called heterogeneous AUV swarm. The fundamental research of heterogeneous AUV swarm is how to search targets, and how to reach a sensing consensus of targets in the given underwater region [5–7].

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This problem had received a consistent attention in the research community over the past decade. In [8], an internal wave detection experiment conducted by the Massachusetts Institute of Technology (MIT) Laboratory of Autonomous Ocean Sensing System (LAMSS) was summarized in August 2010. The goal was to allow AUV swarm to collaborate autonomously through on-board autonomous software and real-time hydroacoustic communication, but no relevant research of search locational optimization has been conducted. A distributed cooperative search strategy for multiple AUVs based on initial target information was introduced in [9]. The process of target searching was decomposed into two stage, in the first stage, the possible range of the target was predicted based on its speed and elapsed time, in the second stage, AUV entered the prediction range, and then updated the target existence probability in real time according to the sensor detection results, and used predictive control to make optimal decisions based on the target existence probability. This method can detect the existence of targets quickly, however, it can cause waste of resources and unnecessary time consumption, and the search region was not divided optimally, because it did not consider the functional differences between the individual AUV of the swarm. A cooperative combat strategy based on simultaneous survey was presented in [10], which can solve the problem of AUV task region partition, however, this method required AUV to meet and exchange information with its neighbors periodically, which increased the difficulty of formation control and the load of the information network. There were also methods such as the multi-AUV target allocation strategy based on improved communication network which was proposed in [11].

In this work, the collaborative search task of heterogeneous AUV swarm was decomposed into two sub-problems: optimal partition of regions and cooperative search of targets.

Different types of AUVs in a heterogeneous AUV swarm usually carry devices and sensors with different performance. Therefore, when dealing with the problem of optimal partition of regions, we address a control protocol involving the voronoi partition with consensus approach, which makes the best of heterogeneous AUV swarm, and then makes all the sub-detection regions of the AUV reach an even distribution.

After the sub-detection region is determined, each AUV reaches its respective search area to perform target search task. A distributed collaborative search method is designed to solve the problem of cooperative search of targets. Through the information exchange between the AUV swarm, each AUV can determine for themselves whether there is duplication of targets, then recounts and sorts the targets until the target information of all AUVs reach consensus.

2 Problem Formulation and Preliminaries

2.1 Voronoi Partition

According to the optimal partition strategy of target region, the region is divided by Voronoi partition principle [12–16]. Compared with other regional partition strategies, Voronoi partition has the characteristics of strong pertinence and high computational efficiency.

Consider a target region S with i AUVs, and the collection of position of AUVs is given by $\eta = \{\eta_1, \eta_2, ..., \eta_n\}$. The density function of the target region is $\phi(\xi)$, ξ is defined as a function to describe the measurement cost of i-th AUV to any point in the target region. The i-th Voronoi cell is thus defined as:

$$V_i = \{\xi \in S | f(\eta_i, \xi) \leq f(\eta_j, \xi), \forall i \neq j, \forall i, j \in n\} \quad (1)$$

Lemma 1. *n AUVs are randomly distributed in target region S, the collection of position of AUVs is given by η. According to the current location information, the optimal region partition principle for heterogeneous AUV swarm is Voronoi partition principle.*

According to Lemma 1, within the target region, the optimal coverage region of each AUV is Voronoi region, which size and shape are not only related to the location information of the AUV and its neighbors, but also related to the measurement cost function of each AUV. Therefore, the measurement cost function of each AUV needs to be predefined. Suppose $f(\eta_i, \xi) = \|\eta_i - \xi\|$ denotes the distance between any AUV and the target point. At present, $f(\eta_i, \xi)$ is used as the cost function by most studies. It is easy to understand and calculate the distance as the measuring cost function of the vehicle.

2.2 Bioinspired Model

On biological cell membranes, there are a series of contents such as voltage, current, signal input and output, and conclusions based on their mutual interaction can solve many problems [17,18].

The bioinspired model was first proposed by British scientist Hodgeto while studying the action potential of squid neurons. The mathematical model of this kind of neuron potential is also called H-H model, The dynamic characteristic equation of the cell membrane voltage V_m in this model is as follows:

$$C_m \frac{dV_m}{dt} = -(E_p + V_m) \cdot g_p + (E_{Na} - V_m) \cdot g_{Na} - (E_K + V_m) \cdot g_K \quad (2)$$

where V_m represents the membrane voltage of the cell membrane, C_m represents the membrane capacitance of the cell membrane; E_p, E_{Na} and E_K represent the energy of the negative current, sodium ion and potassium ion in the cell membrane respectively. g_p, g_{Na}, and g_K are the conductivities corresponding to negative current, sodium ion, and potassium respectively.

