Comparison of Accuracy in Prediction of Radial Strain in Stone Columns Using AI Based Models

Tanwee Mazumder[1](✉) and Ankit Garg[2]

[1] Department of Civil Engineering, Indian Institute of Technology Madras, Chennai 600036, India
[2] Guangdong Engineering Center for Structure Safety and Health Monitoring, Shantou University, Shantou, China
ankit@stu.edu.cn

Abstract. Ground improvement of soft soil with construction of stone columns has been widely adopted. Lateral deformation of stone columns plays a significant role in behavior of columns. This study aims to explore the applicability of different AI techniques/mathematical models in predicting radial strain (ε) (change in radius/original radius of column) in stone columns as a function of significant input parameters viz. diameter (d) of stone column, *l/d* ratio, *s/d* (spacing/diameter) ratio, area ratio (A_r), λ (area of stone column/total area of loading), geosynthetic stiffness (k), β (clearance ratio). The radial strain (ε) in ordinary and encased columns is predicted with the help of linear regression, SVM, GPR and ANN models using Matlab software. The datasets of input parameters are obtained from already published literature. The values predicted by the models are compared to the corresponding true values of radial strain reported in the literature. A comparative analysis of the efficiency of all models is examined in terms of RMSE, R-squared, MSE and MAE values. It was observed that ANN models closely predicted the radial strain in columns with higher accuracy as compared to other models. ANN models may therefore be used to predict radial strain even in larger size columns in the field/in-situ conditions. However, these models are put forward as a complementary technique to evaluate the radial strain in columns and not as a substitute to field tests.

Keywords: AI techniques · ANN · Stone column · Radial strain

1 Introduction

Ground improvement of soft soil sites with construction of stone columns has been widely adopted across the globe. Construction of stone columns helps to increase the load bearing capacity and decrease the settlement of soft ground [1–5]. Numerous studies have been carried out to study the behavior of stone columns with different geometrical patterns and arrangement [4–9]. Various studies on design procedures and methodologies [2, 3, 7–12] have been established to enable a safe design of such columns. Diverse factors like diameter and length of stone columns, spacing between columns, area replacement

ratio [4, 5, 7, 9, 13, 14]; arrangement of columns and area of load application [6–9, 15]; length of encasement and encasement stiffness [3, 4, 9, 14, 16, 17] influence the behavioural characteristics of stone columns.

The bulging of stone columns significantly influence column behaviour and hence considered in design of stone columns. The load bearing capacity of the column is generated from lateral earth pressure mobilised by the surrounding soil against the bulging of columns [1, 2, 13]. In soft clay, the inability to resist bulging due to the extremely low shear strength of the clayey soil causes failure of such columns [4]. The confinement provided by encasements to stone columns constructed in very soft soil ($c_u \leq 15$ kPa) significantly cause reduction in column bulging [3, 7, 18]. Murugesan and Rajagopal [3] have found that with encasement the lateral deformation or bulging of columns reduced significantly and the trend of radial deformation is similar to the pattern of variation of hoop strain in columns. The study observed that maximum bulging occurred over a depth equal to twice the diameter of column and providing encasement over this range of depth would be sufficient to arrest column bulging. The geosynthetic encasement stiffness plays an important role in increasing the column stiffness by increasing the lateral confining stress. With increase in lateral confining stress in columns the lateral stress transferred to the surrounding soil decreased. It was noted that smaller diameter columns perform better than larger diameter columns due to generation of higher confining stress.

The depth of predominant bulging has been reported by various studies and it falls closely in the range of 1–4 times the diameter of columns. Irrespective of being an end-bearing or floating column, long unreinforced columns fail by bulging while short columns fail by punching [4]. The study also found that with an increase in diameter of columns, the failure stress increased with construction of reinforced or unreinforced stone columns. In layered ground it was observed that in case of column area loading the top soft soil stratum influenced significantly the limiting axial stress of the columns. On the contrary, for entire area loading, stiffness and load carrying capacity of column reinforced ground decreased as the thickness of top strata of weaker soil increased [19]. The length of columns invariably affects the column performance and failure. Malarvizhi and Ilamparuthi [15] reported that the load capacity of stone column increased with an increase in length of columns. From the review of literature it was found that stone columns having *l/d* ratio less than 4 fail by punching while bulging failure occurs in the columns having *l/d* > 4 [12]. Therefore, it is noted that a wide variety of factors like diameter and length of columns, arrangement of columns, area of load application, encasement stiffness significantly influence the lateral deformation/bulging of columns. Numerical simulations can be performed to evaluate the influence of each factor. However, for engineering purpose, it is more realistic to develop a simple model that can estimate radial strain as a function of all such factors (diameter and length of columns, spacing between columns, area replacement ratio, area of load application, type of columns i.e. floating or end-bearing nature of columns and encasement stiffness). Such modelling is less time consumable and gives an approximate idea of bulging characteristics. This study aims to predict the radial strain in ordinary and encased stone columns as a function of such factors using different numerical models/Artificial Intelligence (AI) techniques.

