

Analytical Study on Predicting the Shear Resistance of Steel Fiber Reinforced Concrete Beams



Michael Cohen

1 Introduction

The Catastrophic failures such as the collapse of large RC beams in the 1950s at US Air-force warehouses, as well as more recent failures such as the collapse of the “De La Concorde Bridge” have emphasized the importance of properly accounting for shear in the design of reinforced concrete structures. To prevent shear failure, beams are traditionally reinforced with transverse shear reinforcement in the form of stirrups. One alternative to this reinforcement may lie in the use of steel fiber reinforced concrete (SFRC). Extensive research has shown that the addition of randomly oriented steel fibers enhances the properties of traditional concrete including tensile capacity, fracture toughness and crack control. These improvements attribute to the influence of the randomly orientated fibers in arresting cracks and the resulting enhancements in the post-cracking resistance of the concrete. In beams that are shear-critical, SFRC enhances the diagonal tension capacity of concrete and thus can result in significant enhancements in shear capacity. If added in sufficient quantities, the use of steel fibers can replace traditional transverse reinforcement and promote flexural failure and ductility [18].

2 Previous Research on the Behaviour of SFRC Beams

Extensive research over the past three decades has shown that steel fibers can be used to enhance shear resistance in reinforced concrete beams. Adding steel fibers to a concrete beam without transverse reinforcement improves shear behaviour due to

M. Cohen (✉)
University of Waterloo, Waterloo, Canada
e-mail: mcohen@uwaterloo.ca

the ability of SFRC to resist and redistribute diagonal tensile stresses after cracking. In addition, the fiber crack-bridging capabilities delay and control the development of cracks. Thus, very much like traditional transverse reinforcement, the use of fibers results in an increase in the overall shear resistance of the beam and can promote flexural failure and ductility.

Para-Montesinos [16] presented a large database of SFRC beam test results which included results from 16 studies. The database included test results from 147 SFRC beams and 45 companion RC beams having effective depths (d) ranging from 180 to 570 mm, shear-span to depth ratio (a/d) ranging from 1.0 to 6.0, compressive strength (f'_c) between 18 and 104 MPa, longitudinal reinforcement ratios (ρ) ranging from 0.4 to 5%, fiber volume fractions (V_f) ranging from 0.25 to 0.75% and having straight, hooked or crimped fibers. The results showed that all SFRC beams in the database having $V_f \geq 0.5\%$ exhibited shear stress at failure greater than $0.17\sqrt{f'_c}$, equivalent to V_c as defined in the ACI-318 code [1] with beams having $V_f \geq 0.75\%$ failing at shear stresses not less than $0.3\sqrt{f'_c}$. Based on the analysis of the database, Parra-Montesinos recommended that steel fibers, if added in sufficient quantity, can be used as a replacement to minimum shear reinforcement in beams subjected to shear forces equivalent to $0.5V_c(0.085)\sqrt{f'_c}$ and $V_c(0.17)\sqrt{f'_c}$. In addition to material performance criteria, Parra-Montesinos also recommended that minimum steel fiber content, V_f , of 0.75% should be provided in order to achieve adequate shear response. This minimum fiber content has also been recommended previously by ACI subcommittee 318-F. Based on the experimental evidence reported in the literature, the use of steel fibers in flexural members is now permitted in several international design codes (see next section).

3 Review of Predictive Equations and Design Guidelines for SFRC Beams

Over the years, several researchers have proposed equations for predicting the shear resistance of SFRC beams. Examples of models include those proposed by Ashour et al. [4], Imam et al. [8], Khuntia et al. [9], Mansur et al. [12], Sharma [20] and others. As shown in Tables 1 and 2 most of these models can be broadly classified into two categories. The first category of models assumes that fibers improve the concrete shear resistance directly due to the improvement in tensile post-cracking resistance. These models use material tests to quantify the improvement in shear capacity. The second category considers that fibers provide a separate shear contribution that is additional to the concrete shear contribution.