H-H model coefficients are simplified by further improving. Suppose $C_m = 1$, $\partial = E_p + V_m$, $A = g_p$, $B = E_p + E_{Na}$, $D = E_K - E_p$, $f(e_i) = \max(e_i, 0)$, $g(e_i) = \max(-e_i, 0)$, the bioinspired models can be obtained:

$$\frac{d\partial}{dt} = -A\partial_i + (B - \partial_i)f(e_i) - (D + \partial_i)g(e_i) \quad (3)$$

where $f(e_i)$ is the excitatory input and $g(e_i)$ is the inhibitory input. The membrane voltage of neuron ∂_i is the output of the system, and any excitation and inhibition signal can be controlled within the range of [-D,B] to make the output signal ∂_i smooth. The bioinspired model can be used in AUV swarm to solve the problems of speed jump in formation and cooperative operation [19,20].

2.3 Region Partition Model

In this work, a position-probability model is proposed to describe the probability of the target appear in the search region. The larger the probability is, the wider the search area of the AUV swarm is, which means the larger the task payload of this AUV swarm. For any position r in a task region, assuming that there are m suspicious positions in the region. Target occurrence probability utilized by gaussian probability function can be expressed as:

$$\text{target}(r) = \sum_{i=1}^{m} \frac{1}{2\pi} \exp[-\frac{1}{2}(r - r_i)^T K_i (r - r_i)] \tag{4}$$

r_i represents the location where the target is most likely to appear, which is mainly judged according to environmental features and prior knowledge, random value is used in this work. K_i is a diagonal matrix, represents the probability weight of occurrence of r_i. According to the Eq. 4, the probability of occurrence of the target is a continuous function. Assuming that the position information of the ith AUV is η_i, the search mission payload of the ith AUV can be expressed as the sum of the probability of occurrence of all targets in its task partition V_i:

$$task(i) = \int_{r \in V_i} \text{target}(r) dr \tag{5}$$

Consider a convex polygon with N vertices as task region, V_i is also convex polygon. Suppose $\{v_1, v_2, ..., v_N\}$ denotes the vertices, the task region can be divided into N triangles whose vertices are respectively represented as (η_i, v_j, v_{j+1}), where $j \in \{1, 2, ..., N-1\}$. The mission payload of the ith AUV can be expressed by double integral:

$$task(i) = \sum_{j=1}^{N} \int_{s_j} \sum_{i=1}^{m} \frac{1}{2\pi} \exp[-\frac{1}{2}(r - r_i)^T K_i (r - r_i)] dr \tag{6}$$

The optimal partition problem of target region can be described as the consistency of all AUV groups to the mission payload:

$$task(1) = task(2) = \; ... \; = task(n) \tag{7}$$

3 Optimal Region Partition with Consensus Protocol

3.1 Control Strategy Design

In order to realize the reasonable partition of target search region, the dynamic process of search task of heterogeneous AUV swarm is simulated based on the biological competition mechanism in the bioinspired model. Consider the dynamic characteristics, sensor performance, computing power and other factors of the AUV swarm, control force is used to indicate the search ability of each AUV, the main system variables in the model are mapped to the partition problem, the territory is used to represent the search region for each AUV swarm, total resources indicates the task payload of the AUV swarm.

According to the biological competition mechanism model, the greater the control power, the more its total resources. The utilization resource ratio represents the resources occupied by the unit's control power, which is the ratio of total resources to control power. Based on the bioinspired model, combined with the Voronoi partition and the optimal partition problem model for target region search, the task allocation model for the AUV swarm is established.