2 Methodology

2.1 Theory of Artificial Intelligence (AI)

The AI techniques attempts to solve the problems with training/learning from the input and output datasets presented. While training, it generates/devises the subtle functional relationships between the data/training examples without explaining the physical laws or determining the underlying relationships that govern the interdependence between the datasets [20]. Hence, AI models are entirely dependent on the training data to obtain the functional relationships as well as establish the parameters or structure of a system with minimum considerations of the physical laws. In case of models based on physical laws between different parameters, a prior knowledge of the governing relationships between various factors, as well as the assumptions and theories involved is necessary unlike AI techniques. This enhances the utility of such AI techniques as compared to other physically based empirical and statistical models in the application of problems where the target output can be evaluated [20]. Also, it was found that the AI models performed comparably similar or better than the traditional models [20].

2.2 Applications of AI in Geotechnical Engineering

Various studies on applications of AI techniques in geotechnical engineering have been carried out over the past few decades. Goh [21] calculated the friction capacity of piles in soft clay with the help of ANN models. The model was trained with data obtained from field tests. Subsequent studies were carried on application of ANN models to develop a correlation between undrained shear strength of soil, effective overburden pressure, undrained side resistance factor in case of drilled shafts [22]; to predict the axial load carrying capacity of driven piles and drilled shafts based on insitu/field tests; prediction of bearing capacity of stone columns [23] and so on. Studies have also been conducted to calculate the settlement of stone columns with the help of ANN models [24, 25], predict the bearing capacity of unreinforced as well as geogrid-reinforced sand bed constructed on stone columns in soft clay using SVR models [26]. Hence, application of AI based approaches/techniques in geotechnical engineering is wide.

2.3 Consideration of Input Parameters

As mentioned in previous section, various factors play a significant role in influencing the lateral deformation/bulging of stone columns. Encasing of columns to significantly reduce column bulging has been well established. Also *l/d* ratio and *s/d* ratio has a remarkable influence on the lateral deformation of columns. To account for the end-bearing as well as floating columns, β (clearance ratio) (*depth between the bottom of the column and test tank/diameter of column*) is taken as an input parameter. Thus, for end-bearing columns, the clearance ratio is 0. Area ratio (A_r) (area of stone column/total area of loading) were considered as other significant input parameters to take into account the variation in stone column arrangement and area of load application in evaluating the

radial strain. The stiffness of geosynthetic used as encasements for stone columns play a remarkable role in the radial strain developed in stone column. Hence, geosynthetic stiffness (k) was considered as an input parameter to evaluate the maximum radial strain and its corresponding depth.

Data collected from published literature [4, 5, 7, 8, 14–16, 19, 27–36] was extensively used to evaluate the ε of ordinary and encased stone columns in the present study. However, considerably fewer studies have reported the radial strain in columns. The parameters considered in the study include data from experimental/model tests as well as numerical studies. The range of input parameters was taken into consideration taking account of the usual ranges adopted in model studies/large scale tests. The range of variation of each of these parameters is presented in Table 1.

Table 1. Range of input parameters

Range	Diameter (mm)	l/d	s/d	$A_{r\,(\%)}$	λ (%)	Geosynthetic stiffness (kN/m)	Clearance ratio (β)
Maximum	1000	20	12	44	100	10000	12
Minimum	20	1	1	0.7	0.1	0	0

In the present study, Linear Regression, Support Vector Machine (SVM), Gaussian Process Regression (GPR) and Artificial Neural Network (ANN) models were built with the help of deep learning tools/Regression Learner app in *Matlab* software. The different models are described briefly in the following sections.

3   AI Techniques/Mathematical Models

Linear Regression Models
Linear regression models comprise of predictors that are linear in the model parameters. These models are simple to understand and enable quick predictions. However, these usually have low accuracy of prediction. The different linear regression models incorporated in the study include Linear, Interactions linear, Robust Linear and Stepwise Linear model (Matlab [37]).

Support Vector Machine Models
Support vector machine (SVM) analysis is a popular machine learning tool employed for classification and regression analysis. SVM regression is based on kernel functions and hence considered to be a nonparametric technique. SVM models with *Linear*, *Quadratic*, *Cubic* and *Fine*, *Medium* and *Coarse Gaussian* kernel functions are incorporated in this study. Unlike nonlinear SVMs, linear SVMs are simple models and can be easily interpreted. However, nonlinear SVMs have higher predictive accuracy (Matlab [37]).

Gaussian Process Regression Models

In Gaussian process regression, the predictions are based on probability distribution over a range of functions. These models have flexibility in choice of presets in the Model Type gallery to enable a small training error and, protects against over-fitting. These models are mostly non-parametric kernel based probabilistic models. Squared exponential GPR, Matern 5/2 GPR, Exponential GPR, Rational Quadratic GPR (Matlab [37]) models have been included in this study to predict the radial strain.