Most of these models are from empirical formulae derived from regression analysis of limited experimental data. Therefore the results obtained from these models can't be relied on in many cases to evaluate the shear capacity in SFRC beams [14]. In particular, it is noted that the model proposed by Sharma [20], and initially recommended by ACI Committee 544, shows very poor accuracy.

Table 1 Various models to predict shear resistance of SFRC beams (Category 1)

Model	Fiber contribution is		Fiber effect accounted for using			Fiber factor
	Included in concrete contribution	Separate term (additional to concrete contribution)	Split cylinder strength (or MOR)	Fracture mechanics/bending test		
Sharma [20] $v_u = kf \left(\frac{d}{a}\right)^{0.25}$	✓		✓			
Li et al. [11] $v_u = 1.25 + 4.7 \left[(f_f f_{sp})^{\frac{2}{3}} \left(\rho \frac{d}{a}\right)^{\frac{1}{3}} d^{-\frac{1}{3}} \right]$	✓		✓			
Cassanova [5] $V = 0.9bd \cdot \overline{\sigma}_p(w) + V_s$	✓				✓	
Minelli and Pizzari [13] $v_u =$ $\left[\frac{0.18}{\gamma_c} k_c \left(100\rho_1 \left(1 + 2.5 \frac{f_{eq}(0.6-3)}{f_{ct}} \right) f_{ck} \right)^{1/3} + 0.15\sigma_{cP} \right]$	✓				✓	

Table 2 Various models to predict shear resistance of SFRC beams (Category 2)

Model	Fiber contribution is		Fiber effect accounted for using		
	Included in concrete contribution	Separate term (additional to concrete contribution)	Split cylinder strength (or MOR)	Fracture mechanics/bending test	Fiber factor
Imam et al. [8] $v_u = 0.6\Psi\sqrt{\omega}\left[f_c^{0.44} + 275\sqrt{\frac{\omega}{(a/d)^5}}\right]$	✓				✓
Mansur et al. [12] $v_u = \left(0.16\sqrt{f'_c} + 17.2\frac{\rho V_d}{M}\right) + 0.41\left(\tau \cdot V_f \frac{L_f}{D_f}\right)$	✓				✓
Narayanan and Darwish [15] $v_u = e\left(0.24f_{spfc} + 80\rho\frac{d}{a}\right) + 0.41\left(\tau \cdot V_f \frac{L_f}{D_f} d_f\right)$	✓				✓
Ashour et al. [4] $v_u = \left(0.7\sqrt{f'_c} + 7 \cdot V_f \frac{L_f}{D_f} d_f\right)\frac{d}{a} + 17.2\rho\frac{d}{a}$	✓				✓
Swamy et al. [22] $v_u = v_c + 0.37\left(\tau \cdot V_f \frac{L_f}{D_f}\right)$	✓				✓

(continued)

Table 2 (continued)

Model	Fiber contribution is		Fiber effect accounted for using		
	Included in concrete contribution	Separate term (additional to concrete contribution)	Split cylinder strength (or MOR)	Fracture mechanics/bending test	Fiber factor
Shin et al. [21] $v_u = 0.19 f_{sp} + 93 \left(\rho \frac{d}{a} \right) + 0.34 \left(\tau \cdot V_f \frac{L_f}{D_f} d_f \right)$		✓			✓
Khuntia et al. [9] $v_u = \left(0.167\alpha + 0.25 \cdot V_f \frac{L_f}{D_f} d_f \right) \sqrt{f'_c}$		✓			✓
Kwak et al. [10] $v_u = 3.7 e f_{sp}^{2/3} \left(\rho \frac{d}{a} \right)^{1/3} + 0.8 \left(0.41 \tau V_f \frac{L_f}{D_f} d_f \right)$		✓			✓