Assuming that the heterogeneous AUV swarm is randomly distributed in the search region S, and the position is represented as $\eta = \{\eta_1, \eta_2, ...\eta_n\}$, $\eta_i \in \mathbb{R}^2$ (2 D plane), with the partitioning method defined:

$$V_i = \{\xi \in S \,|\, f(\eta_i, \xi) \leq f(\eta_j, \xi), \forall i \neq j, \forall i, j \in n\} \tag{8}$$

The initial detection partition $\{V_1, V_2, ..., V_n\}$ of all AUVs can be obtained, and the mission payload of each AUV can be calculated by combining the target occurrence probability target(r), the sum of target occurrence probability $task(i)$ and mission payload $task(i)$, which is represented as $\{T_{auv1}, T_{auv2}, ..., T_{auvi}\}$. Set a constant for the search capability of each AUV, the ratio of task and capability $R_{auvi} = {}^{T_{auvi}}/_{E_{auvi}}$ of each AUV can be calculated. Therefore, the consistency formula of the mission payload can be represented as below:

$$R_{auv1} = R_{auv2} = ... = R_{auvn} \tag{9}$$

AUV_i will move toward AUV_j, When $R_{auvi} < R_{auvj}$ and $i \neq j$. Since the search task assignment is based on the Voronoi partitioning principle, the search task region of AUV_j will be reassigned to AUV_i. The dynamics of the ith AUV is modelled as an integrator: $\dot{\eta}_i = u_i$, the control input u_i of the system can be defined based on consistency theory and biological invasion mechanism:

$$u_i = -\gamma_i \sum_{j=1}^{n} \vec{n}_{ij} a_{ij} (R_{auvi} - R_{auvj}) \tag{10}$$

where γ_i is the feedback control gain coefficient greater than zero. $\vec{n}_{ij} = \frac{\eta_i - \eta_j}{\|\eta_i - \eta_j\|}$ is the direction vector. a_{ij} represents the degree of coupling between AUV_i and

AUV_j in the search task region. According to the occurrence probability of the target, it can be obtained:

$$a_{ij} = \int_{r \in V_i \cap V_j} \text{target}(r) dr \tag{11}$$

3.2 Consistency Analysis

In order to prove whether the uniform distribution of the search region can be achieved based on the above theory, it is necessary to analyze its consistency.

Lemma 2. *For n variables* $\{\delta_1, \delta_2, ..., \delta_n\}$*, satisfy* $\sum_{i=1}^{n} \delta_i = \Delta$*, where* Δ *is a constant value. Then for a set of positive real numbers* $\{\beta_1, \beta_2, ..., \beta_n\}$*, there exists:*

$$\min(\sum_{i=1}^{n} \frac{\delta_i^2}{\beta_i}) = \frac{\Delta^2}{\sum_{i=1}^{n} \beta_i}$$

And for any variable $\{\delta_1, \delta_2, ..., \delta_n\}$*, it satisfies:*

$$\frac{\delta_i}{\beta_i} = \frac{\Delta}{\sum_{i=1}^{n} \beta_i}$$

According to the Lemma 2, if the minimum value of the objective function can be obtained, the consistency of the variables can be achieved. If the mission payload T_{auvi} of AUV_i is taken as δ_i in the lemma, and the search capability E_{auvi} is taken as β_i, when the search region does not change:

$$T_{auv1} + T_{auv2} + ... + T_{auvn} = T_S \tag{12}$$

where T_S is a constant and represents the sum of the probability of occurrence of targets in the search region. Consider a Lyapunov function candidate:

$$\begin{aligned} V &= V_1 + V_2 + ... + V_n \\ &= \frac{T_{auv1}^2}{E_{auv1}} + \frac{T_{auv2}^2}{E_{auv2}} + ... + \frac{T_{auvn}^2}{E_{auvn}} \end{aligned} \tag{13}$$

Lemma 3. *For a first-order integral system* $\dot{\eta}_i = u_i$*:*
with $u_i = -\gamma_i \sum_{j=1}^{n} \vec{n}_{ij} a_{ij} (R_{auvi} - R_{auvj})$ *as the system input. If and only if lyapunov function achieves minimum value, the task-capability ratio of each AUV in the heterogeneous AUV swarm is consistent. That is the search region is evenly distributed according to the load.*