Artificial Neural Network Models

Artificial neural network is one of the widely adopted Artificial Intelligence techniques. It is a computational technique and tries to imitate the cognitive abilities of a human brain to solve complex problems with a simplistic approach [20]. ANN model architecture consist of three types of layers, i.e. input layer, hidden layers and output layer in each case. The independent variables are fed into the input layer which further transmits information through the hidden layer and the predictions are obtained from the output layer. The system develops a learning algorithm to determine the weights to the input signals as well as adjusts/sets the weights and biases and employs an activation function to develop a learning rule for input-output mapping that yields the smallest possible error [20]. Narrow, medium, wide, Bilayered and Trilayered Neural Network models are incorporated in the present study.

4 Determination of Accuracy of Prediction by Different Models

To determine the best model that make prediction with the closest accuracy, the RMSE, R-squared, MSE and MAE values are compared for each model. Also, the plot of predicted vs true value of radial strain for each model demonstrates the correlation between radial strain reported in various studies and that estimated using different models for the corresponding case.

RMSE (Root Mean Square Error): It is a measure of the difference between observed values and that predicted by a model/estimator. Lower values of RMSE are preferred [37].

R-square (Coefficient of determination): It is always smaller than 1 and mostly greater than 0. It enables comparison of the trained model to the model with a constant response and it equals the mean of the training response. R-squared values close to 1 is preferred [37].

MSE (Mean Squared Error): It is the square of the RMSE, smaller values of MSE are preferred [37].

MAE (Mean Absolute Error): This is similar to the RMSE values but it is insensitive of the outliers. Smaller values of MAE are preferred [37].

5   Results and Discussion

The plots of predicted vs true radial strain obtained from different models are shown in Figs. 1, 2, 3, 4, 5. It is observed from Figs. 1, 2, 3, 4, 5 and Table 2 that the predictions by Neural Networks (Fig. 5(a)–(e)) exhibit higher accuracy as compared to the predictions of Linear Regression (Fig. 1(a)–(d)), SVM (Fig. 2(a)–(c); Fig. 3(a)–(c)) and GPR (Fig. 4(a)–(d)) models. The predictions by neural networks demonstrate a significantly higher value of R-square (0.92–0.97) that shows a close correlation between the predicted and true radial strain reported. Also, RMSE, MSE and MAE values of the ANN models are lower than those of other models. Apart from ANN models, Interactions Linear Regressions model (Fig. 1(b)), Quadratic (Fig. 2(b)) and Cubic SVM (Fig. 2(c)), Squared Exponential GPR (Fig. 4(a)), Matern 5/2 GPR (Fig. 4(b)), Rotational Quadratic GPR (Fig. 4(c)) and Exponential GPR models (Fig. 4(d)) exhibit a high R-squared value in the range of 0.75–0.92. However, the RMSE, MSE and MAE values of these models are higher than
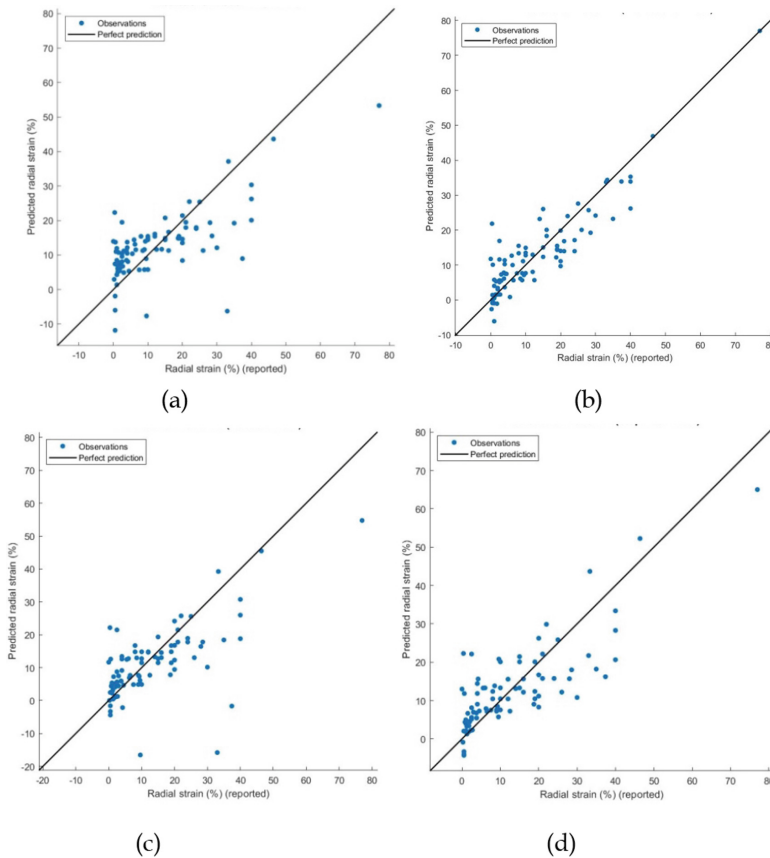