It is noted that the majority of the models outlined previously account for the fiber contribution empirically using a “fiber factor”, F , which is a function of fiber volume fraction (V_f), diameter (D_f) and length (L_f), as shown in Eq. 1, and an additional term to scale this factor based on fiber typology. Nevertheless, the majority of the equations are empirical and do not account for important factors affecting the shear resistance of SFRC beams. As discussed by Minelli [14].

$$F = V_f \frac{L_f}{D_f} \quad (1)$$

4 Proposed Model for Predicting the Shear Resistance of SFRC Beams

This section describes a procedure for predicting the shear resistance of SFRC beams without web reinforcement. As noted previously, several existing models and design guidelines consider the fiber contribution to shear capacity to be additional to the concrete contribution, much in the same way stirrups improve shear capacity. Other models propose that the fibers should modify the concrete contribution directly. In the present model, the former approach is used and the shear resistance of an SFRC beam is assumed to be equivalent to the expected shear strength of a traditional reinforced concrete beam (V_{no}) plus the additional shear resistance provided by the fibers (V_{fib}) as shown in Eq. 2:

$$V_{nf} = V_{no} + V_{fib} \quad (2)$$

4.1 Reinforced Concrete Contribution to Shear Resistance

In order to compute the reinforced concrete (RC) contribution to shear resistance, V_{no} , the general shear design method of the 2004 CSA Standard can be used (see Eq. 3). This method which is based on the Modified Compression Field Theory (MCFT) has been shown to provide accurate estimates of the shear resistance of traditional RC beams.

$$V_{no} = V_c + V_s \quad (3)$$

where, V_c and V_s are the concrete and transverse steel contributions to shear resistance, respectively. The concrete contribution, V_c , is computed using Eq. 4.

$$V_c = \phi_c \beta \sqrt{f'_c} b_w d_v \quad (4)$$

It should be noted that the concrete contribution to shear capacity is generally supported by aggregate interlocking across tension-shear cracks, dowel action and the tensile stress field that becomes militarized in concrete through reinforcement-concrete bond.

In case of using transverse steel reinforcement, its contribution, V_s , can be calculated using Eq. 5.

$$V_s = \frac{\phi_s A_v f_y h d_v \cot \theta}{s} \quad (5)$$

where $b_w d_v$ is the effective shear area represented by the web width multiplied by the effective shear depth of the beam. The factor ϕ_c is a material reduction factor for concrete (can be taken as 1.0 for the purpose of analysis). The value β accounts for the ability of the concrete to transmit tensile stresses between the cracks. The value s_{ze} represents the effective crack spacing parameter, which is a function of the maximum aggregate size. The angle θ is the angle of inclination of the diagonal compressive stresses to the longitudinal strain at mid-depth of the cross section, ϵ_x , as shown in Eqs. 6 and 7, respectively.

$$\beta = \frac{0.40}{1 + 1500\epsilon_x} \times \frac{1300}{1000 + s_{ze}} \quad (6)$$

$$\theta = 29^\circ + 7000\epsilon_x \quad (7)$$

The longitudinal strain at mid-depth of the cross section, ϵ_x , is computed using Eq. 8 for the case of moment and shear.

$$\epsilon_x = \frac{\left(\frac{M}{d_v}\right) + V}{2 \times E_s \times A_s} \quad (8)$$

To determine the shear strength, the shear, V , should correspond to the expected maximum shear resistance of the beam. The moment, M , represents the corresponding moment at the critical section in the beam (for the beams in this analysis the critical section is taken at a distance d_v from the loading point). A_s and E_s are the cross-sectional area and modulus of elasticity of the longitudinal tension reinforcement, respectively.