Taking the derivative of Eq. 13:

$$\begin{aligned} \frac{\partial V}{\partial t} &= \frac{\partial V_1}{\partial t} + \frac{\partial V_2}{\partial t} + ... + \frac{\partial V_n}{\partial t} \\ &= (\frac{\partial V_1}{\partial \xi} + \frac{\partial V_2}{\partial \xi} + ... + \frac{\partial V_n}{\partial \xi}) \frac{\partial \xi}{\partial t} \\ &= \sum_{i=1}^{n} \frac{\partial V}{\partial \eta_i} \frac{\partial \eta_i}{\partial t} \end{aligned} \tag{14}$$

where $\frac{\partial \eta_i}{\partial t} = u_i$, For $j \neq i$, and N_i denote the set of neighbors of the ith AUV:

$$\frac{\partial T_{auvi}}{\partial \eta_j} = 0 \tag{15}$$

Then

$$\frac{\partial V}{\partial \eta_i} = \frac{2T_{auvi}}{E_{auvi}} \frac{\partial T_{auvi}}{\partial \eta_i} + \sum_{j \in N_i} \frac{2T_{auvj}}{E_{auvj}} \frac{\partial T_{auvj}}{\partial \eta_j} \tag{16}$$

T_{auvi} can be represented as:

$$\begin{aligned} \frac{\partial T_{auvi}}{\partial \eta_k} &= \frac{\partial}{\partial \eta_k} \int\limits_{r \in V_i} \text{target}(r) dr \\ &= \int\limits_{r \in V_i} \frac{\partial}{\partial \eta_k} \text{target}(r) dr + \int\limits_{\partial V_i} \text{target}(\mu) n^T(\mu) \frac{\partial \mu}{\partial \eta_k} d\mu \end{aligned} \tag{17}$$

where ∂V_i represents the boundary of V_i, μ is the parameterized expression of the boundary. $n^T(\mu)$ is the outgoing normal line at the boundary, which is the unit vector. Since the probability distribution function target(r) of the occurrence of the target does not depend on the position of η_k, $\int\limits_{r \in V_i} \frac{\partial}{\partial \eta_k} \text{target}(r) dr$ is always zero. So we have the following Eq. 18:

$$\frac{\partial T_{auvi}}{\partial \eta_k} = \int\limits_{\partial V_i} \text{target}(\mu) n^T(\mu) \frac{\partial \mu}{\partial \eta_k} d\mu \tag{18}$$

With $\partial V_i = \sum_{j \in N_i} V_i \cap V_j$, the Eq. 18 can be expressed as:

$$\frac{\partial T_{auvi}}{\partial \eta_k} = \sum\nolimits_{j \in N_i} \int\limits_{V_i \cap V_j} \text{target}(\mu_{ij}) n^T(\mu_{ij}) \frac{\partial \mu_{ij}}{\partial \eta_k} d\mu_{ij} \tag{19}$$

To summarize:

$$\begin{aligned} &\tfrac{\partial V}{\partial \eta_i} = \tfrac{2T_{auvi}}{E_{auvi}} \sum_{j \in N_i} n^T(\mu_{ij}) \int\limits_{V_i \cap V_j} \text{target}(\mu_{ij}) \tfrac{\partial \mu_{ij}}{\partial \eta_i} d\mu_{ij} \\ &+ \sum_{j \in N_i} \tfrac{2T_{auvj}}{E_{auvj}} n^T(\mu_{ij}) \int\limits_{V_i \cap V_j} \text{target}(\mu_{ji}) \tfrac{\partial \mu_{ji}}{\partial \eta_i} d\mu_{ji} \end{aligned} \tag{20}$$

For any two adjacent i and j, μ_{ij} can be expressed as:

$$\mu_{ij} : \frac{\eta_i + \eta_j}{2} + \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \frac{\eta_i - \eta_j}{\|\eta_i - \eta_j\|} \lambda_{ij}, \lambda_{ij} \in [-a_{ij}, b_{ij}]$$

It can be seen that μ_{ij} is the midperpendicular line of η_i and η_j, λ_{ij} represents the boundary length. Since μ_{ij} and $n(\mu_{ij})$ are orthogonal, it can be concluded that:

$$n^T(\mu_{ij}) \frac{\partial \mu_{ij}}{\partial \eta_i} = n^T(\mu_{ij}) \frac{\partial \mu_{ij}}{\partial \eta_j} = \frac{1}{2} n^T(\mu_{ij}) \tag{21}$$

Substituted into the Eq. 21:

$$\begin{aligned}\frac{\partial V}{\partial \eta_i} &= \frac{T_{auvi}}{E_{auvi}} \sum_{j \in N_i} n^T(\mu_{ij}) \int_{V_i \cap V_j} \text{target}(\mu_{ij}) d\mu_{ij} \\ &+ \sum_{j \in N_i} \frac{T_{auvi}}{E_{auvi}} n^T(\mu_{ij}) \int_{V_i \cap V_j} \text{target}(\mu_{ji}) d\mu_{ji} \\ &+ \sum_{j \in N_i} \frac{2T_{auvj}}{E_{auvj}} n^T(\mu_{ij}) \int_{V_i \cap V_j} \text{target}(\mu_{ji}) \frac{\partial \mu_{ji}}{\partial \eta_i} d\mu_{ji}\end{aligned} \tag{22}$$