(a)   (b)

(c)   (d)

Fig. 1. Predicted vs Reported radial strain as obtained from **(a)** Linear **(b)** Interactions linear **(c)** Robust linear **(d)** stepwise linear regression model
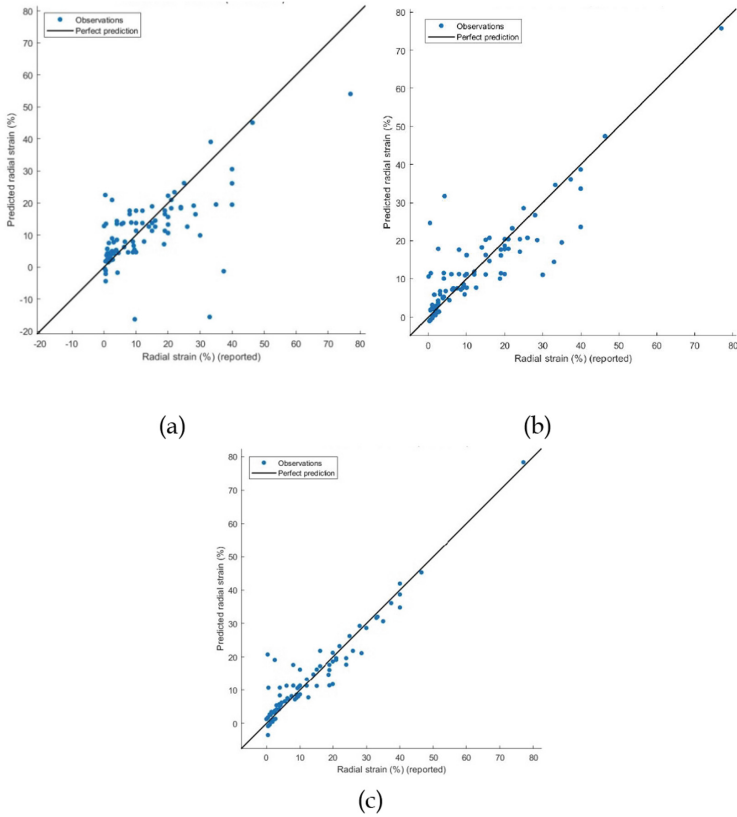
Fig. 2. Predicted vs Reported radial strain as obtained from (a) Linear (b) Quadratic (c) Cubic SVM model

that of ANN models. The values of RMSE, R-square, MSE and MAE of the different models are listed in Table 2.

This observation emphasizes the higher applicability of ANN models in predicting the radial strain.

Comparison of Performance of Different ANN Models

Of all the ANN models adopted in the study, predictions of Wide neural network model (Fig. 5(c)) exhibited the highest accuracy with maximum R-squared value of 0.97 and lowest values of RMSE, MSE and MAE of all the models. The R-squared value of predictions of medium, bilayered and trilayered neural network models were similar to wide ANN model unlike RMSE, MSE and MAE values. The R-squared value of correlation between predicted and reported values was the lowest at 0.92 for narrow neural network. However, the RMSE values of wide and trilayered neural network showed marginal
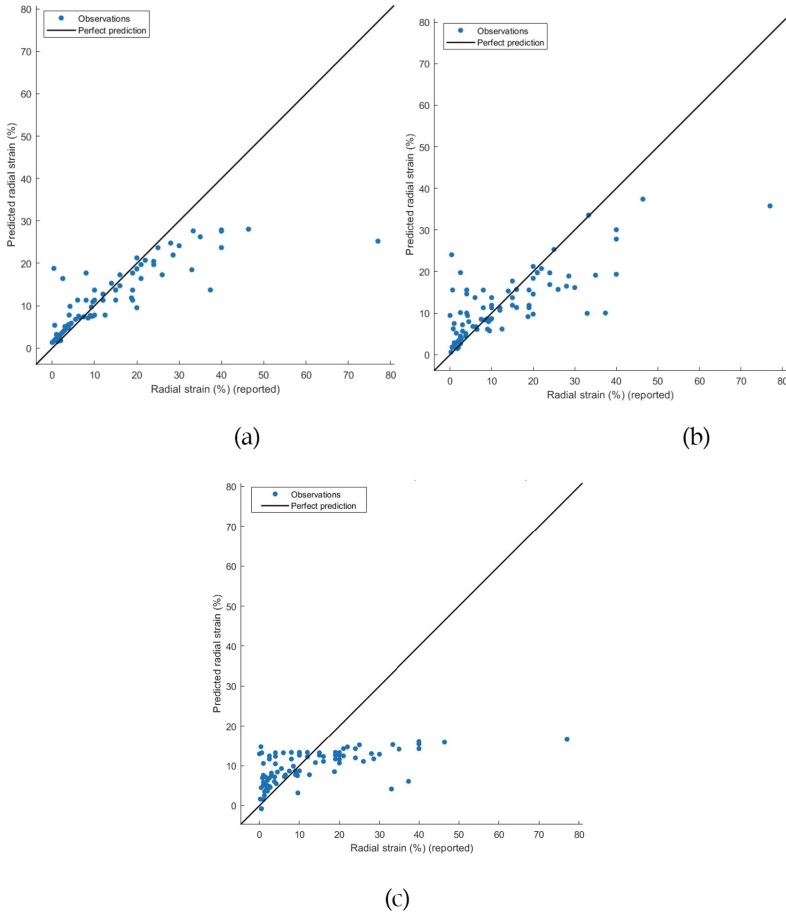
Fig. 3. Predicted vs Reported radial strain as obtained from **(a)** Fine **(b)** Medium **(c)** Coarse Gaussian SVM model