4.2 Pullout Strength of Hooked-End Fibers

Studies have shown that anchorage properties of fibers play an important role in improving the pullout resistance of hooked-end steel fibers [17]. Alwan et al. [2] suggested that the mechanical contribution of the hook, $\Delta P'$, is a function of the cold work needed to straighten the fiber as it is being pulled out from the matrix. The simplified expression for $\Delta P'$ is shown in Eq. 9:

$$\Delta P' = \frac{3.05}{\cos(45^\circ \times \pi/180^\circ)} \left(f_{fy} \times \frac{\pi (D_f/2)^2}{6} \right) \quad (9)$$

The value f_{fy} is the fiber yield strength (in psi) and D_f is the fiber diameter (in inches). As an example for a typical Dramix ZP305 fiber, $\Delta P'$ is found to be 34 lbs (152 N).

Therefore, in the case of hooked-end fibers, taking into account the hook contribution, an estimate of the pullout strength, $F_{pullout}$, can be made using Eq. 10, where the contribution of the hook ($\Delta P'$) is added to the load needed to cause debonding of the fiber:

$$F_{pullout} = \left(\tau_{bond} \times \pi \times D_f \times \frac{L_{f,straight}}{2} \right) + \Delta P' \quad (10)$$

The parameter $L_{f,straight}$ is the length of the straight portion of the fiber.

4.3 Pullout Strength of Crimped Fibers

In order to take into account the enhancement in pullout strength provided by the deformed shape of a crimped fiber, the present model suggests using a “deformation contribution factor”, α_m which scales the expected pullout strength of a straight smooth fiber as shown in Eq. 11. Using data on the pullout strength of crimped fibers in the literature, α_m was taken as the average of the ratio of the bond strength of the deformed fiber (taken as measured pullout load divided by the surface area of the fiber) and the expected bond strength of a straight smooth fiber. Based on this analysis, α_m was estimated to be 2.84 for the case of crimped fibers.

$$F_{pullout} = \tau_{bond} \times \pi \times D_f \times \frac{L_f}{2} \times \alpha_m \quad (11)$$

In the present work the simplified formula shown below was developed to estimate the bond-shear strength, τ_{bond} :

$$\tau_{bond} = 1.11 \times (f_{cr})^{1.354} \quad (12)$$

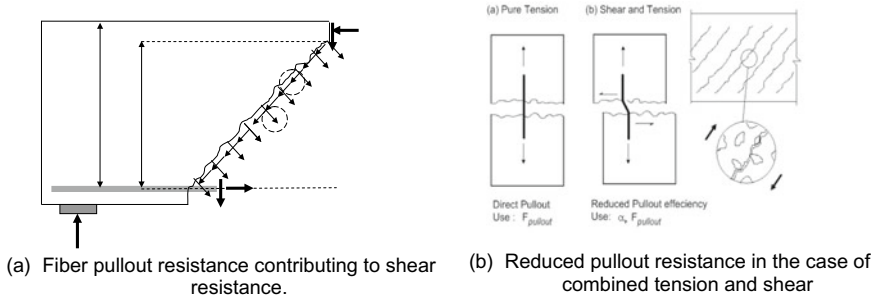


Fig. 1 Fiber pullout resistance mechanisms. Aoude [3], Cohen [6]

where f_{ct} is the tensile strength of concrete and can be found using the following equation:

$$f_{ct} = 0.33\sqrt{f'_c}(\text{MPa}) \tag{13}$$

4.4 Fiber Contribution to Shear Resistance

In order to account for the influence of the fibers on shear strength one can examine the free-body diagram of an SFRC beam with a crack inclination θ (see Fig. 1a). The fiber contribution to shear resistance, V_{fib} , can be related to the pullout strength of the fibers crossing this cracking plane as shown in Eq. 14:

$$V_{fib} = \psi \times K \times (N_{fibers} \times \alpha_v F_{pullout}) \times b_w d_v \cot\theta \tag{14}$$

The effective number of fibers per unit area, N_{fibers} , for fibers randomly oriented in three dimensions can be calculated using Eq. 15:

$$N_{fibers} = \frac{V_f}{A_f} \times \alpha_\theta \times \eta_1 \tag{15}$$

where A_f is the cross-sectional area of the fiber, and where V_f is the volume fraction of fibers in the matrix. The orientation factor, α_θ , is used to account for the random orientation of the fibers crossing any arbitrary cracking plane and is taken as 3/8. The length factor η_1 is used to account for the variability in the fiber embedment length across the cracking plane and is taken as 0.5.