$n^T(\mu_{ij}) = -n^T(\mu_{ji})$, simplify Eq. 22:

$$\frac{\partial V}{\partial \eta_i} = \sum_{j \in N_i} (R_{auvi} - R_{auvj}) n^T(\mu_{ij}) \int_{V_i \cap V_j} \text{target}(\mu_{ji}) d\mu_{ji} \tag{23}$$

Then

$$\frac{\partial P}{\partial t} = -\sum_{i=1}^{n} k_i \left\| \sum_{j \in N_i} (R_{auvi} - R_{auvj}) n(\mu_{ij}) a_{ij} \right\|^2 \tag{24}$$

Since V is continuously differentiable and $\dot{V} \leq 0$, according to the LaSalle invariance principle, if $\dot{V} = 0$, the system state values will converge to the maximum invariant set of the system. According to the Eq. 24, when $\dot{V} = 0$:

$$\sum_{j \in N_i} (R_{auvi} - R_{auvj}) n(\mu_{ij}) a_{ij} = 0_{2\times 1} \tag{25}$$

The matrix form of the Eq. 25 is expressed as:

$$\begin{pmatrix} l_{11} & \ldots & l_{1n} \\ \vdots & \ddots & \vdots \\ l_{m1} & \cdots & l_{mn} \end{pmatrix} \begin{pmatrix} R_{auv1} \\ \vdots \\ R_{auvn} \end{pmatrix} = \begin{pmatrix} 0_{2\times 1} \\ \vdots \\ 0_{2\times 1} \end{pmatrix} \tag{26}$$

where $l_{ij} \in \mathbb{R}^2$, then:

$$l_{ij} = \begin{cases} \sum_{k \in N_i} n(\mu_{ik}) a_{ik}, j = i \\ -n(\mu_{ij}) a_{ij}, j \in N_i \end{cases} \tag{27}$$

Suppose $L_\alpha = L_\alpha^U \otimes \begin{bmatrix} 1 \\ 0 \end{bmatrix} + L_\alpha^D \otimes \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, where L_α^U , L_α^D represent the weighted Laplacian matrix of the system:

$$\begin{cases} L_\alpha^U R_{auv} = 0 \\ L_\alpha^U R_{auv} = 0 \end{cases} \tag{28}$$

where $R_{auv} = \begin{bmatrix} R_{auv1} \cdots R_{auvn} \end{bmatrix}^T$, and R_{auv} is a column vector with the same elements, which is $R_{auv1} = R_{auv2} = \cdots = R_{auvn}$, then the theorem is proved.

4 Collaborative Target Search Based on Consensus Protocol

4.1 Collaborative Target Search Problem Modeling

Assuming that n AUVs of different types have arrived in their respective subregion and started to perform target search tasks, the search range of the ith AUV is centered on itself with radius of D_{auvi}, the target state information detected by each AUV includes the following types:

(1) Position information: $O_{ij}(t) = (x_{ij}(t), y_{ij}(t), z_{ij}(t))$ represents the spatial coordinate of T_j detected by AUV_i in the geodetic coordinate system.
(2) Velocity information: $v_{ij}(t)$ represents the velocity of T_j detected by AUV_i.
(3) Target type: K_{ij} represents the type of T_j detected by AUV_i.
(4) Time information: s_{ij} represents the time stamp of T_j detected by AUV_i.
(5) Number of targets: m_i represents the number of targets detected by AUV_i. Including the number of targets detected by itself and the number of targets obtained from the neighboring AUV.
(6) Target label: n_{ij} represents the label of T_j detected by AUV_i, which does not exceed the value of m_i.