variation. It can therefore be concluded that of Linear Regression, SVM, GPR and ANN models, ANN models (Fig. 5(a)-(e)) give the best prediction of radial strain computed as a function of *d*, *l/d* ratio, *s/d* ratio, λ, *k*, *β*. Also, apart from the narrow ANN model, all the other ANN models yielded close predictions.

Limitations and Future Scope of Work

The models were built with limited number of datasets. Detailed parametric study will be carried out in future and data from such studies may help to better predict the lateral deformation trend/pattern using ANN models. Systematic experimental studies on stone
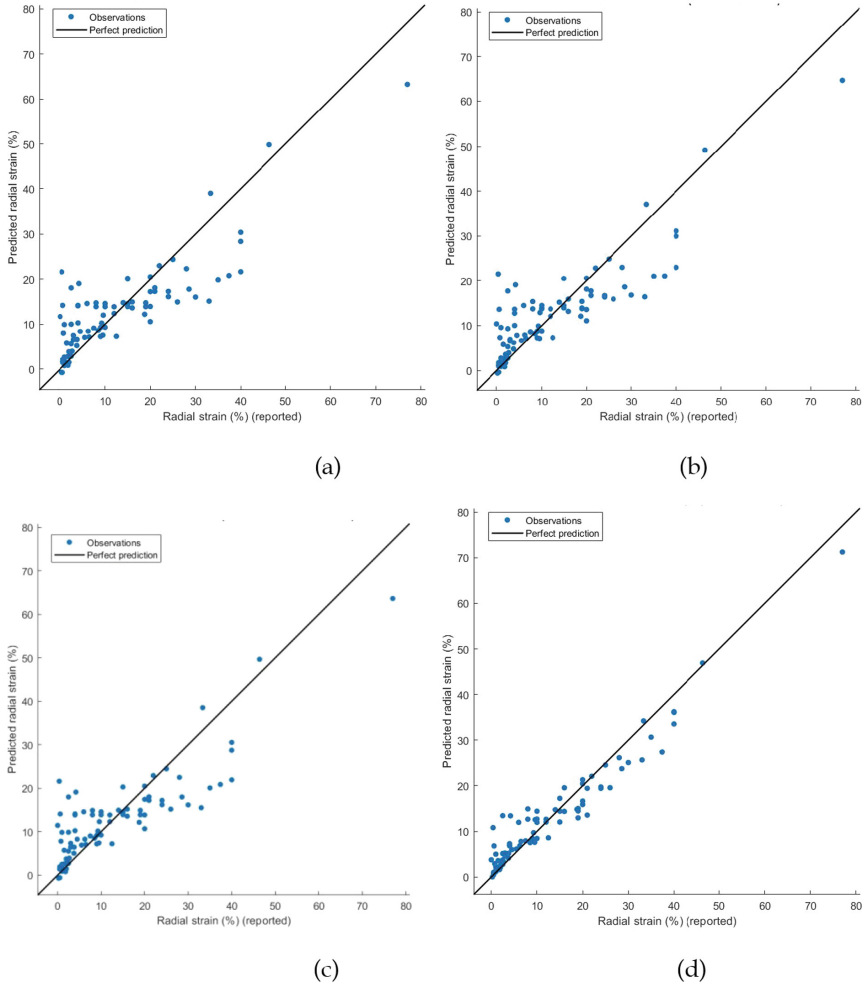
Fig. 4. Predicted vs Reported radial strain as obtained from **(a)** Squared exponential **(b)** Matern 5/2 **(c)** Rotational quadratic **(d)** Exponential GPR model