The pullout strength, $F_{pullout}$, is valid for the situation of pure tension pullout. However, in a beam, the fibers will resist tension along the diagonal cracks while undergoing a shear deformation (see Fig. 1b). Hence, in the proposed model it

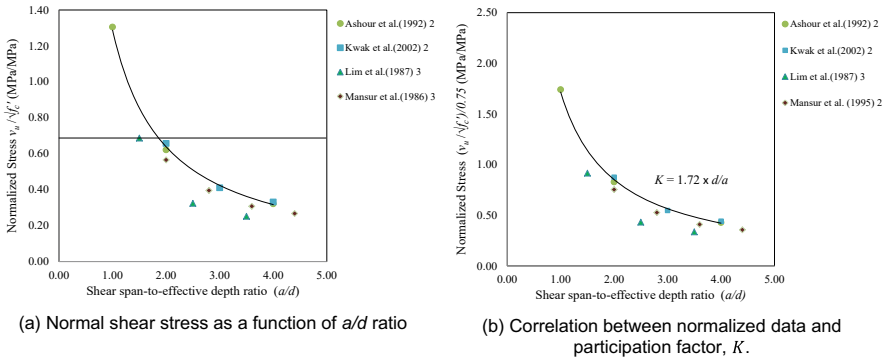


Fig. 2 The calculation of the participation factor, K . Cohen [6]

is assumed that the fibers will have reduced pullout efficiency in the situation of combined tension and shear. Using regression analysis, an empirically determined modification factor “Pullout Reduction Factor” of 0.8, α_v , was used to reduce the pullout resistance of the fibers (the pullout reduction factor was previously proposed in the model of Aoude [3]).

Previous studies have shown that the beam’s shear span-to-effective depth ratio (a/d) has a significant influence on the shear resistance of SFRC beams. In the proposed model a factor, K , is used to account for the effect of the (a/d) ratio as shown in Eq. 16. This expression was determined based on regression analysis using a large database of experimental results from the literature.

$$K = 1.72 \times \frac{d}{a} \tag{16}$$

Figure 2a shows the relationship between normalized shear stress and a/d ratio from four different studies in the literature. It can be seen that as the a/d ratio increases, the normalized shear stress reduces. It can also be seen that at an a/d ratio of 2.0, the normalized shear strength of the beams approaches a value of 0.75. Assuming that the fiber participation is full at an a/d ratio of 2.0 and reduces beyond the point, Fig. 2b plots the relationship of normalized shear strength divided by 0.75 as a function of a/d ratio. It can be seen that the empirically-derived expression for K gives a reasonably good correlation to the experimental data reported in the literature.

Research has shown that the shear stress at failure in RC beams without web reinforcement reduces as the size increase. Some researchers have postulated that this “size effect” is eliminated in SFRC beams. Although there is very limited data on the shear behaviour of larger SFRC beams, various studies in the literature indicate that there may be a size effect in larger SFRC beams. In the proposed model this size effect is taken into account using the size effect factor, ψ . The expression shown in Eq. 17 is the same expression suggested by [23] for accounting for size effect in traditional RC beams:

$$\psi = 1.2 - 0.2 \frac{a}{d} \tag{17}$$

5 Prediction of Shear Strength of SFRC Beams

In this section, the models proposed by other researchers and the author are used to predict the shear capacities of a large database of SFRC beams from the literature. The database includes the results from 173 SFRC beams (120 beams with hooked-end fibers and 53 with crimped fibers), including 137 beams that failed in shear. It must be noted that some of the tests might have stopped earlier due to the premature failure or other reasons, and hence, the obtained shear strength might not be the actual strength of the RC beams. The database includes test results from SFRC beams having effective depths (d) ranging from 180 to 570 mm, shear-span to depth ratio (a/d) ranging from 1.0 to 6.0, compressive strength (f'_c) between 18 and 104 MPa, longitudinal reinforcement ratios (ρ) ranging from 0.4 to 5%, fiber volume fractions (V_f) ranging from 0.25 to 2% and having straight, hooked or crimped fibers. It should also be noted that many of the experimental tests available in the literature present the results of beams that have small a/d ratio. Such beams carry load by strut and tie action and in such cases the strength of the beams is strongly influenced by the details near the supports [7]. From the database of 173 beam test results taken from the literature, 129 beams have a/d ratio greater than 2.3 (with 97 beams in this category failing in shear). Figures 3 and 4 show the predictions of the beams using four different models proposed in the literature and considering all 173 beams in the database. It is very evident that the majority of the models proposed in the literature show high scatter. Finally, Figs. 5a and 6a compare the experimental shear capacities

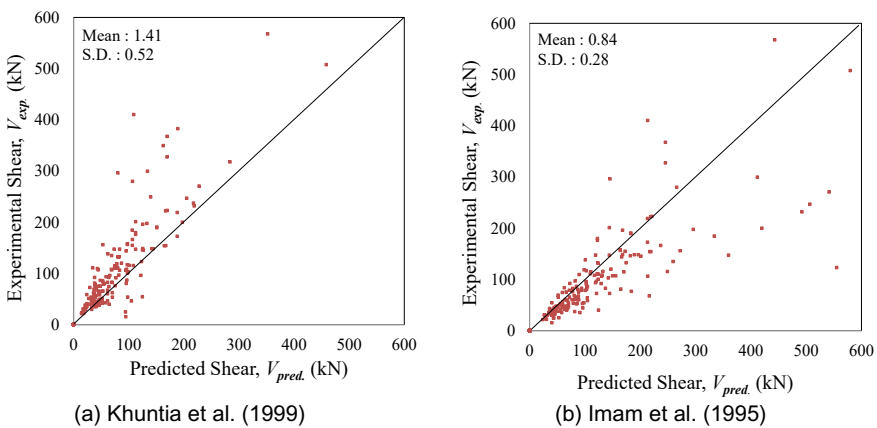


Fig. 3 Experimental versus predicted shear capacities using equations suggested by Khuntia and Imam et al. and including all beams in the database. Cohen [6]

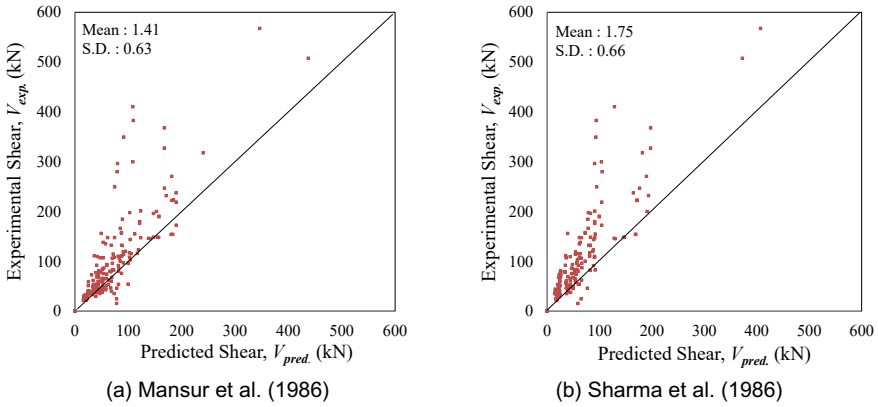


Fig. 4 Experimental versus predicted shear capacities using equations suggested by Mansur and Sharma et al. and including all beams in the database. Cohen [6]