When m_i targets are detected, the status information of AUV_i is established as follows:

$$S_i(t) = (S_{i1}^T(t), S_{i2}^T(t), ..., S_{im_i}^T(t)) \tag{29}$$

where $S_{ij}^T(t) = \{O_{ij}(t), v_{ij}(t), K_{ij}, s_{ij}, m_i, n_{ij}\}$ denotes the set of the status information of target T_j detected by AUV_i. If and only if $\|S_i(t) - S_k(t)\| \to 0$, the system is consistent, which means the AUV swarm completes the cooperative search task. The consistency model of heterogeneous AUV swarms can be expressed as:

$$\begin{cases} |x_{ij}(t) - x_{kj}(t)| < \varepsilon_x \\ |y_{ij}(t) - y_{kj}(t)| < \varepsilon_y \\ |z_{ij}(t) - z_{kj}(t)| < \varepsilon_z \\ |v_{ij}(t) - v_{kj}(t)| < \varepsilon_v \\ K_{ij} - K_{kj} = 0 \\ s_{ij} - s_{ik} = 0 \\ m_i - m_k = 0 \\ n_{ij} - n_{ik} = 0 \end{cases} \tag{30}$$

where ε_x, ε_y, ε_z, ε_v are respectively denote the allowable errors of position information and velocity information of the same target detected by different AUVs.

4.2 Consistency Condition

Consider a heterogeneous AUV swarm composed of n AUVs, which communication network topology is $G = (V, E)$. While $V = \{1, 2, ..., n\}$ is a set of non-empty nodes, and each node represents an AUV. $E \subseteq V \times V$ is a set of connecting edges, represents the communication relationship between AUVs. $G(t) = [g_{ik}(t)]$ is the adjacency relation of the topology, where $g_{ik}(t) = 1$ means that the communication between AUV_i and AUV_k is connected at time t, and $g_{ik}(t) = 0$ means that the communication is interrupted. The Laplace matrix $L = [l_{ik}]$ is:

$$l_{ik} = \begin{cases} \sum_k a_{ik}, i = k \\ -a_{ik}, i \neq k \end{cases} \tag{31}$$

Suppose that AUV_i sends the detected target information T_j to its neighbor AUV_k, and AUV_k compares the received target information T_j with the target information T_q detected by itself, if the following conditions are met:

$$\begin{cases} |x_{ij}(t) - x_{kq}(t)| < \varepsilon_x \\ |y_{ij}(t) - y_{kq}(t)| < \varepsilon_y \\ |z_{ij}(t) - z_{kq}(t)| < \varepsilon_z \\ |v_{ij}(t) - v_{kq}(t)| < \varepsilon_v \\ K_{ij} - K_{kq} = 0 \end{cases} \tag{32}$$

Target T_j and target T_q can be determined as the same target, otherwise, they are marked as different targets. Then, the target is renumbered and updated with n_{kq}.

If T_j and T_k are the same target, compare the timestamp:

(1) If $s_{ij} < s_{kq}$, let $s_{kq} = s_{ij}$.
(2) If $s_{kq} = s_{ij}$, AUV_k reorder the targets according to the detected timestamp.

If T_j and T_q are not the same target, add T_j to the probe list of AUV_k:

$$\begin{cases} m_k = m_k + 1 \\ O_{kq}(t) = O_{ij}(t) \\ v_{kq}(t) = v_{ij}(t) \\ s_{kq} = s_{ij} \end{cases} \tag{33}$$

The target is reordered according to the time stamp, and the sorted result is taken as the new number of the target. If and only if the target information states of all AUVs are identical and tend to be consistent, which means $\|S_i(t) - S_k(t)\| \to 0$. The heterogeneous AUV swarms complete the cooperative search task.

5 Simulation Experiment and Analysis

5.1 Simulation of Optimal Region Partition

The task range is set to 1000×1000. Consider seven AUV swarms, each AUV swarm is represented by the leader AUV within the swarm. The initial position and mission payload are shown in Table 1.

Table 1. Initial position and mission payload

Heterogeneous AUV swarm	The initial position	The mission payload
Swarm 1	[860, 552]	80.1498165
Swarm 2	[428, 527]	32.72320087
Swarm 3	[547, 1]	72.49389006
Swarm 4	[718, 97]	80.03960203
Swarm 5	[368, 687]	33.54609727
Swarm 6	[522, 91]	92.11312154
Swarm 7	[411, 837]	62.61293414

According to the information in Table 1, we calculate the probability of occurrence of the target at each position:

$$\text{target}(x, y) = \sum_{i=1}^{7} \text{target}_i \exp(-10^{-3}((x_i - x)^2 + (y_i - y)^2)) \tag{34}$$

After the initial Voronoi diagram is produced, the state information is updated according to the bioinspired model in this paper until the task loads of all AUV swarms are consistent. The task region generation process is illustrated in Fig. 1.