columns focussing on the failure of columns due to lateral deformation need to be conducted for establishing database for further improvement in ANN model development. It would also enable a reliable sensitivity analysis to understand the principal factors that affect the target parameters. Due to a limited number of datasets sensitivity analysis was not performed in the present study and it is planned to be carried out in further studies.
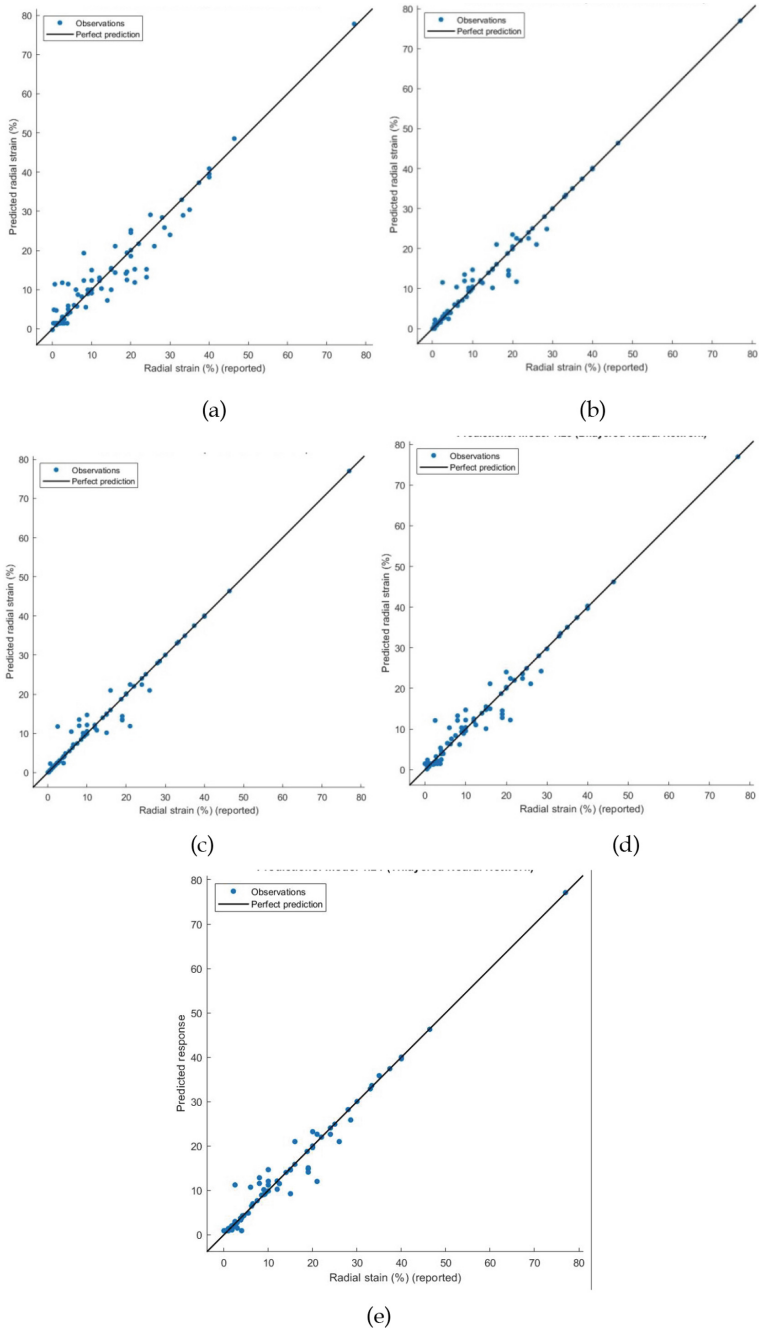
Fig. 5. Predicted vs Reported radial strain as obtained from **(a)** Narrow **(b)** Medium **(c)** Wide **(d)** Bilayered **(e)** Trilayered ANN model

Table 2. Values of RMSE, R-square, MSE and MAE of different models

Model type	RMSE	R-squared	MSE	MAE
Linear regression	9.6588	0.48	93.235	7.1823
Interactions linear regression	5.7451	0.82	33.006	4.2865
Robust linear regression	10.311	0.41	106.31	6.5791
Stepwise linear regression	7.8935	0.65	62.307	5.9342
Linear SVM	10.326	0.41	106.63	6.5033
Quadratic SVM	6.679	0.75	44.609	4.0724
Cubic SVM	4.1814	0.9	17.484	2.6865
Fine Gaussian SVM	8.0808	0.64	65.3	4.2765
Medium Gaussian SVM	8.7979	0.57	77.403	5.4252
Coarse Gaussian SVM	11.917	0.21	142.01	7.7625
Squared exponential GPR	7.1356	0.72	50.916	5.0399
Matern 5/2 GPR	6.785	0.74	46.036	4.7668
Exponential GPR	3.7181	0.92	13.824	2.6707
Rotational quadratic GPR	7.0558	0.72	49.785	4.9707
Narrow neural network	3.6821	0.92	13.558	2.3546
Medium neural network	2.2659	0.97	5.1343	1.0773
Wide neural network	2.2062	0.97	4.8672	0.9548
Bilayered neural network	2.3825	0.97	5.6763	1.3729
Trilayered neural network	2.1939	0.97	4.8133	1.1229

