Interval-Valued Fuzzy Parameterized Multi Fuzzy N-soft Set in Decision-Making

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1 Introduction

The SST [44] is extremely useful in a variety of situations. Molodtsov [44] developed the basic results of SST and successfully applied it to a variety of fields, including the smoothness of operators, operations analysis, Riemann integration, theory of game, probability, and so on. Later, Maji et al. [38] presented several new SST concepts, such as subset, complements, union, and intersection, as well as their implementations in DMPs. Ali et al. [16] identified some new operations on SST and demonstrated that De Morgan's laws apply to these new operations in SST. To solve the DMPs, Maji et al. [39] used SST for the first time. Recently, several authors later looked into the more broad properties and applications of SST [9, 11, 31, 32].

Many academics are interested in hybrid models, as seen by the aforementioned references. Many hybridization options for the recently created NSSs [31] model. This model's primary role is to broaden the scope of SST applications, which deal with qualities that resemble the universe of discourse. Because many real-world examples have insisted on their applicability, this paradigm constitutes a practical expansion of SST (see [11, 31, 32]). In addition, it has demonstrated its theoretical flexibility: the structure is adaptable to admixture with alternative theories of ambiguity and unpredictability. Akram et al. [3–7], Chen et al. [24], Liu et al. [33], and Riaz et al. [50] have built hybrid structures that incorporate other notable properties of approximation knowledge structures. An N-soft structure [49] exists as a natural extension of soft topology and is a natural extension of topological studies [9, 54].

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S. Mukherjee et al. (eds.), *Intelligent Systems for Social Good*,
Advanced Technologies and Societal Change,
https://doi.org/10.1007/978-981-19-0770-8_11

The idea of the FST was started by Zadeh [59], thereafter, many new approaches and ideas have been offered to deal with imprecision and ambiguity, such as the hesitant fuzzy sets (HFSs) [55], MFSs [52], intuitionistic fuzzy set theory (IFST) [19], intuitionistic MFSs [25], IVFSs [62] and so on [1, 2, 20, 21, 43]. FST has a wide range of applications, including databases, neural systems, pattern recognition, medicine, fuzzy modelling, economics, and multicriteria DMPs (see [14, 18]). Maji et al. [40] described fuzzy soft set (FSS) in 2001. An FSS-based DMP was proposed by Roy and Maji [51]. Several authors later looked into the more broad properties and applications of FSS; for example, see [17, 22, 23, 26, 58]. Alcantud and Mathew (2017) [12] recently defined separable FSS and its use in DMPs with both positive and negative qualities. Alcantud et al. [10] developed a methodology for asset assessment using an FSS (flexible FSS) based decision-making technique. Atanassov [19] suggested the notion of IFST as a generalization of FST [59]. Maji et al. [41, 42] presented intuitionistic FSS (IFSS) as an important mathematical method for solving DMPs in an uncertain situation by combining SST [44] with IFST [19]. Several authors later looked into the more broad properties and applications of IFST and IFSS; for example, see [34, 53].

The topic of intertemporal FSS selection was first raised by Alcantud and Muoz Torrecillas [15]. The algorithms for IVFSSs in stochastic multi-criteria DMPs and neutrosophic soft DMM were introduced by Peng et al. [47, 48]. Furthermore, using WDBA and CODAS with the help of new data measures, Peng and Garg [46] suggested unique algorithms for IVFSSs in emergency DMPs. In the case of SST, Zhan and Alcantud [60] provided a rationalized assessment of the parameter reduction novel. One or more of the following constraints limited the majority of SST researchers (for example [37] or other updated hybrid model summaries): The evaluations can either be binary integers between 0 and 1, or real values between 0 and 1, such as FSSs or separable FSSs [40].