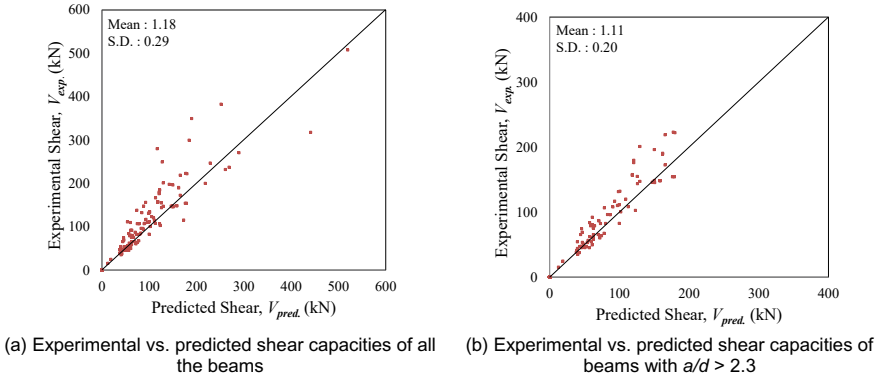
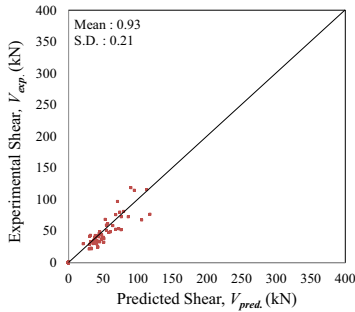


Fig. 5 Experimental versus predicted shear capacities using the proposed model for hooked-end fiber. Cohen [6]

of all 173 beams in the database to those predicted using the analytical models for hooked-end and crimped fibers proposed in this paper. Figures 5b and 6b compare the experimental and predicted shear capacities of only those beams with a/d ratio greater than 2.3. One can see that the predictions using the models proposed in the present study agree reasonably well with the actual capacities for the majority of the beams for the case of slender beams.



(a) Experimental vs. predicted shear capacities of all the beams

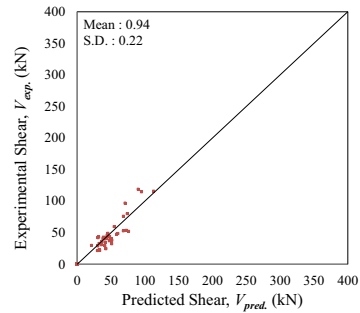
(b) Experimental vs. predicted shear capacities of beams with $a/d > 2.3$

Fig. 6 Experimental versus predicted shear capacities using the proposed model for crimped fiber. Cohen [6]

6 Conclusion

Steel Fiber Reinforced Concrete is a promising material which offers many advantages that can be adopted for structural applications. This paper proposed a reliable method to predict the shear strength of SFRC beams. Based on an evaluation of a large database of results, existing equations in the literature are empirical and many do not accurately predict the shear resistance of SFRC beams, there is thus a need to develop reliable and accurate prediction equations [19]. An analytical model which can predict the ultimate shear resistance of SFRC beams was proposed. The model accounts for: (1) the effect of fiber deformation on pullout strength (with equations developed for both hooked end and crimped steel fibers); (2) the influence of bond-shear strength on pullout strength (with an equation which computes bond strength based on matrix strength); (3) the influence of shear span-to-depth ratio (a/d) on the shear strength of SFRC beams (with a factor that accounts for a/d ratio); (4) the influence of beam size on the shear strength of SFRC beams (with a factor that accounts for the effect of increased beam size). The results show that the analytical model provides accurate and reliable predictions for a large database of SFRC beam test results reported in the literature. The model also provides reasonable predictions for SCFRC beams.