6   Conclusions

The radial strain in OSC and ESC were predicted with the help of Linear Regression, SVM, GPR and ANN models. The variance in column characteristics like the column dimensions, geometry and arrangement, area of load application, including the effect of the presence of a stiff stratum below the columns i.e., end-bearing and floating column, were taken into account by the inclusion of 7 input parameters considered for prediction of target values. The datasets were taken from published literature. Such ANN models are valuable, especially in geotechnical engineering applications, since most problems are dependent on a wide range of factors and determining the physical relationship between such factors involves considerable time. The ANN models are quick to provide an overall estimate of the target parameters in investigation and can work to offer a preliminary idea of the problem and its feasibility/applicability as a solution. However, incorporating basic ideas or prior knowledge of the physical laws that govern the relationship between input parameters that influence the target parameter significantly and the target data would help to ignore noisy data and therefore enhance the efficiency and applicability of such models. Such models will help to predict the radial strain even in larger size

columns adopted in field conditions based on such critical input parameters. However, such AI techniques can be adopted as an additional tool and are not recommended as a substitute to field studies/tests. The conclusions derived from the present study are presented below:

1. As compared to Linear Regression, SVM and GPR models, ANN models exhibited higher efficiency of prediction of radial strain in OSC and ESC columns.
2. ANN models with multiple layered networks like medium, wide, bilayered, trilayered neural networks yielded predictions with higher accuracy as compared to narrow ANN models.
3. Exponential GPR and Cubic SVM models were also found to give close predictions with a considerable R-squared value of 0.90.

References

1. Greenwood, D.A.: Mechanical improvement of soils below ground surface. In: Ground Engineering Proceedings Conference, Institution of Civil Engineers, London, pp. 11–22 (1970)
2. Barksdale, R.D., Bachus, R.C.: Design and Construction of Stone Columns. Report No. FHWA/RD-83/026, Office of Engineering and Highway Operations Research and Development, Federal Highway Administration, Washington, DC (1983)
3. Murugesan, S., Rajagopal, K.: Geosynthetic-encased stone columns: numerical evaluation. Geotext. Geomembr. **24**(6), 349–358 (2006)
4. Ali, K., Shahu, J.T., Sharma, K.G.: Model tests on geosynthetic-reinforced stone columns: a comparative study. Geosynth. Int. **19**(4), 292–305 (2012)
5. Hasan, M., Samadhiya, N.K.: Experimental and numerical analysis of geosynthetic-reinforced floating granular piles in soft clays. Int. J. Geosynth. Ground Eng. **2**(3), 1–13 (2016)
6. Wood, D.W., Hu, W., Nash, D.F.: Group effects in stone column foundations: model tests. Geotechnique **50**(6), 689–698 (2000)
7. Murugesan, S., Rajagopal, K.: Studies on the behavior of single and group of geosynthetic encased stone columns. J. Geotech. Geoenviron. Eng. **136**(1), 129–139 (2010)
8. Dash, S.K., Bora, M.C.: Influence of geosynthetic encasement on the performance of stone columns floating in soft clay. Can. Geotech. J. **50**(7), 754–765 (2013)
9. Miranda, M., Fernández-Ruiz, J., Castro, J.: Critical length of encased stone columns. Geotext. Geomembr. **45**(2), 68–80 (2021). https://doi.org/10.1016/j.geotexmem.2021.05.003
10. Balaam, N.P., Booker, J.R.: Analysis of rigid rafts supported by granular piles. Int. J. Numer. Anal. Meth. Geomech. **5**(4), 379–403 (1981)
11. Priebe, H.J.: The design of vibro replacement. Ground Eng. **28**(10), 31 (1995)
12. IS 15284: Design and Construction for Ground Improvement: Guidelines Part 1: Stone Columns. Bureau of Indian Standards, New Delhi, India (2003)
13. Hughes, J.M.O., Withers, N.J., Greenwood, D.A.: A field trial of the reinforcing effect of a stone column in soil. Geotechnique **25**(1), 31–44 (1975)
14. Rathod, D., Abid, M.S., Vanapalli, S.K.: Performance of polypropylene textile encased stone columns. Geotext. Geomembr. **49**(1), 222–242 (2021)
15. Malarvizhi, S.N., Ilamparuthi, K.: Comparative study on the behaviour of encased stone column and conventional stone column. Soils Found. **47**(5), 873–885 (2007)