Both scenarios are discussed in [13], which includes an examination of partial data. In scenarios such as social assessment structures or the arrangement of ranking positions in ordinary life, however, we encounter data with a different framework which not binary. NSSs [31] are, nonetheless, the accurate formal expression of the idea of parameterized description of the universe of items based on a limited figure of ordered grades, and the other extended structures of SST that have been linked to social choice were mentioned by Fatimah et al. [29, 30]. The idea of parameter reduction in NSSs was recently presented by Akram et al. [8]. When the membership degrees of the alternatives are not uniquely defined, such as due to group knowledge or hesitancy [57, 61], HFSs [55] are useful. Hesitancy is a model that can be combined with other key structures (See [28, 35, 36, 45, 56] for contemporary examples).

Recently, Fatimah and Alcantud [27] introduced the concept of MFNSS and developed a DMM for solving DMPs, but in their DMM, there have some limitations. They used MFNSS evaluated by only one DM, so this method is may not be useful in the modelling of group-DMPs. The new structure combines the advantages of MFSs [52] with those of NSSs [31] and IVFSs [62], these two structures that have received a lot of attention in current years. The constructed method in this chapter is very advantageous for solving group-DMPs. To demonstrate the applicability of our methodology in practical situations, some examples are used.

2 Preliminary

Let us consider Ω represents the starting universe and Q represents a nonempty collection of parameters with P ⊆ Q. Let the power set of Ω is denoted as P(Ω). Let $N \in \{2, 3, 4, 5, ...\}$ and $R = \{0, 1, 2, 3, 4, 5, ..., N - 1\}$.

Definition 2.1 [59] A fuzzy set (FS) Z on Ω having a structure $Z = \{(\mathrm{o}, \mu_Z(\mathrm{o})) : o \in \Omega\}$, where $\mu_z : \Omega \to [0, 1]$ is called membership function, and $\mu_z(\mathrm{o})$ is the membership value for each object $o \in \Omega$.

Assume that, in this chapter FS(Ω) means the collection of all FSs on Ω.

Definition 2.2 [44] A soft set over the nonempty universe Ω is a pair (ψ, P), where ψ is a mapping defined by ψ: P → P (Ω).

Definition 2.3 [53] An MFS Z on Ω is a set with a structure membership value for each object

where $\mu_{\mathrm{k}} : \Omega \to [0, 1][0, 1]$ for $k = 1, 2, 3,\ldots$,q are the real valued functions.

In this chapter, $MFs(\Omega)^q$ means the collection of all MFSs on Ω.

Definition 2.4 [31] A triple (Ψ, P, N) is called NSS on Ω, where $\Psi : \mathrm{P} \to 2^{\Omega \times R}$ is a function, satisfying the condition, for each p ∈ P and o ∈ Ω there exists a unique couple $(o, r_p) \in \Omega \times R$ such that $(o, r_p) \in \Psi(p)$, $r_p \in R$. The object o belongs to the collection of p-approximations of Ω with the grade r_p, according to the interpretation of the couple $(o, r_p) \in \Psi(p)$.

Definition 2.5 [5] A triple (*ψ, P, N*) is called a hesitant N-soft set (simply, H(N)SS) over *Ω,* where $\psi : \mathrm{P} \to 2^{\Omega \times R}$ is a function such that for every p ∈ P, o ∈ Ω there exists at least one couple $(o, r_p) \in \Omega \times R$ such that $(o, r_p) \in \psi(p)$, $r_p \in R$.

Definition 2.6 [5] The collection h satisfying the condition $\phi \neq h \subseteq R = \{0, 1, 2, 3, 4, 5, ..., N - 1\}$ is said to be hesitant N-tuple (simply, H(N)T). Any H(N)SS has a tabular form consisting of a matrix with cells H(N)Ts.

Definition 2.7 [62] An IVFS Γ on Ω having the form $\Gamma = \left\{\left(o, [\mu_\Gamma^L(o), \mu_\Gamma^U(o)]\right) : u \in \Omega\right\}$, where $\mu_\Gamma^L, \mu_\Gamma^U : \Omega \to [0, 1]$ are functions.