As can be seen from the final distribution diagram, the gray value of each region is the same, achieving uniform distribution. The final consistency state is shown in Fig. 2.

In Fig. 2, the algorithm adopted in this paper can effectively realize the average distribution of the task load of different AUV swarms in the specified search region. The final task load ratio R tends to be consistent, and the velocity of each AUV group in the x and y directions also tends to be consistent. The area value of the assigned area eventually tends to be stable.

5.2 Simulation of Cooperative Target Search

Consider that there are 3 AUVs in a swarm, and each AUV carries out target search in a distributed manner. The target information detected by the AUV is shown in Table 2.

Suppose $\varepsilon_x = \varepsilon_y = \varepsilon_z = 0.02$ and $\varepsilon_v = 1$. The simulation was carried out according to the distributed collaborative algorithm proposed in this paper, the simulation results were shown in Table 3.

As shown in Table 3, three AUVs in the group detect six targets. Target 1 and 2 detected by AUV1 were the same as target 4 and 5 detected by AUV2, and target 1, 2 and 3 detected by AUV2 were the same as target 5, 6 and 3 detected by AUV3. The results show that the algorithm can eliminate the repeated target information among each AUV, and it can effectively realize distributed collaborative detection in AUV swarm.

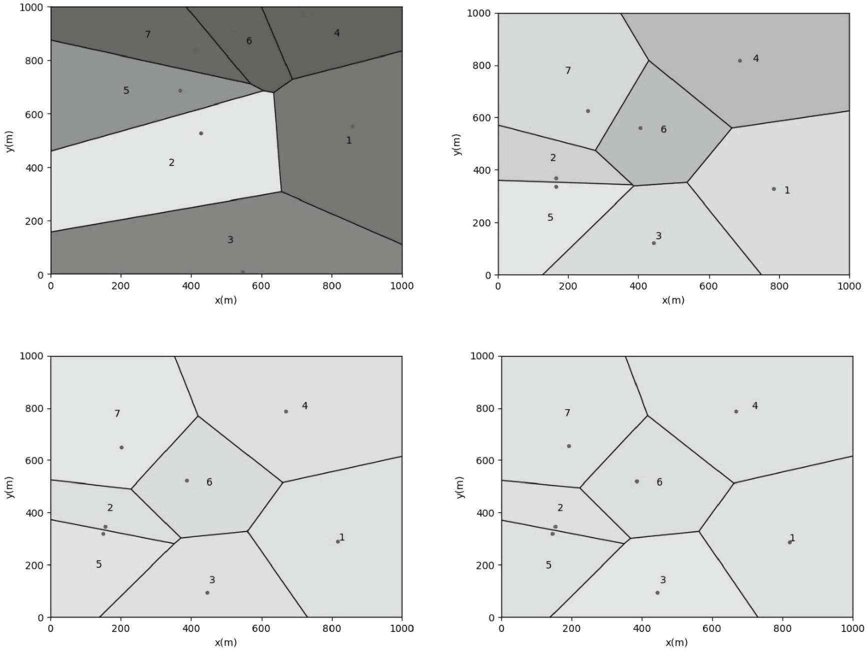

Fig. 1. Task region generation

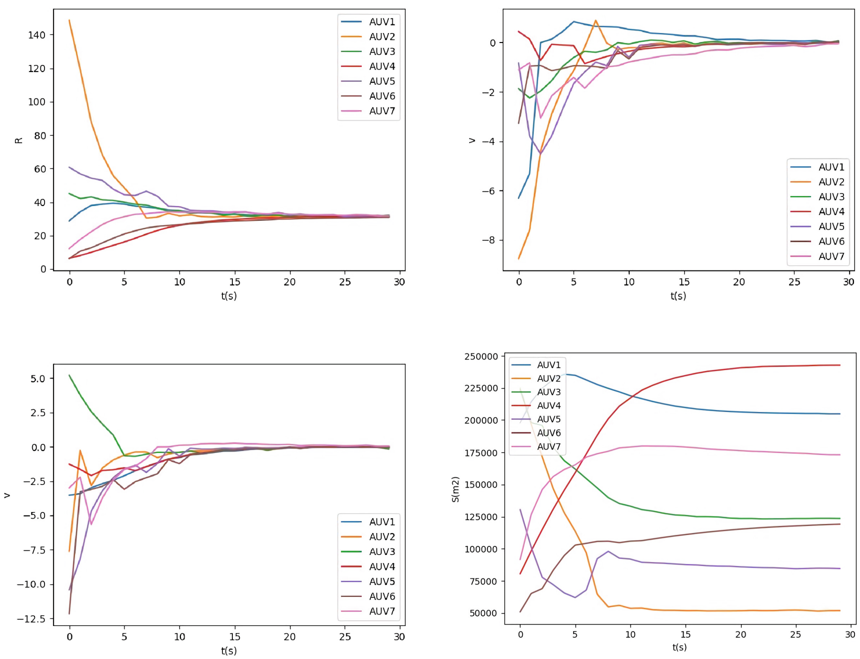

Fig. 2. Consistent state

Table 2. Detecting target information 1

	Target number n_{ij}	Target coordinate $O_{ij}(t)$	The target speed $v_{ij}(t)$	Time information s_{ij}
AUV1	1	[6.32, 2.56, 4.11]	5	08:55:53
AUV1	2	[11.32, 19, 21, 9.31]	5	08:56:28
AUV2	1	[17, 33, 16.21, 3.85]	6	08:55:56
AUV2	2	[5.21, 12.49, 18, 75]	10	08:58:35
AUV2	3	[10.21, 15.45, 9.08]	8	08:58:41
AUV3	1	[1.35, 16.76, 9.75]	5	08:56:00
AUV3	2	[2.49, 6.55, 10.69]	8	08:57:19
AUV3	3	[10.21, 1.33, 16.76]	10	09:05:03
AUV3	4	[8.37, 13.75, 17.44]	8	09:09:53

Table 3. Detecting target information 2

	Target number n_{ij}	Target coordinate $O_{ij}(t)$	The target speed $v_{ij}(t)$	Time information s_{ij}