16. Mazumder, T., Ayothiraman, R.: Numerical study on behaviour of encased stone columns with partial content of shredded tyre chips in soft clay bed. Int. J. Geosynth. Ground Eng. **7**(2), 1–14 (2021). https://doi.org/10.1007/s40891-021-00280-z
17. Mazumder, T., Rolaniya, A.K., Ayothiraman, R.: Experimental study on behaviour of encased stone column with tyre chips as aggregates. Geosynth. Int. **25**(3), 259–270 (2018)
18. Almeida, M.S., Hosseinpour, I., Lima, B.: Field studies of stone columns and geosynthetic-encased columns. In: From Research to Applied Geotechnics, pp. 166–180. IOS Press (2019)
19. Shivashankar, R., Babu, M.D., Nayak, S., Rajathkumar, V.: Experimental studies on behaviour of stone columns in layered soils. Geotech. Geol. Eng. **29**(5), 749 (2011)
20. Shahin, M.A.: State-of-the-art review of some artificial intelligence applications in pile foundations. Geosci. Front. **7**(1), 33–44 (2016)
21. Goh, A.T.: Nonlinear modelling in geotechnical engineering using neural networks. Trans. Inst. Eng. Aust. Civ. Eng. **36**(4), 293–297 (1994)
22. Goh, A.T., Kulhawy, F.H., Chua, C.G.: Bayesian neural network analysis of undrained side resistance of drilled shafts. J. Geotech. Geoenviron. Eng. **131**(1), 84–93 (2005)
23. Das, M., Dey, A.K.: Prediction of bearing capacity of stone columns placed in soft clay using ANN model. Geotech. Geol. Eng. **36**(3), 1845–1861 (2018)
24. Chik, Z., Aljanabi, Q.A.: Intelligent prediction of settlement ratio for soft clay with stone columns using embankment improvement techniques. Neural Comput. Appl. **25**(1), 73–82 (2013). https://doi.org/10.1007/s00521-013-1449-0
25. Chik, Z., Aljanabi, Q.A., Kasa, A., Taha, M.R.: Tenfold cross validation artificial neural network modeling of the settlement behavior of a stone column under a highway embankment. Arab. J. Geosci. **7**(11), 4877–4887 (2013). https://doi.org/10.1007/s12517-013-1128-6
26. Dey, A.K., Debnath, P.: Empirical approach for bearing capacity prediction of geogrid-reinforced sand over vertically encased stone columns floating in soft clay using support vector regression. Neural Comput. Appl. **32**(10), 6055–6074 (2019). https://doi.org/10.1007/s00521-019-04092-1
27. Deb, K., Samadhiya, N.K., Namdeo, J.B.: Laboratory model studies on unreinforced and geogrid-reinforced sand bed over stone column-improved soft clay. Geotext. Geomembr. **29**(2), 190–196 (2011)
28. Elsawy, M.B.D.: Behaviour of soft ground improved by conventional and geogrid-encased stone columns, based on FEM study. Geosynth. Int. **20**(4), 276–285 (2013)
29. Gnandji, R.A., Kalumba, D.: Experimental and numerical analysis of the behaviour of rammed stone columns installed in a South African soft soil. Int. J. Eng. Sci. Innov. Technol. **3**(6), 477–499 (2014)
30. Indraratna, B., Ngo, N.T., Rujikiatkamjorn, C., Sloan, S.W.: Coupled discrete element–finite difference method for analysing the load-deformation behaviour of a single stone column in soft soil. Comput. Geotech. **63**, 267–278 (2015)
31. Gu, M., Zhao, M., Zhang, L., Han, J.: Effects of geogrid encasement on lateral and vertical deformations of stone columns in model tests. Geosynth. Int. **23**(2), 100–112 (2016)
32. Demir, A., Sarici, T.: Bearing capacity of footing supported by geogrid encased stone columns on soft soil. Geomech. Eng. **12**(3), 417–439 (2017)
33. Hong, Y.S., Wu, C.S., Kou, C.M., Chang, C.H.: A numerical analysis of a fully penetrated encased granular column. Geotext. Geomembr. **45**(5), 391–405 (2017)
34. Rajesh, S., Jain, P.: Influence of permeability of soft clay on the efficiency of stone columns and geosynthetic-encased stone columns–a numerical study. Int. J. Geotech. Eng. **9**(5), 483–493 (2015)
35. Pandey, B.K., Rajesh, S., Chandra, S.: 3-D finite element study of embankment resting on soft soil reinforced with encased stone column. In: Latha Gali, M., Raghuveer Rao, P. (eds.) Problematic Soils and Geoenvironmental Concerns. LNCE, vol. 88, pp. 451–465. Springer, Singapore (2021). https://doi.org/10.1007/978-981-15-6237-2_38

36. Xu, Z., Zhang, L., Zhou, S.: Influence of encasement length and geosynthetic stiffness on the performance of stone column: 3D DEM-FDM coupled numerical investigation. Comput. Geotech. **132**, 103993 (2021)
37. Matlab R2021a User Interface (2021). https://in.mathworks.com/help/stats/regression-and-anova.html?s_tid=CRUX_lftnav