IVFS(Ω) denote the collection of all IVFSs on Ω. Let Γ, Ψ ∈ IVFS(Ω). Then.
[i] Union of Γ and Ψ is denoted by Γ ∪ Ψ where

$$\Gamma \cup \psi = \left\{\left(\text{o},\left[\max\left(\mu_{\Gamma}^{L}(\text{o})\mu_{\psi}^{L}(\text{o})\right),\ \max\left(\mu_{\Gamma}^{U}(\text{o})\mu_{\psi}^{U}(\text{o})\right)\right]\right) : o \in \Omega\right\}$$

[ii] Intersection of Γ and Ψ is denoted as $\Gamma \cap \Psi$ where

$$\Gamma \cap \psi = \left\{\left(\text{o},\left[\min\left(\mu_{\Gamma}^{L}(\text{o})\mu_{\psi}^{L}(\text{o})\right),\ \min\left(\mu_{\Gamma}^{U}(\text{o})\mu_{\psi}^{U}(\text{o})\right)\right]\right) : o \in \Omega\right\}$$

3 d(N, q)-Set and Its Theoretical Analysis

In this chapter, we consider Ω represents the starting universe and Q represents a nonempty set of parameters. Let $P \subseteq Q$ and $X = \left\{p^{[\mu_X^L(p),\mu_X^U(p)]} : p \in P\right\}$, be an IVFS on P. Let $q, N \in \{2, 3, 4, 5, ...\}$ be two fixed numbers, where q is the dimension of our new structure and N distinguishes how many degrees of satisfaction with the parameters are permitted, allowing us to utilize $R = \{0, 1, 2, 3, 4, 5, ..., N-1\}$ as a collection of ordered grades.

Definition 3.1 Let us define $MFs(\Omega)^{(N,q)}$ as the set of all q-tuples of triples of objects from $R \times [0, 1] \times [0, 1]$ indexed by Ω, i.e., the collection of all objects having the structure $\left\{\left\langle o, (r_1(o), \mu_1(o)), (r_2(o), \mu_2(o)), ..., (r_q(o), \mu_q(o))\right\rangle : o \in \Omega\right\}$, where $r_k : \Omega \to R$, $\mu_{\text{k}} : \Omega \to [0, 1]$ are functions for $k = 1, 2, 3,\ldots$,q. A *d(N,q)*-set on Ω is a pair (Ψ, X) such that Ψ is a mapping $\Psi : X \to MFs(\Omega)^{(N,q)}$ defined by

$$\forall p^{[\mu_X^L(p),\mu_X^U(p)]} \in X,$$
$$\Psi\left(p^{[\mu_X^L(p),\mu_X^U(p)]}\right) = \left\{\left\langle o, (r_1(o), \mu_1(o)), (r_2(o), \mu_2(o)), ..., (r_q(o), \mu_q(o))\right\rangle : o \in \Omega\right\}.$$

Note: Simply, we denote the collection of all *d(N,q)*-sets on Ω by $d(\Omega, P)^{(N,q)}$ where the parameter set P is fixed.

Example 3.2 Let $\Omega = \{o_1, o_2, o_3\}$ be the universe of candidates and $P = \{pm_1, pm_2, pm_3\}$ is the set of attributes and $X = \left\{pm_1^{[0.5,0.6]}, pm_2^{[0.7,0.8]}, pm_3^{[0.6,0.7]}\right\}$ be an IVFS over *P*. A d(5,2)-set (Ψ, X) on Ω is defined by the assignments (Table 1).

$$\Psi(p_1^{[0.5,0.6]}) = \{\langle o_1, (3, 0.3), (4, 0.5)\rangle, \langle o_2, (2, 0.3), (4, 0.5)\rangle, \langle o_3, (2, 0.4), (3, 0.4)\rangle\}$$