AUV1	1	[6.32, 2.56, 4.11]	5	08:55:53
AUV1	2	[11.32, 19, 21, 9.31]	5	08:56:28
AUV1	3	[1.35, 16.76, 9.75]	5	08:56:00
AUV1	4	[5.21, 12.49, 18, 75]	10	08:58:35
AUV1	5	[10.21, 15.45, 9.08]	8	08:58:41
AUV1	6	[2.49, 6.55, 10.69]	8	08:57:19
AUV2	1	[17, 33, 16.21, 3.85]	6	08:55:56
AUV2	2	[5.21, 12.49, 18, 75]	10	08:58:35
AUV2	3	[10.21, 15.45, 9.08]	8	08:58:41
AUV2	4	[6.32, 2.56, 4.11]	5	08:55:53
AUV2	5	[11.32, 19, 21, 9.31]	5	08:56:28
AUV2	6	[1.35, 16.76, 9.75]	5	08:56:00
AUV3	1	[1.35, 16.76, 9.75]	5	08:56:00
AUV3	2	[2.49, 6.55, 10.69]	8	08:57:19
AUV3	3	[10.21, 1.33, 16.76]	10	09:05:03
AUV3	4	[8.37, 13.75, 17.44]	8	09:09:53
AUV3	5	[17.33, 16.21, 3.85]	6	08:55:56
AUV3	6	[5.21, 12.49, 18, 75]	10	08:58:35

6 Conclusion

In this paper, the problems of search region partition and collaborative target detection faced by heterogeneous AUV swarms in implementing collaborative search tasks are studied. According to different stages of detection, two different types of tasks are divided: region partition and target detection. Firstly, the target occurrence probability model is established for the region partition task, and the initial detection region partition is carried out by using Voronoi partitioning principle. By analyzing the detection capability of each AUV and transforming it into the detection mission load index, a dynamic model of AUV mission load based on bioinspired model is proposed and established. The consistency theory is used to prove that this model can realize the uniform distribution of regional detection tasks under the probability weighted condition of target presence. Secondly, A distributed collaborative detection method based on the principle of consistency is proposed, which realizes the detection of repeated targets in the target detection process. By judging the target information status of all AUVs is all the same and tends to be consistent, the heterogeneous AUV swarms completes the collaborative detection task. Finally, the effectiveness and convergence of the proposed method have been validated by the performance presented in numerical experiments on heterogeneous AUV swarms.

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