References

1. ACI Committee 318 (2008) Building code requirements for structural concrete (ACI 318-08) and commentary. American Concrete Institute, 473, Farmington Hills, Michigan, USA
2. Alwan JM, Naaman AE, Guerrero P (1999) Effect of mechanical clamping on the pullout response of hooked steel fibers embedded in cementitious matrices. *Concr Sci Eng* 1(1):15–25
3. Aoude H (2008) Structural behaviour of steel fiber reinforced concrete members. Ph.D. Thesis, McGill University, Montreal, Canada: Department of Civil Engineering and Applied Mechanics

4. Ashour SA, Hasanain GS, Wafa FF (1992) Shear behaviour of high-strength fiber reinforced concrete beams. *ACI Struct J* 89(2):176–184
5. Cassanova P (1996) Bretons renforcées de fibres métalliques: du matériau à la structure. Etude expérimentale et analyse du comportement de poutres soumises à la flexion et à l'effort tranchant. *Matériaux*, Ecole Nationale des Ponts et Chaussées, Paris, France
6. Cohen M (2012) Structural behaviour of self consolidating steel fiber reinforced concrete beams. Master's Thesis, Civil Engineering Department, Ottawa, Canada: University of Ottawa
7. Collins MP, Mitchell D (1997) *Prestressed concrete structures*. Response Publication, Toronto and Montreal, Canada
8. Imam M, Vandewalle L, Mortelmans F (1995) Shear-moment analysis of reinforced high strength concrete beams containing steel fibers. *Can J Civ Eng* 22(3):462–470
9. Khuntia M, Stojadinovic B, Goel SC (1999) Shear strength of normal and high-strength fiber reinforced concrete beams without stirrups. *ACI Struct J* 96(2):282–289
10. Kwak YK, Eberhard MO, Kim WS, Kim J (2002) Shear strength of steel fiber-reinforced concrete beams without stirrups. *ACI Struct J* 99(4):530–538
11. Li VC, Ward R, Hamza AM (1992) Steel and synthetic fibers as shear reinforcement. *ACI Mater J* 89(5):499–508
12. Mansur MA, Ong KC, Paramasivam AP (1986) Shear strength of fibrous concrete beams without stirrups. *ASCE J Struct Eng* 112(9):2066–2079
13. Minelli F, Plizzari GA (2006) Steel fibers as shear reinforcement for beams. In: *Proceedings of the second fib congress*. Naples, Italy 12
14. Minelli F (2005) Plain and fiber reinforced concrete beams under shear loading. Ph.D. Thesis, Department of Civil Engineering, Italy: University of Brescia
15. Narayanan R, Darwish IY (1987) Use of steel fibers as shear reinforcement. *ACI Struct J* 84(3):216–227
16. Para-Montesinos GJ (2006) Shear strength of beams with deformed steel fibers. *J Am Concr Inst* 28(11):57–66
17. Rahman J, Ahmed KS, Khan NI, Islam K, Mangalathu S (2021) Data-driven shear strength prediction of steel fiber reinforced concrete beams using machine learning approach. *Eng Struct* 233:111743
18. Shahnewaz M, Alam MS (2014) Improved shear equations for steel fiber reinforced concrete deep and slender beams. *ACI Struct J* 111(4):851–860
19. Shahnewaz M, Alam MS (2020) Genetic algorithm for predicting shear strength of steel fiber reinforced concrete beam with parameter identification and sensitivity analysis. *J Build Eng* 29(May):101205
20. Sharma AK (1986) Shear strength of steel fiber reinforced concrete beams. *J Am Concr Inst* 83(4):624–628
21. Shin SW, Oh JG, Ghosh SK (1994) Shear behaviour of laboratory-sized high-strength concrete beams reinforced with bars and steel fibers. *ACI Spec Publ* 142:181–200
22. Swamy RN, Jones R, Chiam AT (1993) Influence of steel fibers on the shear resistance of lightweight concrete I-beams. *ACI Struct J* 90(1):103–114
23. Zaris PD, Papaakis GC (2001) Diagonal shear failure and size effect in RC beams without web reinforcement. *ASCE J Struct Eng* 127(7):733–742