Table 1 The *d(5,2)*-set (Ψ, *X*)

	pm_1 [0.5,0.6]	pm_2 [0.7,0.8]	pm_3 [0.6,0.7]
01	(3,0.3)(4,0.5)	(1,0.4)(3,0.5)	(3,0.4)(2,0.5)
02	(2,0.3)(4,0.5)	(2,0.3)(4,0.5)	(2,0.5)(4,0.6)
03	(2,0.4)(3,0.4)	(3,0.5)(2,0.5)	(2,0.5)(1,0.7)

$$\Psi(p_2^{[0.7,0.8]}) = \{\langle o_1, (1, 0.4), (3, 0.5)\rangle, \langle o_2, (2, 0.3), (4, 0.5)\rangle, \langle o_3, (3, 0.5), (2, 0.5)\rangle\}$$

$$\Psi(p_3^{[0.6,0.7]}) = \{\langle o_1, (3, 0.4), (2, 0.5)\rangle, \langle o_2, (2, 0.5), (4, 0.6)\rangle, \langle o_3, (2, 0.5), (1, 0.7)\rangle\}$$

Definition 3.3 Let us consider two d(N,q)-sets $(\psi, X), (\varphi, Y) \in d(\Omega, P)^{(N,q)}$, such that $\forall p^{[\mu_X^L(p), \mu_X^U(p)]} \in X, p^{[\mu_Y^L(p), \mu_Y^U(p)]} \in Y, \psi\left(p^{[\mu_X^L(p), \mu_X^U(p)]}\right) = \{\langle o, (r_1(o), \mu_1(o)), (r_2(o), \mu_2(o)), \ ..., (r_q(o), \mu_q(o))\rangle : o \in \Omega\}\varphi\left(p^{[\mu_Y^L(p), \mu_Y^U(p)]}\right) = \left\{\langle o, (r_1'(o), \mu_1'(o)), (r_2'(o), \mu_2'(o)), \ ..., (r_q'(o), \mu_q'(o))\rangle : o \in \Omega\right\}$.

Then we say that

[1] **Subset**: $(\psi, X)\tilde{\subseteq}(\varphi, Y)$ if

$$(i).\, \forall p \in P,\ \mu_X^L(p) \le \mu_Y^L(p) \text{ and } \mu_X^U(p) \le \mu_Y^U(p)]$$

$$(ii).\, \forall p \in P,\ \psi\left(p^{[\mu_X^L(p), \mu_X^U(p)]}\right) \subseteq \varphi\left(p^{[\mu_Y^L(p), \mu_Y^U(p)]}\right) \Leftrightarrow r_i(o) \le r_i'(o) \text{ and } \mu_i(o) \le \mu_i'(o)$$
$$\forall o \in \Omega,\ \text{and } i = 1, 2, ..., q$$

[2] **Union**: $(\psi, X)\tilde{\cup}(\varphi, Y) = (\rho, Z)$, where $\forall p^{[\mu_Z^L(p), \mu_Z^U(p)]} \in Z$,

$\rho\left(p^{[\mu_Z^L(p), \mu_Z^U(p)]}\right) = \left\{\langle o, (r_1''(o), \mu_1''(o)), \ ..., (r_q''(o), \mu_q''(o))\rangle : o \in \Omega\right\}$, where

$\forall_o \in \Omega, r_i'(o) = \min\left\{r_i(o), r_i'(o)\right\}, \mu'' = \min\left\{\mu_i(o), \mu_i'(o)\right\}, i = 1, 2, \ldots, q$, and $\mu_Z^L(p) = \min\left\{\mu_X^L(p), \mu_Y^L(p)\right\}, \mu_Z^U(p) - \min\left\{\mu_X^U(p), \mu_Y^U(p)\right\}$

[3] **Intersection**: $(\psi, X)\tilde{\cap}(\varphi, Y) = (\rho, Z)$, where $\forall p^{[\mu_Z^L(p), \mu_Z^U(p)]} \in Z$,

$$\rho\left(p^{[\mu_Z^L(p), \mu_Z^U(p)]}\right) = \left\{\langle o, (r_1''(o), \mu_1''(o)), (r_2''(o), \mu_2''(o)), \ ..., (r_q''(o), \mu_q''(o))\rangle : o \in \Omega\right\},$$
$$\forall o \in \Omega,\ r_i''(o) = \min\{r_i(o), r_i'(o)\},\ \mu_i''(o) = \min\{\mu_i(o), \mu_i'(o)\},\ i = 1, 2, ..., q,$$
$$\text{and } \mu_Z^L(p) = \min\left\{\mu_X^L(p), \mu_Y^L(p)\right\},\ \mu_Z^U(p) = \min\left\{\mu_X^U(p), \mu_Y^U(p)\right\}.$$

Definition 3.4 We consider a *d(N,q)*-set $(\Psi, X) \in d(\Omega, P)^{(N,q)}$. Its induced interval-valued fuzzy parameterized hesitant N-soft set of dimension q (briefly, *h(N,q)*-set) is the pair (H_Ψ, X), where $H_\Psi : X \to P(R)$ is a mapping, such that $H_\Psi(p_j^{[\mu_X^L(p), \mu_X^U(p)]}) = \left\{\left\langle o_i, \left\{r_1^j(o_i), r_2^j(o_i), ..., r_q^j(o_i)\right\}\right\rangle : o_i \in \Omega\right\}$, $\forall p^{[\mu_X^L(p), \mu_X^U(p)]} \in X$.

Definition 3.5 Let us fix $(\Psi, X) \in d(\Omega, P)^{(N,q)}$ and $\alpha = (\alpha_1, \alpha_2, ..., \alpha_m)$, a vector of thresholds with $\alpha_i \in [0, 1]$, i = 1, 2,…m. Then the *h(N,q,α)*-set induced from (Ψ, X) is the triple $\left(H_\Psi, X, \alpha\right)$, where $H_{\Psi} : X \rightarrow P(R)$ is a mapping, such that $H_{\Psi}(p_j^{[\mu_X^L(p),\mu_X^U(p)]}) = \left\{\left\langle o_i, \left\{r_t^j(o_i) : \mu_t^j(o_i) \geq \alpha_j, t = 1, 2, ..., q\right\}\right\rangle : o_i \in \Omega\right\}$.

Definition 3.6 Let X be an IVFS and $\delta, \gamma \in [0, 1]$ such that $\delta+\gamma= 1$. Then a soft fuzzification operator $S_{(\delta,\gamma)}$ on IVFS X is an FS denoted as $S_{(\delta,\gamma)}(X)$ and defined by $S_{(\delta,\gamma)}(X) = \left\{p^{\mu_{S_{(\delta,\gamma)}(X)}(p)} : p \in P\right\}$, where $\mu_{S_{(\delta,\gamma)}(X)}(p) = \delta\mu_X^L(p) + \gamma\mu_X^U(p)$.

By adjusting the value of γ, δ an IVFS can be transformed into any FS. Specially, if $\gamma = 0$, $\delta = 1$, then the pessimistic-pessimistic FS, defined by $S_{(1,0)}(X) = \left\{p^{\mu_X^L(p)} : p \in P\right\}$.

If $\gamma = 1$, $\delta = 0$, then the optimistic-optimistic FS, defined by $S_{(0,1)}(X) = \left\{p^{\mu_X^U(p)} : p \in P\right\}$.

If $\gamma = 0.5$, $\delta = 0.5$, then the neutral–neutral FS, defined by $S_{(0.5,0.5)}(X) = \left\{p^{\frac{\mu_X^L(p)+\mu_X^U(p)}{2}} : p \in P\right\}$.

4 DMM Based on *d*(*N*,*q*)-Sets and *h*(*N*,*q*,*α*)-Sets

Now, we present our machine learning algorithm for solving group DMPs based on d(N,q)-sets and *h(N,q,α)*-sets. The steps of our proposed DMM are listed below:

Algorithm 1

Step 1: Enter a nonempty universe $\Omega = \{o_1, o_2, ..., o_n\}$, a set of parameters $P = \{p_1, p_2, ..., p_m\}$ and a group of DMs {M1, M2,…, Mk}.
Step 2: Enter the corresponding decision maker's opinions (*d(N,q)*-sets) $(\Psi_1, X_1), (\Psi_2, X_2), ..., (\Psi_k, X_k)$ respectively.
Step 3: Enter vector α^{λ} of thresholds corresponding to each DM M_{λ}, λ = 1,2,…,k, i.e., a threshold $\alpha_j^{\lambda} \in [0, 1]$ with respect to every parameter $p_j \in P$.
Step 4: Obtain the resultant *d(N,q)*-set (Ψ, X) (using union or intersection) and resultant threshold $\alpha = \left\{\alpha_j \in [0, 1], j = 1, 2, 3, ..., m\right\}$ where α_j associated with each attribute $p_j \in P$(using any FS operation).
Step 5: Obtain the *h(N,q,α)*-set (H_{Ψ}, X, α) in its tabular form.
Step 6: Obtain the scores $\Gamma(h_j(o_i))$ of all the H(N)Ts by taking any operation (say, arithmetic mean, geometric mean, or harmonic mean), $\forall o_i \in \Omega$, and j = 1, 2, ..., m.
Step 7: Enter the values of and $\delta, \gamma \in [0, 1]$ such that $\delta+\gamma= 1$ obtain $S_{(\delta,\gamma)}(X)$.
Step 8: Compute $u_i = \sum_{j=1}^{m} \mu_{S_{(\delta,\gamma)}(X)}(p_j)\times\Gamma(h_j(o_i))$, $\forall o_i \in \Omega$.
Step 9: The best optimal choice is to select o_s if u_s is maximized.
Step 10: If o_s has many values, any of o_s may be selected.

Remark 4.1 In the 9th-step of our constructed DMM, one can return to the 3rd or 4th steps or 7th step and change the thresholds α^{λ} or operations or the values of γ, δ respectively that he previously used to change the optimal choice, particularly when there are lots of optimal decisions to choose from.

5 Result and Discussions

Example 5.1 Let $\Omega = \{o_1, o_2, o_3, o_4\}$ be the universe of candidates, and $P = \{pm_1, pm_2, pm_3\}$ is the set of attributes. We suppose that there are two DMs such as D1, D2 and their observations $(\psi, X), (\varphi, Y) \in d(\Omega, P)^{(5,3)}$ as in Tables 2 and 3 respectively. Also, let {0.5, 0.5, 0.6} and {0.4, 0.4, 0.5} be thresholds corresponding to the DMs D1 and D2 respectively. The resultant *d(5,3)*-set (Ψ, *Z*) (using union) as in Table 4 and the resultant threshold is α = {0.5, 0.5, 0.6} (using FS-union). We obtain h(5, 3, α) as shown in Table 5. We use the arithmetic score on H(N)Ts. Table

Table 2 The *d*(5,3)-set (*ψ*, *X*)

Ω	pm_1 [0.4,0.5]	pm_2 [0.5,0.6]	pm_3 [0.7,0.8]
01	(3,0.3)(4,0.5)(2,0.6)	(1,0.4)(4,0.3)(2,0.5)	(2,0.5)(3,0.3)(2,0.6)
02	(2,0.3)(1,0.5)(3,0.5)	(2,0.3)(4,0.5)(3,0.5)	(3,0.6)(4,0.4)(2,0.4)
03	(2,0.4)(3,0.4)(1,0.5)	(1,0.5)(2,0.5)(4,0.4)	(3,0.5)(2,0.3)(1,0.5)
04	(1,0.5)(3,0.6)(3,0.4)	(1,0.4)(2,0.6)(3,0.4)	(4,0.3)(2,0.6)(3,0.4)

Table 3 The *d(5,3)*-set (*φ*, *Y*)

Ω	pm_1 [0.5,0.6]	pm_2 [0.6,0.7]	pm_3 [0.6,0.7]
01	(2,0.4)(3,0.6)(2,0.5)	(3,0.4)(2,0.4)(1,0.6)	(4,0.4)(2,0.5)(1,0.5)
02	(3,0.4)(4,0.5)(3,0.6)	(3,0.4)(3,0.6)(2,0.4)	(3,0.5)(4,0.5)(2,0.6)
03	(2,0.3)(4,0.4)(1,0.4)	(3,0.5)(4,0.4)(2,0.4)	(4,0.4)(2,0.4)(3,0.6)
04	(2,0.4)(4,0.5)(3,0.5)	(4,0.6)(2,0.5)(3,0.6)	(4,0.5)(2,0.5)(3,0.4)

Table 4 The *d(5,3)*-set (Ψ, *X*)

Ω	pm_1 [0.5,0.6]	pm_2 [0.6,0.7]	pm_3 [0.7,0.8]
01	(3,0.4)(4,0.6)(2,0.6)	(3,0.4)(4,0.4)(2,0.6)	(4,0.5)(3,0.5)(2,0.6)
02	(3,0.4)(4,0.5)(3,0.6)	(3,0.4)(4,0.6)(3,0.5)	(3,0.6)(4,0.5)(2,0.6)
03	(2,0.4)(4,0.4)(1,0.5)	(3,0.4)(4,0.5)(4,0.4)	(4,0.5)(2,0.4)(3,0.6)
04	(2,0.5)(4,0.6)(3,0.5)	(4,0.6)(2,0.6)(3,0.6)	(4,0.5)(2,0.6)(3,0.4)

Table 5 The $h(5,3,\alpha)$-set $\left(H_{\Psi}, X, \alpha\right)$

Ω	pm_1 [0.5,0.6]	pm_2 [0.6,0.7]	pm_3 [0.7,0.8]
01	{4, 2}	{2}	{2}
02	{4, 3}	{4, 3}	{2}
03	{1}	{3, 4}	{3}
04	{2, 4, 3}	{4, 2, 3}	{2}

Table 6 $S_{(1,0)}(X)$ and the scores $\Gamma\left(h_j(o_i)\right)$ with u_i

Ω	pm_1 0.5	pm_2 0.6	pm_3 0.7	u_i
01	3	2	2	4.1
02	3.5	3.5	2	5.25
03	1	3.5	3	4.7
04	4.5	4.5	2	6.35

6 shows the results of the computations at steps 6, 7, and 8. Step 9 suggests that the option 04 be chosen in the last column.

Another example is one in which the constituent pieces have distinct specifications. This demonstrates Algorithm 1 adaptability and versatility.

6 Comparison Analyses

The DMM given in [27] is good for solving DMPs, but in their DMM they used MFNSS evaluated by only one DM, so these methods are may not be useful in the modelling of group-DMPs, but the constructed method in this chapter is very advantageous for solving group-DMPs. Also, the importance of the weights of parameters is considered by only one DM in [27], but in our constructed DMM, every DM can consider their own weights with the parameters so that every DM can give the importance of parameter selections according to their choice.

7 Conclusions

IVFSs can effectively represent and simulate the uncertainty and diversity of judgment information offered by DMs. In comparison to FSs, IVFSs are highly beneficial for expressing vagueness and uncertainty more accurately. As a result, in this research work, we offer an approach for solving group DMPs with d(N,q)-sets by extending the MFNSS based DMM. The DMM given in [27] is good for solving DMPs, but in their DMM they used MFNSS evaluated by only one DM, so these methods are may not be useful in the modelling of group-DMPs, but the constructed method in this chapter is very advantageous for solving group-DMPs. Also, the importance of the weights of parameters is considered by only one DM in [27], but in our constructed DMM, every DM can consider their own weights with the parameters so that every DM can give the importance of parameter selections according to their choice.

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